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Vibration attenuation control of ocean marine risers with axial-transverse couplings

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ABSTRACT

The target of this paper is designing a boundary controller for vibration suppression of marine risers with coupling mechanisms under environmental loads. Based on energy approach and the equations of axial and transverse motions of the risers are derived. The Lyapunov direct method is employed to formulated the control placed at the riser top-end. Stability analysis of the closed-loop system is also included.

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1. INTRODUCTION

Due to its physical structure, a riser basically is modeled as a tensioned beam [1], [2], [3], and [4]. In [5], an active boundary control that produces a vibration-free for an Euler-Bernoulli beam system was designed. Similar use of distributed control can be found in [6]. In [7], the authors used differential evolution optimization to search for the best controller model structure and its parameters for beam control problem. The proposed controller is able to suppress the beam?s vibration without knowledge of the system. However, the searching process is conducted within a set of predefined control structures, no proof of the effectiveness of the control was given.

With efforts to make voltage-source converter (VSC) more efficient in handling distributed parameter systems, sliding-mode control (SMC) was given extra flexibility by adding a neural network and fuzzy control in [8]. The author yield a control law in the form of a mass-damper-spring system at the boundary of a moving string. However, difficulties in selecting proper fuzzy membership functions and a slow convergence speed due to online-tuning might be troublesome when applying the aforementioned controls. After accepting that SMC is non-analytical in the sliding surface, in the first control structure, a boundary layer was defined that enabled fuzzy control by taking a switching function and its derivative as inputs while SMC was activated outside this boundary to achieve fast transient responses. A series of papers with applications of SMC to flexible system can be found in [9], [10], and [11]. A second attempt was made to design a fuzzy neural network control (FNNC) that also employed switching variables as its inputs. The proposed FNNC conducted an online-tuning process to regulate fuzzy reasoning to compromise system uncertainties. Both controls resulted in a variation of axially moving string tension as the control action.

In [12], a beam model representing a tensioned riser is investigated, and a boundary controller consisting of the top-end rise information is designed to achieve exponential stability. Krstic, *et al.* develop a systematic approach based on backstepping control for beam-type structure in [13] and [14]. In [15], the authors proposed a control assisted by a disturbance estimator to guarantee asymptotic stability of an Euler-Bernoulli beam system subjected to unknown disturbances. He, *et al.* In [16], successfully develop a boundary control for a flexible riser with vessel dynamics. In [17], the authors introduce control based on Lyapunov's approach. Through Lyapunov's direct method, the riser's transverse motion under time-varying distributed loads stability is established. A control problem for a coupled nonlinear riser exhibiting longitudinal-transverse couplings is investigated in [3]. Analogous applications to flexible systems are evidenced in [18], [2], and [19]. Since the surface vessel is always control by a dynamic positioning system in practice [20], [21], [22], [23], [24], and [25] the vessel's motions normally are not considered. The paper deals with the vibration control problem for marine risers under environmental disturbances. In addition, the longitudinal-transverse coupling in the riser motion in taken into account. Different from [26], the control is formulated without the assumption of positive tension. Existence, uniqueness, and convergence of the solutions of the closed-loop system is verified in the paper.

2. MATHEMATICAL FORMULATION

The riser kinetic energy is specified by

$$T = \frac{m_0}{2}\int_0^L \Big[\big(\frac{\partial u(z,t)}{\partial t}\big)^2 + \big(\frac{\partial w(z,t)}{\partial t}\big)^2\Big]\mathrm{d}z, \tag{1}$$

where $u(z,t)$ is transverse displacements in the X direction and $w(z,t)$ is longitudinal displacement in the Z direction. L denote the riser length, $m_0 = \rho A$ is the riser oscillating mass per unit length, A is the riser cross-section area, and ρ represents the mass density of the riser. Assuming that the riser is constrained by constant tension P_0. The riser potential energy is given as

$$P = \frac{EI}{2}\int_0^L \big(\frac{\partial^2 u(z,t)}{\partial z^2}\big)^2\mathrm{d}z + \frac{P_0}{2}\int_0^L \big(\frac{\partial u(z,t)}{\partial z}\big)^2\mathrm{d}z + \frac{EA}{2}\int_0^L \Big[\frac{\partial w(z,t)}{\partial z} + \frac{1}{2}\big(\frac{\partial u(z,t)}{\partial z}\big)^2\Big]^2\mathrm{d}z, \tag{2}$$

where E is the Young's modulus and I is the second moment of the riser's cross section area. The hydrodynamic forces can be given as [26]

$$\begin{aligned} &f_u(z,t) = f_{uD} + f_{uL}, \quad f_v(z,t) = f_{wD} + f_{wL},\\ &f_{uD} = -\Omega_{1D}u_t(z,t), \quad f_{wD} = -\Omega_{2D}v_t(z,t), \end{aligned} \tag{3}$$

where f_{uD}, f_{wD} and f_{uL}, f_{wL} correspond to the distributed damping and external forces. The work done by the hydrodynamic forces acting on the system is calculated as

$$W_f = \int_0^L f_u(z,t)u(z,t)\mathrm{d}z + \int_0^L f_w(z,t)w(z,t)\mathrm{d}z, \tag{4}$$

