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System Identification for Time-Varying Systems under Non-White Excitations

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ABSTRACT: Accurate identification of structural properties such as natural frequency and damping under extreme wind conditions is critical for assessing the structural performances. Full-scale monitoring has witnessed that dynamic properties of structures may change over time under extreme winds. System identification (SI) is non-trivial in this case. While wavelet and short time Fourier transforms have been used in tracking time-varying frequencies, they have seldom been used to identify the time-varying damping ratio. This is because the short window (required to capture the temporal information) will amplify the bandwidth significantly and lead to considerably overestimated damping ratios. To address this challenge, this paper proposes a novel non-stationary SI approach to identify time-varying systems under general non-white excitations by extending an earlier approach proposed by the authors. To solve this problem, this study innovatively adapted theoretical frequency response functions (FRF) of systems for marginal spectra of the wavelet transform by adding short window effects explicitly. In this way, both the natural frequency and damping ratio at each time instant can be identified accurately. However, the initially proposed method assumed the excitation to be white in the vicinity of the structural natural frequencies. This assumption might not be strictly valid in some cases. For example, for wind-excited structures, the spectrum of wind force is a function of frequency derived from the turbulent wind spectrum and aerodynamic admittance function. To better handle these non-white excitations, the method proposed in this paper directly models the non-white spectrum of excitation in the frequency domain. Then both the parameters of the force spectrum and the system properties are identified using the output data. The uncertainties of the SI results are evaluated. The performance of the proposed method is demonstrated by numerical examples considering structures under extreme wind conditions.

1. INTRODUCTION

Civil infrastructure plays a crucial role in modern society and structural aging and deterioration have a serious impact on public safety. Thus, structural health monitoring (SHM) has become an important problem in civil engineering during the past half-century. And system identification (SI) of structural dynamic properties has been developed greatly as a tool for damage detection, condition assessment and performance evaluation in SHM (Doebling et al 1996; Carden et al 2004). Many early SI methods require specifying input data in the analysis (Ljung, 1999). Thereafter, some output-only SI methods, such as random decrement technique (RDT) (Lbrahim, 1997) and stochastic subspace identification (SSI) (Peeters and Roeck, 1999), were developed due to the difficulty of measuring input. However, most of those classical SI methods are based on the assumption of stationary data. Actually, the response signal of civil infrastructure under excitations such as strong winds and traffic loads is non-stationary. Comparing to the large body of literature in stationary SI, publications focusing on SI with non-stationary data (e.g. Li and Chang, 2012) are rather limited.

In response to this research gap, this paper proposes a novel method to identify the dynamic properties of structures under extreme winds. The extreme wind excitation is usually non-stationary and non-white. And the dynamic properties such as natural frequency and damping may change under strong winds (Sohn, 2006). Identifying structural system, in this case, is challenging. This is because a high time resolution (achieved by applying short analysis window in the time domain) needed for capturing the time-varying properties of non-stationary signals will lead to a low frequency resolution according to Heisenberg's uncertainty principle. The low frequency resolution manifests itself as the significantly amplified bandwidth, thus can cause over-estimated damping ratio. Due to this reason, the traditional frequency domain system identification method such as spectral moments method (SMM) (Bashor, 2011) and frequency domain decomposition (FDD) (Brincker et al, 2000; Carassale and Percivate, 2009; Rainieri and Fabbrocino, 2010), although can be used to identify time-varying frequencies and modes shapes in conjunction with short analysis windows, may fail to identify damping ratios from non-stationary data. The same difficulty as a result of short window effects presents itself in the time-frequency approaches such as short-time Fourier transform (STFT) and wavelet transform (Carmona et al., 1998). To address this challenge, the authors proposed an approach that introduces a modified frequency response function (FRF) of the system for marginal spectra of the wavelet transform to explicitly handle short window effects and enable accurate identification of damping from non-stationary structural response.

