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Renormalization of one-pion exchange and power counting

A. Nogga,[1,*] R. G. E. Timmermans,[2,†] and U. van Kolck[3,‡]
[1]*Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany*
[2]*Theory Group, KVI, University of Groningen, The Netherlands*
[3]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*
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The renormalization of the chiral nuclear interactions is studied. In leading order, the cutoff dependence is related to the singular tensor interaction of the one-pion exchange potential. In S waves and in higher partial waves where the tensor force is repulsive this cutoff dependence can be absorbed by counterterms expected at that order. In the other partial waves additional contact interactions are necessary. The implications of this finding for the effective field theory program in nuclear physics are discussed.

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I. INTRODUCTION

It is commonly accepted that QCD is the correct theory for the strong interaction in the energy regime of interest for nuclear physics. At the same time, this is of limited practical value, because in this energy regime QCD needs to be solved nonperturbatively. Lattice simulations, which in principle can deal with this nonperturbative character, are, for systems with $A \geqslant 2$ nucleons, still in their beginning stages [1,2].

A possible way out of this dilemma is the application of chiral perturbation theory (ChPT) to nuclear systems [3,4]. Making use of the spontaneously broken chiral symmetry of QCD, one can formulate an effective field theory (EFT) involving nucleons (N) and the Goldstone bosons related to chiral-symmetry breaking, the pions (π). Chiral perturbation theory is a powerful approach because it relates processes with different numbers of pions. In the purely pionic and one-nucleon sectors, the Goldstone-boson character of pions guarantees that amplitudes can be expanded in powers of momenta [5,6]. In the few-nucleon sector, however, the existence of bound states clearly shows the nonperturbative nature of the problem.

Weinberg [7,8] recognized that this nonperturbative nature is caused by an infrared enhancement in the propagation of two or more nucleons. He suggested that the calculation of a generic nuclear amplitude should consist of two steps. In the first step, one defines the nuclear potential as the sum of "irreducible" subdiagrams that do not contain purely nucleonic intermediate states and truncates the sum according to a simple extension of the standard ChPT power counting. In a second step, the potential is iterated to all orders, which can be done by using the Lippmann-Schwinger (LS) or Schrödinger equations. The potential includes pion exchanges and contact interactions, which represent the contributions of more massive degrees of freedom. By assuming that contact interactions obey naive dimensional analysis, one finds that only a finite number of pion exchanges and contact interactions contribute to the potential at any given order: Two nonderivative two-nucleon contact interactions and one-pion exchange (OPE) in leading order and derivative contact interactions, two- and more-pion exchanges, and few-nucleon interactions in subleading orders.

This power counting naturally explains that two-nucleon interactions are more important than three-nucleon interactions, etc. [9]. The resulting two-nucleon (NN) [10,11] and three-nucleon ($3N$) [12,13] potentials provide a quantitative description of few-nucleon systems [14–23]. In addition, this approach matches well with the Nijmegen energy-dependent partial-wave analysis (PWA) of NN scattering data [24]. In this analysis, a separation of long- and short-distance physics is implemented by solving the partial-wave Schrödinger equation with a long-range potential that consists of OPE and two-pion exchange (TPE) (and the electromagnetic interaction) and a boundary condition with as many short-range parameters as are needed for an optimal description of the observables. The pion mass and OPE parameters [25,26] and even TPE parameters [27,28] could be determined from the NN scattering data, in good agreement with values obtained from pion-nucleon scattering [29–31].

However, Weinberg's power counting has been criticized. As in any EFT, a regularization procedure is required to separate high- and low-energy physics. Since this separation is arbitrary, a consistent power counting should provide sufficient counterterms at each order to absorb any cutoff dependence in the limit of large cutoffs. Because the solution of the LS equation is numerical in character, an explicit check of cutoff independence is challenging. This led Kaplan and coworkers [32–34] to examine a few of the diagrams contributing to the NN T matrix. They identified in two-loop diagrams ultraviolet divergences proportional to the square of the pion mass and of the external momenta, which are present in leading order but cannot be absorbed by the available counterterms. They concluded that pion exchange should not be fully iterated; instead it should be treated in finite order in perturbation theory. Quantitative calculations at higher order showed, however, that this idea fails in some partial waves at momenta comparable to the pion mass [35].

For smaller momenta, one can integrate out the pion and construct a "pionless" EFT, which is very successful within

*E-mail: a.nogga@fz-juelich.de
†E-mail: timmermans@kvi.nl
‡E-mail: vankolck@physics.arizona.edu

its limited range [4]. Nevertheless, a lot of interesting nuclear physics is thought to take place at momenta of the order of the pion mass. (The Fermi momentum of isospin-symmetric nuclear matter, for example, is about 300 MeV.) It seems unavoidable that in this larger momentum range pion exchange has to be iterated.

It is now well known that the renormalization of an EFT is not necessarily the same as that of its perturbative series. This is seen clearly in the three-body problem in the pionless EFT [36–38]. The origin of this feature lies in the renormalization of singular potentials [39,40]. In the specific case of OPE, the singularity is the $1/r^3$ behavior of the tensor force in spin-triplet channels. It has been found that the cutoff dependence of an uncoupled $1/r^n$ interaction in the S wave can be absorbed into one counterterm [40].

The renormalization of OPE in lowest waves was reexamined from the nonperturbative viewpoint in Refs. [41,42]. The problem with the ultraviolet divergence proportional to the pion mass squared in the ${}^1S_0$ channel persists in this context [42]. However, the divergence associated with momenta, present in the ${}^3S_1$-${}^3D_1$ coupled channel, *can* be absorbed into the available leading order counterterm [41,42]. With a further expansion around the chiral limit, Weinberg's power counting seems to be consistent in a *nonperturbative* calculation of these waves [42–44]. (For a different conclusion, see Ref. [45].)

One-pion exchange contributes, however, also in higher partial waves. The naive power counting does not predict leading order counterterms in these partial waves. However, the singularity of the tensor interaction exists in all the spin-triplet channels. In fact, it has been argued that for an uncoupled singular interaction boundary conditions need to be fixed in all waves where the potential is attractive [46]. Therefore, cutoff dependence can be expected in some spin-triplet channels if there are no corresponding counterterms, posing a significant difficulty for Weinberg's power counting.

Another important renormalization issue concerns few-nucleon forces. In the pionless theory it has been shown that consistent renormalization requires a $3N$ force in leading order [38]. This result does not necessarily contradict Weinberg's power counting in the "pionful" EFT, because the $3N$ force in the pionless theory includes contributions that are iterations of the NN force with intermediate-state nucleons of momentum $\mathcal{O}(m_\pi)$ in the pionful theory. The two EFTs have NN interactions with different ultraviolet behaviors. Whether OPE sufficiently softens the asymptotic behavior of the $3N$ LS equation is an issue that remains unresolved.

In practice, the renormalization issue has been sidestepped by choosing rather low cutoffs to regularize the LS equation and by varying the cutoffs only in a very limited range [14–23]. Cutoff dependence has generally been observed in higher partial waves; see, for example, the discussion in Ref. [47] regarding higher orders in the Weinberg expansion. For relatively small cutoff variations, it has been noticed that the resulting variations in the phase shifts decreases with increasing order. It has then been assumed that the observed cutoff dependence is of the order of the error expected from the truncation of the expansion. In fact, it has been argued that the EFT involving nucleons and pions necessarily involves a mild cutoff dependence and that cutoff values exist that are optimal for the convergence of the expansion [48,49].

An in-depth study of the cutoff dependence in higher NN partial waves and in the $3N$ system still needs to be performed. This study is the aim of this work. We seek to quantify the cutoff dependence in lowest order and, if possible, to identify ranges of cutoffs in which only small variations of observables occur. We then discuss how to absorb the cutoff dependence in a finite number of counterterms. We first consider NN scattering, in which case we compare our results with the phase shifts and mixing angles from the energy-dependent Nijmegen PWA93 [24], which provide an optimal representation of the NN database. (The fact that this PWA93 did not yet include TPE [27,28] does not affect our investigation of OPE.) We then extend our analysis to the $3N$ bound state. We will restrict ourselves to total NN angular momentum $j \leqslant 4$, which is sufficient to study the $3N$ binding energy. As we are going to show, our results have significant implications: We present here a modification of power counting that is consistent with all known results and could become a new basis to organize nuclear interactions in the EFT with pions.

Section II describes the interaction and our approach to regularize and solve the NN LS equation. In Sec. III, we identify problematic partial waves, explicitly show their cutoff dependence, and present counterterms that generate cutoff-independent phase shifts in reasonable agreement with the PWA. Section IV is devoted to the $3N$ system. The implications of our findings to power counting in nuclear ChPT are analyzed in Sec. V. Finally, our conclusions and an outlook are given in Sec. VI.

II. REGULARIZATION OF THE LIPPMANN-SCHWINGER EQUATION

We first consider NN scattering in the center-of-mass frame. We denote by μ the reduced mass (where $m_N = 2\mu$ is the nucleon mass), by E the energy, and by $\vec{p}$ and $\vec{p}\,'$ the relative momenta, respectively, before and after interaction; the momentum transfer is $\vec{q} = \vec{p} - \vec{p}\,'$. The relative distance between the two nucleons is $\vec{r}$. The standard Pauli matrices in spin and isospin space are denoted by $\vec{\sigma}_i$ and $\boldsymbol{\tau}_i$, respectively.