The work done by boundary control is

$$W_m = U_u(L,t)u(L,t) + U_w(L,t)w(L,t), \tag{5}$$

where $U_u(L,t)$ and $U_w(L,t)$ are the boundary control forces. The total work done on the system is $W = W_f + W_m$. The extended Hamilton principle is indicated as

$$\int_{t_1}^{t_2} \delta(T - P + W)\mathrm{d}t = 0. \tag{6}$$

For the sake of clear presentation, (z,t) is omitted whenever it is applicable. The kinetic energy variation can be written as

$$\int_{t_1}^{t_2} \delta T\mathrm{d}z = -m_0\int_{t_1}^{t_2}\int_0^L \Big(\frac{\partial^2 u}{\partial t^2}\delta u + \frac{\partial^2 w}{\partial t^2}\delta w\Big)\mathrm{d}z\mathrm{d}t, \tag{7}$$

where $\delta u = \delta v = \delta w = 0$ at $t = t_1, t_2$ have been used. For the riser under consideration, ball joints arranged at both ends (Figure 1) implying that bending free. In addition, the lower end stationed at the well-head. The riser dynamics is yielded

$$
\begin{aligned}
&- m_0 u_{tt} - EI u_{zzzz} + P_0 u_{zz} + \frac{3EA}{2} u_z^2 u_{zz} + EA w_{zz} u_z + EA w_z u_{zz} - \Omega_{1D} u_t + f_u = 0,\\
&- m_0 w_{tt} - EA w_{zz} + EA u_z u_{zz} - \Omega_{2D} w_t + f_w = 0,\\
&- EI u_{zzz}(L,t) + P_0 u_z(L,t) + \frac{EA}{2} u_z^3(L,t) + EA w_z(L,t) u_z(L,t) = U_u(L,t),\\
&EA w_z(L,t) + \frac{EA}{2} u_z^2(L,t) + \frac{EA}{2} v_z^2(L,t) = U_w(L,t),\\
&u_{zz}(L,t) = v_{zz}(L,t) = u_{zz}(0,t) = v_{zz}(0,t) = 0, u(0,t) = v(0,t) = w(0,t) = 0,
\end{aligned} \tag{8}
$$

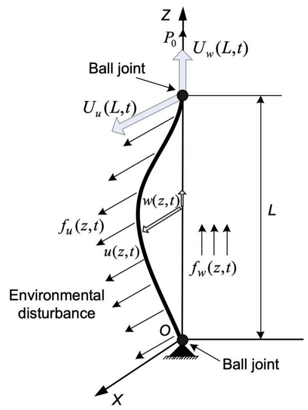

Figure 1. Riser coordinates

3. CONTROL DESIGN

In order to minimize the riser vibration using measured state and applied forces at the top end, we consider the following Lyapunov candidate function

$$
\begin{aligned}
V =& \frac{m_0}{2}\int_0^L (u_t^2 + w_t^2)\mathrm{d}z + \frac{P_0}{2}\int_0^L u_z^2 \mathrm{d}z + \frac{EA}{2}\int_0^L \left(w_z + \frac{u_z^2}{2}\right)^2 \mathrm{d}z + \frac{EI}{2}\int_0^L u_{zz}^2 \mathrm{d}z\\
&+ \rho_1 \int_0^L u u_t \mathrm{d}z + \rho_2 \int_0^L w w_t \mathrm{d}z + \left(k_1 + \frac{k_2\rho_1}{m_0}\right) u^2(L,t) + \left(k_3 + \frac{k_4 \rho_2}{m_0}\right) w^2(L,t).
\end{aligned} \tag{9}
$$

Since $\forall t \geq 0$ and $u(0,t) = w(0,t) = 0$, it can be shown that

$$
\gamma_1\rho_1 \int_0^L u^2 \mathrm{d}z \leq 4L^2\gamma_1\rho_1 \int_0^L u_z^2 \mathrm{d}z, \quad \gamma_2\rho_2 \int_0^L w^2 \mathrm{d}z \leq 4L^2 \gamma_2\rho_2 \int_0^L w_z^2 \mathrm{d}z. \tag{10}
$$

where γ_1 and γ_2 are positive constants, it can be deduced that

$$
-4L^2\gamma_1\rho_1 \int_0^L u_z^2 \mathrm{d}z - \frac{\rho_1}{\gamma_1}\int_0^L u_t^2 \mathrm{d}z \leq \rho_1 \int_0^L u u_t \mathrm{d}z \leq 4L^2\gamma_1\rho_1 \int_0^L u_z^2 \mathrm{d}z + \frac{\rho_1}{\gamma_1}\int_0^L u_t^2 \mathrm{d}z, \tag{11}
$$