The original approach (Guo, 2015) proposed by the authors assumes that the excitation of a structure is white in the vicinity of the structural natural frequencies. However, natural excitations due to wind are non-white. Unlike the constant power spectra density (PSD) of white noise, the PSD of fluctuating wind forces is a function of frequency. Assuming the sufficiently white excitation like in our original non-stationary SI approach or other traditional output-only SI methods might lead to less accurate estimation for damping ratios. In this context, this paper proposes to extend our original approach to address non-white excitations. This can be achieved by directly modeling the non-white spectrum of excitation in the frequency domain. In the following, the methodology of the proposed extended approach is introduced. Then numerical examples are presented to evaluate the accuracy and uncertainties of the proposed method.

2. METHODOLOGY

In this section, the main idea of our original non-stationary SI approach based on wavelet transform is first reviewed. Then the extension to handle non-white excitation is introduced.

In our original non-stationary SI approach (Guo, 2015), the system parameters of a time-varying single-degree-of-freedom (SDOF) system are considered as a constant within a short time window. Accordingly, the identification of a time-varying system is equivalent to the identification of the corresponding time-invariant system within the short windows. The theoretical PSD of a time-invariant system response can be obtained by multiplying magnitude-squared FRF of the system with the PSD of excitation. It is assumed that the non-stationary input is white and the spectrum of excitation is a constant. In this case, the spectrum of system response becomes a scaled version of magnitude-squared FRF.

$$S_{xx}(f) = S_0 \left| H(f;\, f_n, \xi) \right|^2 \tag{1}$$

where S_0 is the spectrum of white excitation, $H(f;\, f_n, \xi)$ is the FRF of the system, S_{xx} is the spectrum of system response and f_n, ξ are the natural frequency and damping ratio, respectively. To allow accurate damping identification from non-stationary signals, the short window effect has been explicitly considered when estimating actual the spectrum of system response, i.e.

$$\hat{S}_{xx}(f;T) = S_0 \left| H(f;\, f_n, \xi) \right|^2 * \left| U(f;T) \right|^2 \tag{2}$$

where T is window length, $U(f;T)$ is Fourier transform of the window function of length T, used in spectral estimation, $\hat{S}_{xx}$ is the estimated spectrum of system response. Actually, it is difficult to remove the window effect and fitting the system response $\hat{S}_{xx}(f;T)$ to a scaled version of the magnitude-squared FRF will cause a large error in the identification of damping ratio. Therefore, instead of making adjustments in system response, the window effect is added into FRF and a modified FRF $\left|H_w(f;f_n,\xi,T)\right|^2$ given below:

$$\left|H_w(f;f_n,\xi,T)\right|^2 = \left|H(f;f_n,\xi)\right|^2 * \left|U(f;T)\right|^2 \tag{3}$$

is introduced to identify the system properties from non-stationary signals. Accordingly, Eq. (2) can be rewritten as:

$$\hat{S}_{xx}(f;T) = S_0\left|H_w(f;f_n,\xi,T)\right|^2 \tag{4}$$

The estimated spectrum of system response becomes a scaled version of the modified FRF and the dynamic properties of the system can be identified by fitting the spectrum of system response to the scaled version of the modified FRF using least squares estimation. In addition, the spectrum of system response and the modified FRF are normalized to eliminate the scale factor S_0.

To extend the original approach to handle the non-white excitation, the spectrum of the excitation is modeled in this paper. An observation of turbulence spectra suggests a linear relationship between frequency and wind force spectra in log-log scale at the frequency range where civil structures typically reside. And Carassale and Percivate (2009) introduced a parameter for the local slope to represent the PSD of wind excitation in the SI method based on FDD. Therefore, a linear function is proposed to model the spectrum of non-white excitation.

