Kernel Regression Estimator for Damage States of Tunnel Lining Concrete

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ABSTRACT: This study evaluates the best estimator for the damage state in the spatial domain and predict the damage growth in temporal domain by the Kernel regression method of conditional random fields. Compared to the classical Kriging of lognormal field approach, the Kernel regression method indicate better performance in specific cases of big data sets.

Tunnel infrastructure provides essential transport functions to the society. Existing highway and rail transit tunnels are gradually becoming functional obsolete and structurally deficient. Efficiently monitoring the structural health of tunnel infrastructure is a socially important challenge.

So far, the monitoring of concrete structure has been done by visual inspections. Inspection operations have been carried out last several decades, such as crack width, crack length and crack expanse in each tunnel lining concrete panel. The severity of concrete lining deterioration is related to the risk of failure, such as a concreate chip fall, leaking of backwater.

Alternative way to prevent such accidents, deteriorated lining surface areas are temporary covered by steel plates or stopgap repair mortars. Regarding the maintenance/repairmen works of tunnel lining concrete, it is required to evaluate the damage degree at unobservable lining concrete panels.

The task of interpolating observed data has a long history. A standard method for estimating of unobserved lining concrete surface is Kriging method. Although Kriging (Perdikaris et.al) is often a highly competitive predictor, it may fail when the sample size of the observations is very small or when the simulation yields non-stationary behavior.

Here we present a new approach to data interpolation which is related to kernel regression estimators and which performs well for small sample sizes in lower dimensions.

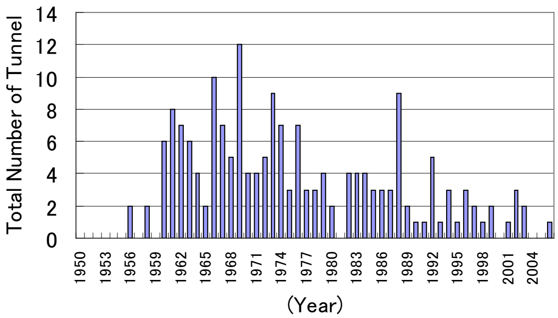


Figure 1: Construction Year in Hokkaido Island

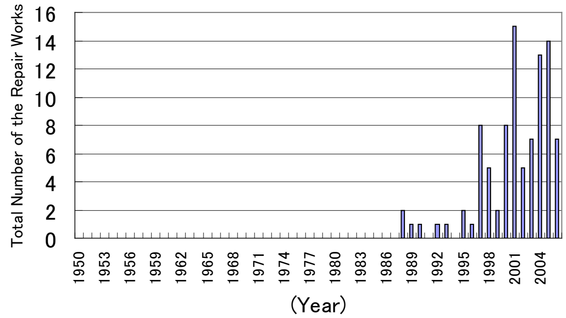


Figure 2: Repair/Maintenance Works

This study evaluates the best estimator for the damage state in the spatial domain and predict the damage growth in temporal domain by the Kernel regression method of conditional random fields. Compared to the classical Kriging of Lognormal field approach, it shows a better performance in specific cases of small data sets and data with non-stationary behavior.

1. TUNNEL LINING INSPECTION

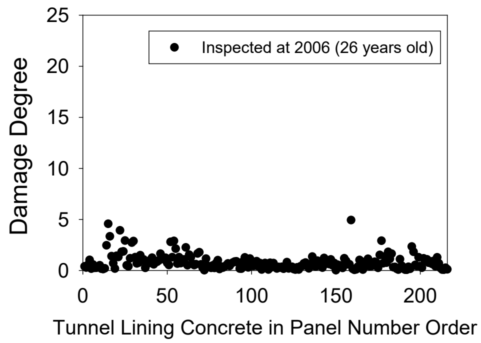
Fig.1 shows total number of tunnels by the construction year in Hokkaido Island. It is observed, many tunnels have been constructed at 1959-1980. Fig.2 shows the total number of repair/maintenance works in each year. It should be noted that recently repair/maintenance works are rapidly increasing. The cost of repairs is expected to increase as these facilities continue to age. It has been done, rehabilitation decisions, scheduling and budgeting were based primarily on the results from visual lining concrete inspections.

Inspection works have been carried out, such as crack width, crack length and crack expanse of over 180 tunnels in Hokkaido Island. The inspection results quantified to the damage degree as 0.0(no damage) to large number (critical damage). For example, Fig.3 shows each lining damage degree of longitudinal direction and probability density function of damage degree in GAMATA tunnel with biennial inspection operations from 2006 to 2012. It is observed that the lognormal distribution conform to the inspection data distribution. The width of a lining concrete is 10m. The total length of the GAMATA Tunnel is over 2 km.

