Neutron Interferometric Observation of Noncyclic Phase

Apoorva G. Wagh* and Veer Chand Rakhecha*
Solid State Physics Division, Bhabha Atomic Research Centre, Mumbai 400085, India

Peter Fischer and Alexander Ioffe†
Berlin Neutron Scattering Center, Hahn-Meitner-Institut, Glienicker Strasse 100, 14109 Berlin, Germany
(Received 11 November 1997)

Using polarized neutrons, we have determined phases as well as interference amplitudes for noncyclic spinor evolutions in static magnetic fields. Both these quantities depend on the angle subtended by the neutron spin with the field. This experiment elucidates the subtle, and widely misunderstood, concepts involved. [S0031-9007(98)07052-5]

PACS numbers: 03.75.Dg, 03.65.Bz

When a quantal system undergoes a cyclic evolution, the initial and final wave functions differ just by a nonzero complex multiplier. The phase acquired in the evolution is then just the argument of the multiplier. In a noncyclic evolution, however, the initial and final states are distinct and the phase prescription is nontrivial. About 40 years ago, a simple, yet brilliant physical deduction that such a phase equals the argument of the inner product of the initial and final wave functions, was made [1]. It has since become known as the Pancharatnam connection [2–4]. Misconceptions (cf., e.g., [5–7]) about the noncyclic phase, however, persist. For a neutron spin precessing about a static magnetic field at an angle θ, for instance, the phase acquired has been widely assumed (cf., e.g., [6,7]) to be one-half the precession angle for all θ, the factor $1/2$ being ascribed to the spin magnitude. For polarized neutrons in rotating magnetic fields, Weinfurter and Badurek [5] mistook the rotation angle of the field to be the noncyclic geometric phase and thereby claimed to have measured this "phase" polarimetrically. Wagh and Rakhecha [8] delineated the correct noncyclic phase for these evolutions and propounded a polarimetric method to measure noncyclic phases. Interferometrically, the noncyclic phase ought to be determined from the *shift* [4,9,10] between U(1) interference patterns recorded without and with the Hamiltonian effecting the required evolution. In this Letter, we present the first observation of the noncyclic phase for neutrons and the associated amplitude of interference.

Thermal neutrons of speed v_0 in an incident polarization state $\psi_0 = \cos(\theta/2)\,|z\rangle + \sin(\theta/2)\,|-z\rangle$, say, subjected to a field $B\hat{\mathbf{z}}$ over a path length l undergo the SU(2) operation $\exp(-i\sigma_z\phi_L/2)$ [11]. This evolution effects a precession $\phi_L = -2\mu Bl/\hbar v_0$ of the unit spin vector $\mathbf{s} = \mathrm{Tr}\rho\boldsymbol{\sigma}$ on a cone of polar angle θ about $\hat{\mathbf{z}}$. Here σ_z denotes the z component of the vector $\boldsymbol{\sigma}$ of the Pauli spin operators, μ signifies the neutron magnetic moment, and $\rho = \psi\psi^\dagger/\psi^\dagger\psi$ stands for the pure state density operator. Mezei's formalism [12], based on the exact evaluation of the phase shift due to the Zeeman term, can also be used to obtain the resultant state. The Pancharatnam connection [1–4] prescribes the phase Φ and interference amplitude $\mathcal{A}$ for this evolution as $\mathcal{A}e^{i\Phi} = \mathrm{Tr}\rho_0 e^{-i\sigma_z\phi_L/2}$, i.e.,

$$\tan\Phi = -\tan\frac{\phi_L}{2}\cos\theta \tag{1}$$

and

$$\mathcal{A} = \sqrt{1 - \sin^2\theta\sin^2\frac{\phi_L}{2}}\,. \tag{2}$$

The phase Φ has a dynamical component [13,14]

$$\Phi_D = \int \mu\mathbf{s}\cdot\mathbf{B}\,dt/\hbar = -\frac{\phi_L}{2}\cos\theta \tag{3}$$

proportional to the integral of the component of the magnetic field along the spin direction and a geometric component [1,3,15–18] $\Phi_G = \Phi - \Phi_D = -\Omega/2$. Here Ω represents the solid angle spanned on the spin sphere by the closed curve obtained by joining the ends of the arc traced on the θ cone by $\mathbf{s}$ with the shorter (than π) geodesic, i.e., a great circle arc here.