$$
-4L^2\gamma_2\rho_2 \int_0^L w_z^2 \mathrm{d}z - \frac{\rho_2}{\gamma_2}\int_0^L w_t^2 \mathrm{d}z \leq \rho_2 \int_0^L w w_t \mathrm{d}z \leq 4L^2\gamma_2\rho_2 \int_0^L w_z^2 \mathrm{d}z + \frac{\rho_2}{\gamma_2}\int_0^L w_t^2 \mathrm{d}z. \tag{12}
$$

The (9) can be lower and upper bounded by

$$V \geq \left(\frac{m_0}{2} - \frac{\rho_1}{\gamma_1}\right)\int_0^L u_t^2 \mathrm{d}z + \left(\frac{m_0}{2} - \frac{\rho_2}{\gamma_2}\right)\int_0^L w_t^2 \mathrm{d}z + \left(\frac{P_0}{2} - 4L^2\gamma_1\rho_1\right)\int_0^L u_z^2 \mathrm{d}z + \left(\frac{EA}{2} - 4L^2\gamma_2\rho_2\right)\int_0^L w_z^2 \mathrm{d}z + \frac{EA}{8}\int_0^L u_z^4 \mathrm{d}z + \frac{EA}{4}\int_0^L w_z u_z^2 \mathrm{d}z + \frac{EI}{2}\int_0^L u_{zz}^2 \mathrm{d}z + \frac{1}{2}\left(k_1 + \frac{k_2\rho_1}{m_0}\right)u^2(L,t) + \left(k_3 + \frac{k_4\rho_2}{m_0}\right)w^2(L,t), \tag{13}$$

and

$$V \leq \left(\frac{m_0}{2} + \frac{\rho_1}{\gamma_1}\right)\int_0^L u_t^2 \mathrm{d}z + \left(\frac{m_0}{2} + \frac{\rho_2}{\gamma_2}\right)\int_0^L w_t^2 \mathrm{d}z + \left(\frac{P_0}{2} + 4L^2\gamma_1\rho_1\right)\int_0^L u_z^2 \mathrm{d}z + \left(\frac{EA}{2} + 4L^2\gamma_2\rho_2\right)\int_0^L w_z^2 \mathrm{d}z + \frac{EA}{8}\int_0^L u_z^4 \mathrm{d}z + \frac{EA}{4}\int_0^L w_z u_z^2 \mathrm{d}z + \frac{EI}{2}\int_0^L u_{zz}^2 \mathrm{d}z + \frac{1}{2}\left(k_1 + \frac{k_2\rho_1}{m_0}\right)u^2(L,t) + \left(k_3 + \frac{k_4\rho_2}{m_0}\right)w^2(L,t). \tag{14}$$

If we select ρ_1, ρ_2, γ_1, and γ_2 such that:

$$\frac{m_0}{2} - \frac{\rho_1}{\gamma_1} = c_1, \quad \frac{m_0}{2} - \frac{\rho_2}{\gamma_2} = c_2, \frac{P_0}{2} - 4L^2\gamma_1\rho_1 = c_3, \quad \frac{P_0}{2} - 4L^2\gamma_2\rho_2 = c_4, \tag{15}$$

where c_i, for $i = 1 \ldots 4$, are strictly positive constants. Differentiating (9) and taking (8) into account yields

$$\dot{V} = \left(u_t(L,t) + \frac{\rho_1}{m_0}u(L,t)\right)\Big(-EIu_{zzz}(L,t) + P_0u_z(L,t) + \frac{EA}{2}u_z^3(L,t) + EAw_z(L,t)u_z(L,t)\Big) + \left(w_t(L,t) + \frac{\rho_2}{m_0}w(L,t)\right)\left(EAw_z(L,t) + \frac{EA}{2}u_z^2(L,t)\right) - (\Omega_{1D} - \rho_1)\int_0^L u_t^2 \mathrm{d}z - (\Omega_{2D} - \rho_2)\int_0^L w_t^2 \mathrm{d}z - \frac{\rho_1 EI}{m_0}\int_0^L u_{zz}^2 \mathrm{d}z - \frac{\rho_1 P_0}{m_0}\int_0^L u_z^2 \mathrm{d}z - \frac{\rho_1 EA}{2m_0}\int_0^L u_z^4 \mathrm{d}z - \frac{EA}{m_0}\left(\rho_1 + \frac{\rho_2}{2}\right)\int_0^L u_z^2 w_z \mathrm{d}z - \frac{\rho_1\Omega_{1D}}{m_0}\int_0^L uu_t \mathrm{d}z + \frac{\rho_1}{m_0}\int_0^L uf_u \mathrm{d}z - \frac{\rho_2 EA}{m_0}\int_0^L w_z^2 \mathrm{d}z - \frac{\rho_2\Omega_{2D}}{m_0}\int_0^L ww_t \mathrm{d}z + \int_0^L u_t f_u \mathrm{d}z + \int_0^L w_t f_w \mathrm{d}z + \frac{\rho_2}{m_0}\int_0^L wf_w \mathrm{d}z + \left(k_1 + \frac{k_2\rho_1}{m_0}\right)u(L,t)u_t(L,t) + \left(k_3 + \frac{k_4\rho_2}{m_0}\right)w(L,t)w_t(L,t). \tag{16}$$