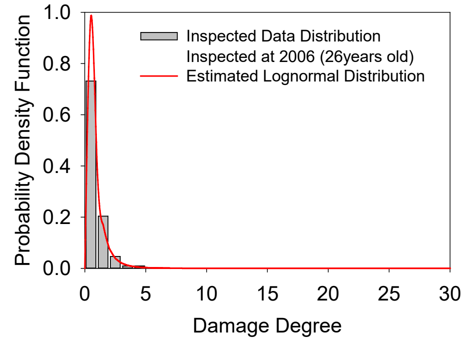
2. STOCHASTIC INTERPOLATION OF A LOGNORMAL FIELD

The deteriorated lining surface areas are temporary covered by steel plates or stopgap repair mortars in order to prevent a rare possibility such as a concreate chip fall, leaking of backwater. This study evaluates the damage degree of the temporary covered lining concrete panels by the stochastic interpolation method of conditional lognormal fields (Maruyama & Hoshiya 2008).

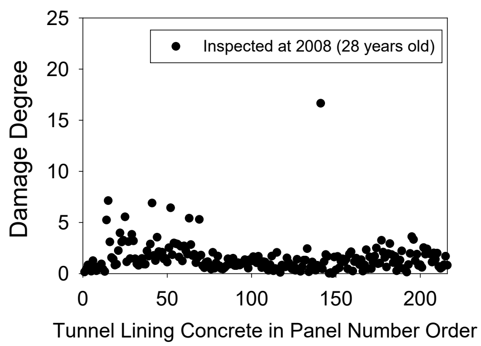
We are concerned with the interpolation of a spatial random field at discrete points. Let damage degree $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots \boldsymbol{x}_i, \cdots, \boldsymbol{x}_n]^T$ be a lognormal variable with the following characteristics: mean $E[\mathbf{X}]$, variance $Var[\mathbf{X}] = \sigma^2$, stationary covariance $C(x_i, x_j) = \sigma^2 \exp(-|x_i - x_j|/a)$. $\mathbf{Z} = \ln \mathbf{X} = [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_n]$ is then a normal variable with the following characteristics, mean $E[\mathbf{Z}] = \ln E[\mathbf{X}] - \sigma'^2/2$, variance $Var[\mathbf{Z}] = \sigma'^2 = \ln(1 + \sigma^2/E[\mathbf{X}]^2)$, statinonary covariance $K(z_i, z_j) = \ln(1 + C(x_i, x_j)/E[\mathbf{X}]^2)$.



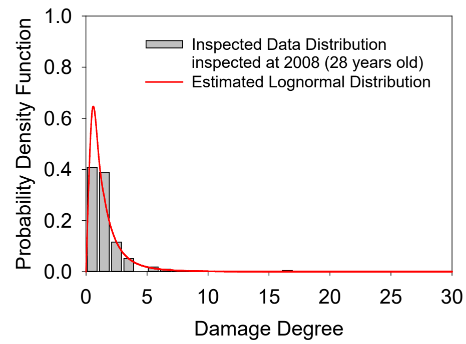
(a) Inspected Data (26 years old)



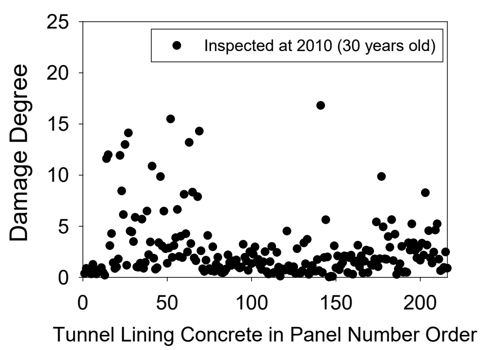
(b) Probability Density Function (26 years old)



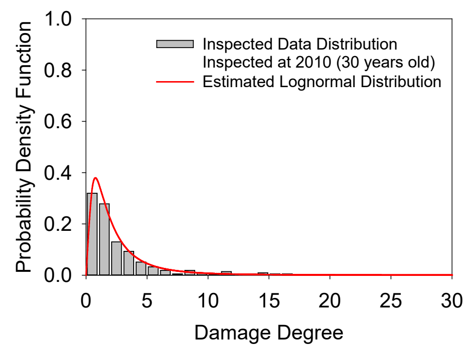
(c) Inspected Data (28 years old)