The amplitude $\mathcal{A}$ equals unity for cyclic evolutions, wherein the final spin coincides with the initial spin $\mathbf{s}_0$. This occurs with $\theta = 0°$ or $180°$ for all precessions ϕ_L and with integral revolutions $\phi_L(\text{degree})/360$ for all θ.

The noncyclic phase Φ and amplitude $\mathcal{A}$ are measured interferometrically [4,18] from the relative shift and attenuation, respectively, between interference patterns

$$I(\chi,\phi_L = 0) \propto D + \cos\chi\,, \tag{4}$$

say, and

$$I(\chi,\phi_L) \propto D + \cos\chi\cos\frac{\phi_L}{2} + \sin\chi\sin\frac{\phi_L}{2}\cos\theta$$
$$= D + \mathcal{A}\cos(\chi + \Phi)\,, \tag{5}$$

recorded without and with the field $\mathbf{B}$, respectively. Here each pattern is generated by varying a U(1) phase χ. If the incident beam has a polarization P less than unity, the observed phase and amplitude are given by

$$\tan\Phi_{\mathrm{obs}} = P\tan\Phi \tag{6}$$

 0031-9007/98/81(10)/1992(4)$15.00

and

$$\mathcal{A}_{\text{obs}} = \mathcal{A}\sqrt{1 - (1 - P^2)\sin^2\Phi}\,. \tag{7}$$

We have determined the phase difference and amplitude ratio between the evolution (θ, ϕ_L) and a reference evolution (θ_R, ϕ_{LR}), say, from their interference patterns.

The experiment was carried out at the V9 interferometry setup [19] in the Berlin Neutron Scattering Center (BENSC) of the Hahn-Meitner-Institut, Germany. A monochromated neutron beam of 2 Å wavelength was polarized by a V-shaped Co-Fe-Si magnetic supermirror based transmission polarizer [20]. The transmitted beam was down polarized, i.e., with the spin oriented antiparallel ($\theta = 180°$) to the vertical ($\hat{\mathbf{z}}$) guide field. By means of a Heusler crystal analyzer downstream of the interferometer and a dc spin flipper (cf. Fig. 1) immediately following the polarizer, the beam polarization P was determined to be about 92%. The spin flipper was Brookhaven-type, ideally suited to operate as a spin rotator. The angle θ subtended by the emergent neutron spin with $\hat{\mathbf{z}}$ varied linearly with the current in the horizontal field coil of the flipper at $-101.6°/\text{A}$ from 180° to $-180°$. A 5 mm wide and 6 mm high polarized neutron beam illuminated the skew symmetric LLL Si (220) interferometer, placed in a guide field of about 45 G, produced by permanent magnet devices.

The z-field gadget for introducing the noncyclic spin precession was fabricated and tested at BARC, Mumbai. It consisted of a coil of 0.9 mm thick enameled copper wire wound on a hollow C-shaped copper tube ending in two 12 mm $\times$ 12 mm square pole pieces about 8 mm apart. The gadget field was proportional to the coil current I and uniform to within a few percent over the required (6 mm high, 8 mm wide) beam cross section. The integrals $\int B_z\, dl$ due to this gadget along the proposed paths **1** and **2** of the interferometer differed by about 23.2 G cm at $I = 1$ A, corresponding to an excess spin precession of $123°/\text{A}$ for 2 Å neutrons. The gadget suspended in path **1** of the interferometer (Fig. 1), consumed about 1.4 W at $I = 2$ A and was maintained at the ambient temperature with a closed-cycle water flow coupled with a programmable fuzzy logic controller. No magnetic material was used in this gadget, since it would distort the ambient guide field. The gadget just added its own field and produced an excess spin precession proportional to the current I.

