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Discrete wavelet transform recursive inverse algorithm using second-order estimation of the autocorrelation matrix

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ABSTRACT

The recursive least squares (RLS) algorithm was introduced as an alternative to least mean square (LMS) algorithm with enhanced performance. Computational complexity and instability in updating the autocolleltion matrix are some of the drawbacks of the RLS algorithm that were among the reasons for the introduction of the second-order recursive inverse (RI) adaptive algorithm. The 2nd order RI adaptive algorithm suffered from low convergence rate in certain scenarios that required a relatively small initial step-size. In this paper, we propose a new second-order RI algorithm that projects the input signal to a new domain namely discrete wavelet transform (DWT) as pre step before performing the agorthim. This transformation overcomes the low convergence rate of the second-order RI algorithm by reducing the self-correlation of the input signal in the mentioned scenatios. Expeirments are conducted using the noise cancellation setting. The performance of the proposed algorithm is compared to those of the RI, original second-order RI and RLS algorithms in different Gaussian and impulsive noise environments. Simulations demonstrate the superiority of the proposed algorithm in terms of convergence rate compared to those algorithms.

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1. INTRODUCTION

Adaptive filtering techniques can promote accurate solutions and high convergence rates in many signal processing problems [1-3]. Some of these well-known problems; such as, noise cancellation [4, 5], channel equalization [6], and system identification [7, 8], have been addressed by many researchers for many decades. The straightforward steps of the least mean square (LMS) adaptive algorithm in weights update together with its fast convergence (if optimum step-size is selected), made it a very popular filtering algorithm. However, its convergence rate is easily affected by the spread of the eigenvalue of the autocorrelation matrix of the tap-input vector [9-13].

The recursive least square (RLS) algorithm [9] was introduced as an alternative to LMS algorithm with a superior performance. Particularly, in highly correlated environments with the possibility of high eigenvalue spread of the autocorrelation matrix. However, the RLS algorithm has its own drawbacks such as; high computational complexity, and updating the inverse autocorrelation matrix that may raise numerical stability problems [14]. To overcome such problems of the RLS algorithm, many other algorithms have been proposed.

The recursive inverse (RI) algorithm [15] has been proposed to overcome some of the above mentioned drawbacks. It has been shown that the RI algorithm performs significantly better than LMS algorithm and its variants. Also, its performance is very comparable to that of the RLS algorithm, in terms of convergence rate and excess mean square error (MSE) [16], in various settings, with less computational complexity. Futher improvement of the performance of the recursive inverse algorithm was achived by considering second-order estimation of the correlations in the update equation of the RI algorithm [17]. Even though the second-order RI converges to a lower MSE, its convergence rate is less than those of the RI and RLS algorithms. This slow convergence is due to second-order estimation of the correlations.

In this paper, we propose the use of discrete wavelet transform (DWT) to improve the performance of the second-order RI algorithm. This domain-based transformation gurentees the reduction of the self-correlation of the input signal that, in turn, helps to overcomes the low convergence rate of the second-order RI algorithm. Hence, we use the advantages of the RI algorithm compared to the RLS algorithm and by the virtue of DWT, the convergence rate is increased. The rest of the paper can be descried as follows: in section 2, DWT is reviewed. In section 3, the proposed algorithm is introduced. In section 4, simulation results that compare the performance of the proposed algorithm to those of the RI, second-order RI and RLS algorithms in different Gaussian and impulsive noise environments in a noise cancellation setting are presented. Finally, conclusions are drawn in section 5.

2. DISCRETE WAVELET TRANSFORM (DWT)

Multi-resolution decomposition theory that was developed by Mallat [18], gives scale-invariant interpretation of signals and images. Wavelet transform is considered to be a powerful approach of multi-resolution analysis to analyse signals that possess both low and high-frequency components. It has been developed to solve the time-frequency resolution problem in short time fourier transform (STFT) [19-21]. DWT decomposes the signal into orthogonal set of wavelets using filter banks. The output of the filter banks is a group of coefficients used to calculate the details and approximations of the signal. Accordingly, the original signal can be reconstructed from the scaling and the wavelet coefficients. A structure of discrete wavelet transform adaptive filter (DWTAF) is shown in Figure 1.

