Phenomenological analysis of K^+-meson production in proton-nucleus collisions

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Total and differential cross sections from literature, on the production of K^+-mesons in pA interactions at projectile energies between $T_p = 0.8$ and 2.9 GeV, covering the transition across the free nucleon-nucleon threshold at 1.58 GeV, have been investigated. From the target-mass dependence of the production cross sections no evidence for the expected change of the dominant reaction mechanism from two step to direct kaon production was found. At $T_p = 1.0$ GeV the A dependences of the total cross sections and of the most recent data from COSY-Jülich, differential cross sections measured under forward angles, are strongly different. The invariant K^+-production cross sections show an overall exponential scaling behavior with the squared four-momentum transfer between the beam proton and the produced K^+ meson for $t < -0.05$ GeV^2, independent of the beam energy and emission angle. The data from COSY-Jülich reveal a strongly different t dependence in the region of $t > 0$ GeV^2. Further data at forward angles and different beam energies should be taken in order to explore this region of kinematically extreme conditions.

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I. INTRODUCTION

The study of subthreshold and nearthreshold proton-induced production of K^+ mesons in nuclei has received considerable interest during the last two decades. Due to the rather high K^+-production threshold in free nucleon-nucleon collisions ($T_{NN} = 1.58$ GeV) and to the large mean free path of K^+ mesons in nuclear matter, one hopes to extract information about the intrinsic properties of the target nuclei and the inmedium properties of kaons. Obviously, in order to obtain this information, it is essential to determine the K^+-production mechanisms.

Total K^+-production cross sections have been measured at the Petersburg Nuclear Physics Institute (PNPI) synchrocyclotron for targets between Be and Pb and projectile energies T_p ranging from 0.8 to 1.0 GeV [1], i.e., at beam energies far below the free nucleon-nucleon threshold. Inclusive differential cross sections have been studied at BEVALAC (Berkeley) [2], SATURNE (Saclay, France) [3] and CELSIUS (Uppsala, Sweeden) [4]. Partial momentum spectra have been obtained at different laboratory emission angles in the range 10°–90°, at projectile energies between 1.2 and 2.1 GeV. Recently, full momentum spectra at forward emission angles $\theta_K \leqslant 12°$ have been measured with the ANKE spectrometer [5] of COSY-Jülich at $T_p = 1.0$ GeV [6]. K^+ production has also been studied at Institute for Theoretical and Experimental Physics (ITEP) [7,8], where kaons with fixed momenta and an emission angle of 10.5° were identified at projectile energies of 1.75 to 2.9 GeV.

The results were discussed in terms of different models [1,9–12], in particular, of single-step or two-step reactions, the latter with the creation of an intermediate pion. It has frequently been argued that the A dependence of the K^+-production cross section is sensitive to the dominant production mechanism. In a naive model for direct K^+ production the A dependence is given by the inelastic scattering cross section of the incident proton that can be parameterized as A^α with $\alpha \sim 0.6-0.7$ at proton energies ranging from 1 to 100 GeV [13–15]. For an opaque nucleus α approaches 2/3 due to absorption of the incident proton on the nuclear surface.

In Sec. II a systematic analysis of the A dependence of the existing data is presented. It is investigated whether conclusions about the reaction mechanisms can be drawn and whether the A dependence of K^+-meson production depends on the reaction kinematics, i.e., beam energy as well as kaon emission angle and momentum. Further measurements at forward kaon emission angles are motivated. In Sec. III data taken at the same beam energy but for different kaon momenta and emission angles are compared by analyzing them as a function of the four-momentum transfer t between the beam proton and the produced K^+ meson. Such cross checks will also be necessary for the comparison and interpretation of data to be obtained with ANKE (as motivated in Sec. II) and the already existing data at larger angles [2–4]. The choice of t as the kinematical variable defining the reference system allows a convenient classification of the experimental data: the maximum positive value of t corresponds to the absolute K^+-production threshold in the coherent reaction $pA \to K^+ {}_YA$ with formation of a hypernucleus in the final state. Smaller values of t indicate projectile fragmentation, whereas large negative values correspond to partial fragmentation of the target nucleus.

II. TARGET-MASS DEPENDENCE

The first step of the analysis is to determine the exponent α by fitting the measured $pA \to K^+X$ cross sections for different target nuclei with an A^α dependence. Available data on K^+ production in pA collisions at beam energies $T_p \leqslant 2.9$ GeV have been analyzed with the exception of Refs. [4],[8] since only a single target material was used in these measurements. A similar analysis of the A dependence for meson production at high energies can be found in Refs. [16–18].

The A dependence can be factorized in terms of the production process and of the kaon propagation and distortion in the nuclear medium. Kaons with relatively low momenta ($p_K \leqslant 1.3$ GeV/c), are not absorbed in nuclear matter after their creation and thus carry the information about the production mechanism. Also quasielastic scattering of K^+ mesons in nuclei is substantially suppressed leading to a large mean free path of $\lambda_{K^+} \simeq 5$ fm.

The *A* dependence for direct kaon production in collisions of the beam proton with a single target nucleon is given by the inelasticity of the *pA* interaction. This also holds for the production of other mesons, since the summation over all possible mesonic inelastic *pA* reaction channels is proportional to the total inelastic *pA* cross section. The data [13,14] on total inelastic proton-nucleus cross sections can be well fitted by an A^α dependence with $\alpha = 0.69 \pm 0.03$ for proton beam energies between 0.84 and 2 GeV. Modifications of the *A* dependence can be due to the intrinsic momenta of the participating nucleons, to Pauli blocking and other nuclear effects, which essentially should not depend on the target mass [19]. Therefore, one expects that the K^+-production cross section is proportional to $\simeq A^{0.7}$ if the kaons are dominantly produced via the direct mechanism.

