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Scaling of the linear response in simple aging systems without disorder

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The time-dependent scaling of the thermoremanent and zero-field-cooled susceptibilities in ferromagnetic spin systems undergoing aging after a quench to a temperature at or below criticality is studied. A recent debate on their interpretation is resolved by showing that for systems with a short-ranged equilibrium spin-spin correlator and above their roughening temperature, the field-cooled susceptibility $\chi_{\rm FC}(t)-\chi_0\sim t^{-A}$, where χ_0 is related to the equilibrium magnetization and the exponent A is related to the time-dependent scaling of the interface width between ordered domains. The same effect also dominates the scaling of the zero-field-cooled susceptibility $\chi_{\rm ZFC}(t,s)$, but does not enter into the thermoremanent susceptibility $\rho_{\rm TRM}(t,s)$. However, there may be large finite-time corrections to the scaling of $\rho_{\rm TRM}(t,s)$ which are explicitly derived and may be needed in order to extract reliable aging exponents. Consistency with the predictions of local scale invariance is confirmed in the Glauber-Ising and spherical models.

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I. INTRODUCTION

The comprehension of the physics of aging phenomena is a topic of much current interest. While aging was originally studied in glassy systems, there are many conceptual problems which are conveniently first studied in simple ferromagnetic systems. In this paper, we shall study a ferromagnet with a critical temperature $T_c>0$, initially prepared in a fully disordered (infinite-temperature) state which is quenched at time $t=0$ to a temperature $T\leqslant T_c$. We shall consider throughout a dynamics with a nonconserved order parameter. Physically, the aging process proceeds via the growth of correlated domains of size $L(t)\sim t^{1/z}$ and the slow motion of the domain boundaries drives in turn the slow temporal evolution of macroscopic observables, see Refs. [1–3] for recent reviews. It has turned out that aging phenomena are more fully revealed through the study of two-time quantities such as the two-time correlator $C(t,s)$ and the two-time linear response function $R(t,s)$, defined by

$$C(t,s)=\langle\phi(t)\phi(s)\rangle,\quad R(t,s)=\left.\frac{\delta\langle\phi(t)\rangle}{\delta h(s)}\right|_{h=0},\qquad(1)$$

where $\phi(t)$ is the time-dependent order parameter, $h(s)$ is the magnetic field conjugate to ϕ, t is referred to as observation time, and s will be called the waiting time. Causality implies that $R(t,s)=0$ for $t<s$.

Aging systems may display dynamical scaling in the long-time limit [1–3]. Specifically, consider the two-time functions in the aging regime $t\gg t_{\rm micro}$, $s\gg t_{\rm micro}$, and $\tau=t-s\gg t_{\rm micro}$, where $t_{\rm micro}$ is some microscopic time. Then one has the scaling behavior

$$C(t,s)\sim s^{-b}f_C(t/s),\quad R(t,s)\sim s^{-1-a}f_R(t/s),\qquad(2)$$

where the scaling functions $f_{C,R}(x)$ have the following asymptotic behavior for $x\to\infty$,

$$f_C(x)\sim x^{-\lambda_C/z},\quad f_R(x)\sim x^{-\lambda_R/z},\qquad(3)$$

and λ_C and λ_R are the autocorrelation [4,5] and autoresponse [6] exponents, respectively. In general, the exponents $\lambda_{C,R}$ and z will take different values for $T<T_c$ and for $T=T_c$. In particular, $z=2$ for $T<T_c$ and a nonconserved order parameter.

The values of the exponents a and b are collected in Table I and depend on the equilibrium spin-spin correlator $C_{\rm eq}$ as follows [7]. If $C_{\rm eq}(\mathbf{r})\sim e^{-|\mathbf{r}|/\xi}$ with a finite ξ, one says that the system is of *class S*, while if $C_{\rm eq}(\mathbf{r})\sim|\mathbf{r}|^{-(d-2+\eta)}$, the system is said to be of *class L*, where η is a standard equilibrium critical exponent. We point out that for systems of class S, the result $a=1/z$ follows from the well-accepted intuitive picture that aging effects come from the slow motion of the domains walls which separate the well-ordered domains in systems undergoing coarsening [1,3,7,8]. A different value for a would invalidate this physical picture.

TABLE I. Nonequilibrium exponents a and b for quenches from a fully disordered state onto and below the critical point $T_c>0$ of simple ferromagnets of the classes S and L defined in the text, according to Ref. [7].

	a	b	Class
$T=T_c$	$(d-2+\eta)/z$	$(d-2+\eta)/z$	L
$T<T_c$	$(d-2+\eta)/z$	0	L
	$1/z$	0	S

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Recently, it has been proposed that aging systems might possess a larger dynamically generated space-time symmetry than mere dynamical scaling [9]. Indeed, for any given value of z, infinitesimal local scale transformations with a space-time-dependent rescaling factor $1+\varepsilon(t,\mathbf{r})$ can be constructed. In particular, the following explicit expression for the response function is obtained from the condition that $R(t,s)$ transforms covariantly under the action of local scale transformations [9,10]

$$R(t,s) = r_0' \left(\frac{t}{s}\right)^{1+a-\lambda_R/z} (t-s)^{-1-a}, \tag{4}$$

and where r_0' is a normalization constant. This prediction has been confirmed in several models, notably the kinetic Ising model with Glauber dynamics [10,11] and several variants of the exactly solvable spherical model [10,12,13], as is reviewed in Refs. [9,14]. Furthermore, predictions exist for the spatio-temporal response. A particular simple result is found for the case $z=2$ [9,15]

$$R(t,s;\mathbf{r}) = \left.\frac{\delta\langle\phi_{\mathbf{r}}(t)\rangle}{\delta h_{\mathbf{0}}(s)}\right|_{h=0} = R(t,s)\exp\left[-\frac{\mathcal{M}}{2}\frac{\mathbf{r}^2}{t-s}\right] \tag{5}$$

($\mathcal{M}$ is a nonuniversal constant) and has been confirmed in the two-dimensional (2D) and 3D Glauber-Ising model quenched to $T<T_c$ [14] (and also in the spherical model for $d>2$). Since the derivation of Eq. (5) depends on the Galilei invariance of the coarsening process, its verification is strong evidence in favor of Galilei invariance as a dynamically generated space-time symmetry of phase ordering.

One would obviously like to be able to test statements of such a general nature as widely as possible, but since there are so few exactly solvable and nontrivial model of aging, quantities such as $R(t,s)$ must in general be obtained numerically. This paper addresses an important conceptual point on the interpretation of the scaling behavior of several commonly used observables related to $R(t,s)$ and we shall try to clear up a debate which has arisen in recent years.

Numerical information on $R(t,s)$ can be obtained using a by-now standard method devised by Barrat [16]. One perturbs the system by a *random* magnetic field $h_{\mathbf{i}}$ with zero mean $\overline{h_{\mathbf{i}}}=0$. We shall use a binary field $h_i=\pm h$ below in our Glauber-Ising model simulation, but a Gaussian random field is also possible. Instead of measuring $R(t,s)$ directly, two common procedures run as follows. Either one quenches the system and turns on the magnetic field after the waiting time s has elapsed and then works with the zero-field-cooled susceptibility $\chi_{\mathrm{ZFC}}(t,s)$, or one may also keep the random field till the waiting time s when it is turned off and then has the thermoremanent susceptibility $\rho_{\mathrm{TRM}}(t,s)$. These are related to $R(t,s)$ as follows:

$$\chi_{\mathrm{ZFC}}(t,s) = \chi(t,s) = \int_s^t du R(t,u) \overset{?}{=} s^{-a} f_\chi(t/s), \tag{6}$$

$$\rho_{\mathrm{TRM}}(t,s) = \rho(t,s) = \int_0^s du R(t,u) \overset{?}{=} s^{-a} f_M(t/s). \tag{7}$$

Here, we straightforwardly used the scaling forms (2) *without paying attention to the conditions of validity* of these and in particular *did not pay any attention as to whether* $t-s \gg t_{\mathrm{micro}}$ *holds true or not*. As we shall show, however, careful consideration of these conditions is crucial in order to obtain valid scaling forms for the integrated responses $\chi_{\mathrm{ZFC}}(t,s)$ and $\rho_{\mathrm{TRM}}(t,s)$.

