

Extracting the Light Quark Mass Ratio m_u/m_d from Bottomonia Transitions

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We propose a new method to extract the light quark mass ratio m_u/m_d using the $Y(4S) \rightarrow h_b \pi^0(\eta)$ bottomonia transitions. The decay amplitudes are dominated by the light quark mass differences, and the corrections from other effects are rather small, allowing for a precise extraction. We also discuss how to reduce the theoretical uncertainty with the help of future experiments. As a by-product, we show that the decay $Y(4S) \rightarrow h_b \eta$ is expected to be a nice channel for searching for the h_b state.

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Although fundamental parameters of the standard model, the masses of light quarks have not yet been well determined. This appears to be a consequence of quark confinement as well as the fact that the light quark masses are significantly lighter than the typical hadronic scale and as such their impact on most of the hadron masses or other properties is very small.

As a consequence of the spontaneous chiral symmetry breaking, the low-energy region of the quantum chromodynamics (QCD) can be described by chiral perturbation theory (CHPT) [1,2]. The most direct way to get information on the light quark mass ratios is to relate the quark masses to the masses of the lowest-lying pseudoscalar mesons, which are the Goldstone bosons of the spontaneously broken chiral symmetry of QCD. To leading order (LO) in the chiral expansion, this gives $m_u/m_d = 0.56$ [3]. Electromagnetic (e.m.) effects have been taken into account using Dashen's theorem [4]. There might be, however, sizable higher order corrections to this LO result, e.g., related to violations of Dashen's theorem, see [5,6]. The up-to-date knowledge of the light quark mass ratio from various sources including recent lattice calculations was summarized in Ref. [7] to be

$$\frac{m_u}{m_d} = 0.47 \pm 0.08. \quad (1)$$

In a completely independent approach it was proposed to use the decays of ψ' into $J/\psi \pi^0$ and $J/\psi \eta$, which break isospin and SU(3) symmetry, respectively [8,9]. It was assumed that these decays are dominated by the emission of soft gluons, and the gluons then hadronize into a pion or an eta. Using the QCD multipole expansion (QCDME), one obtains

$$\frac{\Gamma(\psi' \rightarrow J/\psi \pi^0)}{\Gamma(\psi' \rightarrow J/\psi \eta)} = 3 \left(\frac{m_d - m_u}{m_d + m_u} \right)^2 \frac{F_\pi^2 M_\pi^4}{F_\eta^2 M_\eta^4} \left| \frac{\vec{q}_\pi}{\vec{q}_\eta} \right|^3, \quad (2)$$

where $F_{\pi(\eta)}$ and $M_{\pi(\eta)}$ are the decay constant and mass of the pion (eta), respectively, and $\vec{q}_{\pi(\eta)}$ is the pion (eta)

momentum in the ψ' rest frame. These two decays were widely used in determining the quark mass ratio m_u/m_d [10–13]. Using the most recent measurement of the decay widths from the CLEO Collaboration [14], one gets $m_u/m_d = 0.40 \pm 0.01$, which is much smaller than the one resulting from the meson masses. Using instead the measurement by the BES Collaboration [15], the resulting value $m_u/m_d = 0.35 \pm 0.02$ is even smaller. In Ref. [16], based on a nonrelativistic effective field theory (NREFT) formalism, the striking discrepancy between the values of m_u/m_d extracted from the ψ' decays and from the meson masses was solved by showing that the decay amplitudes of the transitions $\psi' \rightarrow J/\psi \pi^0(\eta)$ are not dominated by the multipole effect as assumed before. Rather, nonmultipole effects via intermediate charmed meson loops are very important, enhanced by $1/v$, $v \approx 0.5$ being the charmed meson velocity, compared with the multipole one. More precisely, the large uncertainty related to the nonmultipole contributions prevents one from an extraction of m_u/m_d from these decays.

In this Letter, we propose a new way to extract the light quark mass ratio using the transitions of the excited bottomonium $Y(4S)$ into $h_b \pi^0$ and $h_b \eta$. Similar to the transitions between charmonium states, the e.m. contribution to the isospin breaking decay $Y(4S) \rightarrow h_b \pi^0$ is negligibly small [17–19]. This provides the possibility of extracting the light quark mass ratio from these decays. It will be shown that the nonmultipole effects from intermediate bottom meson loops are suppressed, and hence the decay amplitudes are proportional to the light quark mass differences.

