Precision Determination of the $d\pi \leftrightarrow NN$ Transition Strength at Threshold

Th. Strauch,[1] F. D. Amaro,[2] D. F. Anagnostopoulos,[3] P. Bühler,[4] D. S. Covita,[2,5,*] H. Gorke,[6] D. Gotta,[1,†] A. Gruber,[4] A. Hirtl,[4,‡] P. Indelicato,[7] E.-O. Le Bigot,[7] M. Nekipelov,[1] J. M. F. dos Santos,[2] S. Schlesser,[7] Ph. Schmid,[4] L. M. Simons,[5] M. Trassinelli,[7,§] J. F. C. A. Veloso,[8] and J. Zmeskal[4]

[1]Institut für Kernphysik, Forschungszentrum Jülich GmbH, D-52425 Jülich, Germany
[2]Department of Physics, Coimbra University, P-3000 Coimbra, Portugal
[3]Department of Materials Science and Engineering, University of Ioannina, GR-45110 Ioannina, Greece
[4]Stefan Meyer Institut for Subatomic Physics, Austrian Academy of Sciences, A-1090 Vienna, Austria
[5]Laboratory for Particle Physics, Paul Scherrer Institut (PSI), CH 5232-Villigen, Switzerland
[6]Zentralinstitut für Elektronik, Forschungszentrum Jülich GmbH, D-52425 Jülich, Germany
[7]Lab. Kastler Brossel (LKB), ENS, CNRS, UPMC-Paris 6, Case 74, 4 place Jussieu, F-75005 Paris, France
[8]I3N, Department of Physics, Aveiro University, P-3810 Aveiro, Portugal
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An unusual but effective way to determine at threshold the $d\pi \leftrightarrow NN$ transition strength α is to exploit the hadronic ground-state broadening Γ_{1s} in pionic deuterium, accessible by x-ray spectroscopy. The broadening is dominated by the true absorption channel $d\pi^- \to nn$, which is related to s-wave pion production $pp \to d\pi^+$ by charge symmetry and detailed balance. Using the exotic atom circumvents the problem of Coulomb corrections to the cross section as necessary in the production experiments. Our dedicated measurement finds $\Gamma_{1s} = (1171^{+23}_{-49})$ meV yielding $\alpha = (252^{+5}_{-11})$ μb.

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Meson production and absorption at low energies plays a key role in developing methods within the framework of effective field theories such as chiral perturbation theory (χPT) [1]. Directly at threshold, experimental access to $NN \leftrightarrow NN\pi$ processes is provided both via the hadronic ground-state broadening Γ_{1s} in pionic deuterium (πD) and pion production in nucleon-nucleon collisions.

Considering only pure hadronic cross sections (denoted by $\tilde{\sigma}$), i.e., with the Coulomb interaction switched off to circumvent the divergence problem at threshold but with the particles keeping their physical mass, the production cross section is parametrized by [2]

$$\tilde{\sigma}_{pp\to\pi^+d} = \alpha\eta + \beta\eta^3 + \ldots \quad (1)$$

with $\eta = p^*_\pi c/m_\pi c^2$ being the reduced pion momentum in the πd rest frame. For $\eta \to 0$ higher partial waves ($\beta, \ldots$) vanish, and only the threshold parameter α contributes owing to pure s-wave production. Directly related is the reaction $np \to \pi^0 d$ because in the limit of charge independence the relation $2\sigma_{np\to\pi^0 d} = \sigma_{pp\to\pi^+ d}$ holds.

Values for α derived from pion production [3–9] and absorption experiments [10] scatter widely even when comparing recent data (Table I and Fig. 1). However, sometimes only statistical errors are given. The fluctuations suggest systematic uncertainties of about 10% possibly stemming from normalization. In particular, the Coulomb corrections, mandatory to obtain the pure hadronic cross section, are a significant source of uncertainty [4,7,14].