On a discrete lattice $\Lambda\subset\mathbb{Z}^d$, the integrated responses (6) and (7) are obtained by measuring the time-dependent magnetization [16]

$$M = \frac{1}{|\Lambda|h}\overline{\left\langle \sum_{\mathbf{i}\in\Lambda} \phi_{\mathbf{i}}(t) h_{\mathbf{i}} \right\rangle}, \tag{8}$$

where $|\Lambda|$ is the number of sites of the lattice Λ. Depending on whether one works in the zero-field-cooled or the thermoremanent protocol, one obtains $\chi_{\mathrm{ZFC}}=M_{\mathrm{ZFC}}/h$ or $\rho_{\mathrm{TRM}}=M_{\mathrm{TRM}}/h$, respectively.

In recent years, Corberi, Lippiello and Zannetti (CLZ) [17,18] have studied in great detail aging in simple ferromagnets such as the Glauber-Ising model. Most notably, and based on MC data on χ_{ZFC} in between one and four dimensions, they have argued that for the phase-ordering kinetics (i.e., $T<T_c$) in the Glauber-Ising model, the exponent a takes a value different from the generally accepted value $a=1/z=1/2$. Their studies consider the two-time autocorrelation $C(t,s)$ and the dynamic susceptibilities $\chi(t,s),\rho(t,s)$ of the O(n) vector model. They separate the autocorrelator into a "stationary"and an "aging" part $C(t,s)=C^{(\mathrm{st})}(t-s)+C^{(\mathrm{age})}(t,s)$, and similarly $\chi(t,s)=\chi^{(\mathrm{st})}(t-s)+\chi^{(\mathrm{age})}(t,s)$, where $\chi^{(\mathrm{st})}(t,s)$ is *defined* such as to satisfy the fluctuation-dissipation theorem with $C^{(\mathrm{st})}(t,s)$ [and similarly for $\rho(t,s)$]. Furthermore, CLZ admit dynamical scaling for the aging part only and further introduce two distinct exponents a_χ and a_ρ according to

$$\chi^{(\mathrm{age})}(t,s) \sim s^{-a_\chi} f_\chi(t/s), \quad \rho^{(\mathrm{age})}(t,s) \sim s^{-a_\rho} f_\rho(t/s). \tag{9}$$

If $\hat{a}_n$ is the value of a_χ in the $O(n)$ model, CLZ propose

$$\hat{a}_1 = \begin{cases} (d-1)/4; & d<3 \\ 1/2; & d>3; \end{cases} \quad \hat{a}_\infty = \begin{cases} (d-2)/2; & d<4 \\ 1; & d>4 \end{cases} \tag{10}$$

for the Glauber-Ising ($n=1$) and spherical model ($n=\infty$), respectively (additional logarithmic factors may occur at $d=3,n=1$ and $d=4,n=\infty$, respectively). In order to account for these results, CLZ invoke a dangerous irrelevant variable and claim that $d=3$ were a critical dimension of coarsening in the Glauber-Ising model. On the other hand, from a log-log plot of $\rho^{(\mathrm{age})}(t,s)$ against s in the Glauber-Ising model "…*no statement on* a_ρ *can be made* …" [18], but roughly $a_\rho\approx 0.55-0.60$ in 2D and $a_\rho\approx 0.65-0.70$ in 3D, see Fig. 6 in Ref. [18]. These conclusions are supplemented by a discussion of the necessary condition $s\gg t_{\mathrm{micro}}$ needed for the validity of the scaling (2) and about the microscopic times t_{micro} which may enter there [18]. However, the crucial role of the additional condition $t-s\gg t_{\mathrm{micro}}$ which must also be satisfied for Eq. (2) to hold is not addressed by CLZ.

Already from a pure phenomenological point of view and given the scaling relations (2) and the definitions of $\chi(t,s)$ and $\rho(t,s)$, the assertion of CLZ of having *distinct* exponents $a_\chi \neq a_\rho$ cannot be maintained. Rather, it is necessary to go beyond a pure scaling analysis in order to understand how the aging behavior can be correctly extracted from integrated response functions.

Our argument runs along the following lines.

(1) We consider the field-cooled susceptibility

$$\chi_{\rm FC}(t) = \chi_{\rm ZFC}(t,s) + \rho_{\rm TRM}(t,s) = \int_0^t du R(t,u). \qquad (11)$$

For ferromagnets of class *S*, one has $\chi_{\rm FC}(t) - \chi_0 \sim t^{-A}$, as we shall show in Sec. II. Here $T\chi_0 = 1 - m_{\rm eq}^2$ is given by the mean equilibrium magnetization. The exponent A is a new exponent without a direct relationship to aging, rather it is related to the roughness of the interface between ordered domains. We stress that since the power-law behavior of $\chi_{\rm FC}(t)$ is *independent* of the waiting time s it has no relation with a possible aging behavior. In particular, we find $A = 1/4$ for the 2D Glauber-Ising model with $T < T_c$. Furthermore, for a fixed scaling variable $x = t/s$ one has, with the scaling function $g(x) \sim x^{-A}$

$$\chi_{\rm ZFC}(t,s) = \chi_{\rm FC}(t) - \rho_{\rm TRM}(t,s) \sim \chi_0 + s^{-A} g(x) + {\rm O}(s^{-a}). \qquad (12)$$

Here we anticipate an important result of Sec. IV, namely $\rho_{\rm TRM}(t,s) \sim s^{-a}$, see also Eq. (13) below. Since for rough interfaces one has $A - a = A - 1/z < 0$, it follows that Eq. (6) cannot be used. Furthermore, the splitting of $\chi(t,s)$ into a stationary and an aging part advocated by CLZ is in contradiction with Eq. (12). Rather, the leading scaling of $\chi_{\rm FC}$ with the waiting time s is unrelated to the aging behavior of the model as described by Eq. (2). Indeed, the terms describing aging only occur as subleading terms in $\chi_{\rm ZFC}(t,s)$ and are therefore difficult to extract. In particular, the exponent A cannot be identified with the aging exponent a. An example is provided by the 2D Glauber-Ising model, where $a = 1/z = 1/2$ but $A = 1/4$.

(2) For ferromagnets of class *L*, we shall show that $A = 0$. We shall explicitly test this in Sec. III for the nonequilibrium critical dynamics (i.e., $T = T_c$) in the 2D Glauber-Ising model and in the kinetic mean spherical model for any $T \leqslant T_c$.

(3) Equation (12) shows that aging effects merely provide a finite-time correction to the scaling of $\chi_{\rm ZFC}$. On the other hand, aging terms are leading in the thermoremanent magnetization which scales more precisely as [7]

$$\rho_{\rm TRM}(t,s) = s^{-a} f_M(t/s) + s^{-\lambda_R/z} g_M(t/s), \qquad (13)$$

where the scaling functions $f_M(x)$ and $g_M(x)$ are related to the response function $R(t,s)$ and can be found explicitly, see Sec. IV. For example, for a system in class *S* with an uncorrelated initial state, one has $\lambda_C = \lambda_R \geqslant d/2$ [19] and since $a = 1/z$, these two terms may be of almost the same order and a simple log-log plot may not be sufficient to yield a precise value of a for times accessible in present simulations. Indeed, this situation occurs in the 2D and 3D Glauber-Ising model. However, subtracting the leading correction according to Eq. (13) allows to reliably determine a and the scaling function $f_M(x)$ [7,11]. We shall describe this in Sec. IV.