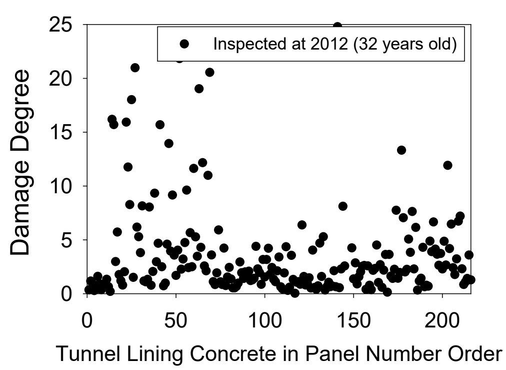
(d) Probability Density Function (28 years old)



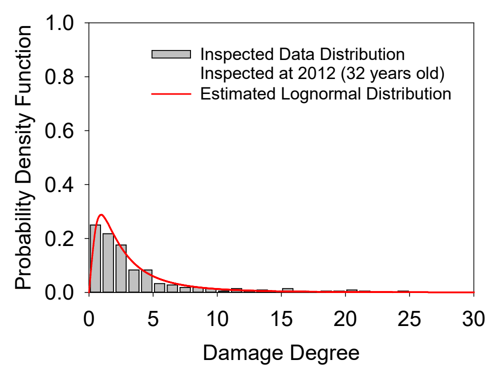
(e) Inspected Data (30 years old)



(f) Probability Density Function (30 years old)



(f) Inspected Data (32 years old)



(b) Probability Density Function (32 years old)

Figure 3: GAMATA Tunnel

Then $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \cdots \mathbf{z}_i, \cdots, \mathbf{z}_n]^T$ is a spatial Gaussian random field with pdf $p(\mathbf{Z})$, where $\mathbf{z}_i$ is a random quantity at the i th discrete spatial point, and the posterior field with pdf $p(\mathbf{Z} \mid \mathbf{Y})$ will be estimated, after an observation $\boldsymbol{Y} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_\ell]^T$ is carried out on $\mathbf{Z}$. The observation is not necessarily made on every corresponding $\mathbf{z}_i$.

Under this condition, we may represent a linear observation equation by

$$\mathbf{Y} = \mathbf{HZ} + \mathbf{w} \tag{1}$$

where $\mathbf{w}$ = observation noise vector of $\ell \times 1$ with pdf $\boldsymbol{r}(\boldsymbol{w})$.

In order to obtain an optimal estimate on the spatial field $\mathbf{Z}$ after the observation of $\boldsymbol{Y} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_\ell]^T$, the following conditional probability $p(\mathbf{Z} \mid \mathbf{Y})$ for updating or filtering after processing the observation data become a basic formulas. For the updating or filtering, the conditional probability law gives

$$p(\mathbf{Z} \mid \mathbf{Y}) = \frac{p(\mathbf{Y} \mid \mathbf{Z}) p(\mathbf{Z})}{\int p(\mathbf{Z}) p(\mathbf{Y} \mid \mathbf{Z}) d\mathbf{Z}} \tag{2}$$

Equation (2) is the basis for fundamental discussions on updating or filtering of stochastic fields. Equation (2) indicates that the conditional pdf $p(\mathbf{Z} \mid \mathbf{Y})$ can be obtained if $p(\mathbf{Z})$ is known, and if $p(\mathbf{Y} \mid \mathbf{Z})$ is evaluated.

Thus, a prior field is assumed to have a Gaussian field, and then a posterior pdf analytically obtained by the updating theory of Gaussian field. $\mathbf{Z}$ be a spatial Gaussian field with the prior mean field $E[\mathbf{Z}]$ and covariance matrix $K(z_i, z_j)$ of $\mathbf{Z}$ are given, and the mean and the covariance of observation noise $\boldsymbol{w}$ are respectively zero and $\boldsymbol{R}$, then conditional Gaussian field of Eq.(2) can be expressed in an explicit expression. An optimum estimate of the state vector is then sought, such as $\mathbf{Z}$ that minimizes the following objective function.

$$\begin{aligned} J = (\mathbf{Z} - E[\mathbf{Z}])^T \mathbf{K}^{-1} (\mathbf{Z} - E[\mathbf{Z}]) \\ + (\mathbf{Y} - \mathbf{HZ})^T \mathbf{R}^{-1} (\mathbf{Y} - \mathbf{HZ}) \end{aligned} \tag{3}$$