With a standard normalization for plane waves, the LS equation for the T matrix reads, in momentum space,

$$T(\vec{p}\,', \vec{p}, E) = V(\vec{p}\,', \vec{p}) + \int d^3p'' V(\vec{p}', \vec{p}'') \times \frac{1}{E + i\epsilon - \frac{\vec{p}''^2}{2\mu}} T(\vec{p}'', \vec{p}, E), \qquad (1)$$

where V is the potential. The OPE potential is

$$V_{1\pi}(\vec{q}\,) = -\frac{1}{(2\pi)^3}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\vec{\sigma}_1 \cdot \vec{q}\,)(\vec{\sigma}_2 \cdot \vec{q}\,)}{\vec{q}^{\,2} + m_\pi^2}, \qquad (2)$$

where m_π is the pion mass. In lowest order the strength of OPE is completely determined by the axial-coupling constant $g_A = 1.26$ and the pion-decay constant $f_\pi = 92.4$ MeV.

In addition to pion exchanges, the EFT contains short-range interactions that represent high-energy degrees of freedom

that have been integrated out. The simplest are two contact interactions,

$$V_c = \frac{1}{4\pi}\frac{1}{(2\pi)^3}(c_s P_s + c_t P_t), \tag{3}$$

where we used the projectors onto spin-triplet and spin-singlet states, P_t and P_s. The two strength parameters c_s and c_t need to be determined from *NN* scattering data, for instance from the scattering lengths in the ${}^1S_0$ and ${}^3S_1$ channels. It is possible to write

$$c_s = C_0 + m_\pi^2 D_2 + \ldots, \tag{4}$$

where the parameters C_0 and D_2 are independent of the quark masses.

For the numerical solution of the LS equation, we need to introduce a regulator $f(p', p)$ that effectively cuts momenta at a cutoff Λ. The regularization procedure is an arbitrary splitting of short-range physics into the high-momentum region of loops and contact interactions. Low-energy physics should, of course, be independent of the choice of regulator (renormalization-group invariance), once the dependence of contact parameters on the cutoff is taken into account, and the cutoff is much larger than the momenta of interest. It is convenient for the partial-wave decomposition to perform the regularization using momentum cutoff functions depending on $\vec{p}$ and $\vec{p}\,'$ rather than on $\vec{q}$. Here we use

$$f(p', p) = e^{-(p^4+p'^4)/\Lambda^4}. \tag{5}$$

This leads to nonlocal interactions in configuration space. However, because the regulator only depends on the magnitude of the relative momenta, it does not influence the partial-wave decomposition. This guarantees that contact interactions act in specific partial waves, independent of Λ. In particular, it implies that V_c only acts in the two *S* waves.

For the following discussion, it is useful to look also at the configuration space expression for OPE,

$$V_{1\pi}(\vec{r}\,) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2[T(r)S_{12} + Y(r)\vec{\sigma}_1\cdot\vec{\sigma}_2], \tag{6}$$

where

$$\begin{aligned} T(r) &= \frac{e^{-m_\pi r}}{m_\pi r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right], \\ Y(r) &= \frac{e^{-m_\pi r}}{m_\pi r}, \end{aligned} \tag{7}$$

TABLE I. Matrix elements of the operator $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\, S_{12}$ for spin-triplet channels with total angular momentum *j*. The matrix elements depend on the isospin *t* and on the incoming and outgoing angular momenta *l* and l'.

t	$s=1$	$l=j-1$	$l=j$	$l=j+1$
$t=1$	$l'=j-1$	$-2\frac{j-1}{2j+1}$	0	$6\frac{\sqrt{j(j+1)}}{2j+1}$
	$l'=j$	0	2	0
	$l'=j+1$	$6\frac{\sqrt{j(j+1)}}{2j+1}$	0	$-2\frac{j+2}{2j+1}$
$t=0$	$l'=j-1$	$6\frac{j-1}{2j+1}$	0	$-18\frac{\sqrt{j(j+1)}}{2j+1}$
	$l'=j$	0	-6	0
	$l'=j+1$	$-18\frac{\sqrt{j(j+1)}}{2j+1}$	0	$6\frac{j+2}{2j+1}$

and the tensor operator is

$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2. \tag{8}$$

The tensor force $T(r)$ of OPE contains a singular interaction $\sim 1/r^3$ that acts in the spin-triplet waves; the tensor force is zero in the spin-singlet channels. Using the partial-wave matrix elements given in Table I one can identify whether the tensor force is attractive or repulsive in specific partial waves. This we will require in the following.

III. NUCLEON-NUCLEON PHASE SHIFTS

Our aim is to study the dependence of observables on the chosen value for the cutoff Λ. We have performed a partial-wave decomposition of the interaction described in the previous section and then solved the LS equation and extracted phase shifts. The explicit expressions are summarized in Appendix A. We study the cutoff dependence of the phase shifts in leading order (LO), or $\mathcal{O}(Q^0)$, in Weinberg's power counting. We consider Λ in a wide range, between 2 and 20 fm^{-1}.

We start with the *S*-wave channels, which were previously examined in Refs. [41–44] with different regularizations. We fit c_s and c_t to the ${}^1S_0$ and ${}^3S_1$ phase shifts at 10 MeV and we confirm the cutoff independence found in Refs. [41,42], as can been seen in Figs. 1 and 2. In Fig. 1 we show the running of c_s with the cutoff Λ and the resulting cutoff dependence of the ${}^1S_0$ phase shifts at various laboratory energies. In Fig. 2 we show the corresponding results for c_t and the ${}^3S_1$ and ${}^3D_1$ phase shifts and the mixing angle ε_1. One sees that the cutoff dependence of the phase shifts is small for $\Lambda \gtrsim 5$ fm^{-1}, but

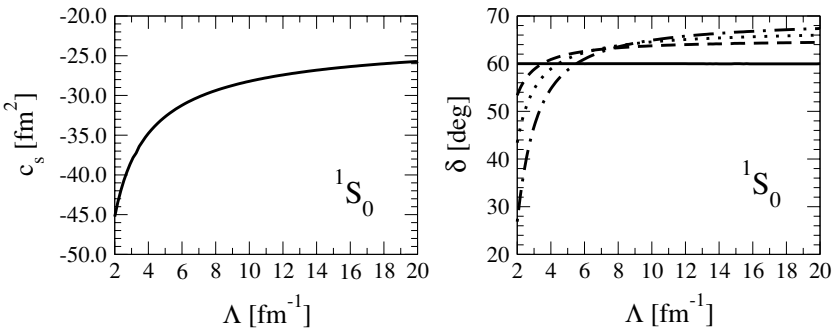

FIG. 1. Fit result for the counterterm c_s as a function of the cutoff, and the resulting cutoff dependence of the ${}^1S_0$ phase shifts at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

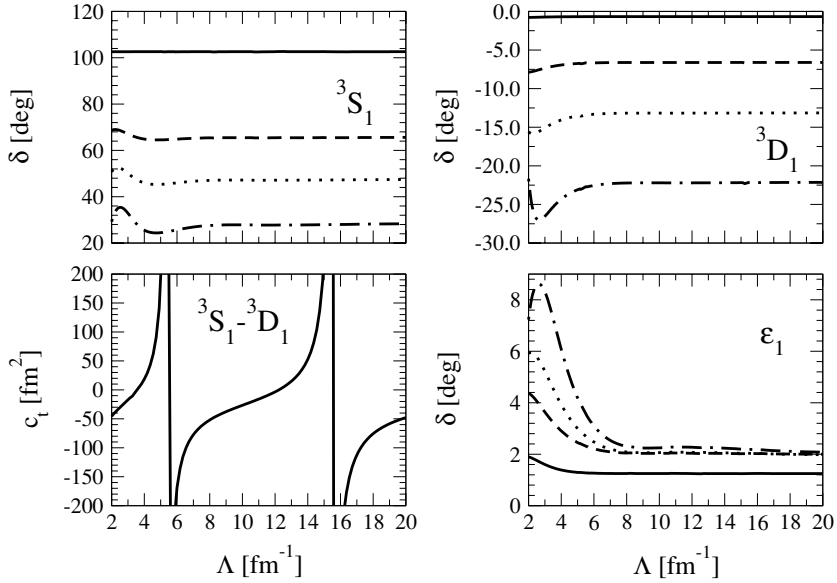

FIG. 2. Fit result for the counterterm c_t as a function of the cutoff, and the resulting cutoff dependence of the $^3S_1$-$^3D_1$ phase shifts and the mixing angle ε_1 at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

it increases for increasing energy, as expected in an EFT. It is interesting to note that $c_t(\Lambda)$ displays a nice limit-cycle-like behavior, similar to the $3N$ force in the $3N$ problem in pionless EFT [38], which is solved using a regulator similar to ours. Since the counterterm strength behaves differently in Ref. [42], where a coordinate-space regulator was employed instead, we conclude that this behavior is regulator dependent. This is in line with a similar recent finding for a central potential [50,51]. Note that the running found here is similar to that observed in Ref. [52] for a counterterm in a different channel at higher orders but with the same regularization of the LS equation. This suggests that the form of the running is perhaps more influenced by the regulator than the specific form of the singularity of the interaction.

The resulting phase shifts as a function of the laboratory energy are shown in Fig. 3. In the $^1S_0$ channel, we recover the known strong deviation of the LO result from the PWA. This is related to the relatively large effective-range parameter in this partial wave [42,53], which cannot be reproduced without a two-derivative contact interaction. This problem is solved once the latter is included in subleading order (see, e.g., Ref. [42]). In the coupled $^3S_1$-$^3D_1$ channels, we find an encouraging agreement with the PWA. The mixing angle ε_1 is underpredicted, when one goes to the limit of high Λ. In this limit the agreement with the PWA has, however, improved compared to the choice $\Lambda \simeq 3\,\mathrm{fm}^{-1}$ used in the literature [16].