Since

$$-\frac{\Omega_{1D}\rho_1}{m_0}\int_0^L uu_t \mathrm{d}z \leq \frac{4L^2\Omega_{1D}\rho_1\gamma_3}{m_0}\int_0^L u_z^2 \mathrm{d}z + \frac{\Omega_{1D}\rho_1}{\gamma_3 m_0}\int_0^L u_t^2 \mathrm{d}z, \tag{17}$$

$$-\frac{\Omega_{2D}\rho_2}{m_0}\int_0^L ww_t \mathrm{d}z \leq \frac{4L^2\Omega_{2D}\rho_2\gamma_4}{m_0}\int_0^L w_z^2 \mathrm{d}z + \frac{\Omega_{2D}\rho_3}{\gamma_4 m_0}\int_0^L w_t^2 \mathrm{d}z, \tag{18}$$

and noted that $-EIu_{zzz}(L,t) + P_0u_z(L,t) + \frac{EA}{2}u_z^3(L,t) + EAw_z(L,t)u_z(L,t) = U_u(L,t)$ and $EAw_z(L,t) + \frac{EA}{2}u_z^2(L,t) = U_w(L,t)$, the boundary controls are designed as follows,

$$U_u = -k_1u(L,t) - k_2u_t(L,t), \quad U_w = -k_3w(L,t) - k_4w_t(L,t), \tag{19}$$

where coefficients k_i, for $i = 1 \dots 4$, are strictly positive constants. Substituting the controls (19) into (16) gives

$$\begin{aligned}\dot{V} \leq & -\frac{k_1\rho_1}{m_0}u^2(L,t) - k_2u_t^2(L,t) - \frac{k_3\rho_2}{m_0}w^2(L,t) - k_4w_t^2(L,t) - \big(\Omega_{1D} - \rho_1 - \frac{\Omega_{1D}\rho_1}{\gamma_3 m_0}\big)\int_0^L u_t^2 \mathrm{d}z \\ & -\big(\Omega_{2D} - \rho_2 - \frac{\Omega_{2D}\rho_2}{\gamma_4 m_0}\big)\int_0^L w_t^2 \mathrm{d}z - \frac{\rho_1 EI}{m_0}\int_0^L u_{zz}^2 \mathrm{d}z - \big(\frac{\rho_1 P_0}{m_0} - \frac{4L^2\Omega_{1D}\rho_1\gamma_3}{m_0}\big)\int_0^L u_z^2 \mathrm{d}z \\ & -\big(\frac{\rho_3 EA}{m_0} - \frac{4L^2\Omega_{2D}\rho_2\gamma_4}{m_0}\big)\int_0^L w_z^2 \mathrm{d}z - \frac{\rho_1 EA}{2m_0}\int_0^L u_z^4 \mathrm{d}z - \frac{EA}{m_0}\Big(\rho_1 + \frac{\rho_2}{2}\Big)\int_0^L u_z^2 w_z \mathrm{d}z \\ & -\frac{\rho_1\Omega_{1D}}{m_0}\int_0^L uu_t \mathrm{d}z + \frac{\rho_1}{m_0}\int_0^L uf_u \mathrm{d}z - \frac{\rho_2\Omega_{2D}}{m_0}\int_0^L ww_t \mathrm{d}z + \int_0^L u_t f_u \mathrm{d}z + \int_0^L w_t f_w \mathrm{d}z \\ & + \frac{\rho_2}{m_0}\int_0^L wf_w \mathrm{d}z. \end{aligned} \tag{20}$$

Remark: It is noted that the authors of [26] use the assumption the riser is alsway stretched in order to conclude that $\int_0^L u_z^2 w_z dt$ is positive. This is not the case in practice since the riser can be bulked or stretched according to external disturbance. Considering the following term

$$\Delta = -\Delta_1 \int_0^L w_z^2 \mathrm{d}t - \Delta_1 \int_0^L u_z^4 \mathrm{d}t - \Delta_3 \int_0^L u_z^2 w_z \mathrm{d}t \tag{21}$$

where

$$\Delta_1 = \big(\frac{\rho_3 EA}{m_0} - \frac{4L^2\Omega_{2D}\rho_2\gamma_4}{m_0}\big), \quad \Delta_2 = \frac{\rho_1 EA}{2m_0}, \quad \Delta_3 = \frac{EA}{m_0}\big(\rho_1 + \frac{\rho_2}{2}\big) \tag{22}$$

