13th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP13
Seoul, South Korea, May 26-30, 2019

Reliability Estimation for Deteriorating Reinforced Concrete Structures using Bayesian Updating

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ABSTRACT: The deterioration mechanisms of reinforced concrete (RC) structures have many kinds of uncertainties, and it is impossible to completely predict the condition of RC structures throughout their lifetime at the initial design stage. Therefore, observation or inspection data, which reflect the actual condition of the existing structures, should be used to reduce the uncertainties of the reliability prediction model of deteriorating RC structures. In this paper, a novel reliability estimation method for deteriorating RC structures using observation data is proposed. Bayesian updating method is used to combine corrosion models with observation data and to update prior probabilistic models, while the structural reliability is calculated with a time-dependent three-dimensional finite element (FE) analysis. Chloride-induced corrosion of reinforcing steels, which is one of the most significant deterioration mechanisms of RC structures, is considered. With Bayesian updating method, the uncertainties in the chloride-ion diffusion model can be reduced, and the probability of corrosion initiation is updated. Finally, as an illustrative case study of the proposed method, the time-dependent structural safety of a box-girder bridge is calculated over its lifetime of 50 yrs. Based on the results of this paper, the structural reliability of deteriorating RC structures can be quantitatively updated, and it can be useful for the appropriate and reasonable decision-making of maintenance.

The structural reliability of reinforced concrete (RC) structures degrades over time due to a variety of factors such as aging, environmental stressors, and natural hazards. Recently, maintaining such deteriorating structures has become a big concern because the number of those structures has increased. For example, according to the report presented by the Ministry of Land, Infrastructure, Transport and Tourism in Japan (MLIT (2018)), approximately 25% of totally about 500 thousand highway bridges were constructed more than 50 yrs. ago, and it is estimated that the number of such bridges will increase to 63% in the next 15 yrs.

The deterioration mechanisms of RC structures present many kinds of uncertainties, and it is impossible to completely predict the condition of RC structures throughout their lifetime at the initial design stage. Recently, large amounts of observation or inspection data are available to comprehend the actual condition of the structures. Therefore, for appropriate maintenance, they should be used to reduce the uncertainties of the reliability prediction model of deteriorating RC structures.

Finite element (FE) analysis has played a significant role in calculating the structural behavior of RC structures. With the development of computer technology, much larger-scale analysis can be conducted within a reasonable time. Therefore, FE analysis can be used to calculate the long-term reliability of the structures in whole or in part.

In this paper, a novel reliability estimation method for deteriorating RC structures using observation data is proposed. Bayesian updating method is used to combine deterioration models with observation data and to update prior probabilistic models, while the structural reliability is calculated with a time-dependent three-dimensional finite element (FE) analysis. Chloride-induced corrosion of reinforcing steels, which is one of the most significant deterioration mechanisms of RC structures, is considered. With the proposed method, the uncertainties in the chloride-ion diffusion model can be reduced, and the probability of corrosion initiation is updated. In the subsequent sections, in the beginning, the general methodology of Bayesian updating is presented. Then, the probabilistic models for chloride-induced corrosion are briefly reviewed, followed by application of Bayesian updating method to the corrosion model. Thereafter, an FE modeling method for corroded RC structures is presented, in which concrete and reinforcing steels are separately modeled, and the degradation in the mechanical behavior of both components is considered. Finally, as an illustrative case study of the proposed method, the time-variant structural safety of a box-girder bridge is calculated over its lifetime of 50 yrs.

1. MODEL UPDATING METHODOLOGY

The deterioration mechanisms of RC structures have many kinds of uncertainties, which can be modeled as random variables $\mathbf{X}$, characterized by their joint probability density functions (PDF) $f(\mathbf{x})$. When some observation data related to the corrosion of reinforcing steels are obtained, the corrosion model can be updated and the uncertainties can be reduced by them. In this paper, Bayesian updating method is used to combine deterioration models with observation data, and to update the prior probabilistic models.