According to DWT theory, reconstruction of the original signal $\mathrm{x}(k)$ can be performed using the following finite sum:

$$\mathrm{x}(k) = \sum_{j=0}^{J-1} \sum_{n\in Z} \theta_{j,n}\, \psi_{j,n}(k) \tag{1}$$

where $\theta_{j,n}$ are the wavelet coefficients and $\psi_{j,n}(k)$ are the wavelet functions that form an orthogonal basis. The purpose of DWT adaptive filter is to generate the discrete reconstruction of $\mathrm{x}_j(k)$ which is the projected discrete form of $\mathrm{x}(k)$ in wavelet subspace. $\mathrm{x}_j(k)$ is given by:

$$\mathrm{x}_j(k) = \sum_{n\in Z} \theta_{j,n}\, \psi_{j,n}(k) \tag{2}$$

if $\mathrm{v}_j(k)$ is the approximation of projected $\mathrm{x}_j(k)$, then

$$\mathrm{v}_j(k) = \sum_{n\in Z} \hat{\theta}_{j,n}\, \psi_{j,n}(k) \tag{3}$$

where $\hat{\theta}_{j,n}$is the discrete approximation of the wavelet coefficients $\theta_{j,n}$,

$$\hat{\theta}_{j,n} = \sum_{l} \mathrm{x}(l)\, \hat{\psi}_{j,n}(l) \tag{4}$$

where $\hat{\psi}_{j,n}(l)$ represents the discrete approximation of the wavelet functions $\psi_{j,n}(l)$ given that,

$$h_j(l,k) = \sum_{n\in Z} \hat{\psi}_{j,n}(l)\, \psi_{j,n}(k) \tag{5}$$

Now, substituting (4) and (5) in (3) results in

$$\mathrm{v}_j(k) = \sum_{l} \mathrm{x}(l)\, h_j(l,k) \tag{6}$$

In (6) is simply the discrete convolution of the input signal $\mathrm{x}(k)$ and the filter coefficients $h_j(l,k)$. Using orthogonality and time-steadiness, filter indices can be rewritten as:

$$h_j(l,k) = h_j(l-k) \tag{7}$$

Therefore,

$$\mathrm{v}_j(k) = \sum_l \mathrm{x}(l)\, h_j(l-k) \tag{8}$$

3. DWT SECOND-ORDER RECURSIVE INVERSE ALGORITHM

Following the structure shown in Figure 1 and using the same notation used in section 2, the updated equation of the second-order RI algorithm [17] can be written as:

$$\mathrm{C}(k+1) = [\mathrm{I} - \mu(k)\mathrm{R}(k)]\mathrm{C}(k) + \mu(k)\mathrm{p}(k) \tag{9}$$

where k is the time parameter ($k = 1, 2, \ldots$), $\mathrm{C}(k)$ represents the filter weight vector calculated at time k, $\mathrm{v}(k) = \mathrm{Wx}(k)$ represents the transformed input signal and W represents the wavelet transform matrix of size $J \times N$. $\mu(k)$ represents the variable step-size [16] which satisfies the convergence criterion [9], the autocorrelation matrix $\mathrm{R}(k)$ represents the estimate of the tap-input vector, and $\mathrm{p}(k)$ represents the estimate of the cross-correlation vector between the desired output signal $d(k)$ and the tap-input vector estimated, recursively, as:

$$\mathrm{R}(k) = \beta_1\mathrm{R}(k-1) + \beta_2\mathrm{R}(k-2) + \mathrm{v}(k)\,\mathrm{v}^T(k) \tag{10}$$

$$\mathrm{p}(k) = \beta_1\mathrm{p}(k-1) + \beta_2\mathrm{p}(k-2) + d(k)\,\mathrm{v}(k) \tag{11}$$

where β_1 and β_2 are positive constants. Choosing the coefficients in (10) and (11) to be equal, i.e. $\beta_1 = \beta_2 = \frac{1}{2}\beta$, will gurentee that the The number of multiplications in the second order update equations will be the same as the first order update equations [16].