The spin-singlet P -wave bottomonium h_b has not been observed yet, however, it is expected to agree in mass with the spin-averaged mass of the spin-triplet P -wave bottomonia χ_{bJ} (see, e.g., Ref. [20]), which is $M_{h_b} = 9900$ MeV. The $Y(4S)$ with a mass of 10579.4 ± 1.2 MeV and width 20.5 ± 2.5 MeV is the first bottomonium above the $B\bar{B}$ threshold, and it decays into $B\bar{B}$ with

more than 96% branching fraction [21]. The mass difference between the $Y(4S)$ and the h_b is about 680 MeV. Hence, both the transitions $Y(4S) \rightarrow h_b \pi^0$ and $Y(4S) \rightarrow h_b \eta$ are kinematically allowed.

Let us consider the multipole decay mechanism with the light meson being directly emitted from the bottomonium first, which is described by a tree-level diagram based on hadronic degrees of freedom. Because the decays are in an S wave and break isospin or $SU(3)$ symmetry, the LO amplitude must scale as the quark mass difference

$$\mathcal{M}^{\text{tree}} \sim \delta, \quad (3)$$

with $\delta = m_d - m_u$ for the transition $Y(4S) \rightarrow h_b \pi^0$ and $\delta = m_s - \hat{m}$, with $\hat{m} = (m_u + m_d)/2$, for the transition $Y(4S) \rightarrow h_b \eta$.

Corrections to the tree-level result arise due to intermediate heavy meson loops and higher order terms in the chiral expansion. The loops can be studied in the framework of the NREFT because the velocity of the heavy meson in the loops is small. The value of the bottom meson velocity for the transitions considered here may be estimated as $v \sim \sqrt{[2M_{\hat{B}} - (M_{Y(4S)} + M_{h_b})/2]/M_{\hat{B}}} \simeq 0.3$ with $M_{\hat{B}}$ the averaged bottom meson mass. This estimate is consistent with determinations of the bottom quark velocity in bottomonium systems based on nonrelativistic QCD (see, e.g., Ref. [22]). For a transition between a P -wave and an S -wave heavy quarkonium with the emission of a pion or an eta, it has been shown that the contribution to the decay amplitude from the intermediate heavy meson loops scales as [23]

$$\mathcal{M}^{\text{loop}} \sim \frac{1}{v^3} \frac{\vec{q}^2}{M_H^2} \Delta, \quad (4)$$

where \vec{q} is the three momentum of the light meson in the rest frame of the decaying heavy quarkonium, M_H is the mass of the intermediate heavy meson, and the meson mass difference Δ encodes the violation of the isospin symmetry for the pionic transition or $SU(3)$ symmetry for the eta transition. Equation (4) arises because the nonrelativistic loop integral measure contains three powers of momentum, and scales as v^3 . After performing the contour integration of the energy, two propagators are left, and each of them scales as $1/v^2$. The P -wave coupling of the light meson to the heavy meson gives a factor of \vec{q} . The coupling of the heavy mesons to the P - and S -wave heavy quarkonia are in S and P wave, respectively. The P -wave vertex provides a momentum in the loop integral, and it must be contracted with the external momentum of the light meson \vec{q} . So the three vertices together provide a factor of \vec{q}^2 . Since we are considering isospin or $SU(3)$ symmetry breaking transitions, the decay amplitude from the loops is nonvanishing because the heavy mesons within the same isospin or $SU(3)$ multiplet have different masses. One may pull out the meson mass difference explicitly to represent the symmetry breaking. Because it is an energy scale and should be

counted as v^2 , one needs to divide it by v^2 for balance. Putting all pieces together, one gets $[v^3/(v^2)^2][\vec{q}^2/M_H^2] \times [\Delta/v^2]$, where $1/M_H^2$ is introduced to match dimensions, and Eq. (4) follows. This kind of nonrelativistic power counting has already been confirmed by explicit calculations of the loops [16,19,23].