K^+ mesons can also be produced in two-step processes with intermediate pion production ($pN_1 \to \pi X$) and subsequent $\pi N_2 \to K^+ X$ reactions on a second target nucleon. Since two nucleons are needed in this case, a stronger *A* dependence is expected. Depending on the beam energy, K^+ production may be due to both, the direct and two-step reaction mechanisms. At low beam energies two-step processes are energetically favored since the intrinsic nucleon motion can be utilized twice.

It has also been suggested that, in particular, at deep subthreshold energies, K^+ production is due to many-body interactions, e.g., the formation of clusters in the target nucleus [10], or is a reflection of a high degree of collectivity in the target nucleus [6]. Such effects are proportional to the number of target nucleons and, therefore, the cross section should scale as $\simeq A^1$.

Total K^+-production cross sections $\sigma_{\rm tot}$ in proton-nucleus collisions at $T_p \leqslant 1.0$ GeV for Be, C, Cu, Sn, and Pb as target nuclei have been measured by Koptev *et al.* [1] at the PNPI. The threshold for K^+ production in free *pN* collisions is $T_p = 1.58$ GeV and, thus, the PNPI measurements were devoted to the study of deep-subthreshold strangeness-production mechanisms. At each beam energy $\sigma_{\rm tot}$ has been fitted by a function $\sigma(A) = \text{const} \cdot A^\alpha$. Figure 1 shows the resulting parameter α as a function of T_p. In the full energy range $0.842 \leqslant T_p \leqslant 0.990$ GeV, the data can be well described by a constant value $\alpha = 1.04 \pm 0.01$ as indicated by the solid line and listed in the first line of Table I. The strong *A* dependence of the total K^+-production cross section has been interpreted [1,9,11] as an indication for the dominance of nondirect K^+-meson production in *pA* collisions at energies far below the free nucleon-nucleon threshold.

To obtain more detailed information about the strangeness-production mechanisms, differential K^+-production cross sections were subsequently measured by several groups. Kaon production induced by 2.1 GeV protons (i.e., above the free *NN* threshold) on NaF and Pb targets has been studied by Schnetzer *et al.* [2] at the Lawrence Berkeley Laboratory (LBL). The kaons were measured at emission angles of $\theta_K = 15°$, 35°, 60°, and 80° and for momenta in the range 0.350–0.750 GeV/c. We have fitted the mass dependence of the double differential cross sections by $d^2\sigma/dp\, d\Omega(A) = \text{const} \cdot A^\alpha$ and show in Fig. 2 the parameter α as a function of the kaon momentum and for the different production angles. Since no dependence of α on the kaon momenta is observed, we fit $\alpha(p_K)$ by constant values. The results are shown in Fig. 2 by the solid lines and in Table I.

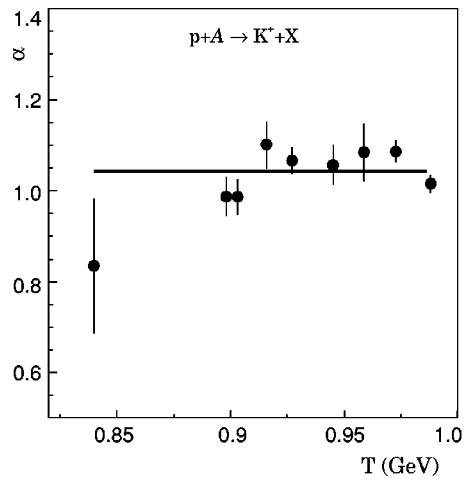

FIG. 1. Dependence of the parameter α on the beam energy T_p evaluated from the PNPI data [1] on total K^+-production cross sections. The solid line indicates the fitted average value $\alpha = 1.04 \pm 0.01$.

The mass dependence is weak ($\alpha = 0.56 \pm 0.05$) for $\theta_K = 15°$, and increases to larger angles ($\alpha = 0.88 \pm 0.08$ for $\theta_K = 80°$). This might be explained by the fact that the direct reaction mechanism dominates at forward laboratory angles due to the Lorentz boost of this production process. It is also possible that the higher values of α, observed at larger angles, are related to a higher transparency of the nucleus in case of particle production with high perpendicular momenta. This phenomenon is well established in many processes and is caused by the small size of the produced particles. In this case, however, it starts at rather low momenta. The dependence of α on the kaon momentum does not indicate a transition from the direct production process to two step or multistep K^+ production. In the full momentum range, the LBL results are consistent with or close to the expectation for direct K^+-meson production and the *A* dependence is weaker than the one of the PNPI data [1].

Double differential cross sections in *p*C collisions at $T_p = 1.2$, 1.5, and 2.5 GeV and in *p*Pb collisions at $T_p = 1.2$ and 1.5 GeV at a production angle $\theta_K = 40°$ have been obtained by Debowski *et al.* [3] at SATURNE. The parameter α evaluated from the data at $T_p = 1.2$ and 1.5 GeV is shown in Fig. 3 as a function of p_K. For both beam energies, the results can be well fitted by a constant value $\alpha = 0.73 \pm 0.04$. In Table I the values for α of the individual fits for 1.2 and 1.5 GeV are given. At 1.2 GeV the large error of α does not allow any conclusions about the reaction mechanisms. At this energy a dominance of two-step kaon produc-

TABLE I. Values for α evaluated from the data on K^+-meson production in pA collisions at various proton-beam energies T_p, kaon momenta p_K, and emission angles θ_K.

