Fatigue reliability using a multiple surface approach

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ABSTRACT: Reliability analysis for offshore wind turbines' fatigue is an effort demanding task. New trends in the design of these systems, such as, the usage of alternative computational fluid dynamics or finite element methods, are expected to further increase the effort needed to design these systems to fatigue. As a result, design techniques that enable practicable fatigue analysis are on demand.
The present paper researches on how to use fatigue damage surfaces in order to assess stress-cycle fatigue reliability. A Gaussian process model is applied as a surrogate of fatigue damage. It allows to enclose multiple normally distributed interpolated surfaces. Probabilistic SN curves are considered, creating a double surface model, where the Gaussian process model is built on top of the curve. Analysis is performed on a 5MW turbine with a monopile foundation, and stress-cycle fatigue is assessed for the tower component.
Results of the implementation show that there is a significant advantage in using surrogates of fatigue damage as only a limited number of time domain simulations is required for design. Fatigue design was assessed using a subset of 25 load cases. Moreover, the predictor surrogates accurately the design procedure within different material probabilistic characteristics, and accounting for loading uncertainty. Fatigue reliability assessment with these models may be performed with approximately 10% to 40% of the binned environmental conditions computational effort, which is of interest in the wind engineering sector.
Nevertheless, the approach implemented may be applied to any component on any system, with the only requirement of defining a representative fatigue indicator.

1. Introduction

Reliability analysis for offshore wind turbine (OWT) structural fatigue is a resource demanding task. Fatigue design requires the assessment of multiple operational scenarios that depend on the different external conditions that load the OWT.

Furthermore, trends in the simulation of OWTs indicate that the complexity in their analysis is expected to increase in the future. Usage of alternative computational fluid dynamics and finite element methods, is expected to increase the effort required to design OWT. In the particular case of fatigue, design techniques that enable practicable fatigue reliability analysis are demanded.

The current paper researches on the usage of stress-cycle(SN) fatigue damage surfaces in order to assess structural fatigue reliability. The SN damage surfaces are built using a Gaussian process model that is capable of enclosing multiple normally distributed interpolated surfaces. These work as probabilistic surrogates of the system's operational SN fatigue. Therefore, implementation of a Gaussian predictor as an interpolator of SN fatigue allows to sample multiple design surfaces, where each generated surface encloses a statistically feasible full design assessment accordingly to Design Load Case (DLC) 1.2 of IEC61400 (IEC, 2005, 2009).

The advantage of applying the methodology researched is related to the need to assess only a limited number of time domain simulations, inferior to the expected number imposed by the standards, in order to assess the OWT fatigue design. These simulations are mainly needed to characterize locally the probabilistic behaviour of the loading.

The Gaussian process model, jointly with the probabilistic SN curve, generates probabilistic designs within different material characteristics. The inherent probabilistic behaviour of the structural fatigue design procedure is replicated, and the reliability of the studied OWT component quantified.

In order to enable the comprehension on how to apply Gaussian process predictors to evaluate the SN fatigue reliability for OWT, in the particular case of the tower component analysis, the following article is organised as follows; Section 2 presents a major overview on how the usage of these meta-models for reliability analysis discussing, the previous works on reliability analysis, the SN fatigue analysis procedure and the OWT modelling; Section 3 presents the theoretical background of the Gaussian process predictors; and Section 4 discusses the main findings of the implementation performed. Finally, the main conclusions of the work developed are presented in Section 5.

2. Meta-modelling in reliability analysis

Gaussian process regression models have recently gained particular interest on structural reliability engineering problems (Forrester et al., 2006; Bichon et al., 2008; Echard et al., 2011, 2014; Yang et al., 2015).

In the case of OWT modelling, the usage of Gaussian process predictors in structural analysis is even more recent.

In this context, Maki et al. (2012) analyses an inland wind turbine using a Gaussian process regression to decrease the effort required to analyse the system. Yang et al. (2015) performs a reliability-based optimization of Tripod foundation OWT using these as surrogates. In Morató et al. (2016) the same models are applied to model the response of an OWT to extreme loading. Teixeira et al. (2017b) discusses the application of Gaussian process models for fatigue design. Teixeira et al. (2018b) uses a similar approach, however investigating the importance of having a search criteria and a notion of improvement in the characterization of the Gaussian process predictor.

