Nonlinear System Dynamic Reliability Analysis Using Equivalent Duffing System Method

Zhenhao Zhang
Associate Professor, School of Civil Engineering, Changsha University of Science & Technology, Changsha, China

Xiao Wang
Postgraduate student, School of Civil Engineering, Changsha University of Science & Technology, Changsha, China

Minhan Liu
Postgraduate student, School of Civil Engineering, Changsha University of Science & Technology, Changsha, China

ABSTRACT: Equivalent linearization method is the main approach for nonlinear structural system random response analysis. But it will generate big error that using the random response results of equivalent linearization method to analyze the structural dynamic reliability. In order to improve the analysis precision of dynamic reliability of nonlinear system, an equivalent nonlinear system method is presented in this paper. In this method general nonlinear systems are converted to equivalent Duffing nonlinear system according to minimum mean square error principle, whose exact analytic solution of steady state of random responses can be worked out by Fokker Planck Kolmogorov equation (FPK equation). Then the exact results of stochastic response processes are used for the analysis of structural dynamic reliability. So it is not only convenient for calculation but also with high degree of accuracy for the results that using the equivalent nonlinear system method to analyze structural dynamic reliability. In addition, the equivalent nonlinear system adopted in this work has a parameterεwhich controls the degree of nonlinear. Thus we can obtain conveniently the analysis results of converting the original system to equivalent nonlinear systems with different degree of nonlinear by changing the value of the parameterε. In particular, when the parameter ε is equal to zero we can obtain the analysis results of equivalent linearization method. It is shown from the example analysis that the analysis results of equivalent nonlinear system method presented in this paper is reliable and the calculation accuracy is higher than equivalent linear system method apparently.

KEYWORDS: nonlinear system, dynamic reliability, equivalent nonlinear system, random response, first excursion mechanism

1. INTRODUCTION

The research of dynamic reliability of nonlinear systems has important theoretical and practical significance. Considering nonlinear factors of structures, the random reaction of structural system under random excitation becomes very complicated, and its dynamic reliability becomes more difficult to be analyzed as well (Yang and Zhang, 2011). In the random vibration theory for nonlinear system, equivalent linear system method is applied most widely to solve the random reaction of nonlinear systems and this method is still developing with time (Su Liang and Wang Yi 2011; Guyader and Iwan 2008; Lin and Miranda 2008; Chen and Liu 2008; Wang, Liu and Zhou 2010).Compared with the exact solution or the numerical simulation result, the accuracy of second moment given by the equivalent linear system method is usually adequate, but other statistics given by this method, such as related functions, extremum, may be unreliable. Therefore, by the equivalent linear system method the statistical result of the times of the values exceeding safety boundary may be seriously wrong; and for nonlinear damping systems, there can be several orders of magnitude between the calculated value and the actual value for the probability of the first exceeding (Zhu, Huang and Suzuki 2001).

For these reasons, scholars have been trying to find better approximation methods, in which the equivalent nonlinear system method had been used essentially. The idea of equivalent nonlinear system method was first proposed by Caughey (1984), but his method is only applied to the case where the original system is quasi-linear. Zhu (1989, 2003) established an equivalent nonlinear system method suitable for solving random reactions of quasi-Lyapunov systems. The "best" equivalence principle adopted in this method is to make the equivalent system have the same law of average energy variation with the original system (the same drift and diffusion coefficient). The equivalent nonlinear system method adopted in this paper preserves the nonlinear characteristics of stiffness as the original system, and the nonlinear damping is linearized equivalently, so that the original nonlinear system will be equivalent to a nonlinear structural system with linear damping and nonlinear stiffness, of which the exact probability distribution of its steady-state reaction process can be obtained by FPK equation method.

2. EQUIVALENT NONLINEAR ANALYSIS OF QUASI-DUFFING SYSTEM

2.1. Equivalent analysis between two nonlinear systems under stationary excitation

The oscillatory differential equation for a nonlinear system with single-degree-of-freedom usually can be expressed as

$$\begin{cases} m\ddot{X}(t) + g(X, \dot{X}) = F(t) \\ X(0) = \dot{X}(0) = 0 \end{cases} \tag{1}$$

where $g(X, \dot{X})$ represents a nonlinear function related to X and $\dot{X}$ in general situation; and the random excitation $F(t)$ is set to a normal white noise with the average of 0 and the spectral density of S_0.

