Isospin-breaking corrections in the pion-deuteron scattering length

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It is shown that isospin-breaking corrections to the pion-deuteron scattering length can be very large, because of the vanishing of the isospin-symmetric contribution to this scattering length at leading order in chiral perturbation theory. We further demonstrate that these corrections can explain the bulk of the discrepancy between the recent experimental data on pionic hydrogen and pionic deuterium. We also give the first determination of the electromagnetic low-energy constant f_1.

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Already in 1977, Weinberg pointed out [1] that the isospin-breaking corrections to certain pion-nucleon scattering amplitudes could become large because the isospin-symmetric contributions to these amplitudes are chirally suppressed. Unfortunately, Weinberg's statement refers to the scattering processes with neutral pions that makes its difficult to verify with present experimental techniques. It turns out, however, that there exists a fascinating possibility to directly observe a large isospin-breaking correction in the negatively charged pion elastic scattering on the deuteron, where the leading-order isospin-symmetric amplitude in chiral perturbation theory (ChPT) is proportional to the isospin-even pion-nucleon scattering length a^+ and is thus very small. Quite surprisingly, such (a rather obvious) phenomenon has not been explored so far. Studies of isospin breaking in the πd system (see, e.g. [2, 3]) include effects coming from the Coulomb field and/or the particle mass differences in the loops. Numerically these effects indeed turn out to be moderate. However, as it is well known (see e.g. [4, 5, 6, 7, 8]), isospin breaking in ChPT at leading order emerges through the direct quark-photon coupling encoded in the electromagnetic low-energy constants (LECs) of the effective chiral Lagrangian, as well as due to the explicit quark mass dependence of the pion-nucleon amplitudes. To the best of our knowledge, neither of these mechanisms have been taken into account in the existing investigations on pionic deuterium.

On the other hand, the presence of large isospin-breaking corrections may have serious implications for the combined analysis of the experimental data on pionic hydrogen and pionic deuterium, which is aimed at the precise determination the S-wave pion-nucleon scattering lengths [9, 10, 11] (see also [12]). As it is well known, in the experiment one measures the ground-state energy shift (ϵ_{1s}) and the width (Γ_{1s}) of pionic hydrogen, as well as the ground-state energy shift of pionic deuterium (ϵ^d_{1s}). Applying a theoretically calculated set of the isospin-breaking corrections, the hydrogen observables are related to the isospin-even and -odd S-wave πN scattering lengths a^+ and a^-

$$\begin{aligned}\epsilon_{1s} &= -2\alpha^3\mu_r^2(a^+ + a^-)(1+\delta_\epsilon)\,,\\ \Gamma_{1s} &= 8\alpha^3\mu_r^2 p_0\left(1+\frac{1}{P}\right)\left(a^-(1+\delta_\Gamma)\right)^2\,, \qquad (1)\end{aligned}$$

whereas the real part of the pion-deuteron scattering length $a_{\pi d}$ is expressed through the measured shift in pionic deuterium

$$\epsilon^d_{1s} = -2\alpha^3\mu_d^2\,\mathrm{Re}\,a_{\pi d}\,. \qquad (2)$$

In the above formulae, μ_r and μ_d denote the reduced mass of the $\pi^- p$ and $\pi^- d$ systems, respectively, $P = 1.546 \pm 0.009$ is the Panofsky ratio, α the fine-structure constant, p_0 is the center-of-mass momentum of the $\pi^0 n$ pair, which emerges in the decay of the pionic hydrogen and δ_ϵ, δ_Γ stand for the isospin-breaking corrections.