In Bayesian analysis, a prior PDF $f'(\mathbf{x})$ is updated to a posterior PDF $f''(\mathbf{x})$ as (Benjamin and Cornell (1970)):

$$f''(\mathbf{x}) = \frac{L(\mathbf{x})f'(\mathbf{x})}{\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} L(\mathbf{x})f'(\mathbf{x})dx_1 \ldots dx_n} \tag{1}$$

where $L(\mathbf{x})$ is the likelihood function reflecting the effect of observation data, which is expressed as a conditional probability of the observation data z given $\mathbf{x}$ as:

$$L(\mathbf{x}) \propto \Pr(z \mid \mathbf{X} = \mathbf{x}) \tag{2}$$

According to Eq. (2), in order to obtain the likelihood function, observation data have to be related to design variables $\mathbf{X}$. When observation data are continuous quantities, they can be expressed with a prediction model $h_i(\mathbf{x})$, which corresponds to the observation data z_i, and the deviation between the prediction model and observation data ε_i, which is caused by observation error and model prediction error as (Straub and Papaioannou (2015)):

$$z_i = h_i(\mathbf{x}) + \varepsilon_{\mathrm{i}} \tag{3}$$

Therefore, when the deviation ε_i is modeled thorough the PDF $f_{\varepsilon_i}(\cdot)$, the likelihood function is expressed as:

$$L_i(\mathbf{x}) = f_{\varepsilon_i}[z_i - h_i(\mathbf{x})] \tag{4}$$

The deviation ε_i is often assumed to follow a normal distribution (Straub and Papaioannou (2015)). Then, when the deviation follows a normal distribution with zero mean, Eq. (4) can be modified as:

$$L_i(\mathbf{x}) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{i}}} \exp\left\{-\frac{(z_i - h_i(\mathbf{x}))^2}{2\sigma_{\mathrm{i}}^2}\right\} \tag{5}$$

In general cases, m observation data are measured. When those data are assumed to be statistically independent each other given the model parameter $\mathbf{x}$, the likelihood functions of each data can be combined as (Straub and Papaioannou (2015)):

$$L(\mathbf{x}) = \prod_{i=1}^{m} L_i(\mathbf{x}) \tag{6}$$

The n-fold integral in Eq. (1) can not be directly evaluated in most cases. Therefore, some sampling methods are required to obtain a post PDF, and simple rejection sampling algorithms are used in this paper.

2. APPLICATION TO CORROSION MODEL

2.1. *Corrosion initiation*

In the past few decades, several models for chloride-induced corrosion of reinforcing steels have been developed and applied to certain standards, such as CEB-FIP (2013). According to CEB-FIP (2013), the chloride concentration $C(x,t)$ at depth x from the concrete surface at time t can be appropriately calculated by Fick's second low as:

$$C(x,t) = C_S\left\{1 - erf\left(\frac{x}{2\sqrt{D_{app} \cdot t}}\right)\right\} \tag{7}$$

where C_S is the chloride concentration on the concrete surface, D_{app} is the apparent coefficient of chloride diffusion in concrete, and $erf(\cdot)$ is the error function. The apparent coefficient of chloride diffusion can be expressed as (Cao et al. (2013)):

$$D_{app}(t) = k_e \cdot k_t \cdot D_c \cdot \left(\frac{t_0}{t}\right)^n \tag{8}$$

where k_e is the environmental factor, k_t is a parameter that considers the influence of test methods, which is assumed to be 1.0 in this study, D_c is the chloride migration coefficient, t_0 is the reference point of time (= 28 days), and n is the aging exponent.

It is assumed that the corrosion of reinforcing steels starts when the chloride concentration on the steel surface reaches the threshold value C_T. Therefore, the time to corrosion initiation t_{ini} is calculated from Eq. (7) and (8) as follows:

$$t_{ini} = \left[\frac{d^2}{4 \cdot k_e \cdot k_t \cdot D_c \cdot t_0^n}\left\{erf^{-1}\left(1-\frac{C_T}{C_S}\right)\right\}^{-2}\right]^{\frac{1}{1-n}} \tag{9}$$

The statistical parameters of each design variable are shown in Table 1 (CEB-FIP (2006), Faber and Straub (2006)).

2.2. *Corrosion rate*

After corrosion initiation, corrosion products on the surface of reinforcing steels gradually increase at a certain rate. The corrosion rate is usually expressed by means of the corrosion current density i_{corr}, which can be estimated as a time-dependent variable as:

$$i_{corr}(t) = i_{corr}(1) \cdot \alpha \,(t - t_{ini})^{\beta} \tag{10}$$

where $i_{corr}(1)$ is the initial corrosion current density, and α and β are the constant coefficients. In general, corrosion rate may be effected by concrete quality and concrete cover depth. Therefore, in this study, the corrosion current density can be expressed by means of w/c rate and concrete cover depth, d, as follows (Vu and Stewart (2000)):

$$i_{corr}(1) = \frac{37.8\,(1-w/c)^{-1.64}}{d} \tag{11}$$

If the corrosion rate is assumed to be constant with time, α and β should be equal to 1.0 and 0.0, respectively. On the other hand, if it is assumed that the corrosion rate can be changed with time, it becomes a time-dependent variable. In Vu and Stewart (2000), it is suggested that the formation of rust products on the steel surface will reduce the diffusion of the iron ions away from the steel surface, and as a result, the corrosion rate will also reduce with time. In this study, it is assumed that α equals to 0.85 and β equals to -0.29. The estimated corrosion current density can be directly related to the loss of reinforcing steels by Faladay's law, that is, $1\,\mu\mathrm{A/cm^2}$ corresponds to $0.0116\,\mathrm{mm/year}$.