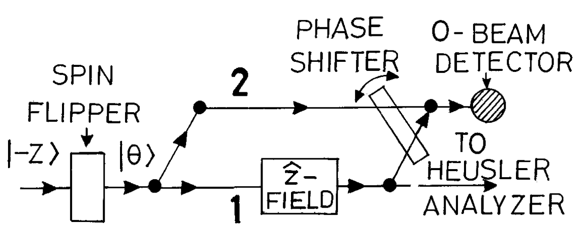

FIG. 1. Experimental arrangement (schematic). A magnetic guide field is applied along $\hat{\mathbf{z}}$, transverse to the plane of the diagram. The spin flipper brings the spin of the incident monochromatic $-\hat{\mathbf{z}}$ polarized neutrons to an angle θ from $\hat{\mathbf{z}}$. An O-beam interference pattern is obtained by rotating the phase shifter for a given additional $\hat{\mathbf{z}}$ field introduced on path **1** of the skew symmetric interferometer.

The intensity of the outgoing O beam was about 1 count/s. The interference patterns were recorded by rotating a 5 mm thick silicon phase shifter (Fig. 1) in the interferometer to vary the scalar (nuclear) phase χ [21,22]. For cyclic evolutions, the interference contrast was about 32% without, and 40% with, background subtraction. Phases were measured for states with $\theta = 0°$, 70.5°, 90°, 109.5°, and 180°.

During each run, the current I in the field gadget and hence the spin precession ϕ_L, was held constant and two interference patterns were generated simultaneously for two incident states, θ and the reference θ_R (0° or 180°). At each angular setting of the phase shifter, the O-beam intensity was measured successively for two preset currents in the flipper coil, appropriate for θ and θ_R. The shift between these interference patterns eliminated U(1) phases and phase drifts if present. Phase shifts much larger than the experimental errors were ensured by setting θ_R at 180° for $\theta = 0°$ and 70.5°, but at 0° for $\theta = 90°$ and 109.5°. Because of the constant current I in each run, the thermal environment of the interferometer remained steady, producing good quality interferograms. Clean interferogram pairs were likewise recorded for a given θ with gadget currents $+I$ and $-I$. Attempts to generate three simultaneous patterns with currents $+I$, 0, and $-I$, however, yielded erratic results (except for the lowest $I = 0.4$ A) due to the thermal disturbances introduced by the switching off and on of the current.

The difference between spin precessions on paths **1** and **2** of the interferometer varies as $\phi_L = \phi_L^0 + CI$, with the gadget current I, ϕ_L^0 being the residual precession arising from the nonuniformity of the guide field. Using ϕ_L^0, C, and the incident polarization P as parameters, we fitted the observed phase shifts between 52 pairs of interference patterns, 38 recorded for θ pairs at fixed I, and 14 at fixed θ for I pairs, to the expected phases [Eqs. (1),(6)]. The least-squares fit yielded the parameter values $\phi_L^0 = 55.9° \pm 1.6°$, $C = 123.7 \pm 1.2$ degree/A and $P = (92.3 \pm 3.5)\%$. The last two parameters are in excellent agreement with the values 123 degree/A and 92% inferred, respectively, from the gadget field mapping and polarization analysis.

The observed phase shifts were corrected for the incomplete incident beam polarization P by numerically inverting the functional relation between shifts in Φ_{obs} and Φ obtained from Eq. (6). For simultaneous interferograms recorded at fixed ϕ_L for θ pairs, the reference evolution ($\theta_R = 0°$ or 180°) being cyclic, the ratio of amplitudes, duly corrected for the deviation of P from unity, equals the interference amplitude $\mathcal{A}$ for the noncyclic evolution (θ, ϕ_L).

The corrected phases and amplitudes for four θ pairs depicted against ϕ_L in Figs. 2 and 3, respectively, agree

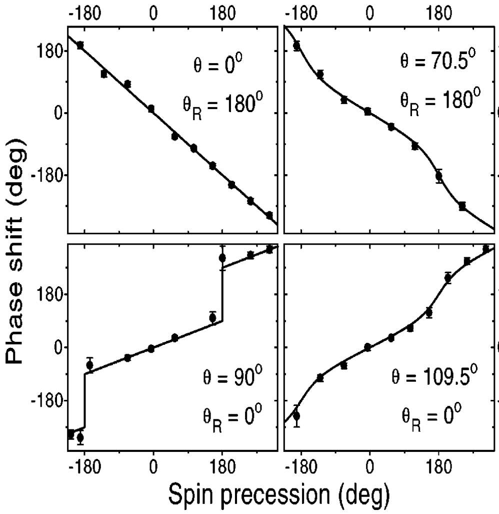

FIG. 2. Corrected phase shifts between incident states with spin angles θ and θ_R as a function of the precession ϕ_L. Solid curves represent the theoretical phases. Note large error bars for noncyclic evolutions near $\phi_L = \pm 180°$ due to the reduced interference amplitude (cf. Fig. 3).