In the Appendix possible implications for the scaling of $R(t, t-\varepsilon)$ for $\varepsilon \to 0$ are discussed.

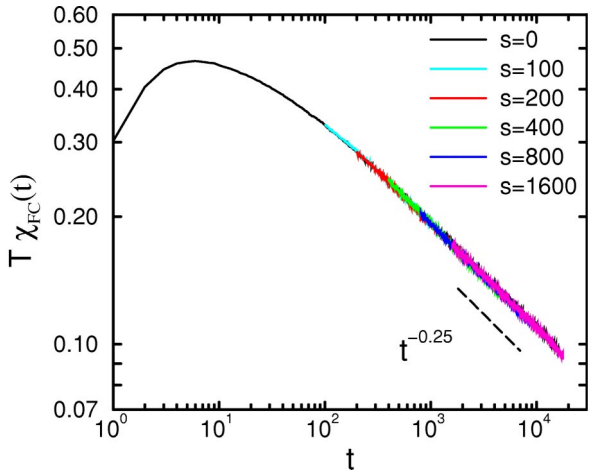

FIG. 1. Field-cooled susceptibility $\chi_{\rm FC}(t) = \rho_{\rm TRM}(t,s) + \chi_{\rm ZFC}(t,s)$ in the 2D Glauber-Ising model at temperature $T = 1.5 < T_c$ and for several waiting times. An alternating field $h_{\rm i} = \pm h$ with $h = 0.05$ was used. The power law $\chi_{\rm FC}(t) \sim t^{-0.25}$ is also shown for comparison. The simulated systems contained 300×300 spins, and the data have been obtained after averaging over at least 5000 different runs.

II. SCALING OF $\chi_{\rm FC}$ FOR SHORT-RANGED EQUILIBRIUM CORRELATORS

We begin by discussing the time-dependent scaling for the field-cooled susceptibility $\chi_{\rm FC}(t)$ for the Glauber-Ising model. We consider the Ising model on a hypercubic lattice with periodic boundary conditions and the equilibrium Hamiltonian $\mathcal{H} = -\Sigma_{(\mathbf{i},\mathbf{j})} \sigma_{\mathbf{i}} \sigma_{\mathbf{j}}$, where the sum is over nearest neighbors only. We use heat-bath dynamics defined through the stochastic rule

$$\sigma_{\mathbf{i}}(t+1) = \pm 1 \quad \text{with probability} \quad \tfrac{1}{2}[1 \pm \tanh(H_{\mathbf{i}}(t)/T)] \qquad (14)$$

with the local field $H_{\mathbf{i}}(t) = \Sigma_{\mathbf{y}(\mathbf{i})} \sigma_{\mathbf{y}}(t)$ and where $\mathbf{y}(\mathbf{i})$ runs over the nearest neighbors of the sites $\mathbf{i}$. Initially, the system is prepared in an infinite-temperature state. Thermoremanent and zero-field-cooled susceptibilities $\rho_{\rm TRM}(t,s)$ and $\chi_{\rm ZFC}(t,s)$ can now be measured by perturbing the model by a binary random field $h_{\mathbf{i}} = \pm h$ with zero mean ($\overline{h_{\mathbf{i}}} = 0$) and using Eq. (8). From these two independent measurements, we obtain the field-cooled susceptibility

$$\chi_{\rm FC}(t) = \rho_{\rm TRM}(t,s) + \chi_{\rm ZFC}(t,s) \qquad (15)$$

In Fig. 1, we show $\chi_{\rm FC}(t)$ so obtained for several values of the waiting time s in the case of the two-dimensional Ising model, at the fixed temperature $T = 1.5 < T_c$. Clearly, $\chi_{\rm FC}$ is

independent of s, which means that we are well inside the linear-response regime. Furthermore, within the times considered, our data are consistent with the scaling behavior,

$$\chi_{\rm FC}(t) \sim t^{-A}, \quad A \approx 0.25. \tag{16}$$

Since this dynamical scaling is completely independent of the waiting time s, it is unrelated to any aging behavior which might occur in this model.

We now give a heuristic argument in order to understand where the scaling form (16) and the value of A come from. Consider a simple ferromagnet in $d>1$ dimensions which is quenched at time $t=0$ from an infinite-temperature initial state to a final temperature $T<T_c$. The dynamics is assumed to be purely relaxational, i.e., without any conservation law. Microscopically, it is well known that the configurations of the system consist of fully ordered domains of spins, of a typical size $L(t)\sim t^{1/z}$ with $z=2$. We now perturb with a random field of zero mean ($\overline{h_{\mathbf{i}}}=0$) and wish to obtain the susceptibility $\chi_{\rm FC}=M_{\rm FC}/h$ from Eq. (8). First consider the case when $T=0$. Then, because the spins deep inside the cluster are ordered, the only nonvanishing contribution to $\chi_{\rm FC}$ comes from the spins from near the interfaces between the ordered clusters. We denote the interface density by $\rho_{\rm I}(t)$ and have $\rho_{\rm I}(t)\sim L(t)^{-1}$, see Refs. [1,3,8]. If $w(t)$ is the interface width, we have

$$\chi_{\rm FC}(t) = \frac{1}{|\Lambda|h^2}\overline{\left\langle \sum_{\mathbf{i}\in\Lambda} \sigma_{\mathbf{i}}(t)h_{\mathbf{i}} \right\rangle} = \frac{1}{|\Lambda|h^2}\overline{\left\langle \sum_{\mathbf{i}\in \text{interfaces}} \sigma_{\mathbf{i}}(t)h_{\mathbf{i}} \right\rangle} \sim L(t)^{-1}w(t). \tag{17}$$

For a finite temperature $T>0$, the order deep inside the clusters is not perfect and there remains a residual contribution to the susceptibility. We then have, for large times

$$\chi_{\rm FC}(t) \simeq \chi_0 + L(t)^{-1}w(t), \quad T\chi_0 = 1 - m_{\rm eq}^2, \tag{18}$$

where $m_{\rm eq}$ is the equilibrium magnetization [for the 2D Ising model, $m_{\rm eq}=\left(1-\sinh(2/T)^{-4}\right)^{1/8}$, see e.g. Ref. [20]]. The dynamics of a $(d-1)$-dimensional interface in a d-dimensional system ($d\geqslant 2$) can be described by the dynamics of a height model [21] of continuous height variables $v_{\mathbf{j}}\in\mathbb{R}$ and the equilibrium Hamiltonian $\mathcal{H}[v]=-(\tau/2)\Sigma_{(\mathbf{j},\mathbf{j}')}(v_{\mathbf{j}}-v_{\mathbf{j}'})^2$ with nearest-neighbor interactions and where τ is the effective interfacial tension [21]. In adopting this description, we tacitly assume that the system is above its "roughening" temperature T_R, see e.g. Ref. [22], such that the fluctuations of the interface are unbounded (rough interface) for $T>T_R$ and with bounded fluctuations for $T<T_R$ (smooth interface). This condition is always satisfied for $d=2$, since then $T_R=0$ and indeed the description adopted here can be derived rigorously [23]. On the other hand, for $d=3$ one has $T_R\approx 0.5\,T_c$ and finally, $T_R=\infty$ for $d\geqslant 4$. If the dynamics of the model is described by a Langevin equation, the squared interface width was shown by Abraham and Upton to scale for large times as $w(t)^2=\langle v_{\mathbf{0}}(t)^2\rangle\sim t^{1/2}$ in 2D and $w(t)^2\sim \ln t$ in 3D [21]. Inserting this into Eq. (17), we find the following leading dynamical scaling for $\chi_{\rm FC}$ in the Glauber-Ising model as $t\to\infty$:

$$\chi_{\rm FC}(t) - \chi_0 \sim \begin{cases} t^{-1/4} & \text{if } d=2 \\ t^{-1/2}(\ln t)^{1/2} & \text{if } d=3, \end{cases} \tag{19}$$

provided that $T>T_R$. For dimensions $d\geqslant 4$, and generically if $T<T_R$, one expects a flat interface with $w(t)\sim$ const, and consequently $\chi_{\rm FC}(t)-\chi_0\sim t^{-1/2}$. Reconsidering Fig. 1, it is easy to check that the final approach of $\chi_{\rm FC}$ towards χ_0 only occurs at times much beyond those accessed by our simulation.

