Charge symmetry breaking as a probe for the real part of η-nucleus scattering lengths

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We demonstrate that one can use the occurrence of charge symmetry breaking as a tool to explore the η-nucleus interaction near the η threshold. Based on indications that the cross section ratio of π^+ and π^0 production on nuclei deviates from the isotopic value in the vicinity of the η production threshold, due to, e.g., π^0-η mixing, we argue that a systematic study of this ratio as a function of the energy would allow to pin down the sign of the real part of the η-nucleus scattering length. This sign plays an important role in the context of the possible existence of η-nucleus bound states.

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I. INTRODUCTION

During the last decade or so the η interaction with nucleons and nuclei has attracted much attention both experimentally and theoretically. One reason for this excitement is the possibility of the formation of η-nucleus bound states. The existence of such so-called η-mesic nuclei was first predicted by Haider and Liu [1] based on the observation that the elementary ηN interaction is attractive and relatively strong [2]. It is expected that the attraction gets increasingly stronger with increasing mass number of the nuclei and eventually should lead to a bound state. However, so far it is unclear for which mass number that actually happens. For example, in the literature one can find speculations that even the ηd system might already form such a bound state [3] which, however, is disputed by other investigations [4]. More conservative estimations consider the η^4He system as the lightest possible candidate [5–7].

The occurrence of a bound state near the reaction threshold will be also reflected in the corresponding scattering length [8]. In such a case the (real part of the) scattering length should be relatively large and negative. (We adopt here the sign convention of Goldberger and Watson [9].) Studies of the η-nucleus interaction near threshold can be used to determine the η-nucleus scattering length, and then, in principle, would permit conclusions on the existence of such η-nucleus bound states. Information on the η-nucleus interaction can be deduced from analyzing the energy dependence of η production reactions such as $pn\to d\eta$, $pd\to{}^3\mathrm{He}\eta$, etc. But, unfortunately, the energy dependence of the production cross section of those reactions itself is not sensitive to the sign of the real part of the scattering length, but only to its magnitude. Therefore, in the present paper, we want to propose a complementary analysis that would then also allow to constrain the sign of the η-nucleus scattering length.

II. EFFECTS OF THE FINAL STATE INTERACTION

Recently, it was suggested that the study of π production in nucleon-nucleus and nucleus-nucleus collisions at energies around the η production threshold could allow to obtain information on charge symmetry breaking effects caused by π-η mixing [10–14]. Specifically, in Refs. [11–13] the authors proposed to measure the cross section ratio for the production of $^3\mathrm{H}\pi^+$ and $^3\mathrm{He}\pi^0$ in pd collisions, i.e., the ratio

$$R=\frac{d\sigma}{d\Omega}(pd\to{}^3\mathrm{H}\pi^+)\Bigg/\frac{d\sigma}{d\Omega}(pd\to{}^3\mathrm{He}\pi^0). \tag{1}$$

Utilizing a simple phenomenological model these authors derived the following result for the ratio R:

$$R\simeq\frac{p_{\pi^+}}{p_{\pi^0}}\frac{|\mathcal{M}_{\pi^+}|^2}{|\mathcal{M}_{\tilde{\pi}^0}+\theta_m\mathcal{M}_\eta|^2}$$
$$\simeq\frac{p_{\pi^+}}{p_{\pi^0}}\frac{2}{1+2\theta_m\mathrm{Re}(\mathcal{M}_\eta\mathcal{M}^*_{\tilde{\pi}^0})/|\mathcal{M}_{\tilde{\pi}^0}|^2}. \tag{2}$$

Here $\mathcal{M}_{\pi^+}$, etc., are the corresponding production amplitudes and the tilded quantity in the denominator indicates that this is the isospin state and not the physical state, i.e., $\mathcal{M}_{\tilde{\pi}^0}=\mathcal{M}_{\pi^+}/\sqrt{2}$. The quantity θ_m is the π^0-η mixing angle. If isospin is conserved then the ratio R should be equal to 2. However, there are indeed experimental indications of significant deviations from this value [11–13]. Note that the quantity $\mathcal{M}_\eta$ should, in principle, have a tilde as well. However, the effect of $\mathcal{M}_\eta$ or $\mathcal{M}_{\tilde{\eta}}$ on the cross section ratio would be the same up to the order in θ_m that we consider. Therefore, we do not distinguish between these two quantities here.