As discussed, for example, by Lensky *et al.* [18,19], phenomenological descriptions [2,13–17] may suffer from an incomplete knowledge of the contributing mechanisms, which in principle is avoided within the χPT approach if enough terms are considered in the expansion. A recent calculation up to next-to-leading order (NLO) terms yields $\alpha^{\rm NLO} = 220$ μb [18] (Fig. 1) and thus ${\rm Im}\,a_{\pi D} = 5.65 \times 10^{-3} m_\pi^{-1}$ [19] for the imaginary part of the πD

TABLE I. Threshold parameter α derived from the hadronic broadening Γ_{1s} in πD and pion-production and -absorption data together with a selection of theoretical approaches.

Pionic deuterium	α (μb)	
$3p \to 1s$	220 ± 45	[11]
$2p \to 1s$	257 ± 23	[12]
$3p \to 1s$	252^{+5}_{-11}	This experiment
Pion production or absorption	α (μb)	
$pp \to d\pi^+$	138 ± 15^{a}	[3]
$pp \to d\pi^+$	240 ± 20^{a}	[4]
$pp \to d\pi^+$	180 ± 20^{a}	[5]
$pp \to d\pi^+$	228 ± 46	[6]
$np \to d\pi^0$	184 ± 14	[7]
$d\pi^+ \to pp$	174 ± 3^{a}	[10]
$p_{\rm pol}p \to d\pi^+$	208 ± 5^{a}	[8]
$pp \to d\pi^+$	205 ± 9^{a}	[9]
Theoretical approach	α (μb)	
Watson-Brueckner	140 ± 50	[2]
Rescattering	146	[13]
Rescattering	201	[14]
Faddeev (Reid soft core)	220	[15]
Faddeev (Bryan-Scott)	267	[15]
Heavy meson exchange	203 ± 21	[16,17]
χPT NLO	220 ± 70	[18]

[a]Experiments reporting statistical uncertainty only.

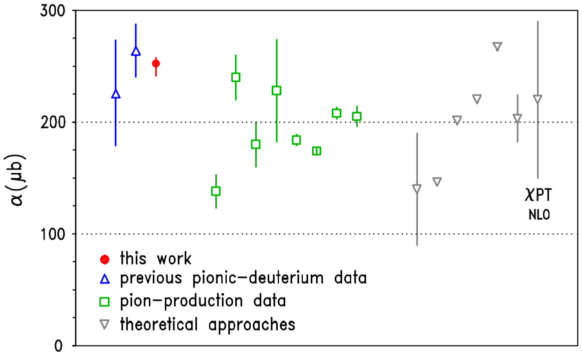

FIG. 1 (color online). Threshold parameter α. Points are shown in the same order as listed in Table I.

scattering length. The uncertainty of about ±30% is expected to decrease to below ±10% by next-to-next-to-leading order (NNLO) calculations [20].

A measurement of Γ_{1s} in pionic deuterium is equivalent to the determination of Im$a_{\pi D}$ [21] being predominantly attributed to true pion absorption $\pi^- d \to nn$. Pion absorption at rest on the isospin $I = 0$ nucleon-nucleon pair of the deuteron induces the transition ${}^3S_1[{}^3D_1](I = 0) \to {}^3P_1(I = 1)$, the inverse of which accounts for s-wave pion production in $pp \to d\pi^+$ [22]. Therefore, Γ_{1s} is a measure of the s-wave pion-production strength.

In contrast to production experiments, the extraction of the threshold parameter α or Im$a_{\pi D}$ from Γ_{1s} avoids the problem of Coulomb corrections to the measured cross sections. However, previous x-ray experiments [11,12] are of limited statistics, and insufficient knowledge on the experimental resolution and cascade-induced broadening prevents a precise extraction of the pure hadronic width Γ_{1s}. Hence, a remeasurement of Γ_{1s} was performed [23] aiming at an accuracy of at least the one expected from the forthcoming χPT calculations.

The complex pion-deuteron scattering length $a_{\pi D}$ is related to the 1s-state shift ϵ_{1s} and width Γ_{1s} in πD by

$$\epsilon_{1s} - i\frac{\Gamma_{1s}}{2} = -\frac{2\alpha^3\mu^2c^4}{\hbar c}a_{\pi D}\left[1 - \frac{2\alpha\mu c^2}{\hbar c}(\ln\alpha - 1)a_{\pi D}\right]. \quad (2)$$

The first term corresponds to the classical Deser formula [21] yielding the scattering length in leading order [24]. The term in brackets corrects for the fact that $a_{\pi D}$ is determined from a Coulomb bound state [25–27]. In Eqs. (2) and (3), α denotes the fine structure constant.