Another nonlinear system equivalent to Eq. (1) is set as the following Duffing system,

$$\begin{cases} m\ddot{X}(t) + c_e\dot{X} + k_e[X + \varepsilon\beta(X)] = F(t) \\ X(0) = \dot{X}(0) = 0 \end{cases} \tag{2}$$

where c_e is the equivalent damping coefficient, k_e is the equivalent stiffness coefficient, and ε is a constant and the structure will degenerate into a linear system when ε is 0; $\beta(X)$ is an odd function, and there is $\lim_{|x|\to\infty}\int_0^x \beta(s)ds = \infty$.

The error between the two systems can be expressed by the difference between Eq. (1) and Eq. (2), and its expression can be presented with $e(t)$ as follows,

$$e(t) = g(X, \dot{X}) - c_e\dot{X} - k_e[X + \varepsilon\beta(X)] \tag{3}$$

where the error item $e(t)$ also follows random process. In order to optimize the equivalent system to approximate the original system, the equivalent criterion determining the equivalent parameters c_e and k_e requires the absolute deviation between the equivalent system and the original system is minimum, and for random process, it requires the mean value of the square of $e(t)$ (the mean square value of $e(t)$) is minimum.

Thus, from Eq. (3), there is

$$E[e^2(t)] = E\{g(X, \dot{X}) - c_e\dot{X} - k_e[X + \varepsilon\beta(X)]\}^2$$

(4)

where $E[e^2(t)]$ can be considered as a two-variate function of the equivalent paraments c_e and k_e. According to the method of finding extremum of multivariate functions, the minimum of $E[e^2(t)]$ will occur when

$$\begin{cases} \dfrac{\partial E[e^2(t)]}{\partial c_e} = 0 \\ \dfrac{\partial E[e^2(t)]}{\partial k_e} = 0 \end{cases} \tag{5}$$

By the above equation, and using the exchangeability between the mathematical expectation and the derivation operations in calculation process, we can finally obtain Eq. (6). And then convert the two equations in Eq. (6) to simultaneous equations of c_e and k_e as Eq. (7).

$$\begin{cases} E[\dot{X}\, g(X,\dot{X})] - c_e E(\dot{X}^2) - k_e\{E(X\dot{X}) + \varepsilon E[\dot{X}\,\beta(X)]\} = 0 \\ E[Xg(X,\dot{X})] + \varepsilon E[\beta(X) g(X,\dot{X})] - c_e\{E(X\dot{X}) + \varepsilon E[\dot{X}\,\beta(X)]\} - k_e E\{[X+\beta(X)]^2\} = 0 \end{cases} \tag{6}$$

$$\begin{cases} c_e = \dfrac{E[\dot{X}\, g(X,\dot{X})] E\{[X+\varepsilon\beta(X)]^2\} - E[Xg(X,\dot{X})]\{E(X\dot{X}) + \varepsilon E[\dot{X}\,\beta(X)]\} - \varepsilon E[\beta(X) g(X,\dot{X})]\{E(X\dot{X}) + \varepsilon E[\dot{X}\,\beta(X)]\}}{E(\dot{X}^2) E\{[X+\varepsilon\beta(X)]^2\} - \{E(X\dot{X}) + \varepsilon E[\dot{X}\,\beta(X)]\}^2} \\ k_e = \dfrac{E[\dot{X}\, g(X,\dot{X})]\{E(X\dot{X}) + \varepsilon E[\dot{X}\,\beta(X)]\} - E[Xg(X,\dot{X})] E(\dot{X}^2) - \varepsilon E[\beta(X) g(X,\dot{X})] E(\dot{X}^2)}{\{E(X\dot{X}) + \varepsilon E[\dot{X}\,\beta(X)]\}^2 - E(\dot{X}^2) E\{[X+\varepsilon\beta(X)]^2\}} \end{cases} \tag{7}$$

From Eq. (7), we find that the mathematical expectations at the right end should be known before solving the equivalent parameters c_e and k_e. These expectations are quite difficult to obtain without making any assumptions, because the calculation requires the joint probability distributions of $X(t)$ and $\dot{X}(t)$ which are unknown.