Δ con be written as

$$\Delta = \Delta_1 \int_0^L w^2 \mathrm{d}t - \big(\Delta_2 - \frac{1}{4}\Delta_3\big)\int_0^L u_z^4 - \Delta_3 \int_0^L \big(w_z + \frac{1}{4}u_z^2\big)\mathrm{d}z \tag{23}$$

To remove the requirement of positive tension, we use the following property [27] that

$$w^2 u_z^2 + \frac{1}{4} \geq 0 \tag{24}$$

From (20), the designed parameters are selected such that

$$\begin{aligned} & \Omega_{1D} - \rho_1 - \frac{\Omega_{1D}\rho_1}{\gamma_3 m_0} = c_5, \quad \Omega_{2D} - \rho_2 - \frac{\Omega_{2D}\rho_2}{\gamma_4 m_0} = c_6, \\ & \frac{\rho_1 P_0}{m_0} - \frac{4L^2\Omega_{1D}\rho_1\gamma_3}{m_0} = c_7, \quad \frac{\rho_3 P_0}{m_0} - \frac{4L^2\Omega_{2D}\rho_3\gamma_4}{m_0} = c_8, \quad \Delta_2 - \frac{1}{4}\Delta_3 = c_9 \end{aligned} \tag{25}$$

where c_1, for $i = 5 \dots 9$, are strictly positive constants. Applying the upper bound of V in (14), (20) can be written as

$$\begin{aligned}\dot{V} \leq & -\frac{k_1\rho_1}{m_0}u^2(L,t) - k_2u_t^2(L,t) - \frac{k_3\rho_2}{m_0}w^2(L,t) - k_4w_t^2(L,t) - cV + \frac{\rho_1}{m_0}\int_0^L uf_u \mathrm{d}z \\ & + \int_0^L u_t f_u \mathrm{d}z + \int_0^L w_t f_w \mathrm{d}z + \frac{\rho_3}{m_0}\int_0^L wf_w \mathrm{d}z, \end{aligned} \tag{26}$$

where

$$c = \frac{\min\Big\{c_5, c_6, c_7, c_8, \frac{\rho_1 EI}{m_0}, \frac{\rho_1 EA}{2m_0}, \beta_1\Big\}}{\max\Big\{\frac{m_0}{2} + \frac{\rho_1}{\gamma_1}, \frac{m_0}{2} + \frac{\rho_2}{\gamma_2}, \frac{P_0}{2} + 4L^2\gamma_1\rho_1, \frac{EA}{2} + 4L^2\gamma_2\rho_2, \frac{EA}{8}, \frac{EI}{2}, \beta_2\Big\}}, \tag{27}$$

where

$$\beta_1 = \Big\{\frac{EA}{m_0}\Big(\rho_1 + \frac{\rho_2}{2}\Big), k_1\frac{\rho_1}{m_0}, k_3\frac{\rho_2}{m_0}\Big\}, \ \beta_2 = \Big\{\frac{1}{2}\Big(k_1 + \frac{k_2\rho_1}{m_0}\Big), \frac{1}{2}\Big(k_3 + \frac{k_4\rho_2}{m_0}\Big)\Big\}. \tag{28}$$

Remark: Different from [16], the control design process is carried out in this chapter without any assumptions on boundedness of time and spatial derivatives of the riser system.
Equation (26) can be written as

$$\dot{V} \leq -k_1\frac{\rho_1}{m_0}u^2(L,t) - k_2u_t^2(L,t) - \frac{k_3\rho_2}{m_0}w^2(L,t) - k_4w_t^2(L,t) - cV + \Delta_c, \tag{29}$$

where

$$\Delta_c = \frac{\rho_1}{m_0}\int_0^L uf_u\mathrm{d}z + \frac{\rho_2}{m_0}\int_0^L wf_w\mathrm{d}z + \int_0^L u_tf_u\mathrm{d}zdz + \int_0^L w_tf_w\mathrm{d}z. \tag{30}$$

An upper bound of Δ_c can be written as

$$\begin{aligned}\Delta_c \leq& \frac{1}{\gamma_5}\int_0^L u_t^2\mathrm{d}z + \gamma_5\int_0^L f_u^2\mathrm{d}z + \frac{4L^2\rho_1}{m_0\gamma_6}\int_0^L u_z^2\mathrm{d}z + \frac{\gamma_6\rho_1}{m_0}\int_0^L f_u^2\mathrm{d}z \\ &2 + \frac{1}{\gamma_7}\int_0^L w_t^2\mathrm{d}z + \gamma_7\int_0^L f_w^2\mathrm{d}z + \frac{4L^2\rho_2}{m_0\gamma_8}\int_0^L w_z^2\mathrm{d}z + \frac{\gamma_8\rho_2}{m_0}\int_0^L f_w^2\mathrm{d}z.\end{aligned} \tag{31}$$