The measurement of this cross section ratio at the COSY facility in Jülich was suggested with the main motivation to quantify the effects from charge symmetry breaking and even to determine the π^0-η mixing angle. We will argue in the present paper that the ratio R defined in Eq. (2) is possibly an even more useful quantity for something else, namely for determining the sign of the η-nucleus scattering length, which in turn is related to the possible existence of η-nuclear quasibound states. The basic observation behind this idea is that the expression on the very right hand side of Eq. (2) should still be valid, if we drop the assumption that the effects from charge symmetry breaking are given by π^0-η

mixing alone. All we assume is, and this is crucial, that the additional piece which causes the ratio to deviate from 2 is strongly energy dependent and should be proportional to the amplitude for η-nucleus scattering.

To be concrete, let us parametrize the η-nucleus production amplitude by

$$\mathcal{M}_\eta = \mathcal{M}_\eta^0 T_\eta = \frac{\mathcal{M}_\eta^0}{1 - i p_\eta a(\eta A)}, \tag{3}$$

which takes into account the well-known fact that the energy dependence of such production reactions is primarily determined by the interaction of the particles in the final state [9]. In the present case, this interaction is given by the η-nucleus scattering amplitude T_η. The T matrix is approximated here by the lowest-order term in the effective-range expansion where $a(\eta A)$ is the complex valued η-nucleus scattering length and p_η is the relative momentum of η with respect to the nucleus. The constant $\mathcal{M}_\eta^0$ parametrizes the overall strength of the production amplitude. For a specific reaction, the constants $\mathcal{M}_\eta^0$ and $a(\eta A)$ can be determined by a fit to corresponding (near-threshold) cross section data. However, the production of a real η as in $pd \to {}^3\mathrm{He}\eta$ is sensitive to $|\mathcal{M}_\eta|^2$ only. As a consequence, it is not possible to pin down the sign of the real part of the scattering length just from fitting to such data. For example, for that particular reaction the values

$$\begin{aligned} |\mathrm{Re}(a(\eta^3\mathrm{He}))| &= (3.8 \pm 0.6)\ \ \mathrm{fm}, \\ \mathrm{Im}(a(\eta^3\mathrm{He})) &= (1.6 \pm 1.1)\ \ \mathrm{fm} \end{aligned} \tag{4}$$

were extracted from the data [15]. Also subsequent analyses of those data within theoretical models did not yield unique results. While Wilkin [16] reported a negative sign for Re $a(\eta A)$, based on an optical potential approach, this was not confirmed by a more refined study later on, using multiple scattering theory, carried out by Wycech *et al.* [5], who arrived at positive values.

In contrast to the total cross section for $pd \to {}^3\mathrm{He}\eta$, the ratio R as defined in Eq. (2) is sensitive to $\mathrm{Re}(\mathcal{M}_\eta \mathcal{M}_{\pi^0}^*)$ and consequently, as we will demonstrate below, also to the sign of Re $a(\eta A)$ and therefore it can provide additional and independent information. Let us write $\mathrm{Re}(\mathcal{M}_\eta \mathcal{M}_{\pi^0}^*)$ as

$$\begin{aligned} \mathrm{Re}(\mathcal{M}_\eta \mathcal{M}_{\pi^0}^*) = |\mathcal{M}_{\tilde{\pi}^0}||\mathcal{M}_\eta^0|[\cos(\phi)\mathrm{Re}(T_\eta) \\ + \sin(\phi)\mathrm{Im}(T_\eta)]. \end{aligned} \tag{5}$$