When addressing fatigue calculations, Echard et al. (2013) was able, through the application of Gaussian process predictors, to reduce the cost of fatigue assessment by approximately a factor of 265. Yang and Wang (2012) compared the performance of a Gaussian process predictors with other meta-model when addressing fatigue of a bending stiffener.

The current paper discusses how reliability analysis for OWT tower can be addressed by using a meta-model that compiles information from multiples sources of uncertainty.

2.1. *OWT modelling*

A 5MW turbine installed on a monopile is considered for the representative analysis on meta-modelling with Gaussian process predictors. This turbine, presented in more detail in Jonkman et al. (2009), is characterized by its wide applicability in OWT research. Some of its main generic characteristics are presented in Table 1.

2.2. *Stress-cycle Fatigue assessment for OWT*

The most widely applied procedure to design OWT to fatigue uses the stress-cycle method. IEC (2005, 2009) certification to structural fatigue involves performing multiple time-domain evaluations of operation, assessing the operational loads, extracting load ranges and cycles and comparing these

Table 1: NREL's Monopile OWT model main generic characteristics.

Horizontal axis OWT type	3/63m blades
Rated Power	5MW
Rated wind speed	11.4 m/s
Cut-in and cut-out speed	3m/s, 25m/s
Hub height	87.6m above mean sea level (MSL)
Tower base height	10m above MSL
Seabed foundation	-20m below MSL
Foundation Tower interface (TP)	Rigid connection
Diameter and Thickness at base of the tower	6m / 0.027m
Control system	Variable-speed (variable blade-pitch-to-feather configuration)

with the support of a pre-specified SN curve by applying the Palmgren-Miner's rule, Equation (1).

$$D_t = \sum_{S_i} = \frac{n_{S_i}}{N_{S_i}} \quad (1)$$

where D_t is the damage generated in a specified period of time t, for which n_{S_i} is the recorded number of cycles, or repetitions, of a S_i load/stress range and N_{S_i} is the allowed number of cycles at S_i given by a pre-specified SN curve. As the assessment is performed in a t shorter than the lifetime T, D_t is referred to as the short term SN damage rate and is used to approximate the life-time fatigue (D_T) for a specified design life T.

3. GAUSSIAN PROCESS PREDICTOR

Gaussian process predictors, also widely known as Kriging models, approach a true function $g(x)$, depending on $x \subset \mathbb{R}^{\mathrm{d}}$ in a d dimensional space, using an approximate regression function $G(x)$ that considers uncertainty within the regression.

Assuming that $g(x)$ can be characterized $\forall x$, $G(x)$ can be defined by using a sample of k support points or observations of the true function. In the context of the Gaussian process predictors, these support points are designated as Design of Experiments (DoE); $DoE = [\boldsymbol{X}, \boldsymbol{Y} == g(X)]$ with $\boldsymbol{X} = [x_1, x_2, \ldots x_n]$ as a vector of realisations of x and $\boldsymbol{Y}$ the respective true evaluations of $g(x)$.

The true response function $g(x)$ is then be approximated with

$$G(x) = f(\beta; x) + Z(x) \quad (2)$$

$$f(\boldsymbol{\beta}; x) = \beta_1 f_1(x) + ... + \beta_p f_p(x) \quad (3)$$

where $f(\boldsymbol{\beta}; x)$ is a deterministic function determined by a regression model with p ($p \in \mathbb{N}^+$) basis trend functions $f_p(x)$ and p regression coefficients $\boldsymbol{\beta}$ to be defined by the known sample $\boldsymbol{X}.Z(x)$ is a Gaussian stochastic process with zero mean that relates to a covariance matrix $\mathbf{C}$ of the support points:

$$\mathbf{C}(x_i, x_j) = \sigma^2 R(x_i, x_j; \boldsymbol{\theta}); \quad i, j = 1, 2, 3, ..., k \quad (4)$$

this matrix relates the $\boldsymbol{X}$ input points using; a process constant varianceσ^2 and a correlation function $\mathbf{R}(x_i, x_j; \boldsymbol{\theta})$.

For the structural analysis *separable* form correlations are widely applied (Roustant et al., 2012), Equation (5). Nevertheless, other types of correlation are available (Rasmussen, 2004) and may be also applied.

$$\mathbf{R}(x_i, x_j; \boldsymbol{\theta}) = \prod_{i=1}^{d} R(h_i; \theta_i), \ \boldsymbol{\theta} \in \mathbb{R}^{\mathrm{d}} \quad (5)$$

The correlation function depends on $h = [h_1, \ldots, \ h_d]$, a set of incremental values of type $x - x_i$ type and $\boldsymbol{\theta}$ hyperparameters.