Despite these positive results, there are potential problems in other waves. The explicit expression in Eq. (6) for $V_{1\pi}$ in configuration space suggests that, because of the singularity of $T(r)$, we can expect a number of bound states—infinite in the limit $\Lambda \to \infty$—in all channels where the tensor force is attractive. A consequence would be cutoff dependence in these waves. We will, therefore, study higher partial waves, starting from P waves, since these, according to Weinberg's

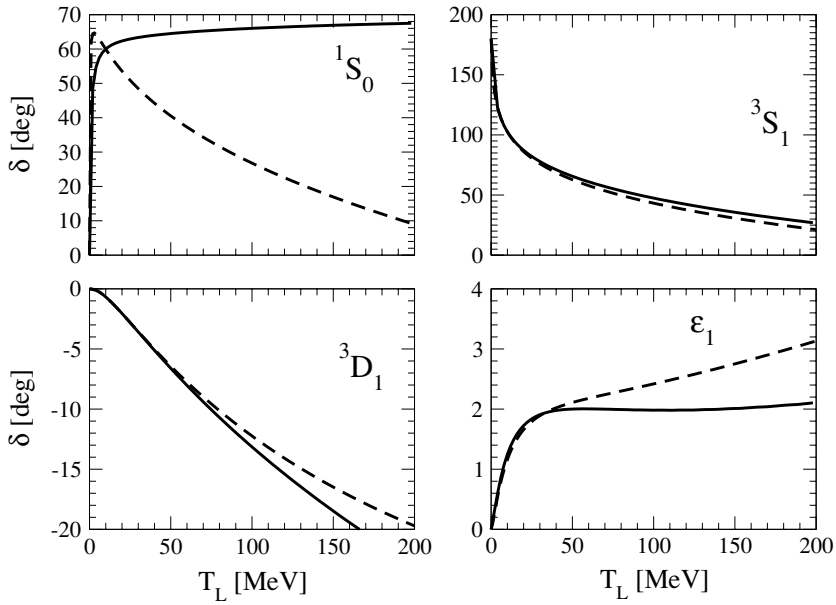

FIG. 3. Comparison of the $^1S_0$ and $^3S_1$-$^3D_1$ phase shifts and the mixing angle ε_1 (as function of the laboratory energy) in lowest order for $\Lambda = 20\,\mathrm{fm}^{-1}$ (solid line) to the Nijmegen PWA (dashed line).

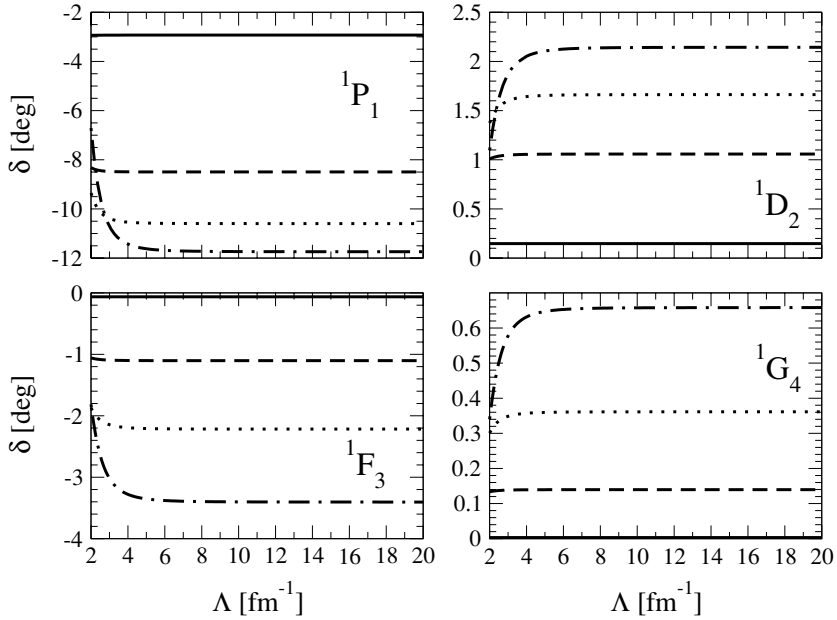

FIG. 4. Cutoff dependence of the singlet phase shifts for various partial waves. Results are given for laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

power counting, should not require counterterms in lowest order.

We start with spin-singlet channels, where the tensor interaction is zero and the potential is nonsingular. The results for the $^1P_1, ^1D_2, ^1F_3$, and $^1G_4$ phase shifts as a function of Λ at four different lab energies are shown in Fig. 4. It is seen that indeed the dependence on Λ becomes smaller with increasing Λ. Even for a rather high energy of 190 MeV, we find in all cases only negligible changes in the phase shifts for $\Lambda \gtrsim 5$ fm^{-1}. This supports the claim that in these channels no inconsistency in Weinberg's power counting exists. Figure 5 compares the resulting phase shifts to the PWA. For the P and D waves the agreement is good below 30 MeV. Above that energy significant higher order contributions are necessary to improve agreement with the PWA. For the F and G waves, where contact interactions are expected at even higher orders, the agreement is much better for energies up to 100 MeV. We note that for the energies below 30–50 MeV, where all singlet phase shifts are described well by the lowest order predictions, the cutoff dependence is already negligible for $\Lambda \gtrsim 2$ fm^{-1}.

In the next step, we look at the triplet channels where the tensor force is repulsive. The $^3P_1$ and $^3F_3$ partial waves belong to this class. The Λ dependence of these phase shifts is shown in Fig. 6 and, again, we obtain Λ independent results for $\Lambda \gtrsim 5$ fm^{-1} even for energies as high as 190 MeV. The comparison with the data for these cases is shown in Fig. 7. The $^3P_1$ result is in much better agreement with the PWA than the corresponding result of the $^1P_1$ singlet channel. The F wave has a comparable accuracy in the triplet and singlet cases. These results confirm that Weinberg's power counting

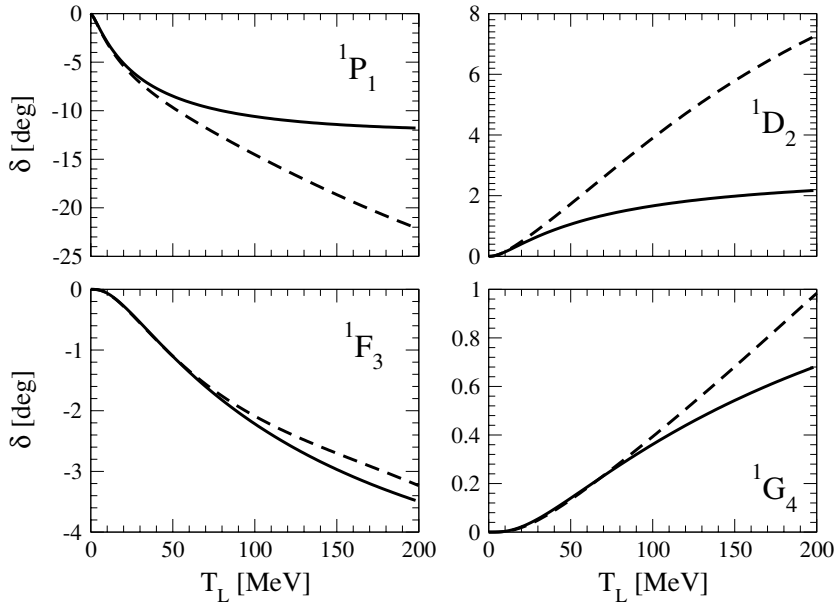

FIG. 5. Comparison of the LO singlet phase shifts (as a function of laboratory energy) for $\Lambda = 20$ fm^{-1} (solid line) to the Nijmegen PWA (dashed line).

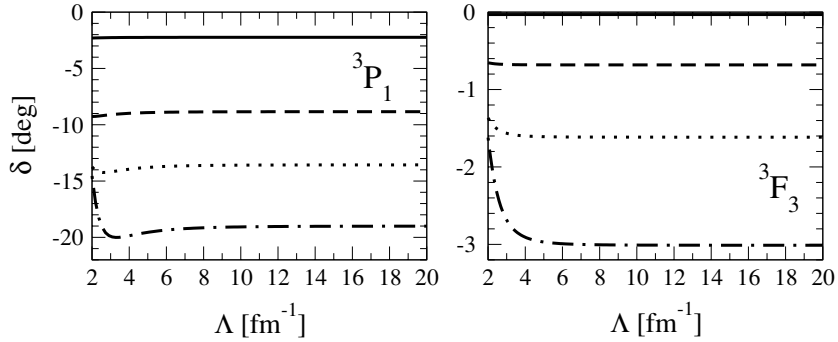

FIG. 6. Cutoff dependence of the repulsive triplet phase shifts for two partial waves. Results are given for laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

is, again, consistent in channels without an attractive singular interaction. The predictions agree well with the low-energy data in all these cases.

Next, we look at the triplet channels in which an attractive singular tensor force acts, and therefore the unregulated problem is not well defined. At a finite cutoff, bound states occur in various waves, and increasing Λ generates more and more of these bound states. For Λ between 2 and 20 fm^{-1}, we find bound states in the $^3P_0$ and $^3D_2$ channels (see Fig. 8). In the $^3P_2$-$^3F_2$ partial waves, we find a bound state for cutoffs just above $\Lambda = 20\,\mathrm{fm}^{-1}$. In higher partial waves the interaction appears to be screened enough by the centrifugal barrier that no bound state occurs in our cutoff range, although bound states should appear at sufficiently higher cutoffs.

Even though the binding energies increase rapidly with the cutoff, the bound states appear at zero energy. It is to be expected that in the cutoff regions where new bound states appear the variations of the phase shifts are strong. This is explicitly shown in Fig. 9 for the phase shifts at 10 and 50 MeV in various attractive channels. Clearly, an inconsistency in Weinberg's power counting shows up in these channels, because there are no counterterms available to remove the cutoff dependence of the observables. This inconsistency has not been previously noticed in the literature.