Here ϕ is the phase between the amplitudes $\mathcal{M}_{\tilde{\pi}^0}$ and $\mathcal{M}_\eta^0$. Pion production around the η threshold involves already many partial waves, as is obvious from a comparision of the data for different proton-pion relative angles given in Fig. 1 of Ref. [11]. Thus, it is clear that the phase ϕ must necessarily depend on the pion production angle. However, and this is important, its variation with momentum (or energy) is very slow and practically negligible compared to the strong energy dependence induced by the η-nucleus interaction in the vicinity of the η production threshold. Therefore, the energy dependence of $\mathrm{Re}(\mathcal{M}_\eta \mathcal{M}_{\pi^0}^*)$ is given entirely by the energy dependence of T_η. Above the η production threshold, the η momentum p_η is real and thus

$$\begin{aligned} \mathrm{Re}(T_\eta) &= \frac{1 + p_\eta a_I}{1 + 2a_I p_\eta + |a(\eta A)|^2 p_\eta^2}, \\ \mathrm{Im}(T_\eta) &= \frac{p_\eta a_R}{1 + 2a_I p_\eta + |a(\eta A)|^2 p_\eta^2}, \end{aligned} \tag{6}$$

where we used $a(\eta A) = a_R + i a_I$. Below the threshold, however, we have to use the analytic continuation for $p_\eta = i\bar{p}_\eta$, where $\bar{p}_\eta$ is a positive real number. Then

$$\begin{aligned} \mathrm{Re}(T_\eta) &= \frac{1 + \bar{p}_\eta a_R}{1 + 2a_R \bar{p}_\eta + |a(\eta A)|^2 \bar{p}_\eta^2}, \\ \mathrm{Im}(T_\eta) &= \frac{-\bar{p}_\eta a_I}{1 + 2a_R \bar{p}_\eta + |a(\eta A)|^2 \bar{p}_\eta^2}. \end{aligned} \tag{7}$$

Thus, when moving from above the threshold to below the threshold the real part and the imaginary part of the η-nucleus scattering length interchange their roles. Because of that also the signs of these two quantities enter in a different way. Since unitarity fixes the sign of the imaginary part, i.e., $a_I \geqslant 0$, this feature opens the unique opportunity to access the sign of the real part of the η-nucleus scattering length by measuring the energy dependence of the cross section ratio (2) around the η threshold.

The only crux in this kind of analysis is the occurrence of the phase ϕ which is unknown. However, we will argue below that the knowledge of ϕ is not necessary for the analysis we propose, i.e., we will show that different signs of $\mathrm{Re}a(\eta A)$ lead to qualitatively different results for the energy dependence of the cross section ratio R so that the two cases can be distinguished experimentally even without knowledge of ϕ.

As should be clear from Eq. (5), a variation in ϕ does not introduce any peculiarities but leads to a rather smooth behavior of $\mathrm{Re}(\mathcal{M}_\eta \mathcal{M}_{\pi^0}^*)$. Therefore, we look only at the dependence of the ratio R on the η momentum for fixed values of ϕ. Thereby, we consider basically the whole range of ϕ. However, we restrict ourselves to those values of ϕ where R is smaller than 2 above the η threshold, as is suggested by the preliminary data from GEM [12,13].

As was mentioned above, the phase ϕ should depend on the pion emission angle. Thus, any possible systematic error introduced by the ϕ dependence could be explored and eliminated by performing the measurement of the energy dependence of R for a variety of pion angles.

Finally, for the $\eta\,{}^3\mathrm{He}$ scattering length we use the values for the real and imaginary parts as given in Eqs. (4), which were extracted from the data in Ref. [15], and we investigate the influence of different choices for the sign of the real part of $a(\eta^3\mathrm{He})$ on R. We should mention at this point, however, that the values of the real and imaginary parts of $a(\eta^3\mathrm{He})$

cannot be independently determined by a fit to the $pd \to {}^3\mathrm{He}\eta$ cross section based on Eq. (3). Rather, there is a correlation between them with the consequence that all values fulfilling the relation (units in fm)