$$\log_{10}(S_{in}) = k\log_{10}(f) + c \tag{5}$$

where S_{in} is the spectrum of wind excitation, k is the slope and typically negative, c is a constant and f is the frequency. In this case, the estimated spectrum of system response can be expressed as:

$$\hat{S}_{xx}(f;T) = \left|H_w(f;f_n,\xi,T)\right|^2 f^k C \tag{6}$$

where C=10^c and a power function of frequency $f^k C$ is used to describe the spectrum of wind excitation. The system properties (natural frequency and damping ratio) can be identified by fitting the right-hand side of equation (6) to the response spectrum $\hat{S}_{xx}(f;T)$ estimated from non-stationary data. Following this idea, a non-stationary SI method based on wavelet transform is proposed. The wavelet transform is adopted due to its prowess in characterizing time-varying frequency contents of non-stationary signals. In this study, the marginal spectrum of the continuous wavelet transforms is employed as an estimator of the response spectrum. The continuous wavelet transform of the signal is given by:

$$\begin{aligned} WT(a,t) &= \frac{1}{\sqrt{a}}\int x(\tau)\cdot\psi^*\left(\frac{\tau-t}{a}\right)d\tau \\ &= \sqrt{a}\int X(f')\cdot\Psi^*(af')e^{j2\pi f't}df' \end{aligned} \tag{7}$$

where $x(\tau)$ is the signal, $\psi(\tau)$ is wavelet, $X(f')$ is the Fourier transform of a signal, $\Psi(f')$ is the Fourier transform of the wavelet, a is the scale factor and * denotes complex conjugate. The marginal spectrum is the integration of wavelet scalogram over the time variable and given by:

$$M_{WT}(a) = \int\left|WT(a,t)\right|^2 dt \tag{8}$$

Now the modified FRF for the marginal spectrum needs to be developed. The expected marginal wavelet spectrum of a time-varying system within a short window is equivalent to the expected marginal wavelet spectrum of the corresponding time-invariant system, given by:

$$E\left[M_{WT}(a,T)\right] = a\int\hat{S}_{xx}(f';T)\cdot\left|\Psi(af')\right|^2 df' \tag{9}$$

Substitute Eq. (6) to Eq. (9):

$$\begin{aligned} &E\left[M_{WT}(a,T)\right] \\ &= a\int\left|H_w(f';f_n,\xi,T)\right|^2 f^k C\cdot\left|\Psi(af')\right|^2 df' \end{aligned} \tag{10}$$

where $f = \dfrac{f_0}{a}$, f_0 is the central frequency and $f^k C$ can be separated from the integrand. Then we define the modified FRF for the marginal spectrum estimator as:

$$\left|H_{WT\_marg}(a; f_n, \xi, T, k, C)\right|^2 = af^k C \cdot H'$$

$$H' = \int \left|H_w(f'; f_n, \xi, T)\right|^2 \cdot \left|\Psi(af')\right|^2 df' \quad (11)$$

Equation (10) can be rewritten as:

$$E\left[M_{WT}(a,T)\right] = \left|H_{WT\_\mathrm{marg}}(a; f_n, \xi, T)\right|^2 \quad (12)$$

Based on Schmidt's (1985a, b, c) theory of spectral estimation, $\left|H_w(f'; f_n, \xi, T)\right|^2$ for acceleration response had been derived in the authors' earlier paper for white noise excitation (Guo, 2015). And the wavelet function used here is Morlet wavelet and given by:

$$\psi_M(t) = e^{j2\pi f_0 t} e^{-t^2/(2\delta^2)} \quad (13)$$

where δ is the window width factor with unit of time. Therefore, the structure properties, natural frequency and damping ratios, can be identified by fitting the new modified FRF $\left|H_{WT\_\mathrm{marg}}(a; f_n, \xi, T, k, C)\right|^2$ to the marginal spectrum of the wavelet transform $E\left[M_{WT}(a,T)\right]$ of the non-stationary response data.

The main difference between the original non-stationary SI method and current extended approach for non-white excitation is that the PSD of non-white excitation is assumed as a function of frequency while the PSD of white excitation is assumed as a constant. Theoretically, the initial PSD of excitation S_0 is replaced by $f^k C$ and the whole procedure does not change much. However, adding an additional parameter *k* will increase the computational efforts in optimization.