T_p (GeV)	p_K (GeV/c)	θ_K (°)	α	Ref.
0.842–0.99	Total	Total	1.04±0.01	PNPI [1]
2.1	0.35–0.75	15	0.56±0.05	LBL [2]
2.1	0.35–0.75	35	0.74±0.05	LBL [2]
2.1	0.35–0.75	60	0.84±0.08	LBL [2]
2.1	0.35–0.75	80	0.88±0.08	LBL [2]
1.2	0.5–0.7	40	0.69±0.3	SATURNE [3]
1.5	0.5–0.7	40	0.73±0.04	SATURNE [3]
1.7–2.91	1.28	10.5	0.54±0.02	ITEP [7]
1.0	0.171–0.507[a]	0–12	0.74±0.05	COSY [6]

[a]Complete momentum spectrum covered.

tion is expected [3,9,11]. At 1.5 GeV the relatively small value of $\alpha=0.73\pm0.04$ is in line with the expected dominance of one-step kaon production [3,9,11].

Double differential cross sections for K^+ production with Be, Al, Cu, and Ta targets were measured by Akindinov *et al.* [7] at the Institute for Theoretical and Experimental Physics, Moscow (ITEP). For $1.65\leqslant T_p\leqslant 2.91$ GeV, kaons with an emission angle $\theta_K=10.5°$ and fixed momentum of $p_K=1.28$ GeV/c were detected. Below $T_p\simeq 2.2$ GeV, such high-momentum kaons cannot be produced in a free NN collision. Thus it is justified to assign these kaons to subthreshold particle production. We have fitted the A^α dependence and show α as a function T_p in Fig. 4. The increase of α at low beam energies is not understood, it maybe either due to statistical fluctuations (indicated by the dashed line) or reflect that below $T_p\simeq 2.2$ GeV there is a transition from direct to two-step K^+ production. Within the experimental uncertainties, the A dependence of the ITEP data is consistent with the LBL and SATURNE results.

The fitted A dependences of kaon production are collected in Table I. The PNPI data on total K^+-production cross sections at beam energies $0.842\leqslant T_p\leqslant 0.990$ GeV indicate a strong, $\simeq A^1$, dependence. The data on differential cross sections available for proton beam energies from 1.2 up to 2.9 GeV yield an A^α dependence with α varying between 0.54 ± 0.02 and 0.88 ± 0.08. Thus, for beam energies $1.2\leqslant T_p\leqslant 2.9$ GeV the average value of $\alpha=0.71\pm0.08$ is in reasonable agreement with the A dependence expected from the direct production mechanism.

The inconsistency between the PNPI [1] data and the other experiments [2,3,7] might be related to the different beam energies. Since the PNPI measurements were performed at $T_p\leqslant 1.0$ GeV, thus substantially below the kaon production threshold in free space, two-step mechanisms or many body effects should be more pronounced. To draw final conclusions one would like to have data on differential cross sections at beam energies close to 1.0 GeV. The available data [2,3,7] indicate that the A dependence does not vary with the K^+ momentum. There is no indication for a transition between direct strangeness production and the two-step mechanism, which should cause a visible change in α.

To clarify the situation, a measurement of K^+-meson pro-

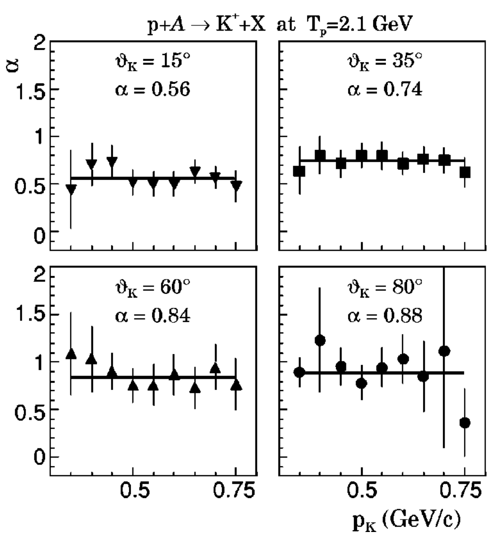

FIG. 2. Exponent α as a function of the laboratory K^+ momentum evaluated from the LBL data [2] on differential K^+-production cross sections. The solid lines show fits by constant values.

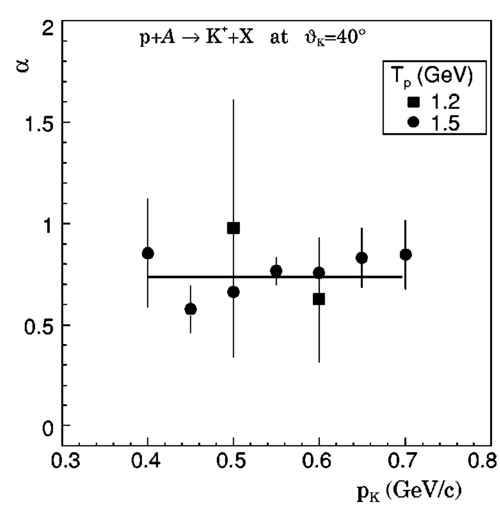

FIG. 3. α as a function of p_K evaluated from the SATURNE data [3] on differential K^+-production cross sections. The different symbols corresponds to different beam energies T_p as indicated in the figure. The solid line shows a fit by $\alpha=0.73\pm0.04$.