$$a_R^2 + 0.449 a_I^2 + 4.509 a_I = 21.44 \tag{8}$$

lead to basically the same χ^2 minimum [16]. In order to explore also the influence of this uncertainty, we employ several values for the $\eta\,{}^3\mathrm{He}$ scattering length. That is, we make a (certainly extreme) assumption that $a_I = 0.5$ fm [the lowest limit for the imaginary part in Eq. (4)], which then leads to $|a_R| = 4.3$ fm [c.f. Eq. (8)], and we look at the other extreme as well by choosing the largest possible value for a_I which is still compatible with the data in Ref. [15] [see Eq. (4)]. Here we get $|a_R| = 2.4$ fm, $a_I = 2.7$ fm. Of course, we also employ the central values of Eq. (4).

The other parameter values used in our analysis are $\theta_m = 0.0015$ [17]; $|\mathcal{M}_{\tilde{\pi}^0}|^2 = 0.06 \mu$b/sr, which is extracted from the $pd \to {}^3\mathrm{He}\pi^0$ amplitude at a proton-pion relative angle of $\theta_{p-\pi} = 180°$ and at energies around the ${}^3\mathrm{He}\eta$ threshold [11]. The value of $\mathcal{M}_\eta^0$ depends on the employed $\eta^3\mathrm{He}$ scattering length. Here we obtained $|\mathcal{M}_\eta^0|^2 = 1.51 \mu$b/sr [for $a_R + i a_I = (\pm 4.3 + i0.5)$ fm], $|\mathcal{M}_\eta^0|^2 = 1.74\ \mu$b/sr [for $(\pm 3.8 + i1.6)$ fm], and $|\mathcal{M}_\eta^0|^2 = 1.93\ \mu$b/sr [for $(\pm 2.4 + i2.7)$ fm], respectively, by fitting to the $pd \to {}^3\mathrm{He}\eta$ cross section data [15].

The results of our investigation are presented in Fig. 1. Though we have explored basically the whole available parameter space, we would like to concentrate here on a few but exemplary cases. Varying the phase ϕ we found examples where there is a very pronounced difference in the energy dependence of the cross section ratio R for the two choices of the sign of a_R and which, therefore, can be easily distinguished in an experiment. On the other hand, there are also cases where the differences in the results around the η production threshold can be very small. Representative results for those "best" or "worst" cases are shown in Fig. 1. It may be noted that all results for positive values of a_R are basically between the dashed curves and for negative ones between the solid curves, respectively, for any choice of ϕ. But one should keep in mind that the bounds alone are not that important. The variation of the ratio R with energy is the main criterion for distinguishing between a positive or negative a_R based on experimental data.

As a more qualitative feature we see that for a_R larger than zero (dashed lines) there is, in general, a cusplike structure of the ratio R at the η production threshold (the corresponding proton momentum is $p_p \approx 1563$ MeV), whereas for a_R smaller than zero (solid lines) one observes a so-called rounded step [18]. Consequently, in the former case $|R-2|$ decreases more or less monotonously below the η production threshold. On the other hand, for a_R smaller than zero, $|R-2|$ increases and, moreover, shows a strong momentum dependence. For instance, in the upper and middle panels of Fig. 1 one can see that the curves with $a_R < 0$ have either a clear bump or a dip (or even both) for some specific momenta below the threshold which can be easily distinguished from the monotonously decreasing curves corresponding to $a_R > 0$.