with theory (smooth curves) to within the error bars. The phase difference between $\theta = 0°$ and $\theta_R = 180°$ states just equals $-\phi_L$. The phase differences for θ angles 70.5°, 90°, and 109.5° also reproduce the predicted nonlinear relations. At $\phi_L = \pm 180°$, $\mathcal{A}$ becomes minimum (cf. Fig. 3), equal to $|\cos\theta|$, implying maximum noncyclicity. This reduced interference contrast near $\phi_L = \pm 180°$ (see also the lower pattern of Fig. 4), lowers the precision of phase determination, and causes relatively large error bars on the measured phase shifts (Figs. 2 and 4) for $\theta = 70.5°$, 90°, and 109.5°.

At $\theta = 90°$, $\mathrm{Tr}\rho_0 \exp(-i\sigma_z\phi_L/2) = \cos(\phi_L/2)$ is real, changing sign across odd integral values of $\phi_L/180$. This corresponds to an amplitude $\mathcal{A} = |\cos(\phi_L/2)|$ (Fig. 3) and a staircase function [4], of 180° high and 360° long steps, for the phase. Here the dynamical phase vanishes identically [Eq. (3)]. The spin precesses along the equator, i.e., a geodesic, spanning the angle ϕ_L on the spin sphere. For $-180° < \phi_L < 180°$, the geodesic traversed is shorter than π and the shorter geodesic between its ends just retraces it. Hence the closed curve encloses no solid angle and yields a null geometric phase. The total phase acquired by the $\theta = 90°$ state over this ϕ_L range is hence zero. At $\phi_L = -180°$ or 180°, an infinite number of geodesics each of length π can be drawn between the ends $-\mathbf{s}_0$ and $\mathbf{s}_0$ of the traversed arc, rendering Ω, Φ_G, and hence Φ indeterminate. Here the initial and final states of the evolution being mutually orthogonal ($\mathcal{A} = 0$), do not interfere. When ϕ_L crosses $-180°$ or 180°, the shorter geodesic closing the arc lies on the other side and completes the equator, enclosing a hemisphere ($\Omega = \pm 2\pi$) to yield a jump [18,23,24] $\mp 180°$ in $\Phi = \Phi_G$. This phase jump manifests itself as a shift between the interferogram pair (Fig. 4) for $\theta = 90°$ recorded with $I = 1.2$ and -1.2 A, i.e., $\phi_L = 204.3°$ and $-92.5°$. The difference between the staircase phase for $\theta = 90°$ and the phase $-\phi_L/2$ for $\theta_R = 0°$ climbs a sloped step function (Fig. 2), as verified in this experiment.

Thus over the θ domain, we have one extreme, viz. $\theta = 0°$ or 180° of cyclic evolutions yielding unattenuated interferograms ($\mathcal{A} = 1$) with dynamical phases $-\phi_L/2$ or $\phi_L/2$. In the other extreme of $\theta = 90°$, the interference pattern just gets modulated by $\cos(\phi_L/2)$, implying an attenuation $\mathcal{A}$ and geometric phase jumps of 180°. At intermediate θ angles, both Φ and $\mathcal{A}$ vary with ϕ_L.

This experiment contradicts the commonly held view that the phase is one-half the Larmor precession angle, for all incident angles θ. Casella and Werner [6] ascribed the interference oscillations as a function of ϕ_L (with $\chi = 0$) to a "phase" $\phi_L/2$. As seen by substituting $\chi = 0$ in Eq. (5), these oscillations vary as [4] $D + \mathcal{A}\cos\Phi$.