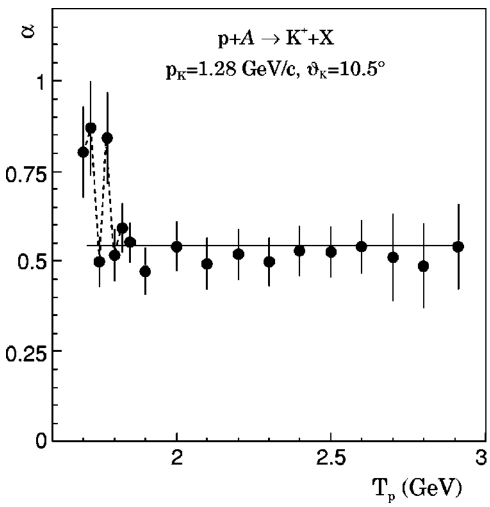

FIG. 4. α as a function of the beam energy T_p evaluated from the ITEP data [7] on differential K^+-meson production cross sections for $p_K = 1.28$ GeV/c and $\theta_K = 10.5°$. The solid line shows a fit by a constant value $\alpha = 0.54 \pm 0.02$. The dashed line only serves to guide the eye.

duction in *pA* collisions at a proton beam energy of 1.0 GeV was performed recently with the ANKE spectrometer [6]. Double differential cross sections were measured at emission angles of $0° \leqslant \theta_K \leqslant 12°$ for C, Cu, and Au targets. The experiment was designed for measurements at forward angles, where the transition between the two-step mechanism and direct K^+ production is expected to be most pronounced [11].

Figure 5 shows the exponent α as a function of the laboratory K^+ momentum evaluated from the ANKE data. The solid line indicates a fit to the data with a constant value $\alpha = 0.74 \pm 0.05$. A comparison with the values for α obtained from the total cross sections [1] ($\alpha = 1.04 \pm 0.01$, see Table I) shows that the ANKE data have a much weaker *A* dependence. The only difference between the PNPI and the ANKE measurements is that at ANKE the K^+ mesons were detected inside the forward cone $0° \leqslant \theta_K \leqslant 12°$, whereas at PNPI angular-integrated spectra were obtained. One can only speculate that at 1.0 GeV, α increases towards larger K^+-emission angles. A similar behavior has been deduced from the LBL data shown in Fig. 2. However, these data were obtained at a significantly higher beam energy where kaon production in single step reactions should always dominate. It would be interesting to check with new data from ANKE whether at 2.1 GeV α is also significantly smaller at angles around 0°.

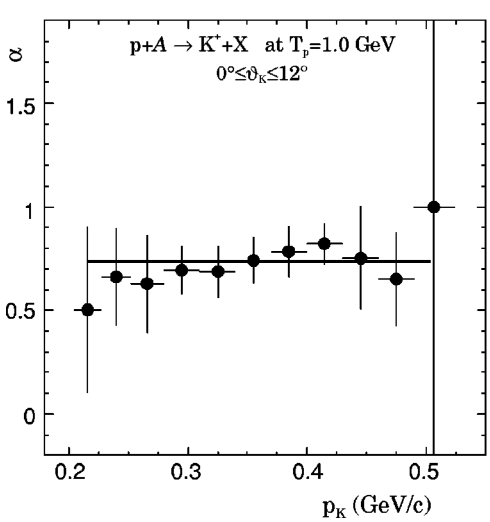

FIG. 5. Dependence of α on p_K evaluated from the ANKE data [6] at $T_p = 1.0$ GeV and $0° \leqslant \theta_K \leqslant 12°$. The solid line shows the fit by a constant value $\alpha = 0.74 \pm 0.05$.

We also find that the momentum-integrated cross section from ANKE for angles $\theta_K \leqslant 12°$ corresponds to roughly 10% of the total cross section, $\sigma_{\text{tot}} = 39$ nb for *p*C collisions at $T_p = 0.990$ GeV [1], whereas the covered solid angle corresponds to less than 1% of 4π [5]. This indicates a strong forward peaking of the produced kaons in the laboratory system.

In contrast to previous studies at higher energies, the ANKE experiment covers the full momentum range of the produced kaons at 1.0 GeV [6]. Figure 5 illustrates that the data do not show a dependence of α on p_K. Thus there is no indication for a change of the *A* dependence due to a transition from the direct to the two-step production mechanism. We cannot provide any conclusive explanation for this observation and for the low values of α, rather than that for the forward subthreshold K^+ production the direct mechanism appears to be dominant. This is in strong contrast to results of model calculations [9–12] where most of the kaons are produced in the two-step mechanism.

The results collected in Table I do not allow an unambiguous conclusion about the reaction mechanisms. There is a strong disagreement between the 1.0 GeV data from ANKE [6] measured under forward angles and the total cross sections from PNPI [1] obtained at the same beam energy. Since at this energy the total cross section is dominated by K^+ production at large angles (90% of the kaons are produced at angles $\theta_K > 12°$), a qualitative explanation of this discrepancy can be obtained if one assumes that in the subthreshold domain the large angle K^+ production is due to reactions of the projectile protons with more than a single nucleon. The *A* dependence of the ANKE data at small angles indicates direct production of K^+ mesons through the interaction of incident protons with surface nucleons. There is no clear indication of two-step production as in the case of the PNPI data.

In order to clarify the situation and to allow a comparison with the data on differential cross sections obtained at higher energies [2,3,7] it is necessary to systematically measure kaon production at forward angles and at higher energies. These data would also help to reconstruct the angular dependences of kaon production in *pA* collisions at the higher energies.

III. SYSTEMATICS OF KAON SPECTRA

At high energies data on hadron production in *pA* collisions are generally analyzed [16–18] in terms of the Feynman variable x_F and the transverse momentum p_t of the produced particle. The Feynman-scaling variable is defined as

$x_F = p_1^*/p_{\max}^*$, where p_1^* is the longitudinal momentum of the produced particle in the center-of-mass system (CMS) of the incident proton and the target nucleon, while $p_{\max}^*$ is the maximum CMS hadron momentum at a given beam energy. However, one cannot apply the commonly adopted x_F analysis of the K^+ data at energies below the free *NN* threshold, since a *pN* CMS with the nucleon at rest is kinematically not allowed. One may select an overall *pA* CMS [4], however then the joint analysis of subthreshold kaon production and the data available above the *NN* threshold becomes rather questionable.