To determine the relative size of the loop amplitude compared to the tree-level one, in order to find out whether the tree-level contribution is dominant, one should compare the meson mass difference Δ and the quark mass difference δ , and estimate the value of the dimensionless prefactor $\vec{q}^2/(v^3 M_H^2)$. The momenta of the pion and eta in the final states of $Y(4S) \rightarrow h_b \pi^0(\eta)$ are 645 MeV and 389 MeV, respectively. Taking $v \simeq 0.3$ for the velocity, the dimensionless factor $\vec{q}^2/(v^3 M_B^2)$ is about 0.6 for the pionic transition and 0.2 for the eta transition. One cannot naively assign the meson mass differences as the same order as the quark ones. In fact, due to destructive interference between the quark mass difference and the e.m. contribution [24], the isospin mass splitting of the bottom mesons B^0 and B^+ is rather small, $M_{B^0} - M_{B^+} = 0.33 \pm 0.06$ MeV [21]. It is one order-of-magnitude smaller than $m_d - m_u$. Together with the dimensionless factor, which is about 0.6, the bottom meson loops contribute to the decay $Y(4S) \rightarrow h_b \pi^0$ for no more than a few percent, and hence are negligible. The situation for the eta transition is somewhat different because $M_{B_s} - \hat{M}_B = 87.0 \pm 0.6$ MeV, where $\hat{M}_B = (M_{B^0} + M_{B^+})/2$, and it is of similar size as $m_s - \hat{m}$. This means that the loop contributions to the $Y(4S) \rightarrow h_b \eta$ as compared to the tree-level decay amplitude are also suppressed, but they might give a nonvanishing correction of about 20%.

An intriguing implication of the suppression of the bottom meson loops in these transitions is that the decay amplitudes are dominated by the quark mass differences, and hence it is possible to extract the light quark mass ratio from the ratio of the branching fractions of the transitions $Y(4S) \rightarrow h_b \pi^0(\eta)$ with good accuracy. It has been demonstrated that the LO results of chiral Lagrangians for the heavy quarkonia transitions can reproduce the LO results of the QCDME [25]. In the QCDME, the transitions between two heavy quarkonia occur through radiating soft gluons, and the soft gluons then hadronize into light mesons [26–28]. In the case of transitions with the emission of a pion or an eta, the gluon operator is $G\tilde{G} \equiv \alpha_s G_{\mu\nu}^A \tilde{G}^{A\mu\nu}$ [9], where α_s is the strong coupling constant, $G_{\mu\nu}^A$ is the gluon field strength tensor, and its dual is $\tilde{G}^{A\mu\nu} = \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^A/2$. For the transitions $Y(4S) \rightarrow h_b \pi^0(\eta)$, we have

$$\frac{\Gamma(Y(4S) \rightarrow h_b \pi^0)}{\Gamma(Y(4S) \rightarrow h_b \eta)} = r_{G\tilde{G}}^2 \left| \frac{\vec{q}_\pi}{\vec{q}_\eta} \right|, \quad (5)$$

where $\vec{q}_{\pi(\eta)}$ is the momentum of the pion (eta) in the rest frame of the $Y(4S)$, and the ratio of the gluon matrix elements is defined as

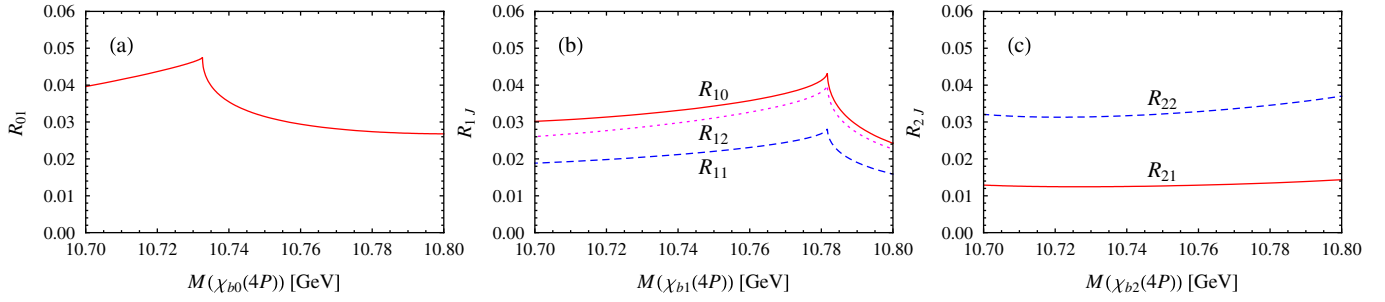


FIG. 1 (color online). Predicted ratios defined in Eq. (9) in the NREFT. The value of g_{1b} has been set to $1 \text{ GeV}^{-1/2}$.