3. NUMERICAL EXAMPLES

The dynamic systems used in the numerical examples are SDOF systems, governed by the following equation of motion:

$$m(t)\ddot{x}(t) + c(t)\dot{x}(t) + k(t)x(t) = f(t) \quad (14)$$

where $m(t)$, $c(t)$ and $k(t)$ are considered as constant over a short time interval. The input, non-white wind excitation, is generated according to the von Karman model and the time history is simulated using the approach proposed by L.E.Wittig (1975).

The proposed new method can be used for the identification of a time-varying system. As mentioned before, the time-varying system can be treated as an equivalent time-invariant system within a short moving window. Therefore, a time-invariant system is firstly used to evaluate the performance of this method. A comparison of the SI accuracy between the original and proposed methods is made to demonstrate the improvement of the proposed method when identifying systems under non-white excitations. Then, the effectiveness of the proposed method is tested for a time-varying system.

3.1 Identification of time-invariant system

Spectra based methods have proven to achieve a good estimation of natural frequency; however, the identification of the damping ratio can be challenging when data segments are short. Therefore, this study focuses on evaluating the effectiveness of the proposed method in identifying the damping ratio. Four time-invariant system models with the same natural frequency 1Hz and different damping ratios ranging from 1% to 5% are established and those systems are excited by the same input wind force. 100 wind force signals of half-an-hour length are generated and applied to the time-invariant system, respectively. The identified natural frequency and damping ratio are shown in Table 1.

As shown in Table 1, this new method can identify the natural frequency and damping ratio very well and the coefficient of variation (COV) of the results of 100 runs are relatively small. As expected, the identification results of natural frequency are accurate and quite stable for all the models. There is a gradual decline in the accuracy of natural frequency identification with the increase of damping ratio and this decline might be caused by the increase of damping's contribution in the modified FRF. However, all the errors are under 2%, hence this slight decline is acceptable. From Table 1, it is found that the proposed method achieved a more accurate and stable estimation

Table1: Identification results of time-invariant systems

	Model	1 (1%)*	2 (1.5%)*	3 (2%)*	4 (5%)*
f_n	Mean (Hz)	0.9973	0.9949	0.9932	0.9809
	Error (%)	0.27	0.51	0.68	1.91
	COV (%)	0.23	0.13	0.16	0.21
ξ	Mean (%)	1.15	1.58	2.10	4.91
	Error (%)	15.04	5.10	4.83	1.88
	COV (%)	26.28	7.26	7.43	4.03

*Note: *the percentage in the brackets is the damping ratio of each model*

of damping for systems with larger damping ratios. The identification result of Model 1 is worse than that of the other models, but this result is still acceptable.

The original method for white excitation was tested in the same conditions for the purpose of comparison. The identification results are listed in Table 2.

Table2: Identification results of time-invariant systems with the original method

	Model	1 (1%)*	2 (1.5%)*	3 (2%)*	4 (5%)*
f_n	Mean (Hz)	0.9923	0.9886	0.9850	0.9647
	Error (%)	0.77	1.14%	1.50	3.53
	COV (%)	1.80	0.21	0.24	0.34
ξ	Mean (%)	0.47	0.70	0.92	2.23
	Error (%)	53.00	53.33	54.00	55.40
	COV (%)	12.21	9.76	8.41	5.76

*Note: *the percentage in the brackets is the damping ratio of each model*

As shown in Table 2, the original method can also obtain a good estimation of natural frequency, but the identification results of the damping ratio are much worse than those by the new method. Comparing Tables 2 and 1, it is found that the damping ratio is under-estimated by the original method. As mentioned before, the PSD of excitation is directly modeled by a power function of frequency in the new method and if the parameter k is assumed to be zero, the two methods will be the same. Since the real parameter k of wind force is negative, assuming k equal to zero will cause overestimation of the modified FRF for frequencies higher than 1Hz and underestimation for frequencies lower than 1Hz. Thus, using a constant to describe the PSD of wind excitation will lead to large errors in damping estimation. Therefore, the PSD of non-white excitation should be directly modeled. This comparison study shows that a power function of frequency used to model the non-white excitation will greatly improve the accuracy of damping ratio estimation under wind excitation.