For a given sample of support points the problem of prediction can then be solved through a generalised least squares formulation, where the estimators for $\boldsymbol{\beta}$ and σ^2 depend on $\boldsymbol{\theta}$.

The prediction for the true realisation $g(u)$ in a point u in the space is then given based on the Kriging expected value μ_G and variance σ_G^2:

$$\mu_G(u) = f(u)^T \ \boldsymbol{\beta} + \mathbf{c}(u)^T \mathbf{C}^{-1}(\boldsymbol{Y} - \mathbf{F}\boldsymbol{\beta}) \quad (6)$$

$$\begin{aligned} \sigma_G(u)^2 = \sigma^2[1 + D(u)^T(\mathbf{F}^T\mathbf{C}^{-1}\mathbf{F})^{-1}D(u) \\ -\mathbf{c}(u)^T\mathbf{C}^{-1}\mathbf{c}(u)] \end{aligned} \quad (7)$$

$$D(u) \equiv \mathbf{F}^T \mathbf{C}^{-1} \mathbf{c}(u) - f(u); \tag{8}$$

with $\mathbf{c}(u) = c(u,x_i),\ i = 1,2,\ldots,k$ is the correlation vector that relates the realisation to be evaluated with the known points and $f(u)$ is the vector of trend functions evaluated at u. $D(u)$ is introduced for the sake of brevity.

One particularity of $G(x)$ is that of the deterministic prediction in X.

In order to account for the uncertainty in the DoE a $\boldsymbol{\tau}^2$ component may be introduced in the formulation of $\mathbf{C}$.

$$\mathbf{C}(x_i,x_j) = \mathbf{C}(x_i,x_j) + \boldsymbol{\delta}\boldsymbol{\tau}.^2 \tag{9}$$

where $\boldsymbol{\tau}^2$ is the vector of variance σ_Y^2 of the realisations of $\boldsymbol{Y} \in g(x)$ used to define the surrogate model. $\boldsymbol{\delta}$ is the identity matrix of size k.

4. SN FATIGUE REALIABILITY ANALYSIS USING META-MODELS

A SN damage surface consists in an interpolation model where SN fatigue indicators and their uncertainty are defined through the application of a Gaussian process predictor. Results for the implemented approach are discussed in the present section.

SN fatigue analysis and its uncertainty, in regard of the loading characterization, is a problem of mean. Sutherland (1999) highlighted before the statistical behaviour of the SN fatigue when analysing wind turbines. SN fatigue design requires the cumulative responses to short-term operational conditions. These are commonly characterized by a loading spectra and, due to their repetitive and random character, a D_t probability distribution. As multiple operational conditions repeat, the distribution gets in-filled in both above and below the mean value. The result is that the cumulative behaviour of the short-term damage rates approaches a sum of the mean value. Therefore, uncertainty in the SN fatigue calculations is highly related to the uncertainty in characterization of the mean D_t caused at a specified operational conditions. This statistical behaviour of the SN fatigue is of interest for the application of meta-models as surrogates of SN fatigue.

A Latin Hypercube Sampling (LHS) scheme is applied in order to define the DoE. The LHS is one of the most widely applied techniques to generate support points for meta-modelling. It allows to efficiently cover the DoE, accounting for the DoE probability distributions. Recorded meteo-oceanographic data, presented in Teixeira et al. (2018a), supported the definition of the LHS sampling space. The correlation of the LHS space was considered using the method presented in Iman and Conover (1982).

Figure 1 presents an example of a meta-model for fatigue calculations that predicts the D_T for the tower component.