2.3. *Formulation for updating*

In this paper, it is assumed that chloride concentration at a certain depth from the concrete surface is measured. The prediction model $h(\mathbf{x})$ can be expressed as Eq. (7), and observation data z can be expressed by substituting Eq.(7) for Eq.(3) as:

$$z(x,t) = C_S\left\{1 - erf\left(\frac{x}{2\sqrt{D_{app} \cdot t}}\right)\right\} + \varepsilon \tag{12}$$

Here, the deviation between the prediction model and observation data ε is assumed to follow a normal distribution with zero mean.

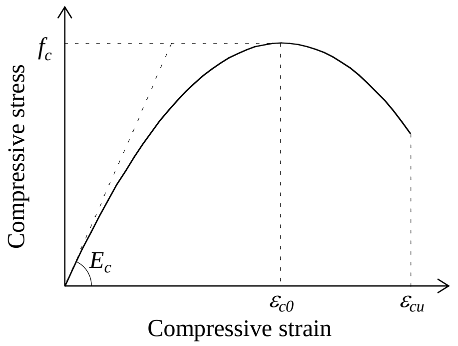
3. FE MODELING FOR CORRODED RC STRUCTURES

In order to apply the presented corrosion model to the design of RC structures, a time-variant three-dimensional FE model for corroded RC structures is proposed in this section. In this paper, concrete and reinforcing steels are separately modeled in the FE analysis. The concrete is modeled by means of three-dimensional solid elements, while the reinforcing steels are modeled by two-node beam elements. Moreover, the bond relation between concrete and reinforcing bars is represented by spring elements in the longitudinal direction of the reinforcing steels. The constitutive models for each component consider nonlinear behavior:

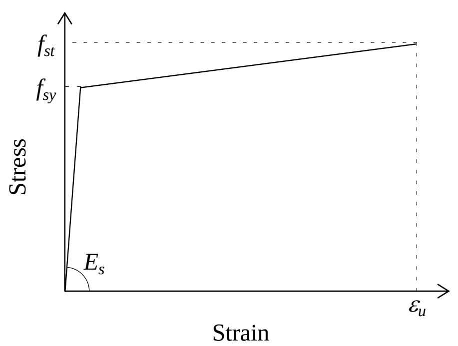
- non-linear stress-strain relation with smeared crack model for concrete(Figure 1a), and
- elastoplastic (bi-linear) stress-strain relation for reinforcing steels (Figure 1b).

Table 1: Statistical parameters of design variables

Parameter	Distribution	Mean	COV
Critical chloride concentration C_T	Beta	0.60 wt.-%/c	0.25
Surface chloride concentration C_S	Normal	1.15 wt.-%/c	0.38
Cover depth d	Normal	45.0 mm	0.30
Environmental factor k_e	Gamma	0.68	0.16
Chloride migration coefficient D_c	Normal	$1.0 \times 10^{-11}\,\mathrm{m^2/s}$	0.20
Aging exponent n	Beta	0.3	0.40



(a) Concrete under compression



(b) Reinforcing steels

Figure 1: Constitutive relations

In order to adapt the corrosion mechanisms to the FE model, the following effects of corrosion are considered:

1. the cross-sectional reduction in reinforcing steels,
2. the reduction of the yield strength of reinforcing steels,
3. the reduction in concrete compressive strength, and
4. the reduction in the bond strength between reinforcing steels and concrete.

3.1. Reinforcing steel

Uniform corrosion is considered and is modeled by reducing the cross-sectional area and the yield strength of corroded reinforcing steels. The remaining cross-sectional area of corroded reinforcing steels can be calculated as follows:

$$A_{res} = \eta A_0 = \frac{\pi \eta D_0^2}{4} \tag{13}$$

where A_{res} and A_0 are the cross-sectional area of corroded and non-corroded reinforcing steels, respectively, and η is the corrosion rate, which is calculated from the mass loss of corroded reinforcing steels.