The present status of the data analysis, which is performed by the Pionic Hydrogen Collaboration at PSI, is the following. Until very recently, one used a set of corrections calculated using a potential model approach [13]: $\delta_\epsilon = (-2.1 \pm 0.5)\cdot 10^{-2}$ and $\delta_\Gamma = (-1.3 \pm 0.5)\cdot 10^{-2}$, with $\epsilon_{1s} = -7.108 \pm 0.013 \pm 0.034$ eV and $\Gamma_{1s} = 0.868 \pm 0.040 \pm 0.038$ eV, whereas the isospin-breaking corrections in the deuteron have been neglected [9]. This led to a coherent

picture which is shown in Fig. 1. In this plot, the constraints on the a^+ and a^-, which emerge from these three different measurements, are shown as three shaded strips denoted as "Hydrogen energy, potential model," "Hydrogen width, potential model" and "Deuteron, no isospin breaking." The first strip corresponds to the value of $a^+ + a^-$ determined from the energy shift measurement in pionic hydrogen, the second strip defines a^- from the hydrogen width and the third strip is obtained from the experimental value $\mathrm{Re}\, a_{\pi d}^{\rm exp} = -(0.0261 \pm 0.0005) M_\pi^{-1}$ [10] by applying the formula

$$\begin{aligned}
\mathrm{Re}\, \bar a_{\pi d} &= 2\,\frac{1+\mu}{1+\mu/2}\,a^+ \\
&+\; 2\,\frac{(1+\mu)^2}{1+\mu/2}\,\left((a^+)^2 - 2(a^-)^2\right)\,\frac{1}{2\pi^2}\,\left\langle\frac{1}{\mathbf{q}^2}\right\rangle_{\rm wf} \\
&+\; 2\,\frac{(1+\mu)^3}{1+\mu/2}\,\left((a^+)^3 - 2(a^-)^2(a^+ - a^-)\right)\,\frac{1}{4\pi}\,\left\langle\frac{1}{|\mathbf{q}|}\right\rangle_{\rm wf} \\
&+\; a_{\rm boost} + \cdots\,, \qquad (3)
\end{aligned}$$

which was derived in Ref. [14] within ChPT under the assumption of exact isospin symmetry (the bar over $a_{\pi d}$ refers to the quantities in the isospin limit). In the above equation, $a_{\rm boost} = (0.00369 \cdots 0.00511) M_\pi^{-1}$ and $\langle 1/\mathbf{q}^2\rangle_{\rm wf} = (12.3 \pm 0.3) M_\pi$ and $\langle 1/|\mathbf{q}|\rangle_{\rm wf} = (7.2 \pm 1.0) M_\pi^2$, where NLO wave functions with the cutoff mass in the interval $\Lambda = (500 \cdots 600)$ MeV [15] have been used in order to evaluate the above wave-function averages. As one immediately sees from the plot, all three strips intersect in a small domain of the (a^+, a^-)-plane shown by a cross. The resulting values of the scattering lengths are $a^+ = (-0.0034 \pm 0.0007) M_\pi^{-1}$ and $a^- = (0.0918 \pm 0.0013) M_\pi^{-1}$ [14].

Recent developments have completely changed the picture. On the experimental side, new measurements have resulted not only in a considerably improved accuracy for the hydrogen width, but in a shift of its central value as well. The latest results are [11]: $\epsilon_{1s} = -7.120 \pm 0.008 \pm 0.009$ eV and $\Gamma_{1s} = 0.800 \pm 0.030$ eV. On the theoretical side, there now exist calculations of the isospin-breaking corrections in ChPT at $O(p^2)$ [7, 8, 16]. Applying these corrections to the latest experimental data, we get two shifted bands which are shown in Fig. 1. Note also that the increase of the uncertainty in the energy shift in [7, 8] was largely due to the unknown $O(p^2)$ LEC f_1, which was first considered in [4] and merely omitted in the potential model calculations. In the case of the energy shift, the correction at $O(p^3)$ is available as well [8]. For consistency reasons however, we use $O(p^2)$ results everywhere. As evident from the plot, the three bands do not have a common intersection domain any more. It could of course be argued that the discrepancy is due to the incomplete treatment of the deuteron structure. Recent investigations (see, e.g. [14, 17, 18, 19, 20]), however, converge to the conclusion that the uncertainty in the three-body calculations can be made rather small – at least, it can not be solely responsible for the large discrepancy which one observes in Fig. 1. With this conclusion, it becomes evident that we encounter a serious problem in the interpretation of the experimental data.