To evaluate the optimum estimate by minimizing J, following equation must be evaluated.

$$\frac{\partial J}{\partial \mathbf{Z}} = \mathbf{M}^{-1}(\mathbf{Z} - \overline{\mathbf{Z}}) - \mathbf{H}^T \mathbf{R}^{-1} (\mathbf{Z} - \mathbf{HZ}) \tag{4}$$

Estimated conditional mean and conditional covariance matrix in Gaussian field are given as

$$\hat{\mathbf{Z}} = \overline{\mathbf{Z}} + \mathbf{PH}^T \mathbf{R}(\mathbf{Y} - \mathbf{H}\overline{\mathbf{Z}}) \tag{5}$$

$$\mathbf{P}^{-1} = \mathbf{M}^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \tag{6}$$

Then, estimated conditional mean $\hat{\mathbf{Z}}$ and conditional variance $\mathbf{P}$ in Gaussian field transformed into the lognormal fileld.

3. KERNEL INTERPOLATION

Many linear models for regression and classification can be reformulated in terms of a dual representation in which the kernel function arises naturally.

$$z(x) = \sum_{i=0}^{M-1} w_i \varphi_i(x) = \mathbf{W}^T \phi(x) \tag{7}$$

We consider a linear regression model whose parameters are determined by minimizing a regularized sum-of-squares error function given by

$$J(w) = \frac{1}{2} \sum_{n=1}^{\ell} \left[\mathbf{W}^T \phi(x_n) - y_n \right]^2 + \frac{\lambda}{2} \mathbf{W}^T \mathbf{W} \tag{8}$$

where $\lambda \geq 0$. If we set the gradient of $J(\mathbf{W})$ with respect to **w** equal to zero, we see that the solution for **w** takes the form of a linear combination of the vectors $\boldsymbol{\varphi}(\mathbf{x}_n)$, with coefficients that are functions of **w**, of the form

$$\mathbf{K} = \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) & \cdots & k(x_1,x_\ell) \\ k(x_2,x_1) & k(x_2,x_2) & \cdots & k(x_2,x_\ell) \\ \vdots & \vdots & \cdots & \vdots \\ k(x_\ell,x_1) & k(x_\ell,x_2) & \cdots & k(x_\ell,x_\ell) \end{bmatrix} \quad (9)$$

where $k(x_i, x_j)$ is the kernel function.

In this study, we employ the Gaussian kernel function as

$$k(x_i, x_j) = \exp(-b\left\|x_i - x_j\right\|^2) \quad (10)$$

We obtain the following solution for kernel based interpolation (Bishop, 2006).

$$z(x) = \mathbf{k}(x)^T (\mathbf{K} + \lambda \mathbf{I}_N)\mathbf{Y} \quad (11)$$

4. NUMERICAL EXAMPLE

A stochastic interpolation of a one-dimensional spatial lognormal field and kernel interpolation method are investigated by using an inspected data in GAMATA tunnel (2012). First, unconditional lognormal field has LN(4.18,4.8) is estimated by inspected data in Figure 3(f), where the first argument 4.18 is the mean, and the second argument 4.8 is the standard deviation. Further, figure 4 shows the estimated spatial correlation coefficient ρ_{ij} of x_i and x_j. In the numerical analysis, no observation noise is considered.

Next, kernel interpolation method applied for the same data. In the numerical analysis, unconditional/apriori information is given by unit matrix as shown in Eq. (11). This means,