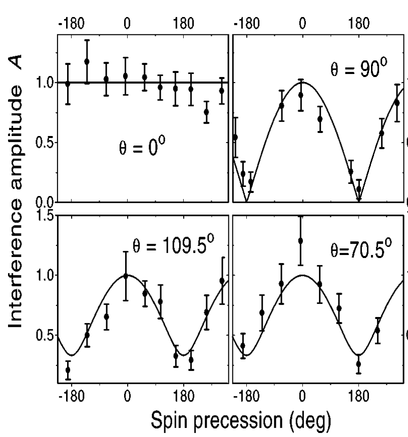

FIG. 3. Corrected interference amplitudes for four incident spin angles θ. The smooth curves are the theoretical predictions.

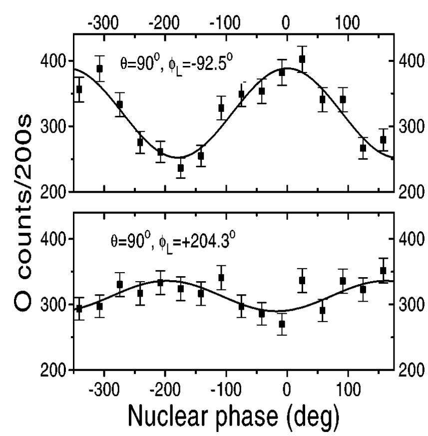

FIG. 4. Interference patterns recorded with $I = 1.2$ and -1.2 A in the z-field gadget for $\theta = 90°$, display a phase shift $197° \pm 17°$ against the expected 180° phase jump occurring across $\phi_L = 180°$. The smooth curves are the best sinusoidal fits.

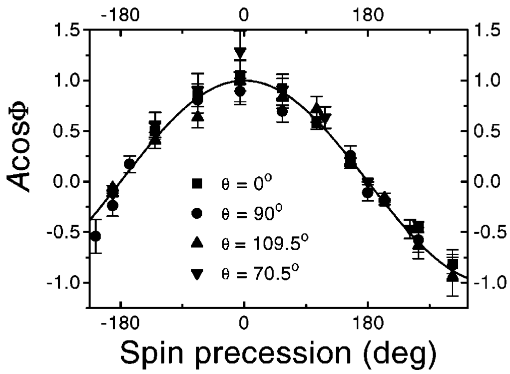

FIG. 5. The products of the observed interference amplitudes and cosines of the corresponding phases for four incident angles θ, all lie close to the single curve $\cos(\phi_L/2)$.

This quantity $\mathcal{A}\cos\Phi = \mathrm{Tr}\ \exp(-i\sigma_z\phi_L/2)/2 = \cos(\phi_L/2)$ has been misinterpreted as "$\cos\Phi$." Though $\mathcal{A}$ and Φ depend individually on θ (Figs. 2 and 3), $\mathcal{A}\cos\Phi$ is θ-independent [4]. The values $\mathcal{A}\cos\Phi$ computed from the observations for $\theta = 0°$, 70.5°, 90°, and 109.5° are plotted in Fig. 5. They all lie close to the single curve representing $\cos(\phi_L/2)$, as expected. This implies that if we record interference oscillations with $\chi = 0$, all states θ will yield a single curve $D + \cos(\phi_L/2)$, though the phase acquired does depend on θ.

An unpolarized incident beam would produce interference patterns (5) identical to those for $\theta = 90°$. However, an unpolarized beam is an *incoherent* mixture in equal proportions of an *arbitrarily* selected pair of orthogonal states. The corresponding interference pattern is the sum of intensities in the individual patterns of equal and opposite phases for the two constituent states [cf. Eq. (5)]. The modulation of the interferogram here originates from the product $\mathcal{A}\cos\Phi = \cos(\phi_L/2)$ for the constitutent states. It was this modulation that the 4π symmetry experiments [25–27] observed with unpolarized neutrons. These experiments did verify the sign change of the spinor wave function for a 360° precession. However, they do *not* constitute a measurement of a phase $\phi_L/2$ for unpolarized neutrons, since no specific wave function ψ_0 and hence no phase Φ can be assigned to unpolarized neutrons [18]. The only phase unpolarized neutrons may acquire is the U(1) phase. Only a pure θ-polarized state can acquire a definite SU(2) phase. Using an incident beam polarized along the magnetic field direction, Badurek *et al.* (cf. Fig. 2 in [28]) effected cyclic evolutions ($\mathcal{A} = 1$). Their $\chi = 0$ interferograms therefore indeed measured a phase $\phi_L/2$.