Because the excitation $F(t)$ is a stationary process, and if we ignore the transition stage of reaction process and only consider steady state reaction, according to the conclusion that a stationary process is always unrelated to its mean square derivative at the same time point, we can find that stationary displacement reaction $X(t)$ and velocity reaction $\dot{X}(t)$ are not related, so there is $E[X(t)\dot{X}(t)] = 0$. As a result, Eq. (7) can be simplified as Eq. (8).

Further, the higher order items of ε can be omitted approximately if the parameter $\varepsilon << 1$, then c_e and k_e.can continue to be simplified into Eq. (9).

$$\begin{cases} c_e = \dfrac{E[\dot{X}\, g(X,\dot{X})] E\{[X+\varepsilon\beta(X)]^2\} - E[Xg(X,\dot{X})]\cdot \varepsilon E[\dot{X}\,\beta(X)] - \varepsilon^2 E[\beta(X) g(X,\dot{X})]\cdot \varepsilon E[\dot{X}\,\beta(X)]}{E(\dot{X}^2) E\{[X+\varepsilon\beta(X)]^2\} - \varepsilon^2 E^2[\dot{X}\,\beta(X)]} \\ k_e = \dfrac{E[\dot{X}\, g(X,\dot{X})]\cdot \varepsilon E[\dot{X}\,\beta(X)] - E[Xg(X,\dot{X})] E(\dot{X}^2) - \varepsilon E[\beta(X) g(X,\dot{X})] E(\dot{X}^2)}{\varepsilon^2 E^2[\dot{X}\,\beta(X)] - E(\dot{X}^2) E\{[X+\varepsilon\beta(X)]^2\}} \end{cases} \tag{8}$$

$$\begin{cases} c_e = \dfrac{E[\dot{X}\, g(X,\dot{X})]E(X^2) + 2\varepsilon E[\dot{X}\, g(X,\dot{X})]E[X\beta(X)] - \varepsilon E[Xg(X,\dot{X})]E[\dot{X}\,\beta(X)]}{E(\dot{X}^2)E(X^2) + 2\varepsilon E(\dot{X}^2)E[X\beta(X)]} \\ k_e = \dfrac{E[Xg(X,\dot{X})]E(\dot{X}^2) - \varepsilon E[\dot{X}\, g(X,\dot{X})]E[\dot{X}\,\beta(X)] + \varepsilon E[\beta(X)g(X,\dot{X})]E(\dot{X}^2)}{E(\dot{X}^2)E(X^2) + 2\varepsilon E(\dot{X}^2)E[X\beta(X)]} \end{cases} \quad (9)$$

In random equivalent analysis, the joint probability density of the equivalent system reaction is usually used for substituting the joint probability density of the original system reaction to determine the mathematical expectations in Eq. (7), Eq. (8) and Eq. (9) derived from the equivalent nonlinear (2) which makes these expressions always contain c_e and k_e. Similar to equivalent linear method, in equivalent nonlinear method, iterative method is a general method to solve the specific values of c_e and k_e, as shown below.

Firstly, set the initial values of c_e and k_e; then take these values into the equivalent equation (2), and use FPK equation method to solve the first moment, two moment and two order joint moment of $X(t)$ and $\dot{X}(t)$; next use any one of the Eq. (7), the Eq. (8) or the Eq. (9) to solve the first iteration values of c_{e1} and k_{e1}; repeat the above steps until the values of c_e and k_e have satisfied the convergence criteria; finally, take the final values of c_e and k_e into the equivalent equation (2) to obtain the final result as the approximate solution of the original nonlinear system.

2.2. *Discussion on the situation under non-stationary excitation*

From Eq. (7), Eq. (8) and Eq. (9), we can find that, under non-stationary random excitation, because c_e and k_e directly relate to the statistical moment participating the reaction and the statistical moment of non-stationary reaction is a function of *t* (Yang, Zhang and Lin 2010), the equivalent parameters must vary with time, and there is

$$c_e = c_e(t)\;,\; k_e = k_e(t) \quad (10)$$

Now, the equivalent damping and the equivalent stiffness, as well as the reaction statistical moment, need to be calculated iteratively from the time of $t_1 = \Delta t$ until the desired moment of $t_k = k\Delta t$ as the above steps.