By taking the expectation of (10), the new equation can be written as:

$$\overline{\mathrm{R}}(k) = \frac{1}{2}\beta\overline{\mathrm{R}}(k-1) + \frac{1}{2}\beta\overline{\mathrm{R}}(k-2) + R_{vv} \tag{12}$$

where $\mathrm{R}_{\mathrm{vv}} = E\{\mathrm{v}(k)\,\mathrm{v}^T(k)\}$ and $\overline{\mathrm{R}}(k) = E\{\mathrm{R}(k)\}$. The poles of the system in (12) can be calcutaed using:

$$\begin{aligned} z_1 &= \frac{1}{4}\left(\beta - \sqrt{\beta^2 + 8\beta}\right) \\ z_2 &= \frac{1}{4}\left(\beta + \sqrt{\beta^2 + 8\beta}\right) \end{aligned} \tag{13}$$

which have magnitudes less than unity if $\beta < 1$. By solving (12) using the initial conditions $\overline{\mathrm{R}}(-2) = \overline{\mathrm{R}}(-1) = \overline{\mathrm{R}}(0) = 0$, it results in,

$$\overline{\mathrm{R}}(k) = \left(\frac{1}{\beta-1} + \alpha_1 z_1^k + \alpha_2 z_2^k\right)\mathrm{R}_{\mathrm{vv}} \tag{14}$$

where,

$$\begin{aligned} \alpha_1 &= \frac{\beta - z_2}{(1-\beta)(z_2 - z_1)}, \\ \alpha_2 &= \frac{\beta - z_1}{(1-\beta)(z_2 - z_1)}. \end{aligned} \tag{15}$$

using,

$$\gamma(k) = \frac{1}{\beta-1} + \alpha_1 z_1^k + \alpha_2 z_2^k \tag{16}$$

then, in the DWT second-order RI algorithm, the variable step-size is selected as:

$$\mu(k) = \frac{\mu_0}{\gamma(k)}, \tag{17}$$

where μ_0 is a constant [16] selected as:

$$\mu_0 < \mu_{max} = \frac{2(1-\beta)}{\lambda_{max}\,\mathrm{R}_{\mathrm{vv}}}$$

where λ_{max} is the maximum eigenvalue of R_{vv}.

The adaptive estimation error can be defined as:

$$e(k) = d(k) - y(k) \tag{18}$$

where,

$$y(k) = \mathrm{v}^T(k)\mathrm{C}(k) = \sum_{j=0}^{J-1} v_j(k)c_j(k) = \sum_{j=0}^{J-1}\sum_l c_j(k)\, h_j(l-k)\, x(l). \tag{19}$$

The major advantage of RI-based algorithms over the RLS-based algorithm is the unnecessity to update the inverse autocorrelation matrix [16]. Such an update of the inverse autocorrelation matrix might cuase numerical instabilities in RLS-based algorithms [22-24]. Fortunately, this is not the case for the RI algorithm and its variants.

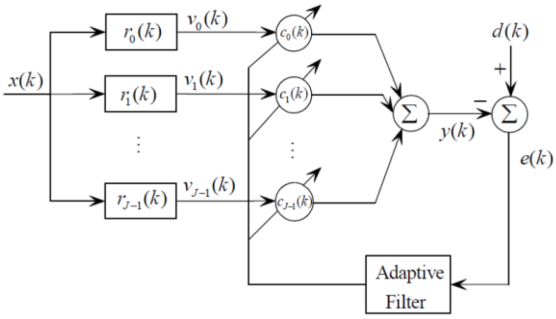

Figure 1. Structure of discrete wavelet transform transversal adaptive filter

4. SIMULATION RESULTS

The proposed algorithm is compared to the RI, second-order RI and RLS algorithms in the noise cancellation setting as shown in Figure 2 in terms of convergence rate and mean square error (MSE). In all conducted expeirments, filter length for all implemenetd algorithms was equal to 16 taps and SNR $= 30$ dB. The received signal is generated using:

$$x_i(k) = 1.79x_i(k-1) - 1.85x_i(k-2) + 1.27x_i(k-3) - 0.41x_i(k-4) + n_0(k),$$

where $n_0(k)$ is a Gaussian process with zero mean and variance $\sigma^2 = 0.15$. The simulation results for Gaussian and impulsive noise are obtained by averaging 1000 independent runs. For all experiments, the algorithms are simulated using the parameters in Table 1.

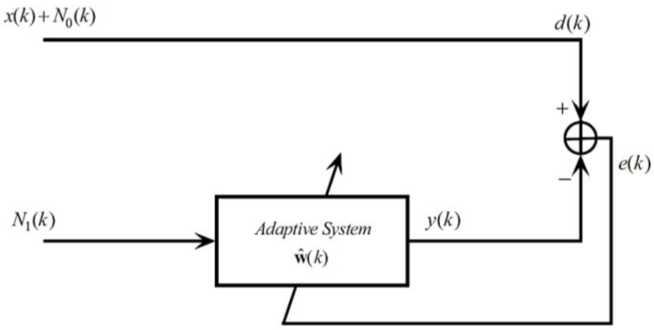

Figure 2. Block diagram of adaptive noise cancellation configuration

Table 1. Parameters used for simulating the proposed, 2^{nd} order RI, RI and RLS algorithms in all experiments

Algorithm	μ_0	β
Proposed	0.1	0.99
2^{nd} order RI	0.1	0.99
RI	0.0005	0. 99
RLS	-	0.99

4.1. Additive Gaussian noise

In order to test the performance of the proposed algorithm, the signal is assumed to be distorted with an additive white/correlated Gaussian noise (AWGN/ACGN) process. The correlated noise is created using AR(1) process ($N_0(k+1) = 0.7N_0(k) + v(k)$) where $v(k)$ respresents a white Gaussian process with mean equals to zero and a variance that maintains a 30 dB SNR. From Figure 3 and Figure 4, it can be seen that the proposed algorithm converges to same MSE (MSE = -30 dB) of all algorithms with faster convergence rate (approximately $170, 350$ and 850 iterations faster than the RLS, 2^{nd} order RI and RI algorithms, respectively).

4.2. Additive impulsive noise

Man-made noise, such as underwater acoustic noise, added to the received signal makes it hard to model the signal using Gaussian distribution. To over come this problem, such type of noise is believed to better modelled using a Gaussian mixture model. The impulsive noise process is generated by the probability density function [25]. $p = (1-\zeta)G(0,\sigma^2) + \zeta G(0,\kappa\sigma^2)$ with variance $\sigma_p^2 = (1-\zeta)\sigma^2 + \zeta\kappa\sigma^2$ where $G(0,\sigma^2)$ is a Gaussian probability density function with zero mean and variance σ^2 that represents the nominal background noise. $G(0,\kappa\sigma^2)$ represents the impulsive component of the noise model, where ζ is the probability and $\kappa \geq 1$ is the strength of the impulsive noise components, respectively.

In order to test the robustness of the proposed algorithm, and to study the effects of the impulsive components (outliers) of the noise process in the noise cancellation setting, an impulsive noise process is generated by the aforementioned probability density function with $\zeta = 0.2$ and $\kappa = 100$. Firstly, the signal is assumed to be distorted by an additive white impulsive noise (AWIN) process. Then, the same experiment is repeated while assuming the signal is corrupted by a correlated impulsive noise created using the aforementionedAR(1). In both Figures 5 and 6 it can be seen that the proposed algorithm converges to same MSE (MSE = -30 dB) of the 2^{nd} order RI and RI algorithms with faster conver-gence rate (approximately 400 and 600 iterations faster than 2^{nd} order RI and RI algorithms, respectively). In addition, it is noted that even though the RLS tries to converge at the beginning, it starts to slowly diverge after almost 800 iterations.