In order to compare kaon spectra measured at different kinematical conditions, i.e., proton beam energies T_p as well as the kaon momenta p_K and emission angles θ_K, we propose a more natural kinematical variable given by the squared four-momentum transfer t between the produced kaon and the incident proton

$$t = m_p^2 + m_K^2 - 2\sqrt{(p_p^2+m_p^2)(p_K^2+m_K^2)} + 2p_p p_K \cos\theta_K, \tag{3.1}$$

where m_p, m_K, p_p, and p_K are the proton and kaon masses and laboratory momenta, respectively, while θ_K denotes the K^+-meson emission angle measured in the laboratory system relatively to the direction of the proton beam. Since t is Lorentz invariant the analysis becomes independent of the choice of the reference CMS, *pN* or *pA*.

A large four-momentum transfer from the incident proton to the target (corresponding to large *negative* values of t) induces excitation and disintegration of the target nucleus. Reactions with small $|t|$ are those where the produced K^+ mesons carry away a substantial part of the momentum and energy of the incident proton. For a $pA \to K^+X$ reaction the minimum and maximum values of t follow from Eq. (3.1) as

$$t_\pm = m_p^2 + m_K^2 - 2\sqrt{(q_p^2+m_p^2)(q_K^2+m_K^2)} \pm 2q_p q_K, \tag{3.2}$$

where the sign of the last term corresponds to kaon production in forward and backward direction relative to the proton-beam momentum in the overall *pA* CMS. q_p and q_K are the proton and K^+-meson three momenta in the *pA* CMS,

$$q_p^2 = \frac{(s - m_p^2 - m_A^2)^2 - 4m_p^2 m_A^2}{4s}$$

$$q_K^2 = \frac{(s - m_K^2 - m_X^2)^2 - 4m_K^2 m_X^2}{4s}, \tag{3.3}$$

where m_A denotes the target mass and s the total invariant energy of the beam proton with T_p and the target

$$s = m_p^2 + m_A^2 + 2m_A(m_p + T_p). \tag{3.4}$$

In Eq. (3.3) m_X represents the invariant energy of the residual final system with respect to the detected kaon. The minimum and maximum values of m_X are given as

$$m_X^{\min} = m_A + m_\Lambda$$

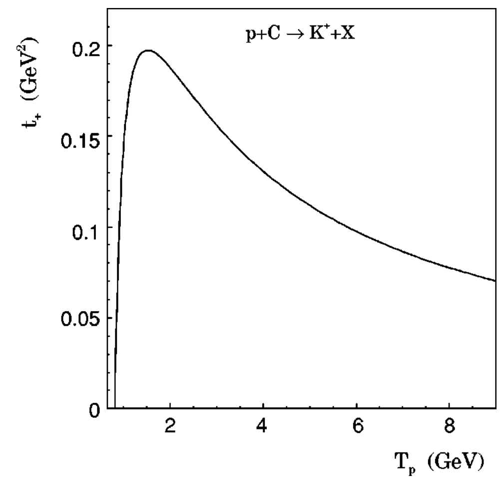

FIG. 6. Values for the maximum positive t_+ for $p^{12}\mathrm{C} \to K^+X$ reactions as a function of the beam energy T_p. t_+ has been calculated for a minimum mass of the residual system $m_X = m_A + m_\Lambda$, neglecting binding energy of the Λ hyperon in the target nucleus.

$$m_X^{\max} = \sqrt{s} - m_K, \tag{3.5}$$

where m_Λ is the Λ-hyperson mass. By the definition of $m_X^{\min}$, we neglect the Λ binding energy, i.e., formation of Λ hypernuclei. $m_X^{\max}$ corresponds to K^+-meson production with transfer of all available collision energy to the invariant mass of the residual system X leading to $q_K = 0$. The maximum and minimum values of t are given by Eq. (3.2) with minimum mass of the residual system $m_X^{\min}$. The maximum positive squared four-momentum transfer t_+ depends both on the beam energy T_p as well as the target mass m_A. Figure 6 shows the T_p dependence of t_+ calculated for a carbon target and $m_X^{\min}$. It is seen that t_+ can be positive and has its maximum at small beam energies around $T_p \simeq 1.5$ GeV. It is of specific interest to check whether processes with such positive values of t_+ are experimentally accessible.

The $pA \to K^+X$ reaction amplitude $\mathcal{A}$ is a function of the invariant collision energy s, the squared four momentum transfer t, and the squared invariant mass m_X^2 of the residual system. Obviously, these Mandelstam variables can be expressed in terms of the laboratory observables as the K^+-emission angle θ_K and momentum p_K. Following Regge theory we factorize the s dependence of the reaction amplitude as

$$\mathcal{A}(s,t,m_X^2) \propto f(t,m_X^2)\exp[\gamma(t)\ln(s/s_0)], \tag{3.6}$$

where functions f and γ are given by the Regge pole with quantum numbers of a strange baryon like a Λ or Σ hyperon and s_0 characterizes the domain of validity of the Regge-pole theory. Assuming a linear t dependence of the Regge trajectory one can expand $\gamma(t) = \gamma_0 + \gamma_1 t$, with γ_0 and γ_1 being constant.