Combining Eqs. (12) and (19), we have therefore reproduced the findings of CLZ in the Glauber-Ising model and have also made clear the physical origin of these results. Besides, a definite prediction for the logarithmic factor in $d=3$ was obtained. In summary, we have for the Glauber-Ising model $A=1/4$ in 2D and $A=1/2$ for all $d\geqslant 3$, up to a known logarithmic correction in 3D.

III. SCALING OF $\chi_{\rm FC}$ FOR LONG-RANGED EQUILIBRIUM CORRELATORS

We now ask whether the heuristic discussion of the scaling of $\chi_{\rm FC}$ presented in the preceding section can be taken over for systems of class L. Indeed, the main physical difference with respect to systems of class S is that although correlated clusters of size $L(t)\sim t^{1/z}$ form, fluctuations do persist in the interior of these clusters on all length scales up to $L(t)$. This means that one should consider an "interface width" scaling as $w(t)\sim L(t)$. This in turn leads to $\chi_{\rm FC}(t)\sim$ const, and $A=0$ (on the other hand, since the clusters should have no "inside," we do not expect a term χ_0 to occur).

We now test this heuristic idea in the exactly solvable mean spherical model and shall also present evidence from the 2D Glauber-Ising model quenched onto criticality.

(1) First, we consider the mean kinetic spherical model, see e.g. [6,12,13,24,25]. To each site $\mathbf{x}$ of a hypercubic lattice $\Lambda\subset\mathbb{Z}^d$ one attaches a continuous spin variable $S_{\mathbf{x}}(t)\in\mathbb{R}$, subject to the mean spherical constraint

$$\sum_{\mathbf{x}\in\Lambda}\langle S_{\mathbf{x}}^2\rangle = 1. \tag{20}$$

By analogy with the Glauber-Ising model, we also add a random magnetic field $h_{\mathbf{x}}$ and the equations of motion read

$$\frac{d}{dt}S_{\mathbf{x}}(t) = \sum_{\mathbf{y}(\mathbf{x})} S_{\mathbf{y}}(t) - [2d - \mathcal{z}(t)]S_{\mathbf{x}}(t) + h_{\mathbf{x}} + \eta_{\mathbf{x}}(t), \tag{21}$$

where the noise and the random field have zero average, $\langle\eta_{\mathbf{x}}\rangle=\overline{h_{\mathbf{x}}}=0$, and the correlators

$$\langle\eta_{\mathbf{x}}(t)\eta_{\mathbf{y}}(t')\rangle = 2T\,\delta_{\mathbf{x},\mathbf{y}}\delta(t-t'), \quad \overline{h_{\mathbf{x}}h_{\mathbf{y}}} = 2\Gamma\,\delta_{\mathbf{x},\mathbf{y}}, \tag{22}$$

where the temperature T and the width Γ are constants and $\mathcal{z}(t)$ is a Lagrange multiplier to be determined below. In addition, the noise and the field are assumed independent, i.e., $\overline{\langle\eta_{\mathbf{x}}(t)h_{\mathbf{y}}\rangle}=0$, and in addition the initial state is uncorrelated in the sense that $\overline{\langle S_{\mathbf{x}}(0)h_{\mathbf{y}}\rangle}=0$. Here and in the following the

average $\langle \mathcal{X} \rangle$ is always taken over the initial conditions and the noise, while the average $\overline{\mathcal{X}}$ is over the random field. The solution of this model follows standard lines [25]. Taking Fourier transforms, the solution of Eq. (21) is

$$\widetilde{S}(\mathbf{q},t) = \frac{\exp[-\omega(\mathbf{q})t]}{\sqrt{g(t)}}[\widetilde{S}(\mathbf{q},0) + \int_0^t dt'\ e^{\omega(\mathbf{q})t'} \times \sqrt{g(t')}[\widetilde{h}(\mathbf{q}) + \widetilde{\eta}(\mathbf{q},t')]], \tag{23}$$

with the dispersion relation $\omega(\mathbf{q}) = 2\Sigma_{i=1}^{d}(1-\cos q_i)$ and $g(t) = \exp[2\int_0^t dt'\ \varkappa(t')]$. The spin-spin correlator $\widetilde{C}(\mathbf{q},\mathbf{q}';t,s) = \overline{\langle \widetilde{S}(\mathbf{q},t)\widetilde{S}(\mathbf{q}',s)\rangle}$ is readily found and we have in direct space the autocorrelator

$$\begin{aligned} C(t,s) &= C_{\mathbf{x},\mathbf{x}}(t,s) \\ &= \frac{1}{(2\pi)^{2d}}\int_{\mathcal{B}^2} d\mathbf{q}d\mathbf{q}' e^{i(\mathbf{q}+\mathbf{q}')\cdot\mathbf{x}}\widetilde{C}(\mathbf{q},\mathbf{q}';t,s) \\ &= \frac{1}{\sqrt{g(t)g(s)}}\left[A\left(\frac{t+s}{2}\right) + 2T\int_0^s du\ f\left(\frac{t+s}{2}-u\right)g(u) \right. \\ &\quad \left. + 2\Gamma\int_0^t dt'\int_0^s ds' f\left(\frac{t+s-t'-s'}{2}\right)\sqrt{g(t')g(s')}\right], \end{aligned} \tag{24}$$

where $\mathcal{B}$ is the Brillouin zone and with the definitions

$$A(t) = (2\pi)^{-d}\int_{\mathcal{B}} d\mathbf{q} e^{-2\omega(\mathbf{q})t}\widetilde{C}(\mathbf{q},0), \tag{25}$$

$$f(t) = (2\pi)^{-d}\int_{\mathcal{B}} d\mathbf{q} e^{-2\omega(\mathbf{q})t} = [e^{-4t}I_0(4t)]^d, \tag{26}$$

and where I_0 is a modified Bessel function. For infinite-temperature initial conditions $A(t) = f(t)$. The mean spherical constraint (20) gives $C(t,t) = 1$, and this leads to the following generalized Volterra integral equation:

$$g(t) = A(t) + 2T\int_0^t du\ f(t-u)g(u) + 2\Gamma\int_0^t du'\int_0^t du'' \times f\left(t - \frac{u'+u''}{2}\right)\sqrt{g(u')g(u'')}, \tag{27}$$

which determines $g(t)$. Finally, the response function is given by the usual equation $R(t,s) = f((t-s)/2)\sqrt{g(s)/g(t)}$. At zero temperature $T=0$, Eqs. (24) and (27) are identical to those found for the spherical spin-glass [24,26].