The imaginary part of $a_{\pi D}$ is given by

$$\mathrm{Im}\, a_{\pi D} = \frac{\hbar c}{2\alpha^3\mu^2c^4}\frac{\Gamma_{1s}/2}{1 - \frac{2\alpha\mu c^2}{\hbar c}(\ln\alpha - 1) \times 2\mathrm{Re}a_{\pi D}}$$
$$= 0.010\,642 m_\pi^{-1}\ \mathrm{eV}^{-1} \times 1.004(\Gamma_{1s}/2). \quad (3)$$

The factor 1.004 stands for the—in this case—small bound-state correction. Hence, it is sufficient to insert the leading order result for the real part of the scattering length in Eq. (3). The value Re$a_{\pi D} = (26.3 \pm 0.6)10^{-3} m_\pi^{-1}$ is taken from a previous experiment [12].

Detailed balance states that [22]

$$\tilde{\sigma}_{\pi^+ d \to pp} = \frac{2}{3}\left(\frac{p_p^*}{p_\pi^*}\right)^2 \tilde{\sigma}_{pp \to \pi^+ d}, \quad (4)$$

where p_p^* and p_π^* are the final state momenta of proton and pion in the center-of-mass (c.m.) system. Assuming charge symmetry, for the transition matrix elements $|\tilde{M}_{\pi^- d \to nn}| = |\tilde{M}_{\pi^+ d \to pp}|$ holds. A small difference in the transition rate, $\tilde{\sigma}_{\pi^- d \to nn}/\tilde{\sigma}_{\pi^+ d \to pp} = p_n^*/p_p^* = 0.982$, must be taken into account because of the slightly larger phase space for $\pi^+ d \to pp$ with $p_{n,p}^*$ being the nucleon c.m. momenta.

In principle, both electromagnetic and hadronic isospin-breaking effects must be considered in view of the different quark contents in the final states of the processes $\pi^- d \to nn$ and $\pi^+ d \to pp$. Their magnitude, however, is assumed to be at most a few per cent [20,28], which is about the precision achieved in this experiment but far below the fluctuations of the pion-production data. The atomic binding energy of the π^-D system is neglected.

Combining optical theorem, charge symmetry, detailed balance, and inserting the s-wave part from (1), the purely hadronic imaginary part of the scattering length $a_{\pi^- d \to nn}$ reads in terms of the threshold parameter α

$$\mathrm{Im}\, a_{\pi^- d \to nn} = \lim_{p_\pi^* \to 0}\frac{p_\pi^*}{4\pi}\tilde{\sigma}_{\pi^- d \to nn} = \frac{1}{6\pi}\frac{p_p^* p_n^*}{m_\pi}\alpha. \quad (5)$$

To relate Im$a_{\pi^- d \to nn}$ to Im$a_{\pi D}$, a correction for final states other than nn must be applied. The measured branching ratios [29–31] yield for the relative strength of true absorption with respect to all other processes $S' = nn/(nn\gamma + nne^+e^- + nn\pi^0) = 2.76 \pm 0.04$. Consequently,

$$\mathrm{Im}\, a_{\pi D} = (1 + 1/S')\mathrm{Im}a_{\pi^- d \to nn}$$
$$= (2.48 \pm 0.01) \times 10^{-5} \times \alpha m_\pi^{-1}\ \mu\mathrm{b}^{-1}. \quad (6)$$

Our πD experiment was performed at the high-intensity low-energy pion beam πE5 of the proton accelerator at PSI by using the cyclotron trap II and a Johann-type Bragg spectrometer equipped with a Si crystal and an array of 6 charge-coupled devices (CCDs) as position-sensitive x-ray detector. The setup for the πD(3p-1s) measurement is similar to the one used for the μH(3p-1s) transition [23,32,33] but with restricting the reflecting area of the crystal of 100 mm in diameter to 60 mm horizontally to keep the Johann broadening small.