Within the beam-energy range $1.0 \leqslant T_p \leqslant 2.9$ GeV, $\ln(s/s_0)$ at $s_0 = 1\ \mathrm{GeV}^2$ varies between 5.14 and 5.37 and, therefore, the reaction amplitude can be considered as a function of t and m_X^2 only. In the following, we investigate whether the t

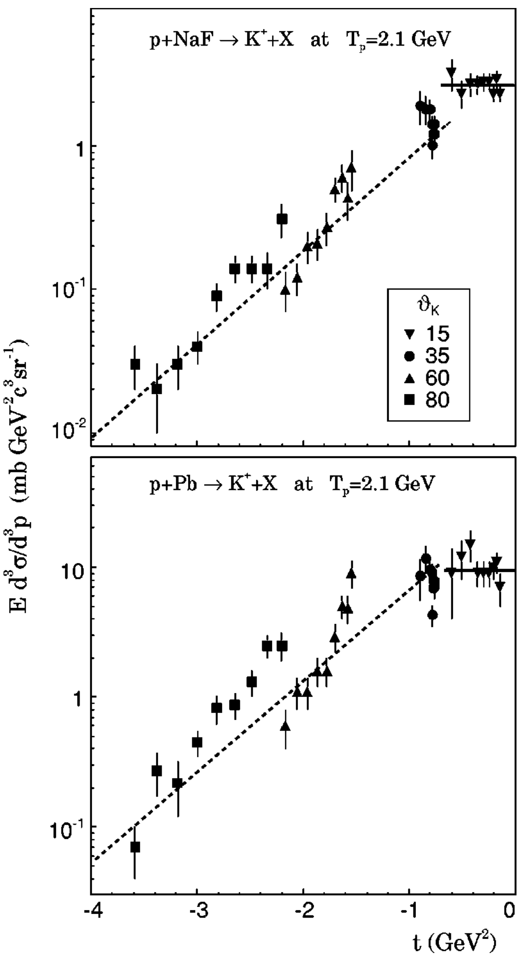

FIG. 7. t dependence of the invariant cross section for NaF (upper) and Pb (lower) targets at $T_p = 2.1$ GeV [2]. The lines show a fit to the data by constant values (solid) and by Eq. (3.7) (dashed) with parameters listed in Table II.

dependence can be factorized out of both the s and m_X^2 dependences for the $pA \to K^+X$ data. According to Eq. (3.6) we describe the invariant K^+-production cross section as

$$E\frac{d^3\sigma}{d^3p} = c_0 \exp[b_0 t], \tag{3.7}$$

with parameters c_0 and b_0 to be fitted to the data. Finally, we intend to investigate the dependence of the slope b_0 on the beam energy T_p and the kaon emission angle θ_K, which both are related to s and m_X.

Figure 7 shows the invariant cross section $Ed^3\sigma/d^3p$ for K^+ production in p(NaF) collisions at $T_p = 2.1$ GeV [2]. The measurements cover a t range from -0.15 to -3.6 GeV^2. For all measured angles and $t \leqslant -0.7$ GeV^2 the invariant cross section can be well fitted by Eq. (3.7) with parameters c_0 and b_0 listed in Table II. At $-0.7 \leqslant t \leqslant -0.15$ the invariant cross section can be reasonably fitted by a constant value c_1, see Fig. 7 and Table II. The exponential dependence of the invariant production cross section is not surprising and quite typical for hadronic reactions. However, the constant t dependence at low squared four-momentum transfer (small kaon emission angles) seems to be unusual and needs further clarification.

The t dependence of the invariant cross section for K^+ production in pPb collisions at $T_p = 2.1$ GeV [2] is shown in the lower spectrum of Fig. 7. Again, the data show an overall exponential scaling behavior in t and can be reasonably described by Eq. (3.7) at $t \leqslant -0.7$ GeV^2, while they are almost constant at small t. The slope b_0 obtained from the pPb data is close to that fitted to the p(NaF) data, see Table II.

The t dependence of the invariant cross section for K^+-meson production in pC collisions at $T_p = 1.2$, 1.5, and 2.5 GeV and in pPb collisions at beam energies of 1.2 and 1.5 GeV [3] is shown in Fig. 8. It is found that the data clearly follow an exponential dependence. The fitted slopes

TABLE II. t dependence evaluated from the data on K^+ production in pA collisions for different proton-beam energies T_p. t_{min} and t_{max} indicate the range of the squared four-momentum transfer measured in the individual experiments. Parameters c_0 and b_0 were fitted with Eq. (3.7) to the data at large $|t|$, while c_1 was evaluated by fitting with a constant value at small $|t|$, see text. c_0 and c_1 are given in units of (mb GeV^{-2} c^{3}sr^{-1}).

T_p (GeV)	A	t_{max} (GeV^2)	t_{min} (GeV^2)	c_0	b_0 (GeV^{-2})	c_1	Ref.
2.1	NaF	-3.58	-0.15	3.7 ± 0.9	1.5 ± 0.1	2.6 ± 0.1	[2]
2.1	Pb	-3.58	-0.15	32 ± 8	1.6 ± 0.1	9.5 ± 0.8	[2]
1.2	C	-0.41	-0.48	$(1.5\pm1.9)\times10^2$	28 ± 3		[3]
1.5	C	-0.59	-0.64	$(9\pm3)\times10^4$	25.5 ± 0.8		[3]
2.5	C	-1.22	-1.65	$(24\pm3)\times10^2$	7.4 ± 0.1		[3]
1.2	Pb	-0.41	-0.45	$(12\pm9)\times10^3$	35 ± 2		[3]
1.5	Pb	-0.58	-0.7	$(55\pm5)\times10^2$	17.3 ± 0.2		[3]
1.2	C	-1.10	-1.25	(53 ± 21)	6.4 ± 0.3		[4]