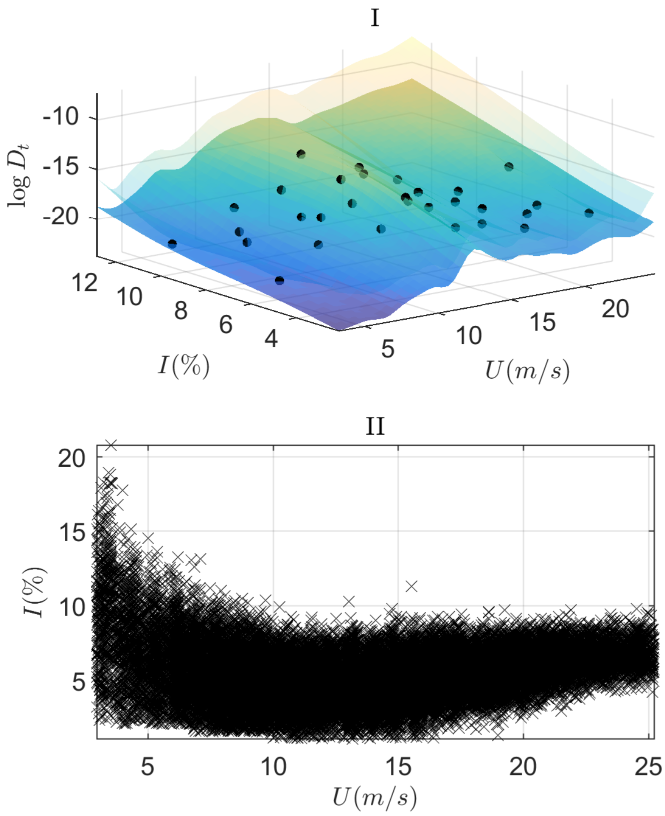

Figure 1: (I) - Surface of damage indicators define by the short term SN fatigue. (II) - Operational prediction points to evaluate the D_T.

In I a meta-model is created using a sample of support points (black markers). The expectation is for the definition of surrogate to be more efficient since only a limited subset of operational points need to be fully assessed. The meta-model acts then as a surrogate of the D_t that is expected in the tower for all different operational conditions. Combined with II the lifetime D_T can be estimated without the need to perform exhaustive evaluations of the OWT.

Two important considerations when characterizing the surrogate model are to, focus on the most important variables that influence the response, and to define the extension of the space of variables to be assessed. Teixeira et al. (2017a) showed that the tower SN fatigue, for the turbine considered, is mostly influenced by the wind components. These are the mean wind speed (U) and the turbulence intensity (I). This occurs due to the relatively high stiffness of the tower component for the turbine considered, allied to the fact that it has no direct interaction with the waves. Additional considerations on the definition of the LHS sample are related to the extent of the sampling space. At low U, computing I at the maximum value above the rated speed (U=11.4 m/s), did not resulted in relevant loss of accuracy on the long term predictions. Most of the SN fatigue life decreases at operational U above the rated speed. If no points are defined in specified x regions, $G(x)$ predictions may be uncertain (have large σ_G^2 or inaccurate μ_G). This is a particular concern when overfitting occurs due to the usage large p values.

In order to implement a Gaussian process predictor for reliability analysis, a representative SN curve from DNV (2014) was considered for validation. A full one-year operational SN fatigue calculation was considered to validate the prediction given by the surrogate. A value of 0.83746 for the R^2 statistic was computed for the cross-validation between the predictions given by $G(x)$ and the full one-year simulated operational data. The D_T prediction given by $G(x)$ diverged with an error of 4.8% when comparing with the value given by the full one-year assessment. In Figure 2 it can be seen that most of the cross-validation divergence in mean value occurs at low D_t values. These have a smaller contribution to D_T. Despite the R^2 being a good measure of the fit, it does not account for the relative importance between evaluated points. The absolute D_T error is more comprehensive measure to evaluate the fit. Nonetheless, it is important to highlight that only in rare occasions a big dataset is available for cross-validation.

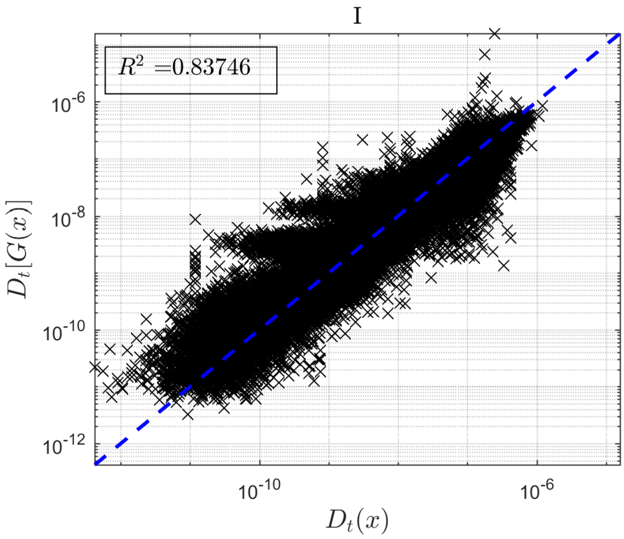

Figure 2: Cross validation of the tower SN fatigue prediction given by $G(X)$ in comparison to a full one year assessment given by 51240 D_t evaluations at different operational conditions. LHS of 25 points was applied to define the surrogate DoE.