$$r_{G\tilde{G}} \equiv \frac{\langle 0|G\tilde{G}|\pi^0\rangle}{\langle 0|G\tilde{G}|\eta\rangle}. \quad (6)$$

Combining CHPT with the $U(1)_A$ anomaly, the next-to-leading order (NLO) expressions for the matrix elements $\langle 0|G\tilde{G}|\pi^0(\eta)\rangle$ were worked out in Ref. [13] in terms of several low-energy constants (LECs) of the $\mathcal{O}(p^4)$ Lagrangian. Moreover, there exists an intriguing relation between the ratio of the matrix elements and a combination of the light quark masses [11,13]

$$\begin{aligned} r_{\text{DW}} &\equiv \frac{m_d - m_u}{m_d + m_u} \frac{m_s + \hat{m}}{m_s - \hat{m}} \\ &= \frac{4}{3\sqrt{3}} r_{G\tilde{G}} \frac{F_\pi}{F_\eta} \frac{F_K^2 M_K^2 - F_\pi^2 M_\pi^2}{F_\pi^2 M_\pi^2} (1 - \delta_{\text{GMO}}) \\ &\quad \times \left[1 + \frac{4L_{14}}{F_\pi^2} (M_\eta^2 - M_\pi^2) \right] \\ &= 10.59(1 + 132.1L_{14})r_{G\tilde{G}}, \end{aligned} \quad (7)$$

where $\delta_{\text{GMO}} = -0.06$ denotes the $\mathcal{O}(p^4)$ deviation from the Gell-Mann—Okubo relation among the Goldstone bosons. Higher order terms in the coupling of the flavor-singlet field that encodes the information of the anomalously broken $U(1)_A$ anomaly are parametrized by the $\mathcal{O}(p^4)$ LEC L_{14} . Therefore, once one has knowledge of the value of the LEC L_{14} , one is able to extract the quark mass ratio from the ratio of branching fractions of the decays $\Upsilon(4S) \rightarrow h_b \pi^0$ and $\Upsilon(4S) \rightarrow h_b \eta$, which can be measured in the future.

There are two main theoretical uncertainties for extracting the value of r_{DW} . The first one is due to the lack of knowledge of the LEC L_{14} . One may use resonance saturation to estimate its value, and it is expected to be in the region [11,13] $L_{14} = (2.3 \pm 1.1) \times 10^{-3}$. From Eq. (7), it gives 11% uncertainty in r_{DW} . The other one is from neglecting the intermediate bottom meson loops of the transition $\Upsilon(4S) \rightarrow h_b \eta$. As already discussed, it gives an uncertainty of 20% in the amplitude, and hence 40% in the decay width. Propagating to the extracted quark mass ratio, the uncertainty is again 20%. Adding them quadratically, the theoretical uncertainty for extracting r_{DW} is 23%, which is comparable to that of Eq. (1).

The uncertainty could be reduced once further information on the size of the loops is available. This kind of information could be provided by high statistics

measurements in the following way: The decay width for $\Upsilon(4S) \rightarrow h_b \eta$ considering only loops can be worked out using the NREFT

$$\Gamma(\Upsilon(4S) \rightarrow h_b \eta)^{\text{loop}} = 0.16g_{1b}^2 \text{ keV}, \quad (8)$$

where the only unknown parameter g_{1b} denotes the coupling of the $1P$ bottomonium states to the bottom mesons, given in $\text{GeV}^{-1/2}$. Although the $1P$ states are below any open bottom threshold, one may extract g_{1b} from the loop dominated transitions involving the $1P$ states. Because the isospin mass splitting of the B mesons is rather small, one should consider the loop dominated transitions with the emission of an η . These are the transitions from excited P -wave bottomonia to the $1P$ states, enhanced by a factor $1/v^3$, using a similar power counting technique presented in Ref. [19]. The best choice are the η transitions from the $4P$ states to the $1P$ states. Based on the quark model calculation of the bottomonium spectrum [29], the $4P$ states have sufficiently large masses to allow for decays into $B^{(*)}\bar{B}^{(*)}$. Because the $4P$ states can decay directly into $B^{(*)}\bar{B}^{(*)}$, the corresponding coupling constant $g_{1b}^{\prime\prime}$ can be obtained by measuring their decay widths. Then, one can extract the value of g_{1b} from any of the transitions $\chi_{b0}(4P) \rightarrow \chi_{b1}\eta$, $\chi_{b1}(4P) \rightarrow \chi_{b0,1,2}\eta$, $\chi_{b2}(4P) \rightarrow \chi_{b1,2}\eta$, and $h_b(4P) \rightarrow h_b\eta$. In Fig. 1, we show the predictions from the NREFT for the following ratios, which depend on g_{1b} only and are proportional to g_{1b}^2 , as a function of the mass of the $4P$ bottomonium state:

$$\begin{aligned} R_{01} &\equiv \frac{\Gamma(\chi_{b0}(4P) \rightarrow \chi_{b1}\eta)}{\Gamma(\chi_{b0}(4P) \rightarrow B^+B^-)}, \\ R_{1J} &\equiv \frac{\Gamma(\chi_{b1}(4P) \rightarrow \chi_{bJ}\eta)}{\Gamma(\chi_{b1}(4P) \rightarrow B^+B^{*-})}, \quad J = 0, 1, 2, \\ R_{2J} &\equiv \frac{\Gamma(\chi_{b2}(4P) \rightarrow \chi_{bJ}\eta)}{\Gamma(\chi_{b2}(4P) \rightarrow B^{*+}B^{*-})}, \quad J = 1, 2. \end{aligned} \quad (9)$$

The result for $\Gamma(h_b(4P) \rightarrow h_b\eta)/\Gamma(h_b(4P) \rightarrow B^{*+}B^{*-})$ is very similar to and slightly larger than R_{10} . The cusps in Figs. 1(a) and 1(b) represent the opening of the $B_s\bar{B}_s$ and $B_s\bar{B}_s^*$ thresholds, respectively. For definiteness, we have used $g_{1b} = 1 \text{ GeV}^{-1/2}$. The dependence on $g_{1b}^{\prime\prime}$ is canceled in the ratios. If any of these ratios were to be measured, one will be able to extract the value of g_{1b}^2 easily. The so extracted coupling g_{1b}^2 bears about 30% uncertainty due

to the loops of higher order. The $4P$ bottomonia should decay dominantly into $B^{(*)}\bar{B}^{(*)}$. For an order-of-magnitude estimate, one may assume g_{1b} to have a similar value as its charm analogue estimated using vector meson dominance, $g_{1c} = -4.2 \text{ GeV}^{-1/2}$ [30]. Then from Fig. 1, one expects the eta transitions have branching fractions of the order of a few percent. Hence, there should be a good opportunity in extracting g_{1b} at the Large Hadron Collider beauty experiment (LHCb). After having measured the partial decay width of $Y(4S) \rightarrow h_b \eta$, one may compare the measured value with the one obtained considering only the bottom meson loops, given in Eq. (8), using g_{1b} determined in the way outlined above as input. Depending on whether the interference between the tree-level and the loop amplitudes is constructive or destructive, one can get two solutions of the width considering only the multipole (tree-level) effect. Then one may insert the resulting $\Gamma(Y(4S) \rightarrow h_b \eta)^{\text{tree}}$ in Eq. (5), reducing the uncertainty from the loop contribution.

Using the same naturalness arguments for the coupling constant in the LO tree-level Lagrangian as that in Ref. [23], we can estimate the branching fractions of the transitions $Y(4S) \rightarrow h_b \pi^0(\eta)$. The branching fraction for the pionic transition is of order 10^{-6} , and the one for the eta transition is of order 10^{-3} . With such a large branching fraction, the latter one even provides a nice option for searching for the h_b . LHCb is expected to have enough events of the $Y(4S)$ to do the measurements.