The field-cooled susceptibility is given by

$$\chi_{\mathrm{FC}}(t) = \int_0^t du R(t,u) = \int_0^t du f\left(\frac{t-u}{2}\right)\sqrt{\frac{g(u)}{g(t)}}. \tag{28}$$

At this point, it is instructive to rederive the equivalence between the definition (28) and Eq. (8), originally proposed [16] for the Glauber-Ising model, in the context of the mean kinetic spherical model. Indeed, we have ($|\Lambda|$ denotes the number of sites of the lattice Λ)

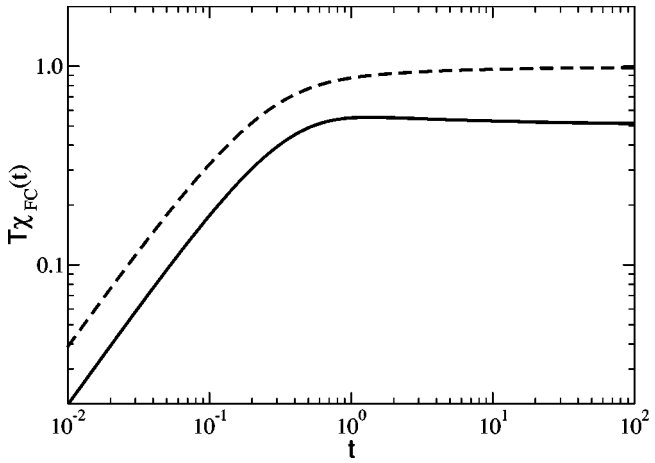

FIG. 2. Field-cooled susceptibility $\chi_{\mathrm{FC}}(t)$ for the 3D kinetic mean spherical model in a Gaussian random magnetic field of width $\Gamma = 0.01$ and at $T = 2 < T_c$ (full curve) and $T = T_c \simeq 3.96$ (dashed curve).

$$\begin{aligned} &\frac{1}{|\Lambda|}\overline{\left\langle \sum_{\mathbf{x}\in\Lambda} S_{\mathbf{x}}(t)h_{\mathbf{x}}\right\rangle} \\ &= \frac{(2\pi)^{-2d}}{|\Lambda|}\sum_{\mathbf{x}\in\Lambda}\int_{\mathcal{B}^2} d\mathbf{q}d\mathbf{q}' e^{i(\mathbf{q}+\mathbf{q}')\cdot\mathbf{x}}\overline{\langle \widetilde{S}(\mathbf{q},t)\widetilde{h}(\mathbf{q}')\rangle}. \\ &= \frac{(2\pi)^{-2d}}{|\Lambda|}\sum_{\mathbf{x}\in\Lambda}\int_{\mathcal{B}^2} d\mathbf{q}d\mathbf{q}' e^{i(\mathbf{q}+\mathbf{q}')\cdot\mathbf{x}} \\ &\times \int_0^t dt' e^{\omega(\mathbf{q})(t'-t)}\sqrt{\frac{g(t')}{g(t)}}\cdot\overline{\widetilde{h}(\mathbf{q})\widetilde{h}(\mathbf{q}')} \\ &= (2\pi)^{-d}2\Gamma\int_0^t dt'\int_{\mathcal{B}} d\mathbf{q} e^{-\omega(\mathbf{q})(t-t')}\sqrt{\frac{g(t')}{g(t)}} \\ &= 2\Gamma\chi_{\mathrm{FC}}(t), \end{aligned} \tag{29}$$

as asserted, and where we used in the second line Eq. (23) and in the third line the field correlator (22).

After these preparations we can test our heuristic picture. Using the techniques described in Ref. [25], we obtain $g(t)$ by solving Eq. (27) numerically. In Fig. 2 we show $\chi_{\mathrm{FC}}(t)$ for the three-dimensional case, starting from an infinite-temperature initial state. We clearly see that $\chi_{\mathrm{FC}}(t)$ saturates rapidly. Consequently, $A=0$ for all temperatures $T \leqslant T_c$. Similar tests can be performed for other values of d as well.

We pause a moment in order to discuss the functional form of the response function. If $\Gamma = 0$, solution of the spherical constraint gives for $T < T_c$ the well-known exact result, valid for all values of d and in the aging regimes $s \gg 1$ and $t-s \gg 1$ is (see Refs. [6,10,12,17,18,27])

$$R(t,s) = r_0'\left(\frac{t}{s}\right)^{d/4}(t-s)^{-d/2}, \tag{30}$$

with $r_0' = (4\pi)^{-d/2}$. From Eq. (2), we read off $a = (d-2)/2$ and $\lambda_R/z = d/4$. We point out that this exact result for a is in contradiction with the claim (10) raised by CLZ. Furthermore, we see that the exact result [Eq. (30)] has precisely the

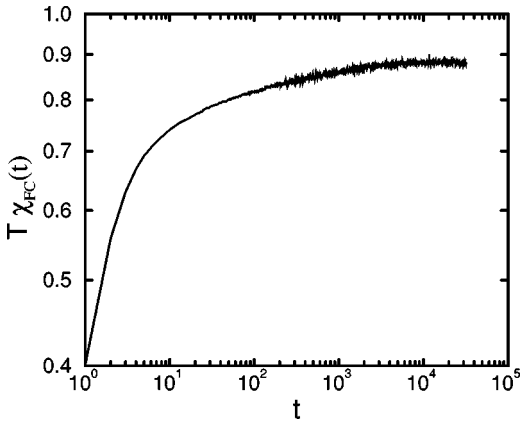

FIG. 3. Field-cooled susceptibility $\chi_{\rm FC}(t)$ in the 2D Glauber-Ising model at the critical temperature $T=T_c\approx 2.269$. An alternating field $h_{\rm i}=\pm h$ with $h=0.05$ was used. Systems with 300×300 spins have been simulated.

form (4) predicted by local scale invariance [9,10]. A similar test can be performed for $T=T_c$ [6,10], or even in a spherical model with spatially long-ranged interactions [13]. It is important to note that the local scale invariance prediction (4) applies to the full response function $R(t,s)$ and not to the part remaining after subtraction of a stationary term, as suggested in Ref. [18].

(2) As a second example, we give numerical evidence that $A=0$ in the 2D Glauber-Ising model quenched to its critical point $T=T_c$. We used a standard heat-bath algorithm and measured the integrated linear response through Eq. (8). Data for $\chi_{\rm FC}(t)$ thus obtained are displayed in Fig. 3 and we see that saturation occurs.

In summary, extending a heuristic argument of Sec. II, we have argued that for ferromagnetic systems in class L, one should find saturation for the field-cooled susceptibility, viz., $\chi_{\rm FC}(t)\sim O(1)$. We have confirmed this expectation in some models.

The heuristic arguments in this and the preceding section can only be applied in $d\geqslant 2$ dimensions. Indeed, the aging behavior of the 1D Glauber-Ising model at its critical point $T_c=0$ is peculiar and will be discussed in the Appendix.

IV. SCALING OF THE THERMOREMANENT RESPONSE

Since we have seen that the terms which describe the physically interesting aging effects are only subleading in the zero-field-cooled susceptibility $\chi_{\rm ZFC}(t,s)$, we discuss in this section the scaling of the thermoremanent susceptibility $\rho_{\rm TRM}(t,s)$. As we shall see, the case where the waiting time s is small needs particular consideration. It is well known [29] that the response with respect to a fluctuation in the initial state scales as

$$R(t,0)\sim t^{-\lambda_R/z}. \tag{31}$$

On the other hand, it can be shown [27] that there is a time scale $t_p\sim s^{\zeta}$, with $0<\zeta<1$, such that if the time difference $\tau=t-s\lesssim t_p$, then the response function is still the one of the equilibrium system $R(t,s)\simeq R_{\rm gg}(\tau)$ while scaling sets in and (2) holds if $t\gtrsim t_p$. For example, in the spherical model one has $\zeta=4/(d+2)$ [27]. We then have

$$\begin{aligned}\rho_{\rm TRM}(t,s) &= \int_0^s du R(t,u)\\ &= \int_0^s d\tau R(t,s-\tau)\\ &= \int_0^{t_p} d\tau R_{\rm gg}(2\tau) + s\int_{t_p/s}^{t_\varepsilon/s} dv R[t,s(1-v)]\\ &\quad + \int_{t_\varepsilon}^{s} d\tau R(t,s-\tau)\\ &\simeq \int_0^{\infty} d\tau R_{\rm gg}(2\tau) + s^{-a}\int_0^1 dv (1-v)^{-1-a} f_R\left(\frac{t/s}{1-v}\right)\\ &\quad + c_\infty t^{-\lambda_R/z}.\end{aligned} \tag{32}$$