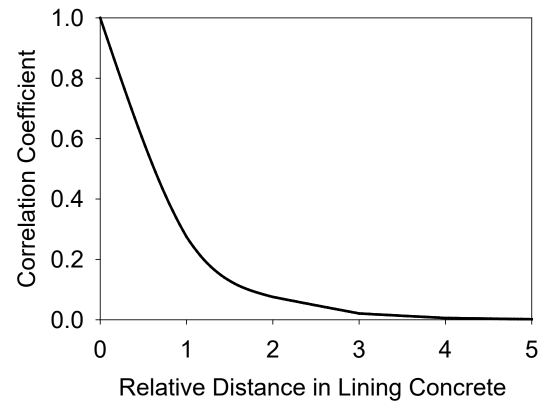


Figure 4: Spatial Correlation

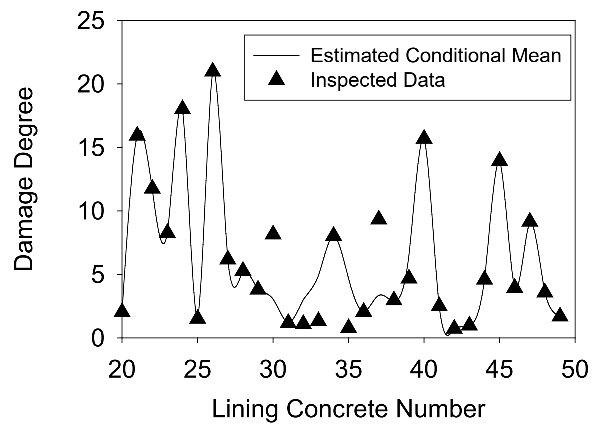


Figure 5: Conditional Mean by Lognormal Kriging

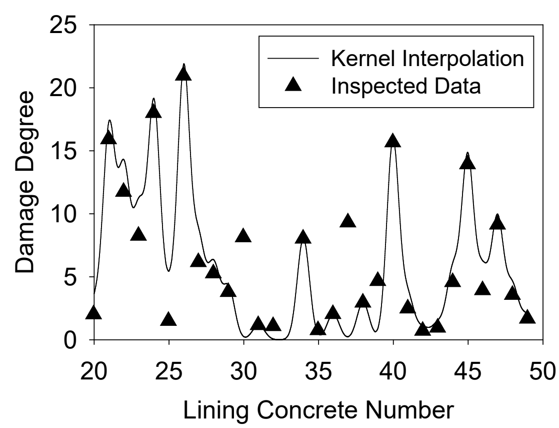


Figure 6: Estimated by Kernel Interpolation

unconditional mean value and unconditional covariance are not consider.

First example is an interpolation of the spatial field in GAMATA tunnel (2012). Figures 5 and 6 show the conditional mean of lognormal field and estimated value of kernel method, respectively. In figures 5 and 6, the total number of interpolation points is assumed to be 30, where observation data taken from #21 to #30, #32, #35, #37, #39, #31 to #49. The results indicated that estimated values by both method consistent with the observation data.

The results unobserved points of kernel method is depend on the kernel function.

Second example is consider the spatial and temporal fields by kernel interpolation method. In this case, observation data taken from all inspected data from 2006 to 2012, then attempt to estimate the after two years damage degree of all lining concrete. Figure 7 shows observation data in which 1 year indicate 2006, in the same way 4 year indicate 2012.

Figure 8 shows an interpolated value indicated black circle in spatial and temporal domain, based on the inspected data form 2006 to 2012. On the other hand red circles are extrapolated/predicted damage state 2013 and 2014 in temporal direction. In the numerical analysis, $\lambda = 0.001, b = 3.0$ is used for the kernel interpolation.

As for the kriging of lognormal method is difficult to evaluate covariance function of temporal domain.

5. CONCLUSIONS

This study compare two methods of evaluate the damage state of tunnel lining concrete. The Kernel regression method indicate better performance in specific cases of big data sets.

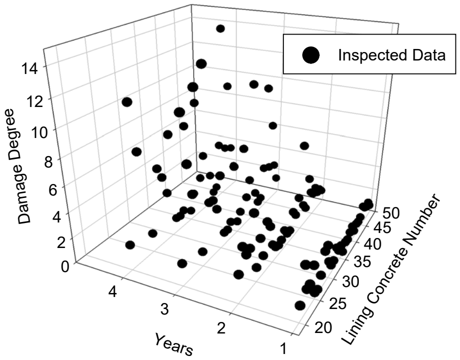


Figure 7: Inspected Data

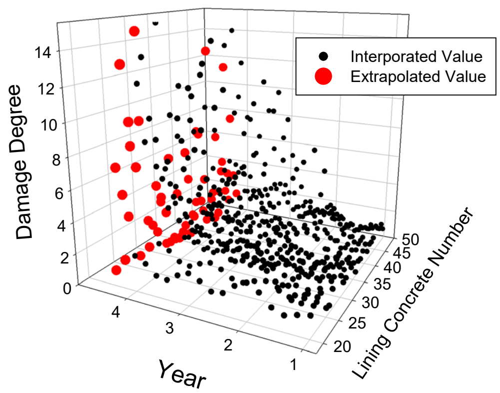


Figure 8: Estimated by Kernel Interpolation

6. REFERENCE

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