There exists a strictly positive constant ξ such that the following inequality holds

$$\begin{aligned}\Delta_c \leq& \xi\Bigg(\int_0^L u_z^2\mathrm{d}z + \int_0^L u_t^2\mathrm{d}z + \int_0^L w_z^2\mathrm{d}z + \int_0^L w_t^2\mathrm{d}z\Bigg) \\ &+ \frac{1}{\xi}\Big(\gamma_5 + \frac{\gamma_6\rho_1}{m_0}\Big)\int_0^L f_u^2\mathrm{d}z + \frac{1}{\xi}\Big(\gamma_7 + \frac{\gamma_8\rho_2}{m_0}\Big)\int_0^L f_w^2\mathrm{d}z.\end{aligned} \tag{32}$$

From the lower bound of V, it is shown that

$$\xi\Bigg(\int_0^L u_z^2\mathrm{d}z + \int_0^L u_t^2\mathrm{d}z + \int_0^L w_z^2\mathrm{d}z + \int_0^L w_t^2\mathrm{d}z\Bigg) \leq \xi\frac{V}{\zeta}, \tag{33}$$

where

$$\zeta = \min\Big\{c_1, c_2, c_3, c_4, \frac{EA}{8}, \frac{EI}{2}, \frac{1}{2}\big(k_1 + \frac{k_2\rho_1}{m_0}\big), \frac{1}{2}\big(k_3 + \frac{k_4\rho_2}{m_0}\big)\Big\}. \tag{34}$$

Substituting (32)and (33) into (29) gives

$$\dot{V} \leq -k_1\frac{\rho_1}{m_0}u^2(L,t) - k_2u_t^2(L,t) - \frac{k_3\rho_2}{m_0}w^2(L,t) - k_4w_t^2(L,t) - \big(c - \frac{\xi}{\zeta}\big)V + \frac{1}{\xi}Q, \tag{35}$$

where

$$Q = \Big(\gamma_5 + \frac{\gamma_6\rho_1}{m_0}\Big)Q_1 + \Big(\gamma_7 + \frac{\gamma_8\rho_3}{m_0}\Big)Q_2, \tag{36}$$

and

$$Q_1 = \max_{t\geq 0}\int_0^L f_u^2\mathrm{d}z, \quad Q_2 = \max_{t\geq 0}\int_0^L f_w^2\mathrm{d}z. \tag{37}$$

If ξ is picked such that $\bar{c} = c - \frac{\xi}{\zeta}$ is strictly positive, then:

$$\dot{V} \leq -\bar{c}V + \frac{1}{\xi}Q. \tag{38}$$

Inequality (38) implies that $V(t)$ exponentially converges to nonnegative constant $\frac{1}{\xi}Q$. Using Inequality A.2 [26], it can be conclude that all terms $|u(z,t)|$ and $|w(z,t)|$ are bounded and exponentially converge to a non-negative constant defined be the value of external disturbances.

4. NUMERICAL SIMULATIONS

At this stage, we illustrate the advantages of the proposed control through a set of simulations. The marine riser system parameters are given as in Table 1 The linear current velocity vector in a form of $V = [\frac{1}{L}s, \frac{0.5}{L}s, 0]^T$ is employed in numerical simulations. The hydrodynamic forces can be given as [26]. Simulations are carried out without the proposed control and with the control by set $k_1 = k_2 = 500$. The riser displacements in the X and Z directions for uncontrolled and controlled cases are plotted in Figure 2 and Figure 3, respectively. It can be observed that when the control is activated, displacement magnitudes in all directions (X and Z) are reduced. The reduction in displacement magnitudes illustrates the effectiveness of the proposed control in driving the riser to the vicinity of its equilibrium position. It also can be observed in Figure 4 that the control forces required to drive the risers are reasonable for the riser under consideration.

Table 1. The marine riser parameters

Nomenclature	Description	Value
L	Length	1000m
D_0	Diameter	0.61m
D_i	Diameter	0.575m
D_H	Diameter	0.87m
ρ_w	Density	1025kg/m^3
ρ_m	Density	1205kg/m^3
E	Young's modulus	$2 \times 10^{10}\text{kg/m}^2$
P_0	Tension	$2.15 \times 10^6\text{N}$