Here, following Refs. [3,27], we have introduced a third time scale t_ε such that $s-t_\varepsilon=O(1)$. In the third line, the limit $s\to\infty$ is taken and the last term is estimated from the mean-value theorem, where c_∞ is a constant. Since we are perturbing with a random magnetic field, the system is not driven to a new equilibrium state and consequently, the first term $\rho_\infty=\int_0^\infty d\tau R_{\rm gg}(2\tau)$ vanishes if the initial mean magnetization was taken to be zero. As a result, we have the scaling form, already announced in Ref. [7]

$$\rho_{\rm TRM}(t,s)=s^{-a}f_M(t/s)+s^{-\lambda_R/z}g_M(t/s), \tag{33}$$

where $f_M(x)$ and $g_M(x)$ are scaling functions. These results only depend on the assumption of dynamical scaling. If, in addition, local scale invariance applies, the form of the response function $R(t,s)$ is given by Eq. (4) and therefore [7,9]

$$f_M(x)=r_0 x^{-\lambda_R/z}{}_2F_1\left(1+a,\frac{\lambda_R}{z}-a;\frac{\lambda_R}{z}-a+1;\frac{1}{x}\right),$$

$$g_M(x)\simeq r_1 x^{-\lambda_R/z}, \tag{34}$$

and where $r_{0,1}$ are nonuniversal constants and ${}_2F_1$ is a hypergeometric function [28].

The importance of taking this finite-time correction into account is illustrated in Fig. 4. For a fixed value of $x=t/s$ and $T<T_c$, we plot data for the Glauber-Ising model in 2D and 3D, respectively, and the exact solution of the Langevin equation of the spherical model, with $\Gamma=0$. A fully disordered initial state was used. We compare the data with the leading scaling form $\rho(t,s)\sim s^{-a}$ which for the times accessible does not fully describe the data, but inclusion of the second term in Eq. (33) gives a very good fit. From this fit we find the nonuniversal values of $r_{0,1}$ listed in Table II. In order to achieve this, however, a must take the values given in Table I. Specifically, for the Glauber-Ising model with $d\geqslant 2$ (which is in class S), we must take $a=1/2$, while if we had chosen $a=1/4$ as advocated by CLZ, only a fit of very low quality is obtained, see Refs. [7,30]. This provides further evidence against the proposed Eq. (10). We point out

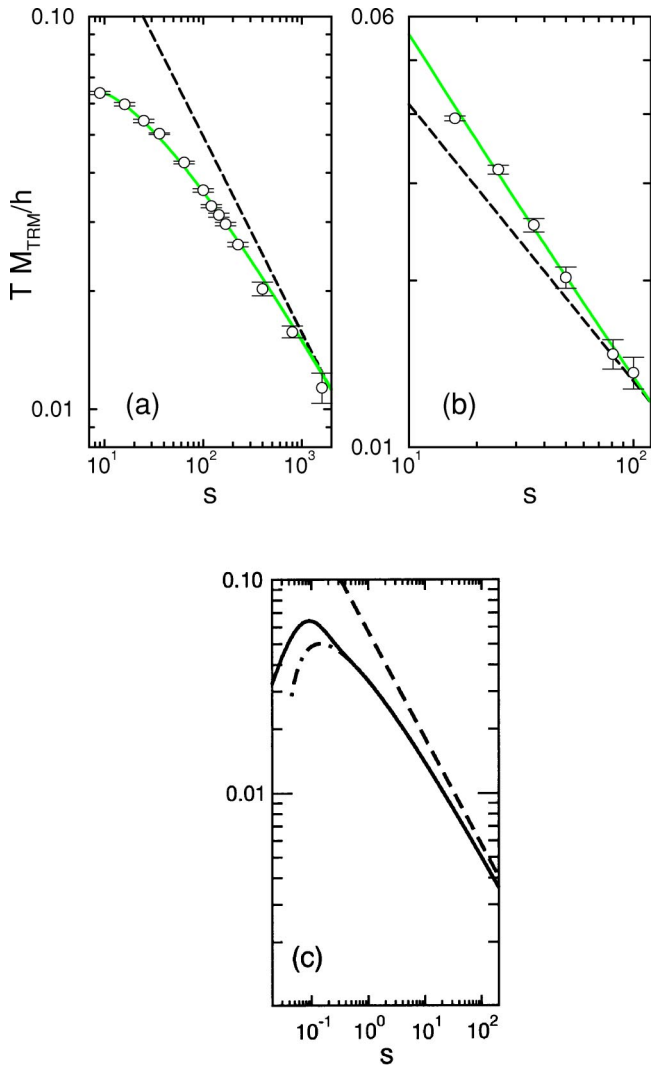

FIG. 4. Scaling of the thermoremanent susceptibility $\rho(t,s) = M_{\mathrm{TRM}}(t,s)/h$ as a function of the waiting time s for a fixed value of $x=t/s$ for three models. (a) The 2D Glauber-Ising model at $T=1.5$ and with $x=7$ (points) is compared with the scaling prediction, Eq. (33) (full gray curve), and with the simple scaling form $\rho(t,s) \sim s^{-1/2}$ (dashed curve). (b) Same comparison for the 3D Glauber-Ising model at $T=3$ and with $x=5$. (c) The 3D spherical model at $T=2$ and $x=5$. The full curve is the exact solution of the Langevin equation, the dash-dotted line the scaling prediction [Eq. (33)], and the dashed line the simple form $\rho(t,s) \sim s^{-1/2}$.

that having fixed $r_{0,1}$ for a given value of x, the scaling of $\rho_{\mathrm{TRM}}(t,s)$ is completely fixed and does not contain any further tunable parameter. Repeating the comparison between the data and Eq. (33) for other values of x therefore allows to test the predictions of local scale invariance and this has been carried out in detail for the 2D and 3D Glauber-Ising model [7,11]. While in the 2D Glauber-Ising model the curvature of the data, see Fig. 4(a), already suggests the presence of strong finite-time corrections to scaling, in 3D it might appear at first sight that a reliable value of the exponent a could be derived, see Fig. 4(b). However, the inclusion of the leading correction term from Eq. (33) together with the study of the x dependence of the coefficients, as carried out in Ref. [7], is required in order to avoid a systematic error in the determination of a. In this way the expected value $a=1/z=1/2$ is reconfirmed. Carrying out the analysis of the scaling functions as described in Ref. [7], we can conclude that local scale invariance holds in the 2D and 3D Glauber-Ising model with $T<T_c$.

TABLE II. Values of the autoresponse exponent λ_R and of the parameters r_0 and r_1 in the Glauber-Ising model in two dimensions at $T=1.5$ and in three dimensions at $T=3$ and of the mean spherical model at $T=2$ in several dimensions. An infinite-temperature initial state was used. The dynamic exponent $z=2$.

Model	d	λ_R	r_0	r_1
Glauber-Ising	2	1.26	1.76 ± 0.03	-1.84 ± 0.03
	3	1.60	0.10 ± 0.01	0.20 ± 0.01
Spherical	3	1.50	0.180 ± 0.01	-0.081 ± 0.002
	3.5	1.75	0.20 ± 0.01	-0.126 ± 0.003
	4.5	2.25	-0.056 ± 0.003	0.095 ± 0.002

A closer inspection of Table II shows that in the spherical model with $d>4$ and $T<T_c$, the signs of r_0 and r_1 change and that the term which was treated as a correction in Eq. (33) becomes dominant. Indeed, the exactly known exponents $a=(d-2)/2$ and $\lambda_R/z=d/4$ cross at the upper critical dimension $d^*=4$. We conclude that the scaling argument used in Ref. [7] to derive the relation $a=(d-2+\eta)/z$ for class L systems should rather be viewed as a hyperscaling argument.