The reduced yield strength of reinforcing steels is generally expressed as follows:

$$f_{sy,res} = (1 - \alpha_{sy}\eta)\, f_{sy} \tag{14}$$

where $f_{sy,res}$ and f_{sy} are the reduced and the initial yield strength of reinforcing steels, respectively, and α_{sy} is the empirical coefficient. The coefficient α_{sy} has been obtained from several experimental results, and it is assumed to be 0.5 (Du et al. (2005)).

3.2. Concrete

The concrete cover may be cracked by the expansion of corrosion products. The effect of concrete cracking is taken into account by reducing the concrete compressive strength of the concrete cover. The reduced concrete compressive strength can be calculated as (Coronelli and Gambarova (2004)):

$$f_{ck,res} = \frac{f_{ck}}{1 + K\varepsilon_1/\varepsilon_{c0}} \tag{15}$$

where $f_{ck,res}$ and f_{ck} are the reduced and the initial compressive strength of concrete, respectively, K is the coefficient related to the roughness and diameter of reinforcing steels (for medium-diameter ribbed bars, $K = 0.1$), ε_{c0} is the compressive strain at peak compressive stress and ε_1 is the average tensile strain in the cracked concrete. The average tensile strain ε_1 is evaluated as (Coronelli and Gambarova (2004)):

$$\varepsilon_1 = n_{bars} w_{cr} / b_0 \tag{16}$$

where n_{bars} is the number of reinforcing bars on the compressive side, w_{cr} is the total crack width, which can be expressed as $w_{cr} = 2\pi x$ (Molina et al. (1993)), and b_0 is the section width of the structure.

The reduction model for concrete compressive strength generally applies to the whole concrete cover. However, the corrosion pattern of cover concrete depends on the arrangement of reinforcing steels. Therefore, in this study, the reduction model for concrete compressive strength is only applied to the concrete cover surrounding reinforcing steels as shown in Figure 2.

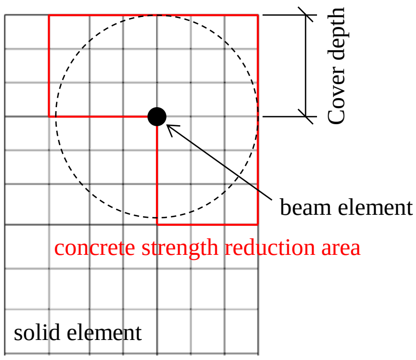


Figure 2: Concrete strength reduction area

3.3. *Bond strength*

In this paper, the local bond stress-slip relation model proposed by CEB-FIP (2013), as shown in Figure 3, is adopted for non-corroded structures.

The bond stress-slip relation of corroded RC structures is complex and difficult to formulate. A wide range of experiments related to bond strength degradation have been conducted, and the following equation is proposed based on such experimental results in Bhargava et al. (2008):

$$\tau_b(s) = \beta \tau_{b0}(s) \tag{17}$$

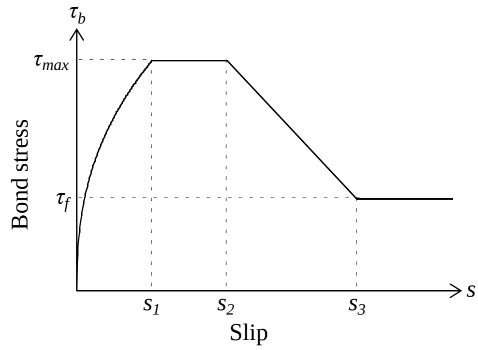


Figure 3: Local bond stress-slip relation

where $\tau_b(s)$ is the bond stress between corroded reinforcing steels and concrete, $\tau_{b0}(s)$ is the bond stress between un-corroded reinforcing steels and concrete presented above, and β is the ratio of the bond strength at a certain corrosion rate to the initial (un-corroded) bond strength, as follows:

$$\beta = \begin{cases} 1.0 & \eta \leq 1.5\% \\ 1.192e^{-11.7\eta} & \eta > 1.5\% \end{cases} \tag{18}$$

4. Case study

As an illustrative case study of the proposed method, a box-girder bridge, which has a cross-section as shown in Figure 4, is considered. Considering symmetry, only half of the cross-section is considered. It is reinforced, and only the outer reinforcing steels (red lines in Figure 4, concrete cover depth is 45 mm) are considered corroded. The compressive strength of concrete is assumed $40\,\mathrm{N/mm^2}$, and the yield strength of reinforcing steels is assumed to be $345\,\mathrm{N/mm^2}$. The FE model is shown in Figure 5. It has 84,760 nodes and 84,028 elements in total. The structural safety is calculated over 50 yrs. at 10 yr. intervals. Therefore, the FE analysis has six time steps including the initial (un-corroded) step.