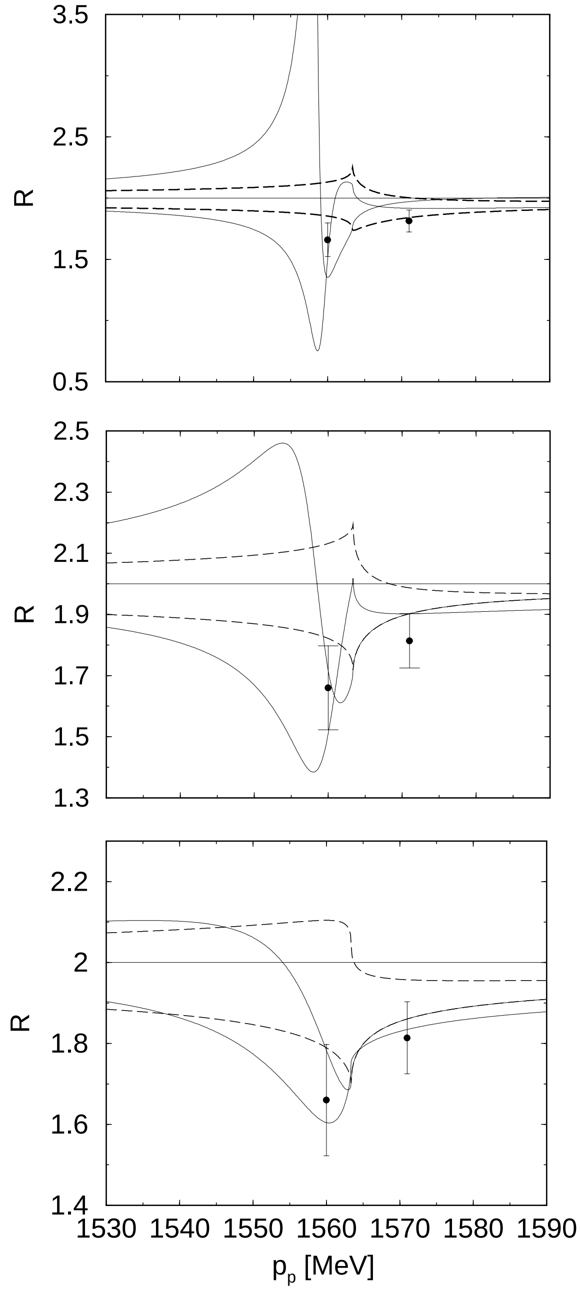

FIG. 1. Predictions for the cross section ratio R for different values and different signs of $\mathrm{Re}(a(\eta^3\mathrm{He}))$. The η^3He scattering length is $(\pm 4.3 + i0.5)$ fm (upper panel), $(\pm 3.8 + i1.6)$ fm (middle panel), $(\pm 2.4 + i2.7)$ fm (bottom panel). The curves are for individually selected values of the phase ϕ, cf. discussion in the text, where the dashed lines correspond to a positive real part of the scattering length and the solid lines correspond to a negative real part. The horizontal solid line indicates the value of 2 for the ratio predicted by isospin symmetry. Note that the scale is different for different panels. The experimental results are those of "run B" taken from Ref. [12].

A detailed inspection of Fig. 1 reveals that in some cases it is insufficient to determine the cross section ratio only in a small energy range around the η threshold—despite the fact that the momentum dependence is strikingly different for different signs of a_R for all values of the angle ϕ. Such a situation can be seen in the middle panel of Fig. 1. One of the sample results for $a_R<0$ (solid curve) exhibits a dip very close to the threshold which would be difficult to distinguish from the cusplike structure of similar magnitude produced by a calculation using $a_R>0$ (dashed curve)—given the present accuracy of the experimental data—if one looks only into a very narrow energy range. Here measurements over a wider energy range are necessary. It is obvious from this figure that measurements at 5–20 MeV/c below the threshold will allow to distinguish the different scenarios.

We also observed some cases where seemingly only a rather high experimental accuracy would allow to distinguish between the two scenarios. An example for this can be found in the lower panel of Fig. 1. Here we see a sample result with $a_R<0$ where the dip is still fairly close to the threshold and where also the momentum dependence of R below the threshold is similar to the one produced by a corresponding calculation based on $a_R>0$.

In this context, let us emphasize, however, that increasing the experimental accuracy is not the only option one has. Further measurements performed at different angles between proton and pion should be also helpful, since then the phase ϕ is changed as well and could be shifted to a different range of values where a discrimination between the two signs for a_R is much better feasible.