The uncertainty quantification model for the stress-cycle curve presented in Sørensen et al. (2008) was adopted in the current study to replicate the randomness of the SN curve. In order to merge the SN curve probabilistic behaviour with the uncertainty given by the SN fatigue design process (related to the procedure and loading estimation), a double surface approach is implemented.

Two main types of uncertainty are enclosed in the DoE points, the uncertainty in the mean value of D_t due to the SN curve uncertainty, and the uncertainty in the D_t due to the sample size used to converge the loading distribution. IEC (2005, 2009) recommends 6 simulations with different seeds to estimate the SN contribution from loading at each operational environmental conditions. In the current assessment, 10 seeded simulation were used. The increase of the number of seeded simulations is a direct benefit of using $G(x)$, which reduces the computational effort of the assessment. Nevertheless, it is noted that further sources of uncertainty may be considered in the analysis, such as the ones described in Sørensen et al. (2008). These should be quantified when defining the indicators that support the characterization of the surrogate.

Figure 3 presents a cumulative density function (CDF) that characterizes the probabilistic behaviour of D_T. As the number of samples increases the density in the tail region also increases, and relatively large values of D_T may be expected (when comparing with the mean value). Despite the SN curve uncertainty being modelled with a Normal distribution, D_T is better approached with a lognormal model. Nevertheless, the lognormal approximation is not very accurate for tail region predictions. It may be of interest to truncate the data-set in the tail region in order to improve the accuracy of the probability of failure calculations. This may be particularly relevant for low probability of failures that are challenging to characterize.

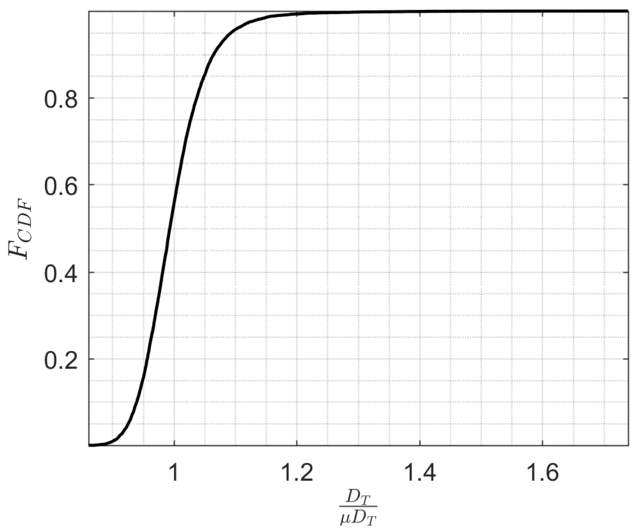

Figure 3: Cumulative density function for D_T, SN curve considered from DNV (2014) with $\log K_1$ = 12.164 and $\log K_2$ = 15.606. Distribution function was characterized using 10000 samples.

As highlighted before, a double slope (m) curve with different transition load range ΔS_N was considered, adopted from Sørensen et al. (2008). Table 2 presents the probabilistic SN curve model.

The conversion from load to stress was assumed to be linearly dependent on the tower section. A finite-element model may be applied to define D_t. In alternative, the fatigue curve can be specified as a load-cycle curve Freebury and Musial (2000). to simplify the analysis, a uncertainty coefficient may be also considered to account for load stress conversion.

Table 2: Random variables considered for the stress-cycle curve. $\log K_1$ and $\log K_2$ are fully correlated. ΔS_N is the point of slope change for the double slope SN curve. For the implementation considered, this load range was expected to occur at 5×10^6

Variable	Distribution	$\mathbb{E}[\,]$	σ
m_h	D	3	-
$\log K_1$	N	$f(\Delta S_N)$	0.20
m_l	D	5	-
$\log K_2$	N	$f(\Delta S_N)$	0.25
D - Deterministic; N - Normal			

According to Sørensen et al. (2008), it is common to consider in fatigue design a value of $T = 60$ years. Therefore, a $T = 60$ is used to characterize the limit state for which fatigue failure is expected to occur. Failure occurs when D_T in 60 years is larger than 1.

The probability of failure was calculated considering different SN curve characteristics. As the SN curve model applied is dependent on δS_N, this variable was applied to research on the variability of the reliability index (β) for different curves. Figure 4 presents the results for tower's β depending on the ΔS_N.