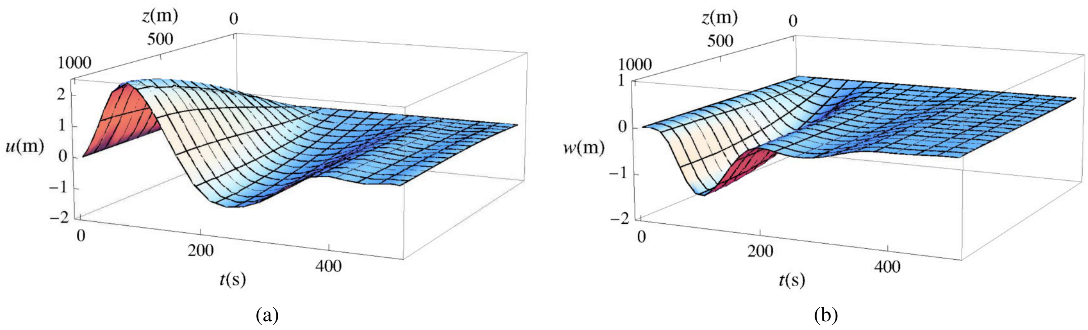

Figure 2. The riser's motions without control: (a) $u(z,t)$ and (b) $w(z,t)$

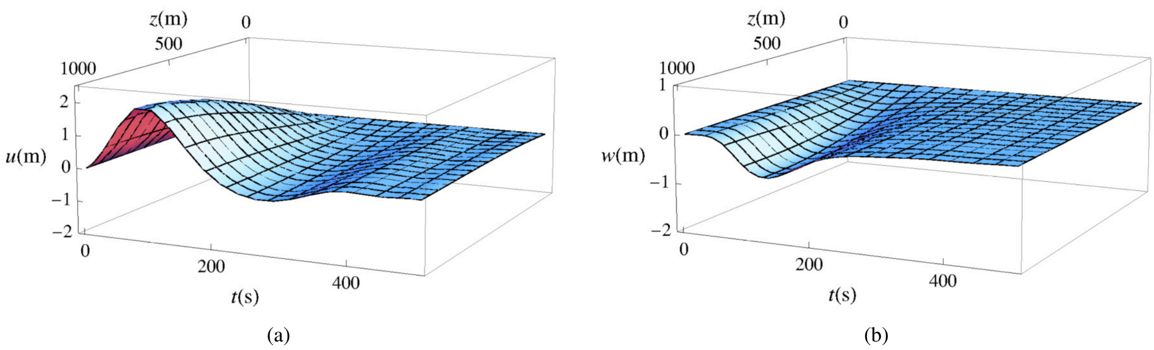

Figure 3. The riser's motions with control: (a) $u(z,t)$ and (b) $w(z,t)$

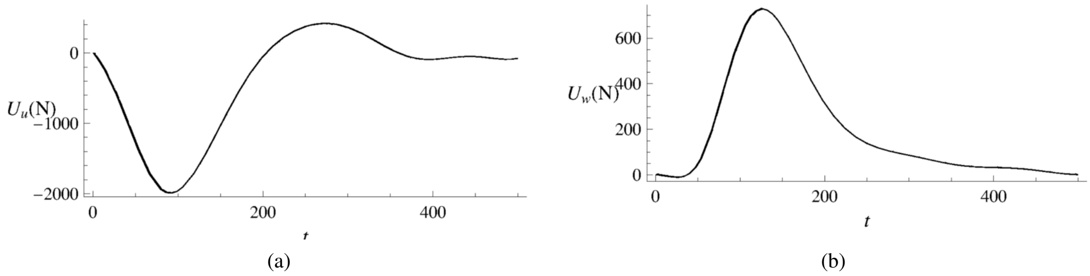

Figure 4. Control input: (a) $U_u(L,t)$, and (b) $U_w(L,t)$

5. CONCLUSIONS

The paper copes with minimizing vibration of the marine riser. After deriving the set of equations specifying the riser dynamics, the boundary controller applied at the riser top end is designed thank to Lyapunov's direct method without the assumption of positive tension applied to the riser. The ability in stabilizing the riser at its equilibrium position of the boundary control is validated analytically and illustrated numerically.

REFERENCES

[1] S. V. Gosavi and A. G. Kelkar, "Modelling, identification, and passivity-based robust control of piezo-actuated flexible beam," *Journal of Vibration and Acoustics*, vol. 126, no. 2, pp. 260–271, 2004.

[2] W. He, S. Zhang, and S. S. Ge, "Boudary control of a flexible riser with the application to marine installations," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 12, pp. 5802–5810, 2013.

[3] S. S. Ge, W. He, B. V. E. How, and Y. S. Choo, "Boundary control of a coupled nonlinear flexible marine riser," *IEEE Transactions on Control Systems Technology*, vol. 18, pp. 1080 – 1091, 2010.

[4] T. L. Nguyen, K. D. Do, and J. Pan, "Global stabilization of marine risers with varying tension and rotational inertia," *Asian Journal of Control*, vol. 17, no. 1, pp. 1–11, 2015.