To summarize, we have measured phases as well as interference amplitudes for noncyclic evolutions of polarized neutrons in magnetic fields. This experiment unfolds the physics of interference between distinct quantal states.

We thank Professor H. Rauch of Atominstitut, Vienna for loaning the skew symmetric interferometer used in this experiment. P. F. and A. I. like to thank Professor F. Mezei for fruitful discussions on the subject. A. G. W. acknowledges local hospitality and partial travel support received from BENSC, HMI.

*Electronic address: nintsspd@magnum.barc.ernet.in
†On leave from St. Petersburg Nuclear Physics Institute, Gatchina, Leningrad distr., 188350 Russia.

[1] S. Pancharatnam, Proc. Indian Acad. Sci., Sect. A **44**, 247 (1956).
[2] M. V. Berry, J. Mod. Opt. **34**, 1401 (1987).
[3] J. Samuel and R. Bhandari, Phys. Rev. Lett. **60**, 2339 (1988).
[4] A. G. Wagh and V. C. Rakhecha, Phys. Lett. A **197**, 107 (1995).
[5] H. Weinfurter and G. Badurek, Phys. Rev. Lett. **64**, 1318 (1990).
[6] R. C. Casella and S. A. Werner, Phys. Rev. Lett. **69**, 1625 (1992).
[7] R. C. Casella, Phys. Rev. Lett. **73**, 2941 (1994).
[8] A. G. Wagh and V. C. Rakhecha, Phys. Lett. A **197**, 112 (1995).
[9] A. A. Michelson and E. W. Morley, Am. J. Sci. **34**, 333 (1887); Philos. Mag. **24**, 449 (1887).
[10] A. G. Wagh and V. C. Rakhecha, Phys. Rev. Lett. **78**, 1399 (1997).
[11] A. G. Wagh, Phys. Lett. A **146**, 369 (1990).
[12] F. Mezei, Physica (Amsterdam) **151B**, 74 (1988).
[13] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).
[14] A. G. Wagh and V. C. Rakhecha, Phys. Lett. A **148**, 17 (1990).
[15] M. V. Berry, Proc. R. Soc. London A **392**, 45 (1984).
[16] *Geometric Phases in Physics*, edited by A. Shapere and F. Wilczek (World Scientific, Singapore, 1989).
[17] M. Berry, Phys. Today **43**, No. 12, 34 (1990).
[18] A. G. Wagh and V. C. Rakhecha, Prog. Part. Nucl. Phys. **37**, 485 (1996).
[19] G. Drabkin, A. Ioffe, S. Kirsanov, F. Mezei, and V. Zabijakin, Nucl. Instrum. Methods Phys. Res., Sect. A **348**, 198 (1994).
[20] A. Ioffe, P. Fischer, T. Krist, and F. Mezei, J. Phys. Soc. Jpn. Suppl. A **65**, 80 (1996).
[21] G. Badurek, H. Rauch, A. Zeilinger, W. Bauspiess, and U. Bonse, Phys. Rev. D **14**, 1177 (1976).
[22] J. Summhammer, G. Badurek, H. Rauch, U. Kischko, and A. Zeilinger, Phys. Rev. A **27**, 2523 (1983).
[23] R. Bhandari, Phys. Lett. A **157**, 221 (1991).
[24] A.G. Wagh, V.C. Rakhecha, J. Summhammer, G. Badurek, H. Weinfurter, B.E. Allman, H. Kaiser, K. Hamacher, D.L. Jacobson, and S.A. Werner, Phys. Rev. Lett. **78**, 755 (1997).
[25] H. Rauch, A. Zeilinger, G. Badurek, A. Wilfing, W. Bauspiess, and U. Bonse, Phys. Lett. **54A**, 425 (1975).
[26] S.A. Werner, R. Colella, A.W. Overhauser, and C.F. Eagen, Phys. Rev. Lett. **35**, 1053 (1975).
[27] A.G. Klein and G.I. Opat, Phys. Rev. Lett. **37**, 238 (1976).
[28] G. Badurek, H. Rauch, and J. Summhammer, Physica (Amsterdam) **151B**, 82 (1988).