V. CONCLUSIONS

In this paper, we have analyzed the long-time behavior of integrated response functions in aging spin systems without disorder. While from the scaling form (2), one might have expected a simple scaling $\chi(t,s) \sim s^{-a}$ of the integrated response, we have shown that matters are more complicated. Conceptually, the issue can be clarified by studying the scaling of the *field-cooled* susceptibility $\chi_{\mathrm{FC}}(t)$ and we have seen that two broad classes of systems must be distinguished, called classes S and L, according to whether their equilibrium spin-spin correlator shows short-ranged or long-ranged spatial decay, respectively [7]. Specifically, we have found the following.

(1) For systems of class S, we have

$$\chi_{\mathrm{FC}}(t) - \chi_0 \sim t^{-A}, \tag{35}$$

where $T\chi_0 = 1 - m_{\mathrm{eq}}^2$ and m_{eq} is the equilibrium magnetization. The exponent A is related to the interface width exponent κ. If $w(t) \sim t^{\kappa}$, we have seen that

$$A = \frac{1}{z} - \kappa \quad (\text{class } S), \tag{36}$$

but provided that the temperature $T>T_R$ is above the roughening temperature T_R.

(2) For systems of class L, Eq. (35) still holds with $\chi_0 = 0$, but with

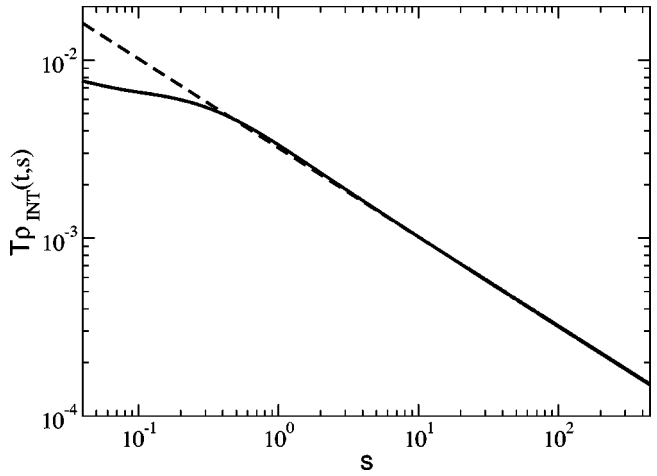

FIG. 5. Integrated linear response $\rho_{\mathrm{Int}}(t,s)$ according to the intermediate protocol for the 3D mean spherical model with $T=2$, an infinite-temperature initial state, and $x=20$. The full curve is the exact solution, the dashed line is a power-law fit $\sim s^{-0.5}$.

$$A=0 \quad (\text{class } L). \tag{37}$$

(3) The scaling of the thermoremanent susceptibility $\rho_{\mathrm{TRM}}(t,s)$ may be affected by a relatively large finite-time correction term, see Eq. (33) where the associated scaling functions can be found explicitly from local scale invariance.

(4) Consequently, the leading time dependence of the field-cooled susceptiblity $\chi_{\mathrm{ZFC}}(t,s)-\chi_0 \sim \chi_{\mathrm{FC}}(t)-\chi_0 \sim t^{-A}$, whereas the aging terms expected from integrating Eq. (2) merely arise as finite-time corrections.

Our results reproduce the entirety of known results in aging ferromagnetic spin systems with $d \geqslant 2$ but do not require to postulate an upper critical dimension $d=3$ in the Glauber-Ising model. For systems of class S, the exponent relation $a=1/z$ is reconfirmed while the proposed Eq. (10) is invalidated.

If questions of simulational efficiency play no role, it might be technically easier to avoid both the zero-field cooled (ZFC) and the thermoremanent (TRM) protocol, as already suggested in Ref. [18]. For example, we propose the "intermediate" protocol, which runs as follows: quench the system at $t=0$ without a magnetic field and fix a waiting time s. At time $s/2$, turn on a random magnetic field and keep it on until the waiting time s. Then turn the field off again and measure the magnetization at the observation time $t>s$. The *intermediate integrated response* is

$$\rho_{\mathrm{Int}}(t,s) = M_{\mathrm{Int}}(t,s)/h: = \int_{s/2}^{s} du R(t,u)$$
$$= s^{-a} f_{\mathrm{Int}}(t/s)[1+o(s^{-\lambda_R/z})] \tag{38}$$

and should be free of the leading term coming from the interface roughness as well as the finite-time correction of order $O(s^{-\lambda_R/z})$. We illustrate this for the mean spherical model in Fig. 5, where it can be seen that already for times much shorter than those in Fig. 4(c), the *linear* response (28) obtained from the exact solution of the Langevin equation (with $\Gamma=0$) converges to the expected power law, with $a=0.5$ in 3D. It would be interesting to see if a recent method to calculate $\chi_{\mathrm{ZFC}}(t,s)$ directly in the Glauber-Ising model [31] [which in turn is based on a method to estimate $R(t,s)$ in an Ising model with a different dynamics modified from Glauber dynamics [32]] could be generalized to find $\rho_{\mathrm{Int}}(t,s)$ as well.

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APPENDIX

In the main text, we studied the time-dependent scaling of the integrated response $\chi_{\mathrm{FC}}(t)$. We shall now attempt to extract further information on the scaling of $R(t,s)$ when $t \approx s$. Note that the *scaling* form Eq. (2) cannot be used close to the upper integration limit, since there the condition $\tau=t-s \gg 1$ needed for the validity of Eq. (2) does not hold. Rather, one might argue that there should be some time scale t^* such that Eq. (2) holds for waiting times $s \lesssim t^*$. If in addition $t-t^*$ is some "small" constant Δt^* for large times, we may write

$$\chi_{\mathrm{FC}}(t) = \int_0^{t^*} ds R(t,s) + \int_{t^*}^{t} ds R(t,s)$$
$$\simeq t^{-a} \int_{t/t^*}^{\infty} dv v^{a-1} f_R(v) + (t-t^*) R(t,t')$$
$$\simeq t^{-a} \int_1^{\infty} dv\ v^{a-1} f_R(v) + \Delta t^* R(t,t'), \tag{A1}$$

where we used the mean-value theorem and $t' \in [t^*, t]$. If the second term in Eq. (A1) is the leading one, then comparison with the scaling of χ_{FC} gives $R(t,t) \simeq 1+O(t^{-A})$, provided the conditions stated above are satisfied. Here the value of the constant is fixed from the physical consideration that for $\tau=t-s \ll 1$ the system should still be in quasiequilibrium.

We can now try and see to what extent this argument applies in specific models and to what extent it might be justified. The only model of class S we studied here is the 2D Glauber-Ising model at $T<T_c$. Then $a=1/2$ and $A=1/4$, and the contributions close to the upper integration limit are indeed dominant. One should therefore expect $\lim_{\varepsilon \to 0} R(t, t-\varepsilon) \simeq \mathrm{cste.} + O(t^{-1/4})$ for large t.

On the other hand, we have considered in Sec. III several examples of systems of class L, where we have found $A=0$ throughout. First, for the spherical model, the exact expression for $R(t,s)$ gives $R(t,t-2\varepsilon)=f(\varepsilon)\sqrt{g(t-2\varepsilon)/g(t)} \simeq 1$

$+\varepsilon[f'(0)-g'(t)/g(t)]+O(\varepsilon^2)$. Since for large t one has $g(t) \sim t^{\psi}$ with a known exponent ψ [6,12], we obtain for $\varepsilon \to 0$ $R(t,t-2\varepsilon) \simeq \text{cste.} + O(t^{-1})$, and have indeed $A=0$, since the constant is different from unity. Second, a similar conclusion should also hold true for the 2D Glauber-Ising model at criticality. Third, an analogous result can also be obtained in the 2D linear voter model at criticality [33], where the exact response function $\lim_{\varepsilon \to 0} R(t+\varepsilon,t) \sim (\ln t)^{-1}$.