It is assumed that chloride concentration at six different depths in the concrete cover is measured after 30 yrs. Figure 6 shows three sets of the assumed observation data. These data are obtained by reference to the original variables in Table 1, which are calculated with Monte Carlo simulation. The slopes of each data set correspond to the coefficient of chloride diffusion, and that of Case 1 is almost the same with that of the original model. In

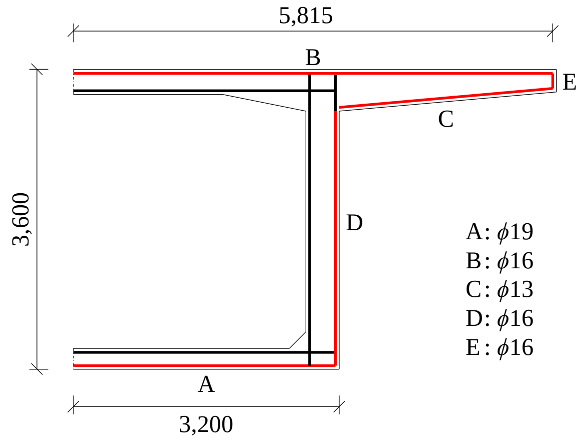


Figure 4: Cross-section of bridge deck

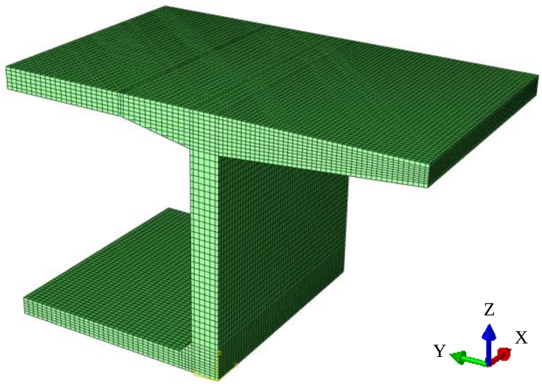


Figure 5: FE model

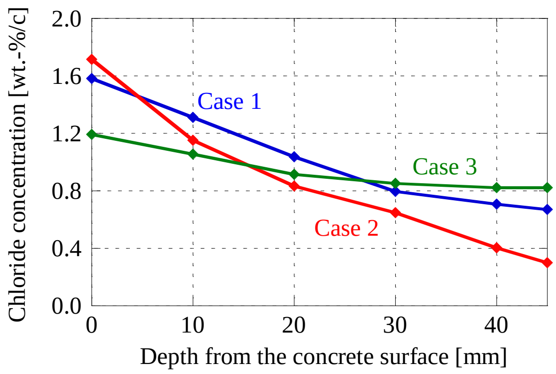


Figure 6: Assumed observation data (chloride concentration at different depth)

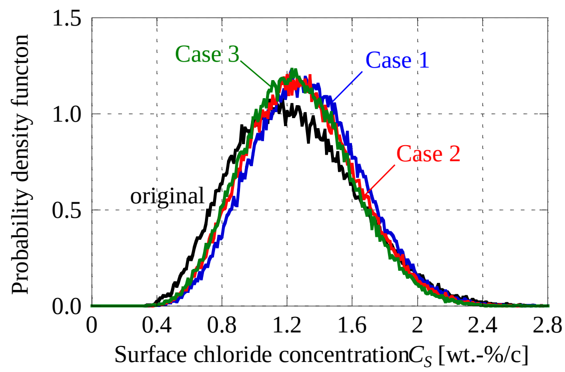


Figure 7: Probability density function of sampling (Surface chloride concentration C_S)

Table 2: Statistical parameters of sampling (Surface chloride concentration C_S)

	Mean [wt.-%/c]	STD [wt.-%/c]	Reduction ratio [%]
Original	1.24	0.373	-
Case 1	1.32	0.333	10.8
Case 2	1.28	0.334	10.5
Case 3	1.26	0.326	12.8

Case 2, the difference of chloride concentration between the concrete surface and the steel surface is larger than Case 1, that is, the coefficient of chloride diffusion is smaller than Case 1, while in Case 3, the difference of chloride concentration between the concrete surface and the steel surface is smaller than Case 1, that is, the coefficient of chloride diffusion is larger than Case 1.