The x-ray energy spectrum is obtained by projection of the Bragg reflection onto the axis of dispersion (Fig. 2). The granularity of the CCDs having a pixel size of 40 μm allows an efficient background rejection by means of pattern recognition. Together with a massive concrete shielding, x-ray spectra with outstanding peak-to-background ratio are achieved. The stability of the mechanical setup

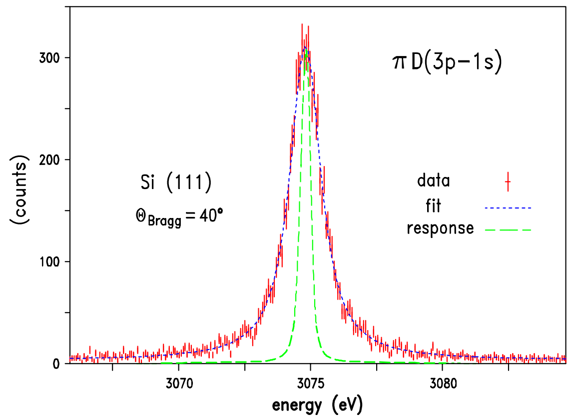

FIG. 2 (color online). Sum spectrum of the πD(3p-1s) transition measured at 10 and 17.5 bar equivalent density in first order with a silicon (111) Bragg crystal having a bending radius of $R = 2982.2 \pm 0.3$ mm. The narrow structure inside the πD line represents the spectrometer response function.

was monitored by two inclinometers. Details may be found elsewhere [34].

πD data were taken at equivalent pressures of 3.3, 10, and 17.5 bar (STP) in order to identify or to exclude any x-ray line broadening due to radiative deexcitation from molecular states [35–37] by means of an energy dependence of the πD(3p-1s) energy. In total, about 1450, 4000, and 4900 πD events were collected corresponding to count rates of 12, 35, and 40 per hour. As in the case of pionic [32] and muonic hydrogen [33], no evidence was found for radiative decay after molecule formation within the experimental accuracy. The D_2 density was adjusted by the temperature of the target gas.

The πD line shape is determined by the spectrometer response (i), the natural width of the x-ray transition (ii), and Doppler broadening from Coulomb transitions (iii).

(i) The spectrometer response was determined using the narrow M1 x-ray line from heliumlike argon at 3.104 keV as outlined in [38,39]. When scaled to the πD(3p-1s) energy (3.075 keV), it corresponds to a resolution of 436 ± 3 meV (FWHM) (Fig. 2), which is close to the theoretical value of 403 meV as calculated for an ideal flat crystal (code XOP [40]).

(ii) The natural linewidth is practically given by the hadronic broadening Γ_{1s}. The 3p-level width is dominated by radiative decay (28 μeV). Nuclear reactions are estimated to be <1 μeV. Likewise, based on calculated transitions rates [41] the induced width due to $3p \leftrightarrow 3s$ Stark mixing turns out to be as small as 1 μeV.

(iii) Coulomb transitions may occur when excited exotic hydrogen atoms penetrate the electron cloud of target atoms, and the energy release of the deexcitation step is converted into kinetic energy shared by the πD system and another D atom [42]. Coulomb deexcitation generates peaks in the kinetic-energy distribution, which are at 12, 20, 38, and 81 eV for the πD $\Delta n = 1$ transitions $(7-6)$, $(6-5)$, $(5-4)$, and $(4-3)$, respectively. Therefore, subsequent x-ray transitions may be Doppler broadened. Acceleration, however, is counteracted by elastic and inelastic scattering, which may lead to a continuum below the peak energies or even complete deceleration.

Cascade calculations have been extended to follow the velocity change during the deexcitation cascade and, therefore, can provide kinetic-energy distributions at the instant of x-ray emission from a specific atomic level (extended standard cascade model ESCM [41]). At present, only calculations for μH and πH are available [41,43,44].