It is interesting to note that in some cases we can identify cutoff regions in which the results are stable and all the bound states are deep. Some clear plateau regions occur, especially at the lower energies. At 50 MeV, the $^3D_2$ phase shift in the plateau region is $\simeq 9^\circ$, which agrees very well with the Nijmegen PWA (8.97°). The corresponding $^3D_3$ phase shift, however, is too small. The situation is even worse in the P waves. At the same energy, the cutoff dependence of the $^3P_0$ and $^3P_2$ phase shifts remains visible in any region of Λs.

This cutoff dependence is related to the singularity of the interaction. It is known that an attractive singular central potential requires a boundary condition in each partial wave [46]. Therefore, we propose to add to each of the problematic triplet channels a counterterm and fit it to experiment, say to the phase shift at a certain (low) energy. We then show that the cutoff dependence indeed vanishes also for other energies. In the following, we will illustrate this explicitly for the $^3P_0$, $^3D_2$, and $^3P_2$-$^3F_2$ partial waves, which are the most problematic cases, because bound states exist or are close to appearing in the cutoff range that we examined. Our results extend the work of Refs. [41–44] to channels beyond S waves (and to our choice of regulator).

As argued, we add contact interactions in the $^3P_0$ ($i = 1$) and $^3P_2$-$^3F_2$ channels ($i = 2$) of the form

$$V_i = \frac{1}{4}\frac{c_i}{(2\pi)^3}\,p'\,p, \tag{9}$$

which in Weinberg's power counting appear only at next-to-leading (NLO) order, or $\mathcal{O}(Q^2)$. The first D-wave counterterms are supposed to be of even higher order: N^{3}LO, or $\mathcal{O}(Q^4)$. In the $^3D_2$ channel, we use

$$V_d = \frac{c_d}{(2\pi)^3}\,p'^2\,p^2. \tag{10}$$

Figure 10 shows our result for the $^3P_0$ partial wave. The value of c_1 was determined by a fit of the phase shift for a laboratory energy of 50 MeV. Since the size of the counterterm is not bounded, we varied this constant by orders of magnitude, but we could not find any further solution that describes the phase shifts equally well. The cutoff dependence of c_1 exhibits a nice limit-cycle-like behavior, similar to that of c_t. Figure 10 also demonstrates that the resulting phase shifts

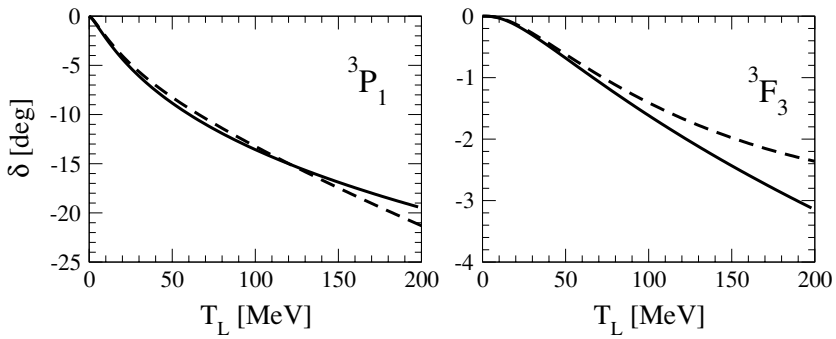

FIG. 7. Comparison of LO repulsive triplet phase shifts (as a function of laboratory energy) for $\Lambda = 20$ fm^{-1} (solid line) to the Nijmegen PWA (dashed line).

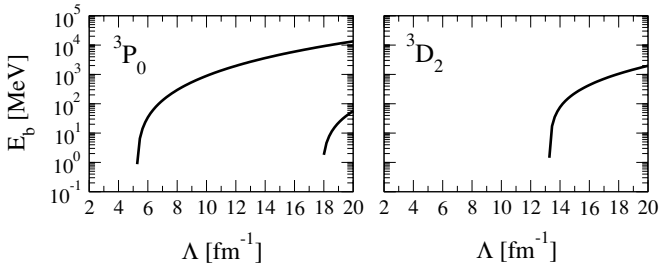

FIG. 8. Binding energies of the spurious bound states in selected attractive triplet channels, before the introduction of the required counterterms.

at other energies are cutoff independent for $\Lambda \gtrsim 8$ fm^{-1}. Figures 11 and 12 summarize the analogous results for the $^3P_2$-$^3F_2$ and $^3D_2$ partial waves, respectively. The fits were performed using the $^3P_2$ phase shift at 50 MeV and the $^3D_2$ phase shift at 100 MeV. We confirm the cutoff independence (for large Λ) in all phase shifts and mixing parameters.

An alternative to absorbing the cutoff dependence in the various P waves individually would be to employ one counterterm with tensor structure. Unfortunately, we have not been able to implement this idea without introducing cutoff dependence in the $^3P_1$ wave.

After removing the cutoff dependence by adding appropriate counterterms, we still find spurious bound states in the $^3P_0$,$^3D_2$, and also the $^3S_1$-$^3D_1$ channels. However, the cutoff dependence of the binding energies is now completely different, as shown in Fig. 13. As desired, only $^3S_1$-$^3D_1$ has a shallow bound state, the deuteron, which is cutoff independent over almost the entire Λ range; the deuteron binding energy is predicted to be 1.92 MeV in this LO calculation. The bound states in the other channels are all very deep. A new bound state appears with infinite binding energy around the cutoff at which the corresponding counterterm is singular, and then approaches a constant, large binding energy for increasing Λ. These bound states are beyond the range of the EFT, and they are irrelevant for the low-energy physics.

With the added counterterms, we obtain a very decent description of the phase shifts. Figure 14 shows that our $^3P_0$ result follows the energy dependence of the Nijmegen PWA remarkably well. Obviously, the addition of the counterterm is here supported by the experimental data. In the coupled $^3P_2$-$^3F_2$ channels the agreement with the PWA below 50 MeV is still satisfactory. We emphasize that the $^3F_2$ phase and the mixing parameter ε_2 are predictions. Choosing a high cutoff Λ clearly does not compromise the description of these observables

For the $^3D_2$ phase (see Fig. 15), we find again a good agreement with the PWA. Here, we also included the prediction based on a calculation without a counterterm, for $\Lambda = 8.0$ fm^{-1} in the plateau region of Fig. 9. For low energies below 50 MeV, the results are comparable. The deviations from the PWA become significant toward higher energies, where the plateau seen in Fig. 9 is more and more tilted. For these higher energies, the counterterm again improves the predictions.

Our overview is completed in Figs. 15 and 16 with the $^3D_3$-$^3G_3$,$^3F_4$-$^3H_4$, and $^3G_4$ channels. In these partial waves there is a relatively small cutoff dependence in the Λ range that we studied (although presumably cutoff dependence will become significant at cutoffs high enough to bring in spurious bound states). In all cases the agreement with the PWA is improved when we increase the cutoff from the traditional values around 2.5 fm^{-1} [16] to our higher values. This is especially true for the $^3D_3$ partial wave, which, for our higher cutoffs, becomes attractive for higher energies.

After these encouraging results, we examine the $3N$ bound state in the next section.

IV. THREE-NUCLEON BOUND STATE

The power of EFT comes to bear when more nucleons are considered. The $3N$ system is the first extension to

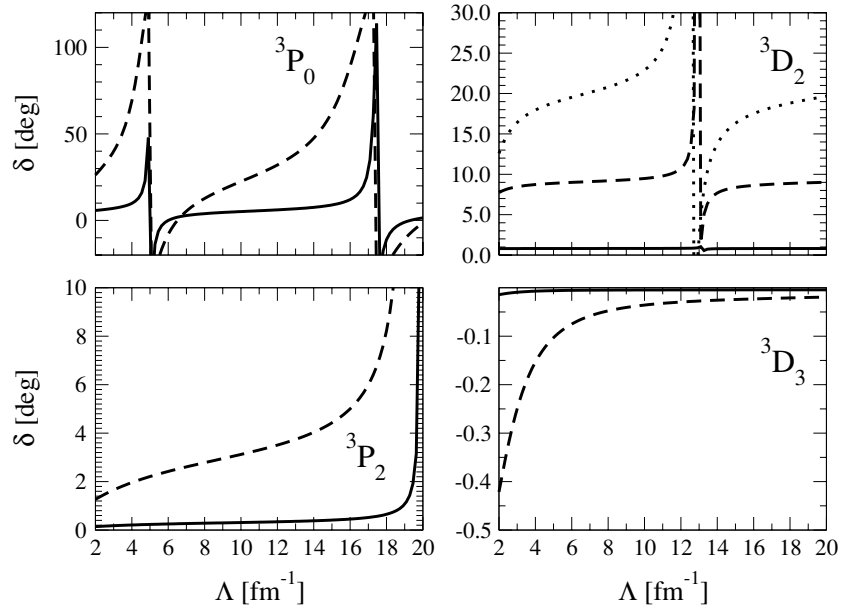

FIG. 9. Cutoff dependence of phase shifts in attractive triplet channels at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), and 100 MeV (dotted line).