3.2 Identification of time-varying system

The SI results of the time-invariant system have demonstrated that the proposed method works well under non-white wind excitation. The time-varying system can be treated as a series of time-invariant systems within a short window, hence the proposed method can be used to identify the time-varying system under non-white winds. However, unlike SI for the time-invariant system, the length of time window T is critical in this case. A short window length is desired for capturing the time-varying damping ratio and natural frequency. The system natural frequency ranges from 1.88 to 2.14 Hz and damping ratio ranges from 0.02 to 0.05 in this example. Three cases with different window lengths are selected for comparison. The lengths of the short moving window in Cases 1 to 3 are 30s, 60s, 150s, and the corresponding time shift is 10s, 20s, and 50s. A total of 100 simulations were conducted for the three cases. Figures 1-3 show the SI results for the time-varying system in different cases.

It is seen that the time-varying natural frequency and damping ratio are effectively identified in all three cases. In Case 3, the SI results do not seem to be as accurate as the first two cases in the section where natural frequency or damping ratio vary, due to the relatively long time window used in spectra estimation. The results of Cases 2 and 3 show a similar level of accuracy, but Case 2 is more stable than Case3. A comparison of the three cases suggests a shorter time window allows better tracking the time-varying features of the system, but meanwhile leads to larger uncertainties in SI results. Since high resolution in the time