With the observation data in Figure 6, the uncertainties of the design variables are reduced, and as a result, the corrosion rate of reinforcing steels is updated. For example, the probability density functions and the statistical parameters of samplings of surface chloride concentration are shown in Figure 7 and Table 2, respectively. According to these results, the deviation of the design variable is reduced about 10 %.

The mean values for the original and updated corrosion rate of reinforcing steels with the diameter of 19 mm, which is estimated with the Monte Carlo method using the statistical data given in Table 1, are shown in Figure 8. In all cases, the more realistic, that is, the less uncertain corrosion rate is smaller than it was predicted at the initial design stage.

The failure probability of the bridge deck is calculated at the joint of the web and the overhanging slab, as shown in Figure 9. The ultimate limit state for flexural strength is assumed to calculate the fail-

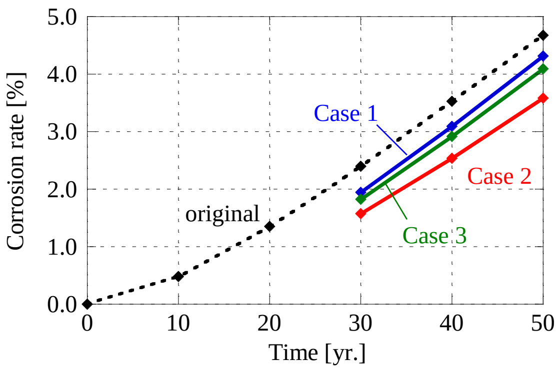


Figure 8: Updated corrosion rate (ϕ19)

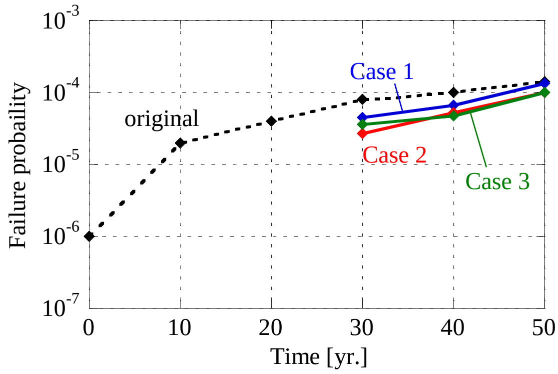


Figure 9: Updated probability of failure

ure probability as:

$$g = \gamma M_u - M \tag{19}$$

where M_u is the ultimate capacity of the target structural section, M is the bending moment obtained from the FE analysis, and γ is a safety factor. To simplify, it is assumed that only the core (uncorroded) concrete and reinforcing steels are considered in the calculation of bending resistance, that is, the corroded concrete cover does not contribute to flexural strength. It is obvious that in all cases, the failure probability becomes smaller than the original one because the corrosion rate becomes smaller. Therefore, from the view of the maintenance of existing structures, it can be noted that when the target reliability level is assumed to be 10-4, which corresponds to that of the ultimate limit state provided in CEB-FIP (2013), according to the original design, the structure may lose the required capacity over the course of 40 yrs., and should be maintained, while in other cases, the maintenance actions are not required at that time. In conclusion, with the proposed method, the current state of existing structures can be quantitatively estimated using observation data, and therefore, it is efficient for decision-making of maintaining existing structures, which requires some comparable decision criteria.

5. CONCLUSIONS

A new probabilistic design method for reducing the uncertainties of corrosion prediction model of RC structures and updating structural reliability of them by observation data was proposed. In the proposed methods, the uncertainties of the design variables for estimating the corrosion initiation time of reinforcing steels can be reduced with Bayesian updating method, and the structural reliability can be updated using a time-dependent three-dimensional FE analysis. In the FE model, concrete and reinforcing steels were separately modeled with solid elements and beam elements, respectively, and the bond behavior between concrete and reinforcing steels was represented with spring elements in the longitudinal direction of reinforcing steels. Finally, the illustrative case study, in which the reliability of a box-girder bridge was calculated, signified that the suggested FE analysis is efficient for estimating the structural reliability of existing structures.

With the proposed method, the structural reliability of existing structures can be quantitatively estimated. As a next stage, it should be applied to the decision-making scheme of maintaining existing structures. Moreover, this study focused on only the initial probabilistic conditions of the corrosion models. However, statistical parameters such as mean values and standard deviations may change during the lifetime of structures. Therefore, the presented methods can be extended to account for changes in such parameters.

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