Nevertheless, it is obvious that the possibility to distinguish between the two scenarios depends to a certain extent on the magnitude of $|a_R|$, and the differences in the cross section ratio caused by a positive or negative sign are getting smaller with decreasing value of $|a_R|$. As we discussed above and as can be seen in the lower panel of Fig. 1, already in the case of $a(\eta^3\mathrm{He})=(\pm 2.4+i2.7)$ fm it is somewhat tricky to discriminate between the two signs for a_R, and the situation will be even more involved should $|a_R|$ be still smaller.

But even in such a situation interesting conclusions can be drawn from the cross section ratio. In order to understand that we need to remind the reader that in case of a complex scattering length, the condition $a_R<0$ alone is not sufficient for having a bound state. Here there is an additional constraint, namely that $|a_I|<|a_R|$ [7]. The results presented above indicate that the possibility to distinguish between the two scenarios for the sign of a_R is getting more and more difficult just in such cases where this constraint is not fulfilled anymore. Therefore, even if the measured cross section ratio shows features such as those in the lower panel of Fig. 1—which would make it difficult if not impossible to determine the sign of a_R—it would still allow to rule out a bound state.

Finally, we want to mention that we have assumed in our analysis for simplicity reasons that there is only one spin amplitude contributing to the cross section. But there are actually two possibilities, namely total spins 1/2 and 3/2. This would change the denominator in Eq. (2) as

$$\mathrm{Re}(\mathcal{M}_\eta \mathcal{M}^*_{\tilde{\pi}^0})/|\mathcal{M}_{\tilde{\pi}^0}|^2 \rightarrow \mathrm{Re}(\mathcal{M}^1_\eta \mathcal{M}^{1*}_{\tilde{\pi}^0}+\mathcal{M}^3_\eta \mathcal{M}^{3*}_{\tilde{\pi}^0})/ (|\mathcal{M}^1_{\tilde{\pi}^0}|^2+|\mathcal{M}^3_{\tilde{\pi}^0}|^2), \tag{9}$$

where the superscripts 1 and 3 denote these spins. However, and this is the important point, the η^3He final state interaction factor T_η defined and extracted is still the same for both spin amplitudes. Consequently, the discussion formulated here for a single spin amplitude as illustration will hold also in the case of two amplitudes. Nonetheless, in principle it is thinkable that the two amplitude products $\mathcal{M}^s_\eta \mathcal{M}^{s*}_{\tilde{\pi}^0}$ in Eq. (9) cancel to a large extent, and then the signal of the η^3He final state interaction may be largely washed out. Also if s-wave η production would take place in only one spin amplitude, the signal could be diminished by a factor of 1/2 or even more. However, there is no particular reason for these incidents to occur.

III. CONCLUSION

To summarize, we have demonstrated that charge symmetry breaking can be used as a tool to get direct access to the real part of the η-nucleus scattering length and specifically to its sign. The knowledge of this sign is important for drawing conclusions about the possible existence of η-nucleus bound states. In the present paper, we outlined the general idea and strategy for a corresponding analysis and exemplified its feasibility for the reactions $pd\rightarrow {}^3\mathrm{H}\pi/{}^3\mathrm{He}\pi$. With the same initial state one can also look at $NN\rightarrow d\pi$ with one nucleon being a spectator. Again, in the case of charge symmetry, R defined analogously to Eq. (1) will be 2. However, close to the η threshold a significant deviation from this value should be observed allowing one to determine the sign of $\mathrm{Re}(a(\eta d))$. In the same way, bombarding a tritium target with protons allows access to $\mathrm{Re}(a(\eta\alpha))$ and so on. All these experiments are presently feasible, e.g., at the CELSIUS as well as COSY accelerators. In addition to the pion cross section ratio, of course, the corresponding η cross section should be measured to high accuracy. Only a profound knowledge of the energy dependence of the η cross section allows to sufficiently constrain the magnitudes of the relevant η-nucleus scattering lengths so that an analysis along the lines suggested becomes practicable.

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