The β presented should be interpreted as follows; DoE of $G(x)$ is defined considering multiple SN curves accordingly to the probabilistic SN curve model, $G(x)$ is characterized using the mean and the standard deviation of the DoE, reliability calculations consider sampling of design surfaces. The design surfaces are sampled from $G(x)\forall x$ and used to predict operational D_T. Each design surface is a deterministic realisation of $G(x)$. As the surrogate encloses uncertainty due to the SN curve and the loading sample, the damage surfaces sampled and used to predict D_T replicate its uncertainty. This sampling approach is no different than designing to SN fatigue accordingly to (IEC, 2005, 2009). Every sampled damage surface realisation replicates a design procedure, as if the designer would perform 10 simulations at each environmental loading conditions and assess SN fatigue using one of the potential SN curves within the uncertainty considered.

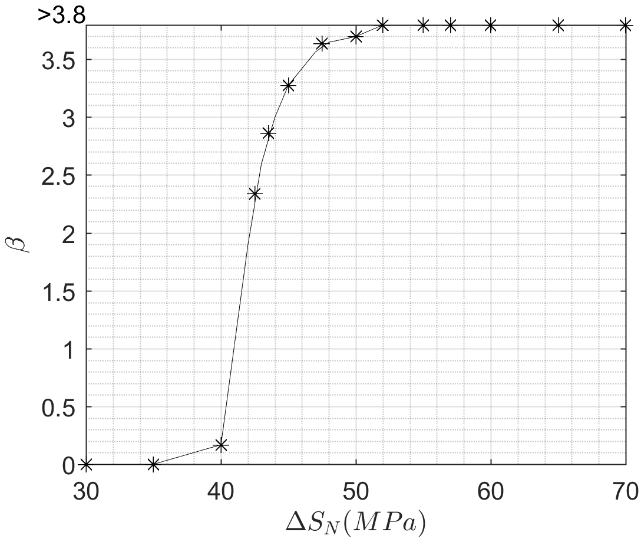

Figure 4: Reliability index of the tower function of ΔS considering a T of 60 years. 100000 samples were applied to converge the D_T distribution for each value of ΔS_N. $\beta = 3.8$ is equivalent to a probability of failure of 1 in 10000.

Other variables of interest could be applied in order to characterize β. The sample size applied to characterize the DoE points is one example of a variable within the model build. In the implemented example the main interest was to present how $G(x)$ may be applied for efficient reliability assessments.

The notorious advantage of using the $G(x)$ predictor for reliability analysis is mainly related to the computational cost. For the space considered, if bins of value 1 were used to divide the environmental conditions for U and I, 253 load cases would be needed to characterize the SN fatigue design. With bins of value 2, this number would decreases to 72 load cases. For the current application, only 25 load cases were assessed to design the OWT tower component to SN fatigue, 10% to 40% of the binned cases.

Moreover, all the probabilistic information about the problem is focused on a model that is able to predict operation while enclosing uncertainty.

To conclude, it is of relevance to highlight the universal character of the approach here presented. It may be applied to design any component of any system, other than OWT. The only requirement is to be able to define a representative indicator to build the meta-model, such as D_t.

5. CONCLUSIONS

Application of Gaussian process predictors as surrogates of stress-cycle fatigue was researched. Gaussian process predictors were applied before as meta-models to mitigate the cost of the stress-cycle fatigue analysis. In the present implementation these models are applied also to allow efficient reliability assessment. Their capability to account for uncertainty as a Gaussian variable is of interest for reliability calculations.

Two main probabilistic variables were considered in the characterization of the meta-model, these related to the material resistance and loading spectra definition. The main purpose of the present assessment was to show how to apply Gaussian process predictors in fatigue reliability calculations. Hence, it is important to highlight that other additional sources of uncertainty may be considered.

Results showed that Gaussian process predictors are efficient and accurate surrogates of the researched turbine's tower fatigue design. Their implementation allowed to reduce the computational time of the assessment from 251 to 25 load cases with minimum loss of accuracy. Moreover, their definition may enclose uncertainty in the design of experiments points, which may be then interpolated over all the operational points allowing efficient reliability assessments.

With the definition of the model, cumulative distributions of the long-term stress-cycle fatigue are sampled with limited computational cost. This allows to research on the design variables and on the probability of failure, enabling comprehensive designs.

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