[5] N. Tanaka and H. Iwanmoto, "Active boundary control of an euler-bernoulli beam for generating vibration-free state," *Journal of Sound and Vibration*, vol. 304, no. 3-5, pp. 570–586, 2007.

[6] E. Scholte and R. D. Andrea, "Active vibro-acoustic control of a flexible beam using distributed control," *Proceedings of the American Control Conference*, 2003.

[7] M. S. Saad, H. Jamaluddin, and I. Z. M. Darus, "Active vibration control of flexible beam using differential evolution optimisation," *World Academy of Science, Engineering and Technology*, vol. 62, 2012.

[8] J. Huang, P. C. P. Chao, R. Fung, and C. Lai, "Parametric control of an axially moving string via fuzzy sliding-mode and fuzzy neural network methods," *Journal of Sound and Vibration*, vol. 264, no. 1, pp. 177–201, 2003.

[9] M. Itik, M. U. Salamci, , F. D. Ulker, and Y. Yaman, "Active vibration suppression of a flexible beam via sliding mode and h-infinity," *Proceeding of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005*, vol. 195, December 2005.

[10] X. Hou, "A variable structure control for a flexible euler-bernoulli beam," *IEEE International Conference on Automation and Logistics (ICAL)*, 2010.

[11] W. Yim, "Variable structure adaptive force tracking control of a cantilever beam," *Journal of Vibration and Control*, vol. 6, no. 7, pp. 1029–1043, 2000.

[12] M. P. Fard and S. I. Sagatun, "Exponential stabilization of a transversely vibrating beam via boundary control," *Journal of Sound and Vibration*, vol. 240, pp. 613–622, 2001.

[13] M. Krstic, A. A. Siranosian, A. Smyshlyaev, and M. Bement, "Backstepping boundary controllers and observers for the slender timoshenko beam: Part ii-stability and simulations," *Proceedings of the 45th IEEE Conference on Decision and Control*, pp. 3938–3943, 2006.

[14] M. Krstic, A. A. Siranosian, and A. Smyshlyaev, "Backstepping boundary controllers and observers for the slender timoshenko beam: Part i-design," *Proceedings of the 2006 American Control Conference*, pp. 2412 – 2417, 2006.

[15] B. Guo and W. Guo, "Stabilization and parameter estimation for an euler-bernoulli beam equation with uncertain harmonic disturbance under boundary output feedback control," *Nonlinear Analysis*, vol. 61, no. 5, pp. 671–693, 2005.

[16] W. He, S. S. Ge, and B. V. E. How, "Robust adaptive boundary control of a flexible marine riser with vesseld ynamics," *IEEE Transactions on Control Systems Technology*, vol. 47, no. 4, pp. 722–732, 2011.

[17] B. V. E. How, S. S. Ge, and Y. S. Choo, "Active control of flexible marine risers," *Journal of Sound and Vibration*, vol. 320, no. 4-5, pp. 758–776, 2009.

[18] M. S. D. Queiroz, M. Dawson, S. Nagarkatti, and F. Zhang, "Lyapunov-based control of mechanical systems," *Birkhauser*, 2000.

[19] W. He, B. V. E. How, S. S. Ge, and Y. S. Choo, “Boudary control of a flexible marine riser with vessel dynamics,” *Proceeding of American Control Conference*, vol. 57, no. 1, pp. 1532–1537, 2010.

[20] J. V. Amerongen, “Adaptive steering of ships- a model reference approach,” *Automatica*, vol. 20, no. 1, pp. 3–14, 1984.

[21] A. Sorensen, S. Sagatun, and T. Fossen, “Design of a dynamic positioning system using model-based control,” *Control Engineering Practice*, vol. 4, no. 3, pp. 359–368, 1996.

[22] T. Fossen and A. Grovlen, “Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping,” *IEEE Transactions on Control Systems Technology*, vol. 6, no. 1, pp. 121–128, 1998.

[23] T. Nguyen, A. Sorensen, and S. T. Quek, “Design of hybrid controlled for dynamic positioning from calm to extreme sea conditions,” *Automatica*, vol. 43, no. 5, pp. 768–785, 2007.

[24] W. Dong and Y. Guo, “Nonlinear tracking control of underactuated surface vessel,” *American Control Conference*, June 2005.

[25] J. Ghommam, F. Mnif, A. Benali, and N. Derbel, “Asymptotic backstepping stabilization of an underactuated surface vessel,” *IEEE Transactions on Control Systems Technology*, vol. 14, no. 6, pp. 1150–1157, 2006.

[26] T. L. Nguyen, K. D. Do, and J. Pan, “Boundary control of coupled nonlinear three dimensional marine risers,” *Journal of Marine Science and Applications*, vol. 12, pp. 72–88, 2013.

[27] K. D. Do, “Boundary control of transverse motion of flexible marine risers under stochastic loads,” *Ocean Engineering*, vol. 155, pp. 156–172, 2018.