Additional studies to test these conclusions further would be welcome.

The heuristic arguments presented in this paper assume $d \geqslant 2$ space dimensions throughout. As an illustration what can happen in one dimension, we now consider the 1D Glauber-Ising model at its critical temperature $T_c=0$. This model has been investigated countless times and we shall not repeat the details of its definition (see, e.g., Refs. [34,35] and references therein) here. The exact autoresponse function is

$$R(t,s) = e^{-(t-s)} I_0(t-s)[1 - C_1(s)], \tag{A2}$$

where $C_1(s)$ is the equal-time correlator of two spins on neighboring sites. For an infinite-temperature initial state, one has [35]

$$\begin{aligned} C_1(s) &= e^{-2s}\left[I_1(2s) + 2\sum_{n=1}^{\infty} I_{n+1}(2s) \right] \\ &= 1 - e^{-2s}[I_0(2s) + I_1(2s)], \end{aligned} \tag{A3}$$

where in the second step the Bessel-function identity $e^z = I_0(z) + 2\Sigma_{n=1}^{\infty} I_n(z)$ was used. It has recently been shown that for any initial correlators of the power-law form $C_{\text{ini}}(r) \sim r^{-\nu}$ with $\nu \geqslant 0$, the leading scaling behavior of $C_1(s)$ for s large is unchanged [35]. Therefore, for any power-law correlated initial state we have for large t

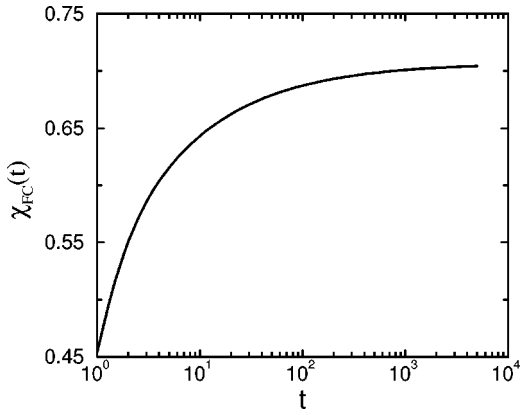

FIG. 6. Field-cooled susceptibility $\chi_{\text{FC}}(t)$ for the 1D Glauber-Ising model at $T=0$.

$$\lim_{\varepsilon \to 0} R(t,t-\varepsilon) \simeq \frac{1}{\sqrt{\pi}} t^{-1/2}. \tag{A4}$$

Comparison of Eqs. (A1) and (A4), if admissible, would then give $A=1/2$. Such a result would appear to be natural in the setting of Sec. II if we recall that at $T=0$ there are fully ordered domains where the width of the domain walls is one lattice constant, thus $w(t)=\text{cste}$.

On the other hand, integrating Eqs. (A2) and (A3), the field-cooled susceptibility becomes for an uncorrelated initial state

$$\chi_{\text{FC}}(t) = \int_0^t ds\ e^{-t-s} I_0(t-s)[I_0(2s) + I_1(2s)], \tag{A5}$$

and we show this in Fig. 6. Clearly, there is saturation for large times and in the spirit of Sec. III one would conclude $A=0$. Therefore, a naive application of the arguments valid for $d \geqslant 2$ would lead to two different values of A in this 1D model. We leave open the question how this apparent inconsistency might be resolved.

[1] See the reviews by J. P. Bouchaud and A. J. Bray, in *Soft and Fragile Matter*, edited by M. E. Cates and M. R. Evans (IOP Press, Bristol, 2000).

[2] C. Godrèche and J.-M. Luck, J. Phys.: Condens. Matter **14**, 1589 (2002).

[3] L. F. Cugliandolo, in *Slow Relaxation and Non Equilibrium Dynamics in Condensed Matter*, Les Houches, Lectures, edited by J.-L. Barrat, J. Dalibard, J. Kurchan, and M. V. Feigel'man (Springer, Heidelberg, 2003).

[4] D. S. Fisher and D. A. Huse, Phys. Rev. B **38**, 373 (1988).

[5] D. A. Huse, Phys. Rev. B **40**, 304 (1989).

[6] A. Picone and M. Henkel, J. Phys. A **35**, 5575 (2002).

[7] M. Henkel, M. Paeßens, and M. Pleimling, Europhys. Lett. **62**, 664 (2003).

[8] L. Berthier, J. L. Barrat, and J. Kurchan, Eur. Phys. J. B **11**, 635 (1999).

[9] M. Henkel, Nucl. Phys. B **641**, 405 (2002).

[10] M. Henkel, M. Pleimling, C. Godrèche, and J.-M. Luck, Phys. Rev. Lett. **87**, 265701 (2001).

[11] M. Henkel and M. Pleimling, Phys. Rev. E **68**, 065101(R) (2003).

[12] C. Godrèche and J. M. Luck, J. Phys. A **33**, 9141 (2000).

[13] S. A. Cannas, D. A. Stariolo, and F. A. Tamarit, Physica A **294**, 362 (2001).

[14] M. Henkel, A. Picone, M. Pleimling, and J. Unterberger, e-print cond-mat/0307649.

[15] M. Henkel and J. Unterberger, Nucl. Phys. B **660**, 407 (2003).

[16] A. Barrat, Phys. Rev. E **57**, 3629 (1998).

[17] F. Corberi, E. Lippiello, and M. Zannetti, Eur. Phys. J. B **24**, 359 (2001); Phys. Rev. E **65**, 046136 (2002); Phys. Rev. Lett. **90**, 099601 (2003).

[18] F. Corberi, E. Lippiello, and M. Zannetti, Phys. Rev. E **68**, 046131 (2003).

[19] C. Yeung, M. Rao, and R. C. Desai, Phys. Rev. E **53**, 3073 (1996).

[20] R. J. Baxter, *Exact Solved Model in Statistical Mechanic* (Aca-

demic Press, London, 1982), Chap. 7.
[21] D. B. Abraham and P. J. Upton, Phys. Rev. B **39**, 736 (1989).
[22] J. Villain, in *Structures et Instabilités*, edited by C. Godrèche (Les editions de physique, Paris, 1986).
[23] D. B. Abraham, Phys. Rev. Lett. **47**, 545 (1981).
[24] L. F. Cugliandolo and D. S. Dean, J. Phys. A **28**, L453 (1995).
[25] M. Paeßens and M. Henkel, J. Phys. A **36**, 8983 (2003).
[26] L. Berthier, L. F. Cugliandolo, and J. L. Iguain, Phys. Rev. E **63**, 051302 (2001).
[27] W. Zippold, R. Kühn, and H. Horner, Eur. Phys. J. B **13**, 531 (2000).
[28] CLZ also attempt to calculate $f_M(x)$, but in their calculation they take the large-x limit $f_R(x) \sim x^{-\lambda_R/z}$ before integrating and approximate the small-x behavior of $f_M(x)$ by a constant [see Eq. (83) in Ref. [18]]. This is too rough an approximation to be useful.
[29] K. Humayun and A. J. Bray, J. Phys. A **24**, 1915 (1991).
[30] M. Henkel and M. Pleimling, Phys. Rev. Lett. **90**, 099602 (2003).
[31] F. Ricci-Tersenghi, Phys. Rev. E **68**, 065104(R) (2003).
[32] C. Chatelain, J. Phys. A **36**, 10739 (2003).
[33] F. Sastre, I. Dornic, and H. Chaté, Phys. Rev. Lett. **91**, 267205 (2003).
[34] C. Godrèche and J. M. Luck, J. Phys. A **33**, 1151 (2000).
[35] M. Henkel and G. M. Schütz, J. Phys. A **37**, 591 (2004).