Therefore, an approach independent of a cascade model was used to extract the Doppler broadening directly from the data, which was applied successfully in the μH x-ray analysis. The kinetic-energy distribution was modeled by boxes of a few eV width corresponding to the peaks generated by the Coulomb transitions. Their number, width, and position are preset but adjustable parameters of the analysis code. The hadronic width Γ_{1s}, the total intensity, and the background level are free parameters of the fit, as are the relative intensities, whose sum is normalized to one.

The line shape was constructed by convoluting the crystal response, imaging properties of the bent crystal, and Doppler contributions with the natural linewidth by means of Monte Carlo ray tracing. Following the experience of the μH(3p-1s) analysis [33], one tries to identify consecutively individual Doppler contributions starting with one single box only and moving it through the range of possible kinetic energies [34]. A χ^2 analysis using the MINUIT package [45] shows the necessity of a low-energy contribution. It was found that the upper bound of this box must not exceed 8 eV, a result obtained independently for the spectra taken at 10 bar and the 17.5 bar equivalent density. The result for Γ_{1s} turned out to be insensitive to the upper boundary of the kinetic-energy box for values ≤ 8 eV. The low-energy component was set to the range 0–2 eV in further analysis.

Searches for any contributions of Coulomb transitions leading to higher energies failed—also for the sum of the spectra measured at 10 and 17.5 bar equivalent density. Tentatively, we used the kinetic-energy distribution from the ESCM calculation for the πH(3p-1s) case, after scaling to πD energies. This distribution, where a fraction of 25% has energies above 15 eV, is unable to describe the line shape, which is in strong disagreement with the findings for muonic [33] and pionic hydrogen [32,46], where sizeable contributions from higher energy Coulomb transitions are mandatory. There is no explanation yet for this different behavior in pionic deuterium.

Detailed Monte Carlo studies have been performed to quantify which amount of high-energy components may be missed for the statistics achieved. It was found that a contribution of 25% can be excluded at the level of 99% for the component around 80 eV corresponding to the $(4-3)$ Coulomb transition. The chance to identify a 10% contribution is about $2/3$ corresponding to 1σ. The value for Γ_{1s} itself hardly varies with the width of any assumed high-energy box.

Omitting any high-energy contribution yields the upper limit for Γ_{1s}. Defining the limit of sensitivity to 10%—according to the above-mentioned 1σ criterion—results in a lower bound 43 meV lower than the upper limit and, hence, in an asymmetric systematic error for Γ_{1s}.

Systematic discrepancies (bias) as arising in maximum likelihood fits [47] have been studied according to the statistics of the measured spectra. The bias has been determined to be -32 ± 2 meV (3.3 bar data) and -2 ± 2 meV (sum of 10 and 17.5 bar). For each set of conditions 400 Monte Carlo generated spectra were analyzed.

The weighted average for the hadronic broadening,

$$\Gamma_{1s} = (1171^{+23}_{-49})\ \text{meV}, \qquad (7)$$

is in good agreement with previous measurements which found $\Gamma_{1s} = 1020 \pm 210$ [11] and 1194 ± 105 meV [12].

This result leads to an imaginary part (3)

$$\text{Im}\, a_{\pi D} = (6.26^{+0.12}_{-0.26})10^{-3} m_{\pi}^{-1}. \qquad (8)$$

The corresponding value for α is given in Table I.

In summary, the $\pi D(3p\text{-}1s)$ x-ray transition in pionic deuterium has been studied to determine the strong-interaction broadening of the $1s$ state and from that the threshold pion-production strength α. The accuracy of 4.2% achieved reaches the expected (5–10)% uncertainty of forthcoming NNLO χPT calculations. It is noteworthy that at the 10% level no components from high-energetic Coulomb transitions could be identified.

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*Present address: I3N, Department of Physics, Aveiro University, P-3810 Aveiro, Portugal.
†Corresponding author: d.gotta@fz-juelich.de
‡Present address: Universitätsklinik für Nuklearmedizin, Medizinische Universität Wien, 1090 Vienna, Austria.
§Present address: Inst. des NanoSciences de Paris, CNRS UMR7588 and UMPC-Paris 6, F-75015 Paris, France.

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