The only loophole left in the theoretical treatment of the πd scattering length lies in the assumption of exact isospin invariance. In the real world isospin symmetry is broken due to two distinct sources: the electromagnetic interactions and the u, d quark mass difference. It is convenient to to treat these two effects on the same footing, introducing a formal parameter δ so that $\alpha \sim (m_d - m_u) \sim \delta$. In ChPT the parameter δ is counted as $O(p^2)$. Further, in the isospin-symmetric world, by convention, the masses of the pions and the nucleons coincide with the charged pion mass $M_{\pi^\pm} \doteq M_\pi$ and the proton mass m_p, respectively (see e.g. Ref. [21] for more details). The πd scattering length is given by

$$\begin{aligned}
\mathrm{Re}\, a_{\pi d} &= \mathrm{Re}\, \bar a_{\pi d} + \Delta a_{\pi d}\,, \\
\Delta a_{\pi d} &= A_1 \alpha + A_2 (m_d - m_u) + O(\delta^2)\,, \qquad (4)
\end{aligned}$$

and A_1, A_2 can be further expanded in quark masses in ChPT. The expansion of the isospin-breaking contribution starts at order p^2

$$\Delta a_{\pi d} \;=\; \Delta a_{\pi d}^{\rm LO} + O(p^3)\,, \qquad (5)$$

i.e. at the same order as the leading isospin-conserving term in Eq. (3). For this reason, it is no wonder that the isospin-breaking corrections turn out to be large.

Let us now consider different pion-nucleon scattering amplitudes in the physical particle basis: $\pi^- p \to \pi^- p$ (labeled hereafter as "p"), $\pi^- n \to \pi^- n$ ("n") and $\pi^- p \to \pi^0 n$ ("x"). In the absence of virtual photons, the threshold amplitudes are defined according to

$$\langle \pi' N' | T | \pi N \rangle\big|_{\rm thr} = \bar u(N') \mathcal{T}_{p,n,x} u(N)\,, \qquad (6)$$

where we use the normalization $\bar u(N) u(N) = 2 m_N$ and N denotes the proton or the neutron. In the presence of virtual photons, the definition of the threshold amplitude is modified: prior to approaching the threshold, one has to subtract one-photon exchange contribution and to remove the (infrared-divergent) Coulomb phase [7, 8]. Below we do not consider these complications, because we work at $O(p^2)$ in ChPT and the virtual photons are still not present at this order. Further, using the Condon-Shortley phase convention, we may write

$$\begin{aligned}
\mathcal{T}_p &= 4\pi(1+\mu)(a^+ + a^-) + \delta\mathcal{T}_p\,, \\
\mathcal{T}_n &= 4\pi(1+\mu)(a^+ - a^-) + \delta\mathcal{T}_n\,, \\
\mathcal{T}_x &= 4\pi(1+\mu)(\sqrt{2} a^-) + \delta\mathcal{T}_x\,, \qquad (7)
\end{aligned}$$

where $\mu = M_\pi/m_p$ and $\delta\mathcal{T}_{p,n,x} = O(\delta)$. At $O(p^2)$ in ChPT we get [5, 6, 7, 8, 16]

$$\delta\mathcal{T}_p \;=\; \frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2}\,c_1 - \frac{e^2}{2}\,(4 f_1 + f_2) + O(p^3)\,,$$