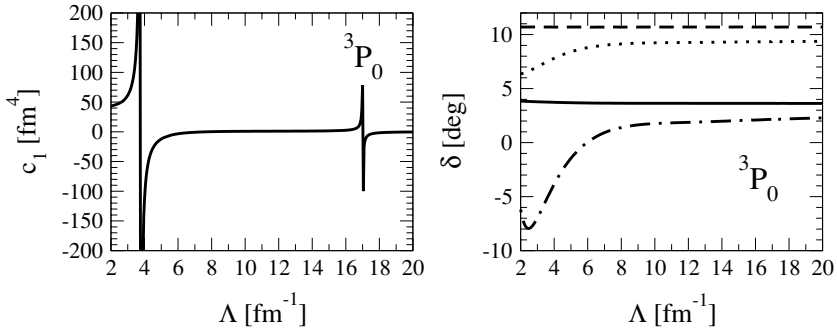

FIG. 10. Fit result for the counterterm c_1 as a function of the cutoff, and the resulting cutoff dependence of the $^3P_0$ phase shift at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

few-nucleon systems to consider. According to Weinberg's power counting, $3N$ forces should be subleading in the pionful theory, even though there is a leading-order $3N$ force in the pionless EFT [38]. Calculations of the triton properties using traditionally low values for Λ have been published in Refs. [21–23] for NLO and N^{2}LO in Weinberg's power counting, with and without $3N$ interactions. So far, a LO calculation was omitted, because of the unsatisfactory description of the NN phase shifts, especially $^1S_0$. Here, our main goal is to assess the ordering of $3N$ counterterms. For this purpose we do not require a high-quality description of the NN phase shifts, but instead we need to study the dependence of the triton binding energy on a larger range of cutoffs.

The $^1S_0$ problem cannot be addressed in this work, but a calculation of the triton binding energy (E_t) is of interest to compare the renormalization of the $3N$ system in the pionful theory with that in the pionless EFT. We will demonstrate that no $3N$ counterterms are necessary to ensure cutoff independence in LO in the pionful theory, once the LO calculation has been modified according to the previous section. This result will be important in future studies of the convergence of the chiral expansion in few-nucleon systems. The appearance of $3N$ interactions at N^{2}LO, the parameters of which are generally adjusted to the experimental value of E_t, makes predictions in LO and NLO especially important in this respect. We defer a study of subleading orders and Nd scattering states to a later publication.

We calculate E_t by solving the Faddeev equations for the $3N$ system. The techniques were recently described, for example, in Ref. [54]. Here we have to deal with the additional complication that the NN interaction supports deep, spurious bound states in various partial waves, which we remove as described in Appendix B. We confirmed the accuracy of this prescription by comparing our results for the energy to the expectation value of the Hamiltonian using the unaltered interaction. Both values agreed within several keV.

The cutoff dependence of E_t is shown in Fig. 17. We see a plateau region starting around $\Lambda = 8$ fm^{-1}. To extract the converged result from this calculation and to confirm that only terms of order $\mathcal{O}(Q^2/\Lambda^2)$ and higher are missing, we fitted the function $E(x) = E_0(1 + (C/\Lambda)^x)$ to our numerical results. We obtained the converged binding energy $E_0 = -3.6$ MeV, $C = 2.54$ fm^{-1}, and $x \simeq 1.8$. The exponent is in reasonable agreement with the expectation $x = 2$. The quality of this approximation to the cutoff dependence can be observed in the Lepage plot [48] of Fig. 18, where we show $E-E_0$ versus Λ on a double-logarithmic scale. The data follow the fitted results nicely. For high values of Λ the slope seems to change slightly. This could be a numerical artifact, because $E-E_0$ is rather

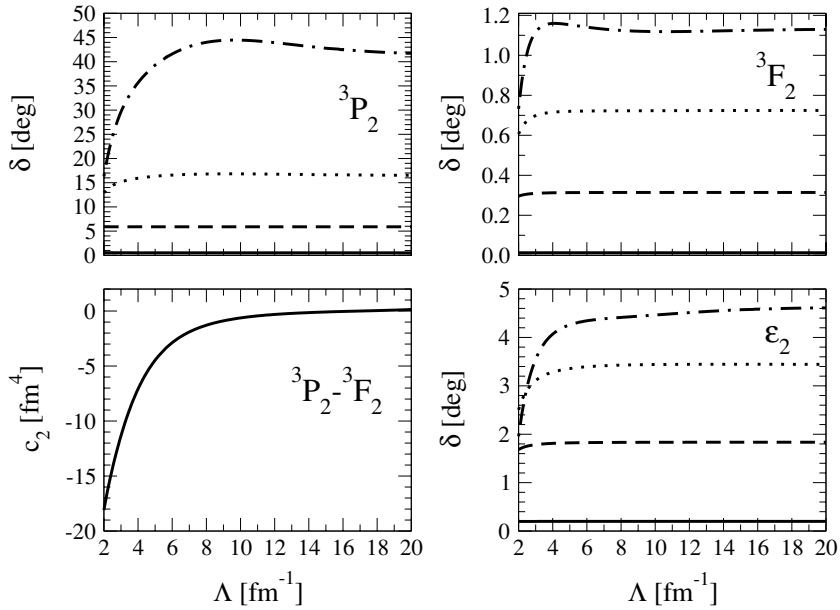

FIG. 11. Fit result for the counterterm c_2 as a function of the cutoff, and the resulting cutoff dependence of the $^3P_2$-$^3F_2$ phase shifts and the mixing angle ε_2 at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

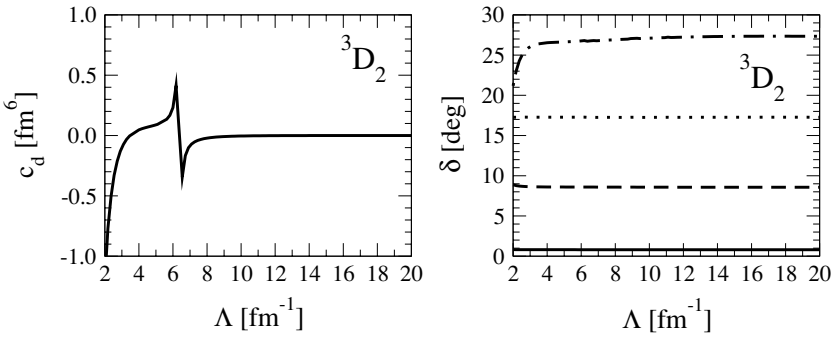

FIG. 12. Fit result for the counterterm c_d as a function of the cutoff, and the resulting cutoff dependence of the $^3D_2$ phase shift at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

small in this range, which increases the relative numerical uncertainty.

A cutoff dependence as occurs in the $3N$ system within the pionless EFT [38] does not appear here. We stress that, in contrast, for the same range of Λ we found considerable variations in several NN phase shifts before adding counterterms. This gives us confidence that our cutoff range is large enough to draw conclusions about the order of the counterterms. In particular, we conclude that the finite range of OPE prevents the Thomas collapse of the $3N$ bound state [55].

Our LO results imply a rather large underbinding of the triton in the limit of high Λ. In the region around $\Lambda = 2.5$ fm^{-1}, our prediction is in much better agreement with the experimental value $E_t = -8.48$ MeV. For those cutoffs the Weinberg NLO predictions are very similar [21]. This suggests that the NLO result might be less cutoff dependent than the LO result and that the plateau region starts at lower values of Λ for higher orders. Our result is probably influenced significantly by the unrealistic description of the $^1S_0$ NN phase shift, which will improve in a NLO calculation. Binding energies are more sensitive to higher orders, because in a theory with pions the potential energy is to a large extent canceled by the kinetic energy. In view of this, we are not very concerned about our LO plateau value. It will be interesting to analyze the NLO results in a similar way. If our expectations are verified, they would reconcile our observations with the good results obtained with a fixed, low cutoff in Refs. [19,23].

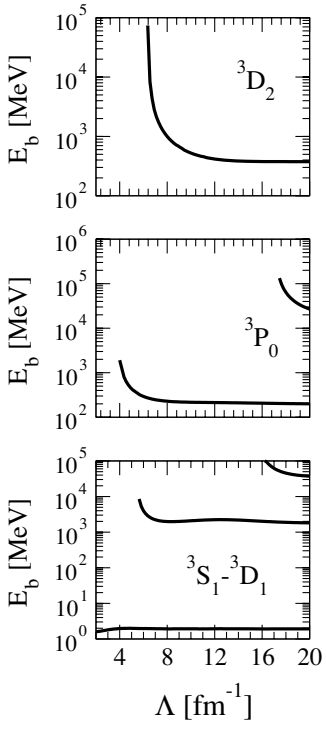

FIG. 13. Binding energies of bound states found in various partial waves as a function of the cutoff. The shallow bound state in the $^3S_1$-$^3D_1$ coupled channels corresponds to the deuteron; the other, deep, bound states are outside the range of applicability of the EFT.

V. LESSONS FOR POWER COUNTING

It has been realized for some time now that EFT power counting is more complicated for nonperturbative than for perturbative processes. In particular, one has to consider the effects of infrared enhancements in the running of counterterms, which can invalidate naive dimensional analysis. In this section, we discuss some of the implications of our findings to power counting in the pionful EFT.

The existence of shallow (real and virtual) bound states in both NN S waves is a clear sign that nonperturbative physics is taking place, in contrast to the situation in ordinary ChPT. We can describe this in the same language used to discuss power counting in ChPT [5,7,8]: We represent typical nucleon momenta by Q and the characteristic scale of QCD in the hadronic phase by $M_{\rm QCD}$. The effect of iterating an interaction in the kernel of the T matrix is twofold. First, one has an extra three-dimensional momentum integral and an extra NN Schrödinger propagator. Second, one has an extra factor of the potential. After the cutoff dependence is removed by renormalization, the contribution to the NN T matrix from an NN intermediate state is expected to be $\mathcal{O}(m_N Q/4\pi)$. This is a factor $m_N/Q \gg 1$ larger than in analogous states in ordinary ChPT, and it is due to the small energy of intermediate states containing nucleons only. If the interaction has an effective strength $v_{l'l}$ connecting waves of orbital angular momenta l and l', one iteration then roughly brings a dimensionless factor $\mathcal{O}(m_N Q v_{l'l}/4\pi)$. For $Q \gtrsim 4\pi/m_N v_{l'l}$ this factor is $\gtrsim 1$ and the interaction has to be iterated to all orders, potentially leading to bound states.