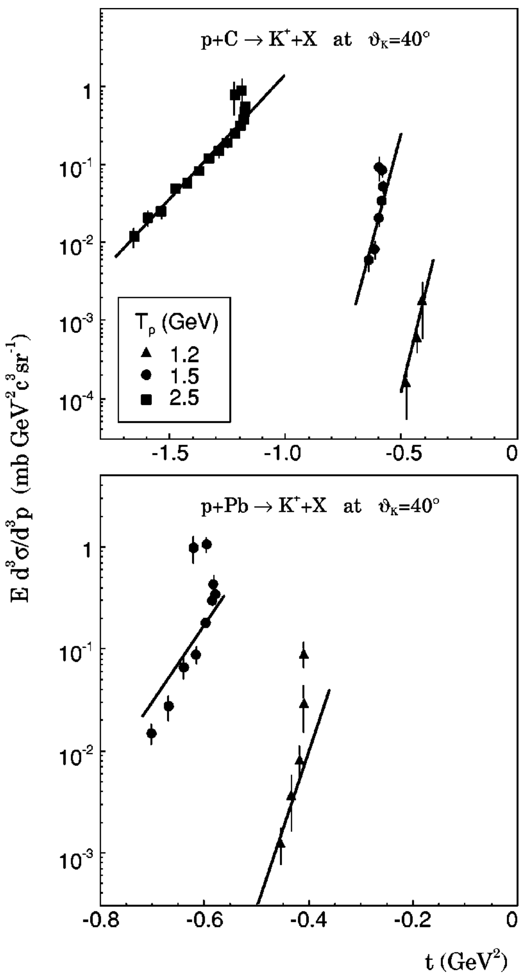

FIG. 8. t dependence of the invariant cross section for pC $\to K^+X$ reactions at $T_p = 1.2$, 1.5, and 2.5 GeV (upper) and pPb $\to K^+X$ reactions at $T_p = 1.2$ and 1.5 GeV (lower). The data were measured at $\theta_K = 40°$ and are taken from [3]. The lines show a fit to the data using Eq. (3.7) with parameters listed in Table II.

b_0, see Table II, substantially depend on the proton-beam energy. At small T_p the t dependence is steeper. However, when comparing the data with those from Ref. [2], the different slopes cannot completely be attributed to the variation of T_p, see first and fifth line of Table II. The slopes for the data from Ref. [2] were obtained by averaging over K^+ emission angles $\theta_K = 15° - 80°$, while the data from Ref. [3] were measured at a fixed emission angle $\theta_K = 40°$. As can be seen from Fig. 7, the slopes for the individual angles can be significantly larger than the averaged values, closer to the values for the data from Ref. [3]. Thus, the slope parameter b_0 is not constant but depends (at fixed s) on the production angle or, equivalently, on m_X and the excitation energy of the residual nucleus. This observation is very interesting since it shows that in inclusive K^+ production, the function $\gamma(t)$ from Eq. (3.6) also depends on m_X and/or $f(t, m_X^2)$ exponentially depends on t. This problem needs further investigation.

Figure 9 shows the t dependence of the invariant K^+-production cross section from pC collisions at $T_p = 1.2$ GeV and $\theta_K = 90°$ [4]. Again, the data can be well fitted by an exponential function with parameters given in Table II.

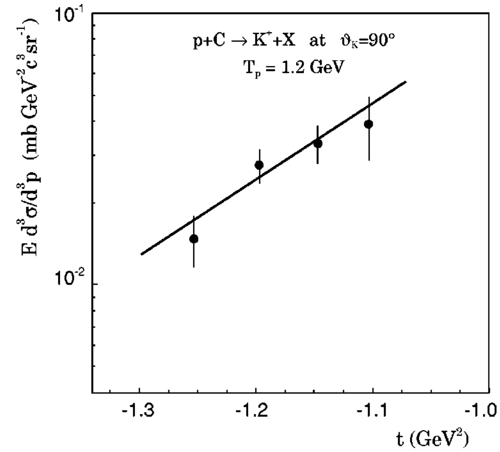

FIG. 9. t dependence of the invariant $p\mathrm{C} \to K^+X$ cross section at $T_p = 1.2$ and $\theta_K = 90°$ [4]. The line shows a fit to the data by Eq. (3.7) with parameters listed in Table II.

We also analyzed the data from Ref. [7], where the T_p dependence of K^+ production in pA collisions was studied for fixed kaon momentum $p_K = 1.28$ GeV/c and production angle $\theta_K = 15°$. The invariant cross section for pBe collisions is shown in Fig. 10 as a function of t. An important feature of the data is that they essentially cover positive values of t. In principle, the measurements probe the region around the maximum positive squared momentum transfer t_+.

As discussed above, the t dependence cannot be trivially factorized from the T_p dependence. For the data from Fig. 10, each experimental point corresponds to a different beam energy T_p which, for fixed p_K and θ_K, can be calculated with Eq. (3.1). Also t_+ is a function of T_p, and different for each of the data points, which makes the analysis of the data rather ambiguous. Thus, with the data from Ref. [7] we cannot verify the results for the data from Ref. [2] indicating a constant t dependence of the invariant kaon production cross section at $t \simeq 0$. We do not show the results of Ref. [7] for the Al, Cu and Ta targets, which have similar t dependence as the data for pBe collisions.

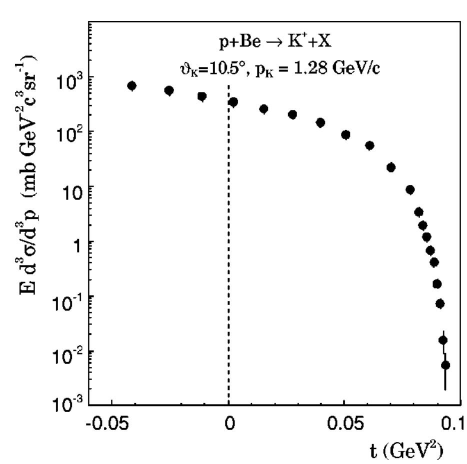

FIG. 10. t dependence of the invariant $p\mathrm{Be} \to K^+X$ cross section [7] measured at $p_K = 1.28$ GeV/c and $\theta_K = 10.5°$ at different beam energies.