In summary, we have proposed a new method for extracting m_u/m_d . We demonstrated that the transitions $Y(4S) \rightarrow h_b \pi^0(\eta)$ can be used to determine the value of r_{DW} with an acceptable theoretical uncertainty, which is about 23%. Using information of \hat{m}/m_s from other sources, one is then able to extract m_u/m_d . The transitions $Y(4S) \rightarrow h_b \pi^0$ and $Y(4S) \rightarrow h_b \eta$ are expected to have branching fractions of order of 10^{-6} and 10^{-3} , respectively. Therefore, they can be measured at LHCb based on a large number of $Y(4S)$ events. The uncertainty can be reduced to obtain a more accurate extraction of the quark mass ratio by measuring the partial decay widths of the $4P$ bottomonium to the $1P$ bottomonium with the emission of an eta. These transitions with branching fractions of order of a few per cent can also be measured at LHCb. As a by-product, the decay $Y(4S) \rightarrow h_b \eta$ is expected to be a nice channel for searching for the h_b state.

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- [1] S. Weinberg, *Physica (Amsterdam)* **96A**, 327 (1979).
- [2] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984); *Nucl. Phys.* **B250**, 465 (1985).
- [3] S. Weinberg, in *A Festschrift for I.I. Rabi*, edited by L. Motz [Trans. N.Y. Acad. Sci. 38, No. 185 (1977)].
- [4] R.F. Dashen, *Phys. Rev.* **183**, 1245 (1969).
- [5] J. Bijnens and J. Prades, *Nucl. Phys.* **B490**, 239 (1997).
- [6] B. Moussallam, *Nucl. Phys.* **B504**, 381 (1997).
- [7] H. Leutwyler, in Colloquium in Memory of Jan Stern, Paris, France, 2009 (unpublished).
- [8] B.L. Ioffe and Yad. Fiz. **29**, 1611 (1979) [*Sov. J. Nucl. Phys.* **19**, 827 (1979)].
- [9] B.L. Ioffe and M.A. Shifman, *Phys. Lett. B* **95**, 99 (1980).
- [10] J. Gasser and H. Leutwyler, *Phys. Rep.* **87**, 77 (1982).
- [11] J.F. Donoghue, B.R. Holstein, and D. Wyler, *Phys. Rev. Lett.* **69**, 3444 (1992).
- [12] H. Leutwyler, *Phys. Lett. B* **378**, 313 (1996).
- [13] J.F. Donoghue and D. Wyler, *Phys. Rev. D* **45**, 892 (1992).
- [14] H. Mendez *et al.* (CLEO Collaboration), *Phys. Rev. D* **78**, 011102 (2008).
- [15] J.Z. Bai *et al.* (BES Collaboration), *Phys. Rev. D* **70**, 012006 (2004).
- [16] F.-K. Guo, C. Hanhart, and Ulf-G. Meißner, *Phys. Rev. Lett.* **103**, 082003 (2009); **104**, 109901(E) (2010).
- [17] J.F. Donoghue and S.F. Tuan, *Phys. Lett. B* **164**, 401 (1985).
- [18] K. Maltman, *Phys. Rev. D* **44**, 751 (1991).
- [19] F.-K. Guo, C. Hanhart, G. Li, U.-G. Meißner, and Q. Zhao, [arXiv:1008.3632](https://arxiv.org/abs/1008.3632).
- [20] S. Godfrey and J.L. Rosner, *Phys. Rev. D* **66**, 014012 (2002).
- [21] C. Amsler *et al.* (Particle Data Group), *Phys. Lett. B* **667**, 1 (2008), and 2009 partial update for the 2010 edition.
- [22] G.P. Lepage, L. Magnea, C. Nakhleh, U. Magnea, and K. Hornbostel, *Phys. Rev. D* **46**, 4052 (1992).
- [23] F.-K. Guo, C. Hanhart, G. Li, Ulf-G. Meißner, and Q. Zhao, *Phys. Rev. D* **82**, 034025 (2010).
- [24] F.-K. Guo, C. Hanhart, and U.-G. Meißner, *J. High Energy Phys.* **09** (2008) 136.
- [25] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, *Phys. Lett. B* **309**, 163 (1993).
- [26] K. Gottfried, *Phys. Rev. Lett.* **40**, 598 (1978).
- [27] M.B. Voloshin, *Nucl. Phys.* **B154**, 365 (1979).
- [28] Y.P. Kuang, *Front. Phys. China* **1**, 19 (2006).
- [29] B.Q. Li and K.T. Chao, *Commun. Theor. Phys.* **52**, 653 (2009).
- [30] P. Colangelo, F. De Fazio, and T.N. Pham, *Phys. Rev. D* **69**, 054023 (2004).