The pionful theory is relevant for momenta $Q \gtrsim m_\pi$. In this case, we estimate the effect of OPE as $v_{l'l} \sim \alpha_{l'l}/f_\pi^2$, where $\alpha_{l'l}$ is a dimensionless angular-momentum factor. Therefore, we

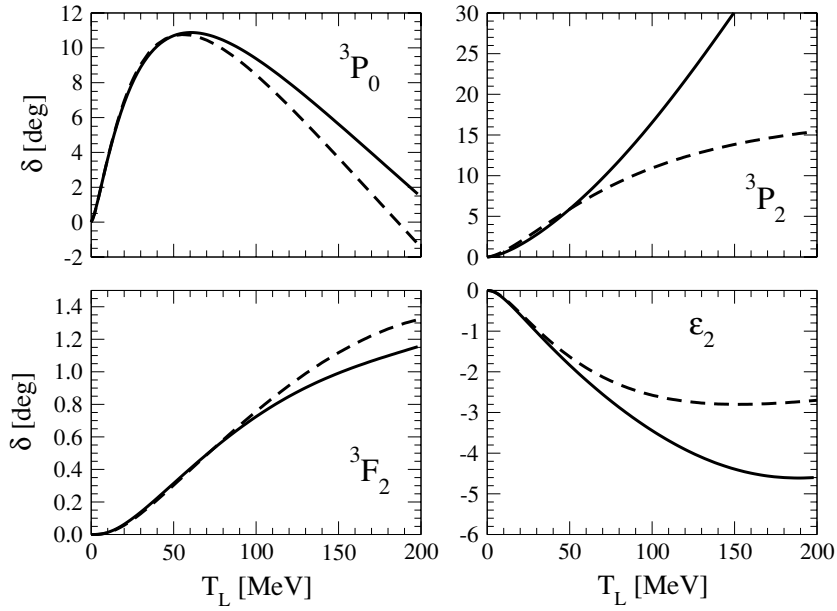

FIG. 14. Comparison of attractive triplet phase shifts (as a function of laboratory energy) for $\Lambda = 20$ fm^{-1} (solid line) to the Nijmegen PWA (dashed line).

can expect OPE to be nonperturbative at $Q \gtrsim 4\pi f_\pi^2/m_N\alpha_{l'l}$. Ignoring the angular-momentum factor, we get $Q \gtrsim 100$ MeV for the momentum where pions need to be iterated. This is in agreement with what is observed in a perturbative calculation of low waves, where twice-iterated pion exchange seems to overcome OPE in various channels for momenta around 100 MeV [35].

In spin-singlet channels, OPE goes as $1 - m_\pi^2/q^2 + \ldots$ at high momentum. When iterated, the first term by itself introduces cutoff dependence in the S wave only, which can be removed by a chirally symmetric counterterm C_0. The interference between the iteration of this counterterm and the second term in OPE generates further cutoff dependence in the $^1S_0$ wave, which in turn can be removed [32,42] by a chiral-breaking counterterm $m_\pi^2 D_2$ [cf. Eq. (4)]. This counterterm is enhanced with respect to naive dimensional analysis, and it should be promoted to LO if pion-mass effects are kept in LO, as seems to be most efficient. In this paper we have checked explicitly that OPE is well behaved in the higher spin-singlet waves. (Higher spin-singlet waves have also been recently discussed in Ref. [56].)

In spin-triplet channels, the situation is complicated by the tensor operator, which retains angular dependence even asymptotically. As shown in Refs. [41,42] and confirmed here, the cutoff dependence introduced by iteration in the $^3S_1$-$^3D_1$ coupled channel can be dealt with by a single chirally symmetric, momentum-independent counterterm, as prescribed by Weinberg's power counting. The deuteron and $^1S_0$ virtual state have, for the observed value of the quark masses, binding momenta somewhat smaller than our crude

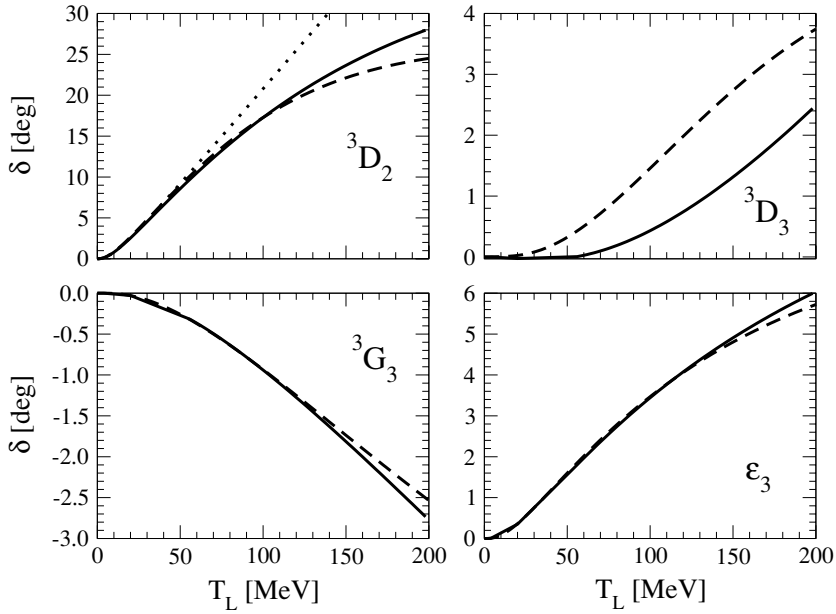

FIG. 15. Comparison of attractive triplet phase shifts (as a function of laboratory energy) for $\Lambda = 20$ fm^{-1} (solid line) to the Nijmegen PWA (dashed line). For $^3D_2$ the result for $\Lambda = 8.0$ fm^{-1} without the counterterm is also given (dotted line).

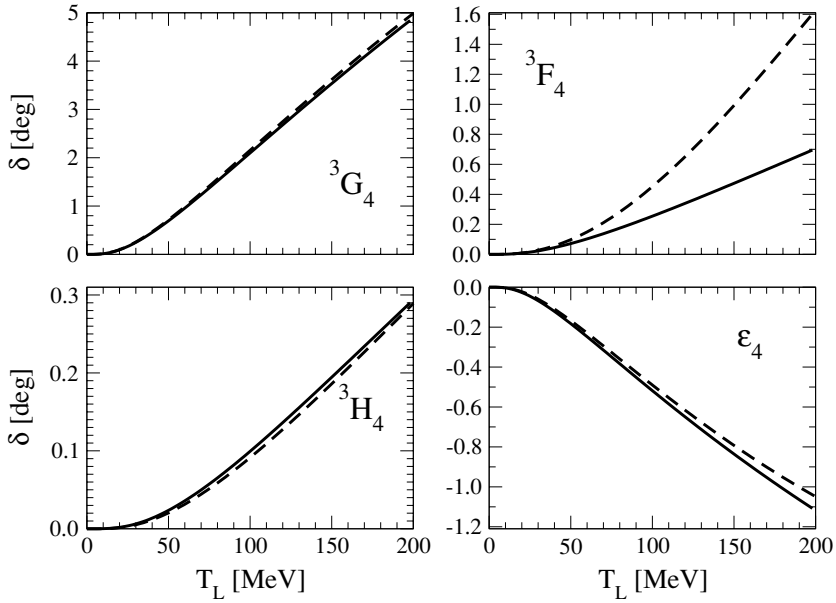

FIG. 16. Comparison of attractive triplet phase shifts (as a function of laboratory energy) for $\Lambda = 20$ fm^{-1} (solid line) to the Nijmegen PWA (dashed line).

estimate $Q \sim 4\pi f_\pi^2/m_N$, indicating certain amount of fine tuning. However, if one varies the pion mass the momentum scales for the bound states acquire more natural values [42,57–59].

Yet, as we have shown here, iterated OPE produces spurious bound states and cutoff dependence in all waves where the tensor force is attractive. This undesired feature can only be removed by additional counterterms in the corresponding waves. Since Weinberg's power counting only prescribes counterterms in the S waves, our results clearly upset Weinberg's power counting.

Weinberg's power counting was based on naive dimensional analysis. Contact interactions are necessary to remove divergences from loops that do not involve purely nucleonic intermediate states. These loops are not infrared enhanced, and it is reasonable to assume that they scale—as in ordinary ChPT—with powers of $(Q/4\pi f_\pi)^2$. This has in fact been confirmed by explicit calculation [10,60,61]. Implicit in Weinberg's power counting is the assumption that loops in the LS equation do not bring significant new cutoff dependence. The parameters of contact interactions with derivatives or powers of the pion mass would thus be suppressed by powers of a large mass scale, $M_{\rm QCD}$, and the effects of derivatives would scale as $Q/M_{\rm QCD}$.

However, we now see that Weinberg's implicit assumption is not correct. The short-range parameters needed to renormalize iterated OPE do not obey naive dimensional analysis because their renormalization-group running is enhanced in the infrared. These counterterms are driven by pion parameters, and the effects of derivatives can scale as Q/f_π. (If we take $m_N \sim 4\pi f_\pi$, there is no dimensionful parameter at LO other than f_π.) Taken at face value, this implies that all these counterterms must be considered in leading order. This is not a complete disaster, as there is still some predictive power left, for example in the energy dependence of each attractive partial wave and in the repulsive waves. However, it would put few-nucleon observables that include significant contributions

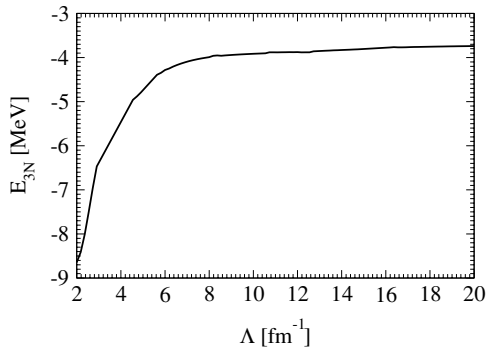

FIG. 17. Cutoff dependence of the triton binding energy.