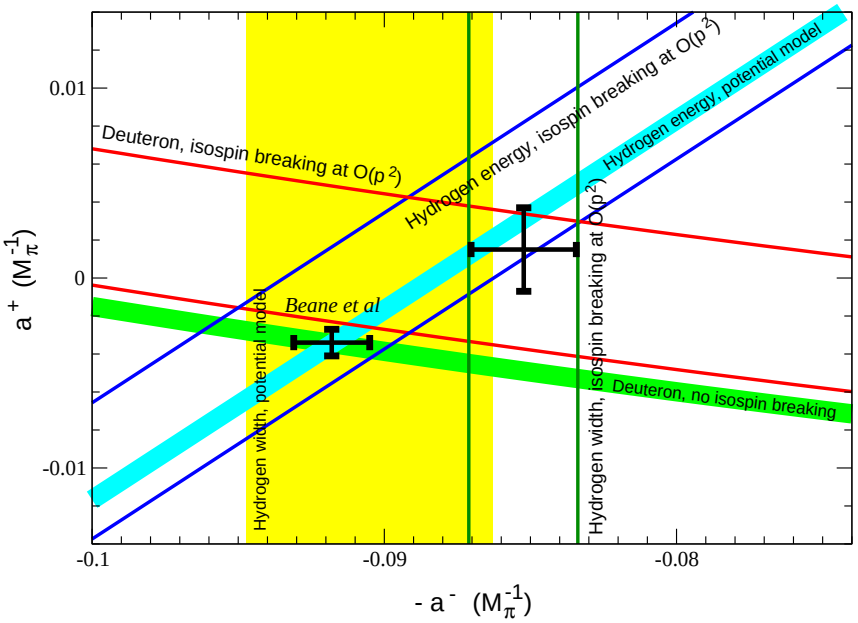

FIG. 1: Determination of the πN S-wave scattering lengths a^+ and a^- from the combined analysis of the experimental data on the pionic hydrogen energy shift and width, as well as the pionic deuterium energy shift (details in the text). The cross denoted as *Beane et al* is taken from Ref. [14]. The second cross corresponds to the scattering lengths given in Eq. (11).

$$\delta\mathcal{T}_n = \frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2}\, c_1 - \frac{e^2}{2}\,(4f_1 - f_2) + O(p^3)\,,$$
$$\delta\mathcal{T}_x = -\sqrt{2}\left(\frac{g_A^2(M_\pi^2 - M_{\pi^0}^2)}{4m_p F_\pi^2} + \frac{e^2 f_2}{2}\right) + O(p^3)\,, \quad (8)$$

where $F_\pi = 92.4$ MeV is the pion decay constant, $g_A = 1.27$ denotes the axial-vector charge of a nucleon and c_1 and f_1, f_2 stand for the $O(p^2)$ strong and electromagnetic LECs, respectively. In the numerical calculations we take $c_1 = -0.9^{+0.5}_{-0.2}\,\mathrm{GeV}^{-1}$ [22], $f_2 = -(0.97\pm0.38)\,\mathrm{GeV}^{-1}$ [8]. Note that the errors on the LEC c_1 are most conservative. The largest uncertainty in the results is introduced by the constant f_1, whose value at present is unknown and for which the dimensional estimate $|f_1| \leq 1.4\ \mathrm{GeV}^{-1}$ has been used. Note also, that the hydrogen energy band, which is shown in Fig. 1 corresponds to the new value of c_1 given above.

At the leading order, the isospin-breaking correction to the πd scattering length is given by

$$\Delta a_{\pi d}^{\rm LO} = (4\pi(1+\mu/2))^{-1}(\delta\mathcal{T}_p + \delta\mathcal{T}_n)\,. \quad (9)$$

One sees that the leading-order isospin-breaking correction is independent on the deuteron structure, which enters in the subsequent terms through the wave-function averages. Substituting numerical values for the various low-energy constants one obtains that the correction is extremely large

$$\Delta a_{\pi d}^{\rm LO} = -(0.0110^{+0.0081}_{-0.0058})\, M_\pi^{-1}\,, \quad (10)$$

that is $\Delta a_{\pi d}^{\rm LO}/\mathrm{Re}\, a_{\pi d}^{\rm exp} = 0.42$ (central values). Moreover, one can immediately see that the correction moves the deuteron band in Fig. 1 in the right direction: the isospin-breaking corrections amount for the bulk of the discrepancy between the experimental data on pionic hydrogen and deuterium. Including the corrections $\Delta a_{\pi d}^{\rm LO}$, all bands now have a common intersection area in the a^+, a^--plane, see Fig. 1. The resulting values for the πN scattering lengths are:

$$a^+ = (0.0015 \pm 0.0022)\, M_\pi^{-1}\,,$$
$$a^- = (0.0852 \pm 0.0018)\, M_\pi^{-1}\,. \quad (11)$$

Further, using the hydrogen energy shift to estimate the LEC f_1, we obtain

$$f_1 = -2.1^{+3.2}_{-2.2}\ \mathrm{GeV}^{-1}\,. \quad (12)$$

Note that the error displayed here does not include the uncertainty coming from the higher orders in ChPT and should thus be considered preliminary. We also wish to point out that the central value of f_1 (large and negative) agrees with a recent model-based estimate [23].