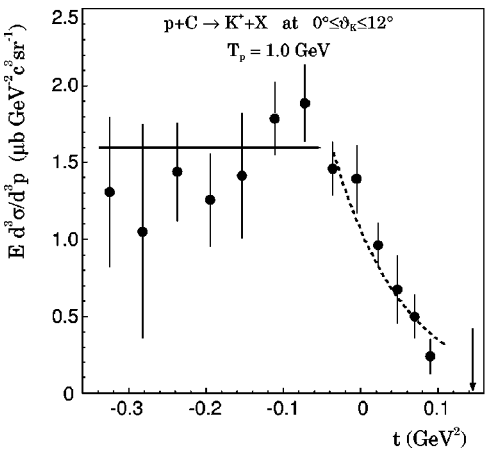

FIG. 11. t dependence of the invariant $pC \to K^+X$ cross section [6] at beam energy $T_p = 1.0$ GeV and kaon emission angles $0 \leqslant \theta_K \leqslant 12°$. The lines show the fit to the data by Eq. (3.7) (dashed) and by a constant value (solid) with the parameters from Eq. (3.7). The arrow shows the maximum possible value t_+, corresponding to kaons with the highest momenta. Note that the data are shown in a linear scale.

Finally, the most recent results [6] on K^+ production in pC collisions at $T_p = 1.0$ GeV and $0° \leqslant \theta_K \leqslant 12°$ are shown in Fig. 11. It can be seen that the measurements probe the region of $t \simeq 0$, as well as close to t_+ from Fig. 6. The data confirm that at small $|t|$ the invariant K^+-production cross section is almost independent of t. Within the range $-0.32 \leqslant t \leqslant -0.07$ GeV^2, we fit the data by a constant value $c_1 = 1.6 \pm 0.1\ \mu\text{b GeV}^{-2}\,\text{c}^3\,\text{sr}^{-1}$.

For positive values of t the dependence of the invariant K^+-production cross section changes drastically and becomes very steep in the range $-0.031 \leqslant t \leqslant 0.09$ GeV^2 where it can be fitted by Eq. (3.7) with

$$c_0 = 1.07 \pm 0.08 (\mu\text{b GeV}^{-2}\,\text{c}^3\,\text{sr}^{-1}),$$

$$b_0 = -11.1 \pm 1.7 (\text{GeV}^2). \tag{3.8}$$

The strong decrease of the cross section towards t_+ is not surprising since this region corresponds to the formation of hypernuclei which characteristically has very small cross sections, see, e.g., Ref. [20].

Table II shows that all available data taken at subthreshold energies cover only very limited ranges of t, making systematical analyses, e.g., the extraction of the remaining T_p and θ_K dependences, impossible. More data are needed at different proton-beam energies T_p, as well as kaon angles different from the previously measured ones. According to the reaction kinematics shown by Eq. (3.2), the extreme values $t_\pm$, corresponding to large t intervals, are accessible at forward laboratory angles $\theta_K \simeq 0°$. In this respect the ANKE spectrometer [5] is particularly useful since it allows to measure kaon production at $0° \leqslant \theta_K \leqslant 12°$, and offers a wide kaon-momentum coverage for beam energies in the range $1.0 \leqslant T_p \leqslant 2.3$ GeV.

IV. SUMMARY

Our analysis of the target-mass dependence of K^+-production cross sections in pA collisions at $T_p \leqslant 2.9$ GeV show that, based on the existing data [1–3,6,7], it is not possible to draw unambiguous conclusions about the underlying reaction mechanisms. The whole set of differential data at beam energies $T_p \geqslant 1.2$ GeV is in line with direct kaon production. Only the data on total cross sections at $T_p \leqslant 1.0$ GeV indicate kaon production on more than one nucleon. This is in contrast to the results of the most recent measurements of K^+ production at forward angles and $T_p = 1.0$ GeV by the ANKE collaboration. The A dependence of these data is in accordance with direct kaon production. The observed discrepancy might be caused by the angular variation of the K^+-production mechanism.

The data on differential cross sections show that the parameter α only weakly depends on the beam energy T_p and kaon emission angle θ_K and is independent of the kaon momentum p_K.

We conclude that further K^+-momentum spectra should be measured at forward angles and beam energies $T_p \geqslant 1.2$ GeV. The A dependence of such data might be sensitive to a transition from one- to two-step reactions. In combination with the existing differential data obtained at larger angles, the new data will also allow to study the angular dependence of the production cross sections.

The invariant K^+-production cross sections measured for different kinematical conditions show an overall exponential scaling behavior with the four-momentum transfer between the beam proton and the produced kaon for $t < -0.05$ GeV^2. Thus the Regge model is applicable at large negative values of t even for subthreshold kaon production down to $T_p = 1.2$ GeV. The dependence on t becomes steeper for small beam energies and kaon emission angles θ_K. This implies a significant dependence of the exponential slope on the excitation energy or the residual nucleus. It would be interesting to check by future measurements whether for $t < -0.05$ GeV^2 the exponential t scaling is violated at $\theta_K \simeq 0°$. It can be speculated that here the t dependence becomes very strong.

At small negative values, $t > -0.05$, the invariant cross sections are almost independent of t, if measured at the same beam energy T_p. For $t > 0$ the data show a strong falloff towards the kinematical limit corresponding to the formation of hypernuclei.

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