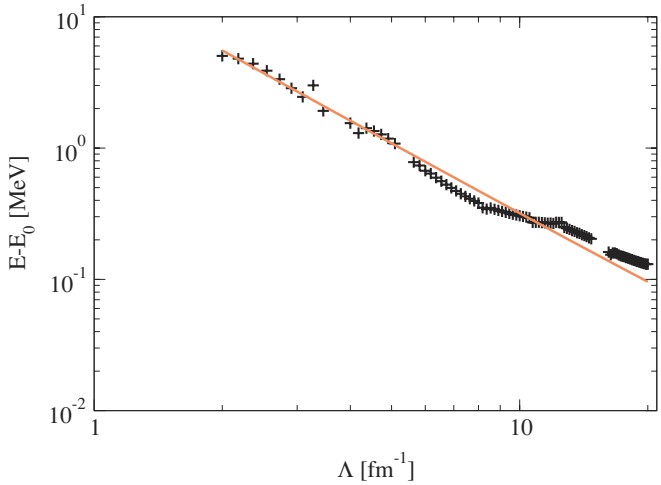

FIG. 18. (Color online) Numerical results (crosses) and fit (solid line) for the deviation of the triton binding energy from its converged value, as a function of the cutoff. See text for details.

from many partial waves, such as the triton binding energy, out of reach.

In the following, we want to demonstrate that this complication can be avoided in higher partial waves, because these are still perturbative. We therefore consider $\alpha_{l'l}$, which involves a kinematic suppression that accounts for the repulsive effect of the centrifugal barrier. The appropriate counterterms will make OPE well defined, by selecting in the region $r \sim 1/f_\pi$ the correct combination of solutions for the long-range potential [40,42]. Therefore, the kinematic suppression can be estimated as for a regular potential. In the case of a central potential, it can be shown [62] that for $l \gg Qd$, where d is the range of the interaction, the l-wave phase shift is given by $\tan\delta_l \sim [Qd/(l+1/2)]^{2l+1} \ll 1$ (barring the exceptional case of a fine tunning that generates a T-matrix pole near the origin of the complex momentum plane). This is consistent with the expectation that in the classical limit there is little scattering when the impact parameter l/Q is much larger than the range d. The ratio of the T matrix, and thus the potential, between $l+1$ and l is $\mathcal{O}(Q/lm_\pi)^2$, for large l. For $Q \sim m_\pi$, we are led to $\alpha_{l,l} = \mathcal{O}(1/l!^2)$. In the case of the tensor force, there must be two different factors $1/l!$ and $1/l'!$, in addition to the elements in Table I, that do depend on l and l' but approach constant values for large l, l'. Although at small l and l', $\alpha_{l'l}$ can be very complicated, we expect that $\alpha_{l'l} = \mathcal{O}(1/l'!l!)$ for large l, l'. For this argumentation we assumed that the pion mass is finite. We note that it does not apply in the chiral limit.

This qualitative argument suggests that the effects of the corresponding higher derivative counterterms are suppressed by a large (for large l) scale lf_π. Obviously, there might be other dimensionless factors that we miss here, but the fact that factors of l suppress OPE and its required counterterms in high-l waves must hold. There are several implications of this new counting that seem to be supported by existing results.

The kinematic suppression of higher waves makes the strength of OPE weaker with increasing l. In high waves, OPE and probably all subleading interactions can be treated in perturbation theory even for momenta of the order of m_π. We therefore find explicitly that high partial waves can be treated perturbatively. The perturbative nature of OPE in high waves is part of nuclear folklore and has been checked in EFT explicitly [60,61,63].

For sufficiently large l, the suppression factor in counterterms becomes dominated by $M_{\rm QCD}$ (rather than lf_π), representing omitted QCD degrees of freedom, and the size of the counterterms is that assumed in Weinberg's power counting.

However, for a finite number of low partial waves, for example, ${}^3P_0$ and ${}^3P_2$-${}^3F_2$, we find that perturbation theory is insufficient for momenta of the order of m_π. This is caused by the lack of enough suppression from l and by unnaturally large $|\alpha_{ll'}|$ in these cases. Resummation is necessary and needs to be performed numerically. Our numerical analysis showed that the cutoff dependence can be absorbed by one counterterm per partial wave. The favorable agreement of our LO calculation with the data indicates that we did not introduce additional inconsistencies and that the resummation includes the most important diagrams.

We conjecture that this mixture of perturbative treatment of higher partial waves, resummation of lower partial waves, and promotion of a finite number of counterterms is the most consistent approach to ChPT for nuclear systems. In subleading orders, it would naturally suggest a perturbative treatment of the subleading interactions, if no unnaturally large $\alpha_{ll's}$ are present. The NLO interactions consist in principle of TPE and counterterms with two more derivatives than LO. Subsequent orders are constructed by the inclusion of successive powers of $Q/M_{\rm QCD}$. The most effective organizational scheme for subleading interactions probably relies on taking into account an explicit delta-isobar field [12,14,15,61,64]. The correctness of our modified power counting needs, of course, to be checked in future studies of higher orders.

Nonetheless, for practical reasons, it might be convenient to perform the resummation in all partial waves that are taken into account. Iterating something small causes only a small error, so one might decide to iterate OPE in all waves, as done automatically when solving the LS for OPE. This is again part of nuclear folklore. If we do this without the corresponding counterterms, however, cutoff dependence is introduced. By increasing the cutoff, the iteration of OPE can be made arbitrarily large, and at some point bound states appear. The kinematic suppression suggests that the cutoffs for which bound states appear increase with l, which is consistent with what we observe in the cutoff window we studied. Existing calculations based on Weinberg's counting should exhibit [47] some of the problems we point out here. In particular, as the cutoff is increased, partial waves without the required counterterms become unwieldy.

In that case, one should consider cutoffs in a limited range, for LO between, say, 5 to 10 fm^{-1}. Variation of the cutoff in this limited range would not exhibit any of the drawbacks pointed out here, because it leads to an effective suppression of the higher order terms. Therefore, reasonable fits to the data can be achieved. This explains the success of existing fits [16–20] over a limited cutoff range.

The variation of the cutoff within a given range will generate a band of values for observables. The error in a fit based on Weinberg's counting is likely dominated by the lowest partial wave without the required counterterm. As one goes to higher orders in Weinberg's counting, one acquires more counterterms, pushing the error to higher waves. The l suppression then ensures that the bands for the observables shrink, as observed [16–20]. This does not, however, imply that Weinberg's power counting is correct.

At any given order, our modified power counting has more short-range parameters than the same order in Weinberg's power counting. In this context, it is interesting to note that existing results in Weinberg's power counting already suggest that short-range parameters are relatively more important than the long-range physics that is supposed to be of the same order. For example, Refs. [17,18] considered N^{2}LO in Weinberg's power counting plus D-wave counterterms and found good fits. These calculations have a couple of counterterms more than what we advocate here, but they come pretty close to our N^{2}LO.

A similar observation can be made about the Nijmegen PWA when we look at it from the point of view of chiral

EFT, with the PWA short-range parameters playing the role of counterterms. The long-range strong interaction consists of OPE and (leading and subleading) TPE [27,28] and thus corresponds to N^{2}LO in Weinberg's power counting. In the Nijmegen PWA, short-range parameters are added for the various partial waves until the fit to the data, up to 350 MeV laboratory energy, is optimal. (The point is nicely illustrated by Fig. 1 of Ref. [24], where the quality of the $^3P_0$ phase shift is shown for an increasing number of short-range parameters; see also Figs. 2 and 3.) The number of "counterterms" needed per partial wave is larger, however, than in Weinberg's power counting in N^{2}LO. In fact, if we assume that some parameters are needed for "fine tuning" to the data (cf. again see Fig. 1 of Ref. [24]), or that they would not be needed if one were fit to the data only up to a lower energy (say, 250 MeV), then it appears that the number per partial wave is closer to what Weinberg's power counting would prescribe in N^{3}LO. It would be very interesting to make this analogy between the Nijmegen PWA and chiral EFT more precise by fitting the data with a number of counterterms mandated by the different power-counting schemes.

Another related point, which deserves further attention, is that a good description of the *NN* data was obtained by the N^{3}LO interactions in Weinberg's counting [19,20]. These calculations automatically include the counterterms in all partial waves that we consider to be nonperturbative. It is encouraging to see that a good description of the data is obtained. However, lower cutoffs were employed than our LO study would suggest. This surprising fact can be understood if the range of cutoffs for which converged results are obtained increases toward lower Λ in higher orders of the expansion. This would be consistent with the observation that the triton binding energy is very well described for small cutoffs in Weinberg's NLO. This needs to be studied more carefully in the future.

We have taken here the minimalist point of view that only counterterms that are infrared enhanced with respect to naive dimensional analysis should be promoted. Since we have found no significant cutoff dependence in the $3N$ system in LO, $3N$ forces are not infrared enhanced at this level and could be considered subleading. The same is true of the effective range in the $^1S_0$ *NN* channel. For both the $^1S_0$ *NN* phase shift and the triton binding energy, our results are internally consistent. However, they are also somewhat unsatisfactory when compared to experiment. Given that the $^1S_0$ *NN* phase shift is well described in subleading orders in perturbation theory (see, e.g., Ref. [42]), we expect that agreement should be improved in subleading orders, when these and all other subleading effects are included. An alternative, less-conservative approach would be to invoke a promotion of one or both of these interactions to LO on the basis of fine tuning. The relatively large value of the effective range supports this viewpoint. However, it is not obvious that a global improvement can be achieved in this way. This is largely an issue of efficiency that we leave to later investigation.