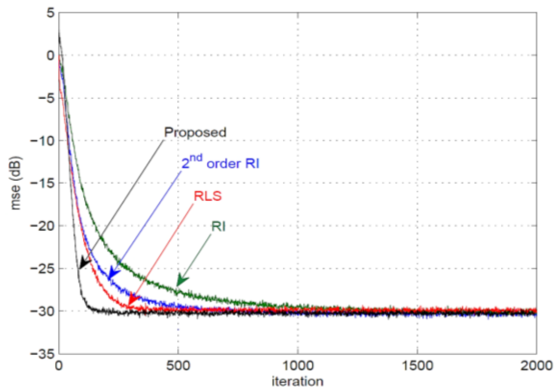

Figure 3. The ensemble MSE for the proposed, RI, 2nd order RI and RLS algorithms in AWGN

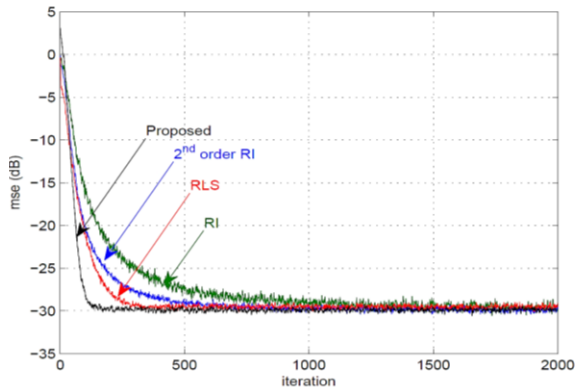

Figure 4. The ensemble MSE for the proposed, RI, 2nd order RI and RLS algorithms in ACGN

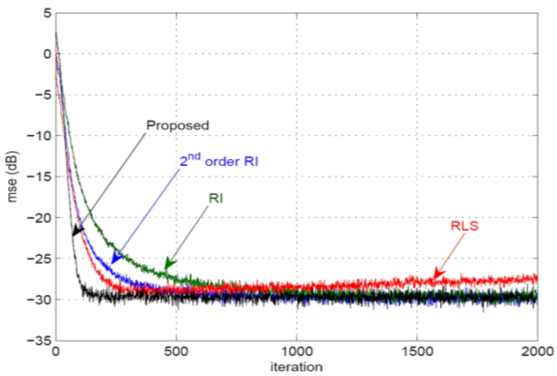

Figure 5. The ensemble MSE for the proposed, RI, 2nd order RI and RLS algorithms in AWIN

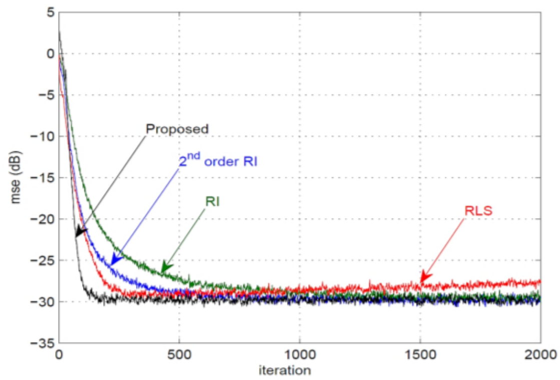

Figure 6. The ensemble MSE for the proposed, RI, 2nd order RI and RLS algorithms in ACIN

5. CONCLUSIONS

In this paper, a new domain transform based second-order RI algorithm was proposed. Applying DWT at the input signal has highly improved the performance of the original second-order RI algorithm. The performance of the proposed algorithm was evaluated using noise cancellation setting. It was compared to those of the RI, 2nd order RI and RLS algorithms in different Gaussian and impulsive noise environments. Condcuted experiments demonestrated that the proposed algorithm has superior convergence rate compared to those algorithms.

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