As we see, the presence of the $O(p^2)$ LECs in the expressions for the isospin-breaking corrections leads to a sizeable increase of the uncertainty in the output. In order to gain precision, in the fit one might also use those particular linear combination(s) of the experimental observables that do not contain f_1 and c_1. To carry out such a combined analysis with the required precision one would first have to evaluate the isospin-breaking corrections with a better accuracy.

Up to now, we have restricted ourselves to the leading-order isospin-breaking correction in ChPT. Below, we

briefly consider higher orders, where a full-fledged investigation has not been done yet. Calculations at $O(p^3)$ exist only for the hydrogen energy shift and yield $\delta_\epsilon = (-7.2 \pm 2.9) \cdot 10^{-2}$ [8] (using $c_1 = (-0.93 \pm 0.07)\ \mathrm{GeV}^{-1}$). The corrections to $O(p^2)$ result are sizable (the energy band in Fig. 1 will be shifted further upwards), but the uncertainty, which is almost completely determined by the $O(p^2)$ LECs, remains practically the same. On the other hand, consistent studies at $O(p^3)$ imply the treatment of the scattering process in the three-body system in the effective field theory with virtual photons. To the best of our knowledge, such investigations have not been yet carried out. At the moment, it would be plausible to put forward the conjecture that all isospin-breaking effects in $\mathrm{Re}\, a_{\pi d}$ at $O(p^3)$ still emerge from the first term in the multiple-scattering series, which contains only $\mathcal{T}_p$ and $\mathcal{T}_n$, whereas the corrections that depend on the structure of the deuteron, start at $O(p^4)$. From this we expect that in order to extract more precise experimental values of the scattering lengths, it might suffice to obtain a full set of isospin-breaking corrections to the πN amplitudes $\delta\mathcal{T}_{p,n,x}$ at $O(p^3)$ in ChPT. Of course, the discussion given here can not be a substitute for a rigorous proof in the framework of EFT, which in the light of the above discussion, is urgently needed.

As mentioned above, isospin-breaking corrections at $O(p^4)$ depend on the details of the NN interactions and the deuteron structure. In practice, it might prove rather difficult to evaluate these corrections with sufficient accuracy. On the other hand, in order to get a feeling of how large the $O(p^4)$-contributions could be, let us consider a typical correction which emerges from the double-scattering term in the multiple-scattering series, Eq. (3)

$$\begin{aligned} \Delta a_{\pi d}^{\text{double scat.}} &= \frac{1+\mu}{4\pi^3(1+\mu/2)} \left\langle \frac{1}{\mathbf{q}^2} \right\rangle_{\text{wf}} \\ &\times \left\{ (a^+ - a^-)\delta\mathcal{T}_p + (a^+ + a^-)\delta\mathcal{T}_n - \sqrt{2}a^-\delta\mathcal{T}_x \right\} \quad (13) \\ &= (-0.023 + 0.028 - 0.002)\, \mathrm{Re}\, a_{\pi d}^{\text{exp}} = 0.003\, \mathrm{Re}\, a_{\pi d}^{\text{exp}}\,. \end{aligned}$$

As an input, we have used the scattering lengths from Eq. (11). The individual terms in this equation amount to a few percent of the isospin-symmetric contribution (with additional cancellations in the sum) that may serve as a rough order-of-magnitude estimate for the isospin-breaking corrections at $O(p^4)$.

Last but not least, we note that in our opinion, the above discussion clearly justifies the need for an improved measurement of the energy shift in pionic deuterium. As we have seen, the uncertainties, which emerge in the treatment of the deuteron structure within effective field theory are much smaller than the uncertainties due to the presence of the $O(p^2)$ LECs (although, there is still some room for the improved higher-order calculations in the isospin-symmetric sector). On the other hand, it is evident that a major effort is needed on the theoretical side: in order to extract the scattering lengths to a good precision, the isospin-breaking corrections should be evaluated at least at $O(p^3)$ in ChPT.

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