The LO results in our power counting are the ones given here. As we argued, there already exists some evidence that this power counting is consistent with previous results. A calculation beyond LO is in progress [65].

VI. CONCLUSIONS

In conclusion, we have reanalyzed the predictions of chiral perturbation theory in the *NN* system using Weinberg's original power counting in LO. We have identified that the singularity of the tensor interaction is responsible for a significant cutoff dependence in partial waves where it acts attractively. Furthermore, we have shown that the addition of one counterterm in each of these partial waves removes this cutoff dependence. It also improves the description of the data, which we see as a confirmation of our approach.

For the $3N$ binding energy, we found cutoff-independent results in the limit of large cutoffs. We conclude that the finite range of the interaction prevents the Thomas collapse of the $3N$ bound state. Meaningful predictions for the $3N$ binding energy, which are not possible using pionless effective field theory, are possible here. We emphasize that this does not invalidate or compromise the pionless EFT approach; rather, it reflects the stronger physical constraints built into ChPT.

Our approach and its relation to the previous work can be best exemplified in the $^3D_2$ partial wave. We found that the addition of a counterterm ensures cutoff independence and increases accuracy. But we also found that in some ranges of the cutoff the low-energy description is equally good without this additional counterterm. This reconciles the traditional approach with our new results. It seems nevertheless advisable to promote counterterms in some partial waves. This ensures that all partial waves are cutoff independent for the same Λ and improves the description of the data over a wide range of energies.

We have discussed how these results can be understood from a power counting that includes angular-momentum suppression. This improved power counting suggests an ordering of interactions that is similar to Weinberg's power counting, except for the infrared enhancement of a few of the counterterms that contribute to lower partial waves.

Our study clearly has to be followed by further investigation in at least two directions. First, one would like to understand in more detail the interplay of scales in infrared-enhanced counterterms. A more detailed analysis of the renormalization-group running and limit-cycle-like behavior of these interactions could shed light on this issue. Second, one would like to carry out a similar investigation for the new NLO and N^{2}LO interactions. One would like to confirm that a good description of *NN* data can be obtained already at N^{2}LO. Few-nucleon systems should also be reexamined, as the triton binding energy, for example, does not come out well in LO. We consider these to be important remaining issues that need to be studied for a consistent understanding of the application of ChPT to nuclear systems.

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APPENDIX A: PARTIAL-WAVE DECOMPOSITION OF OPE

The operator form of the OPE potential is

$$V_{1\pi}(\vec{p},\vec{p}\,') = -\frac{1}{(2\pi)^3}\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{\vec{q}^{\,2}+m_\pi^2}, \quad \text{(A1)}$$

where $\vec{q}=\vec{p}-\vec{p}\,'$ is the momentum transfer.

The isospin operator separates easily. It is given by

$$\langle t||\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2||t'\rangle = (2t(t+1)-3)\,\delta_{tt'}. \quad \text{(A2)}$$

The spin-orbital part can be decomposed into spin and orbital tensor operators using obvious basis states, starting with

$$\begin{aligned}&\langle p(ls)jm|\frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{\vec{q}^{\,2}+m_\pi^2}|p'(l's')j'm'\rangle\\ &\quad= 3\langle p(ls)jm|\frac{\{\sigma_1 q\}^{00}\{\sigma_2 q\}^{00}}{\vec{q}^{\,2}+m_\pi^2}|p'(l's')j'm'\rangle, \quad \text{(A3)}\end{aligned}$$

where we used the representation $\vec{a}\cdot\vec{b}=-\sqrt{3}\{ab\}^{00}$ of the scalar product of two vectors $\vec{a}$ and $\vec{b}$. With $(\vec{q}\,)_\lambda = q\sqrt{\frac{4\pi}{3}}Y_{1\lambda}(\hat{q})$, we can recouple

$$\begin{aligned}&\langle p(ls)jm|\frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{\vec{q}^{\,2}+m_\pi^2}|p'(l's')j'm'\rangle\\ &\quad=\sum_f\frac{3\sqrt{\hat{f}}}{\sqrt{4\pi}}\begin{Bmatrix}1&1&f\\1&1&f\\0&0&0\end{Bmatrix}(11f,00)(j'0j,m'0m)\\ &\qquad\times\sqrt{\hat{j}'}\begin{Bmatrix}j&j'&0\\l&l'&f\\s&s'&f\end{Bmatrix}\sqrt{\hat{l}\hat{s}}\langle pl||\frac{4\pi q^2}{q^2+m_\pi^2}Y_f(\hat{q})||p'l'\rangle\\ &\qquad\times\langle s||\{\sigma_1\sigma_2\}^f||s'\rangle \quad \text{(A4)}\end{aligned}$$

thus separating spin and orbital parts.

The orbital part is

$$\begin{aligned}&\langle plm|\frac{4\pi q^2}{q^2+m_\pi^2}Y_{f\mu}(\hat{q})|p'l'm'\rangle\\ &\quad=\sum_{\lambda_1+\lambda_2=f}\sqrt{\frac{4\pi\hat{f}!}{\hat{\lambda}_1!\hat{\lambda}_2!}}p^{\lambda_1}(-p')^{\lambda_2}\sum_k g_k^f(pp')\\ &\qquad\times\sqrt{\hat{f}}\begin{Bmatrix}k&k&0\\\lambda_1&\lambda_2&f\\l&l'&f\end{Bmatrix}\hat{k}\sqrt{\hat{\lambda}_1\hat{\lambda}_2}(k\lambda_1 l,00)(k\lambda_2 l',00)\\ &\qquad\times(l'fl,m'\mu m)(-)^{l'}\sqrt{\frac{\hat{f}}{\hat{l}}} \quad \text{(A5)}\end{aligned}$$

where the angular dependence of the propagator was expanded in Legendre polynomials using

$$g_k^f(pp') = \frac{\sqrt{\hat{k}}}{2}(-)^k 4\pi\int_{-1}^{1}dx\,P_k(x)\frac{q^2}{q^2+m_\pi^2}\frac{1}{q^f}. \quad \text{(A6)}$$

This confirms that the orbital part is a tensor operator of rank f.

With the spin matrix element

$$\langle s||\{\sigma_1\sigma_2\}^f||s'\rangle = 6\sqrt{\hat{s}'\hat{f}}\begin{Bmatrix}s&s'&f\\1/2&1/2&1\\1/2&1/2&1\end{Bmatrix}, \quad \text{(A7)}$$

one obtains the complete matrix elements of the OPE:

$$\begin{aligned}&\langle p(ls)jm|\frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{\vec{q}^{\,2}+m_\pi^2}|p'(l's')j'm'\rangle\\ &\quad=\sum_f\frac{3\sqrt{\hat{f}}}{\sqrt{4\pi}}\begin{Bmatrix}1&1&f\\1&1&f\\0&0&0\end{Bmatrix}(11f,00)(j'0j,m'0m)\\ &\qquad\times\sqrt{\hat{j}'}\begin{Bmatrix}j&j'&0\\l&l'&f\\s&s'&f\end{Bmatrix}\sqrt{\hat{l}\hat{s}}\\ &\qquad\times\sum_{\lambda_1+\lambda_2=f}\sqrt{\frac{4\pi\hat{f}!}{\hat{\lambda}_1!\hat{\lambda}_2!}}p^{\lambda_1}(-p')^{\lambda_2}\sum_k g_k^f(pp')\sqrt{\hat{f}}\\ &\qquad\times\begin{Bmatrix}k&k&0\\\lambda_1&\lambda_2&f\\l&l'&f\end{Bmatrix}\hat{k}\sqrt{\hat{\lambda}_1\hat{\lambda}_2}(k\lambda_1 l,00)(k\lambda_2 l',00)\\ &\qquad\times(-)^{l'}\sqrt{\frac{\hat{f}}{\hat{l}}}6\sqrt{\hat{s}'\hat{f}}\begin{Bmatrix}s&s'&f\\1/2&1/2&1\\1/2&1/2&1\end{Bmatrix}. \quad \text{(A8)}\end{aligned}$$

APPENDIX B: REMOVAL OF SPURIOUS *NN* BOUND STATES

To remove a spurious bound state in the *NN* system we change the interaction V to

$$\bar{V} = V + |\chi\rangle\lambda\langle\chi|, \quad \text{(B1)}$$

where $|\chi\rangle$ is the wave function of the spurious bound state, and λ is an energy parameter, which determines a shift of the binding energy for the spurious state. The limit $\lambda\to\infty$ removes the spurious state, and $\bar{V}$ is phase-shift equivalent to V.

If t and $\bar{t}$ solve the LS equations for V and $\bar{V}$, respectively, they are related by

$$\bar{t} = t + |\eta\rangle N\langle\eta|, \quad \text{(B2)}$$

with

$$|\eta\rangle = |\chi\rangle + tG_0|\chi\rangle \tag{B3}$$

and

$$N = \frac{\lambda}{1 - \lambda\langle\chi|G_0|\eta\rangle}. \tag{B4}$$

Here G_0 is the free, two-particle Schrödinger propagator. This formulation allows us to perform the limit $\lambda \to \infty$ analytically, and we end up with the t matrix

$$\bar{t} = t - |\eta\rangle \frac{1}{\langle\chi|G_0|\eta\rangle} \langle\eta|, \tag{B5}$$

which then enters our calculations for the triton binding energy. The accuracy of this prescription can be checked numerically by comparison with the expectation value of the Hamiltonian H using the original potential V.

This procedure can easily be generalized to two or more spurious bound states.

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