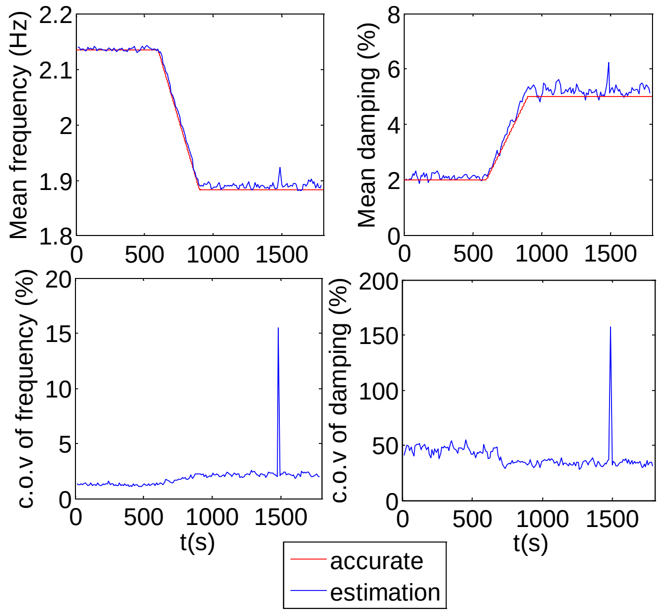

Figure 1: SI results for the time-varying system in case 1

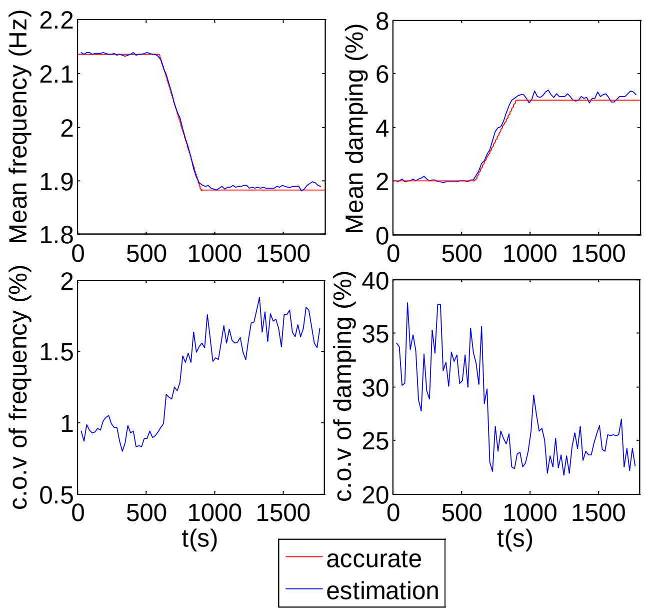

Figure 2: SI results for the time-varying system in case 2

domain and frequency domain cannot be achieved at the same time, there must be a compromise between the ability to track time-vary information and the accuracy to estimate spectra. Among these three cases, Case 2 achieved a good balance between capturing of time-vary information and the uncertainty of estimates. Table 3 shows the percentage errors of natural frequency and damping ratios at different time instants for Case 2.

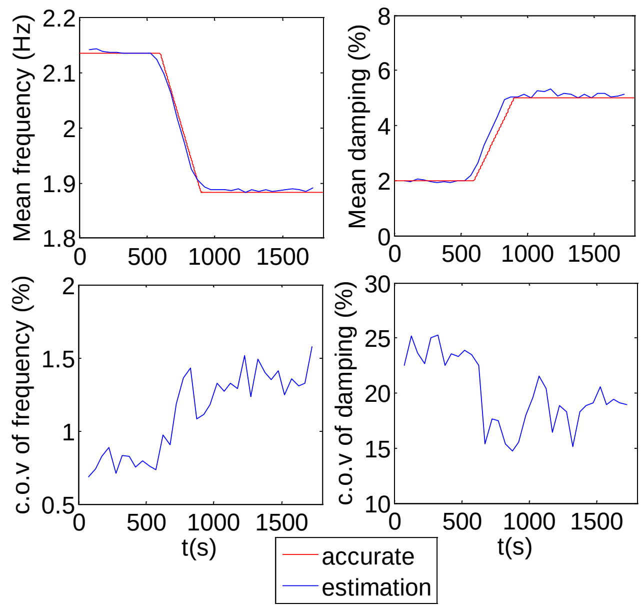

Figure 3: SI results for the time-varying system in case 3

*Table*3: *The identification errors in case 2*

Time(s)	330	610	710	790	890	1330
Error of f_n (%)	0.09	0.03	0.05	0.16	0.23	0.15
Error of ξ (%)	2.49	4.72	2.37	3.10	3.62	0.56

In practice, the length of the short window should be determined based on the structure, environment and the application needs. The SI results of the time-varying system proved that the method proposed in this paper can successfully treat the short window effect and obtain good estimations even for a very short moving window like Case 1.

4. CONCLUDING REMARKS

This paper proposed a new SI method for a time-varying system under non-white excitation by modifying the original non-stationary SI method earlier proposed by the authors. This method is spectra based and a power function of frequency is introduced to model the PSD of wind force. The accuracy and uncertainties of the SI method are investigated through numerical examples of both time-invariant and time-varying systems. The conclusions are drawn as follows:

1. The results of numerical examples show that assuming the wind force as white noise may cause large errors in damping estimation. The direct modeling of the excitation in the proposed method significantly improves SI accuracy for structural systems under non-white winds.

2. The proposed SI method achieved good SI accuracy and can track the time-varying information effectively. The percentage errors of natural frequency and damping ratios in the numerical examples are less than 6% except for the system with 1% damping.

3. The power function of frequency used to model the PSD of wind force is derived based on the characteristics of turbulent wind spectrum and aerodynamic admittance function and proves to be accurate and effective for reflecting the property of the PSD of wind force. In this way, good accuracy of natural frequency and damping ratio can be achieved using output data only.

4. The length of the moving window has an effect on the uncertainties of damping estimation and the method's ability to track time-varying features. Selecting an appropriate window length, which permits tracking the time-varying system and reduces uncertainty, can improve the SI results.

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