3. STRUCTURAL DYNAMIC RELIABILITY

After transforming the original nonlinear system to the Duffing system by equivalent nonlinear analysis, the joint probability density of the system can be obtained by FPK equation, and the dynamic reliability of system can be obtained conveniently by the classical Poisson process method.

The basic equation of dynamic reliability obtained by the Poisson process method based on the mechanism of first transcendence failure can be expressed as follows (ANG and TANG. 2007),

$$P_s(b_1, -b_2) = \exp\{-\int_0^T [v_{b1}^+(t) + v_{b2}^-(t)]dt\} \quad (11)$$

where b_1 and $-b_2$ are the safety limits on either end; and T is the time duration; $v_{b1}^+(\tau)$ and $v_{b2}^-(\tau)$ are the intersection rate between the reaction process and the safety boundary which can be calculated by Rice formula (ANG and TANG 2007),

$$v_b(t) = \int_{-\infty}^{\infty} \left|\dot{x}\right| f_{X\dot{X}}(b, \dot{x}, t)\, d\dot{x} \quad (12)$$

where $f_{X\dot{X}}(x, \dot{x}, t)$ is the joint probability density function of the reaction process $X(t)$ and its derivative process $\dot{X}(t)$.

4. EXAMPLE

Considering Van der Pol oscillator stimulated by Gaussian white noise,

$$\ddot{X}+\varepsilon'(-1+X^2)\dot{X}+X=\sqrt{\varepsilon'}W(t) \quad (13)$$

where $W(t)$ is the Gaussian white noise with spectral density of S_0; ε' is a constant parameter, and based on the mechanism of first transcendence failure, we will try to analyze the dynamic reliability of the nonlinear system.

The nonlinear system equivalent to Eq. (13) is constructed as,

$$\ddot{X}(t)+c_e\dot{X}+k_e(X+\varepsilon X^3)=\sqrt{\varepsilon'}W(t) \quad (14)$$

where c_e and k_e are the equivalent parameters which are determined based on the criterion of minimum error between the two systems, and they have definite physical significances in Eq. (14), namely, c_e and k_e are respectively the damping coefficient and the stiffness coefficient at the time of $\varepsilon=0$ and at that time the Duffing system degenerates into a linear system.

4.1. Solutions of equivalent parameters

According to the expressions of c_e and k_e derived from Eq. (9), and through comparison of the two expressions, we can easily know that,

$$g(X,\dot{X})=\varepsilon'(-1+X^2)\dot{X}+X\ ,\ \beta(X)=X^3$$

According to the computational properties of mathematical expectations, the expectations in Eq. (9) can be obtained as,

$$E[\dot{X}g(X,\dot{X})]=-\varepsilon' E(\dot{X}^2)+\varepsilon' E(X^2\dot{X}^2)$$

$$E[X\beta(X)]=E(X^4)$$

$$E[Xg(X,\dot{X})]=\varepsilon' E(X^3\dot{X})+E(X^2)$$

$$E[\dot{X}\beta(X)]=E(X^3\dot{X})$$

$$E[\beta(X)g(X,\dot{X})]=-\varepsilon' E(X^3\dot{X})+\varepsilon' E(X^5\dot{X})+E(X^4)$$

where the various order moments that need to be solved are $E(X^2)$, $E(\dot{X}^2)$, $E(X^4)$, $E(X^2\dot{X}^2)$, $E(X^3\dot{X})$ and $E(X^5\dot{X})$. For a high order moment, the first second-order moment of X or X and its $\dot{X}$ can be expressed by normal reduced order method (Li and Chen 2009)

$$E(X^4)=3[E(X^2)]^2$$

$$E(X^2\dot{X}^2)=E(X^2)E(\dot{X}^2)$$

$$E(X^3\dot{X})=3E(X\dot{X})E(X^2)=0$$

(for stationary reaction, $E(X\dot{X})=0$)

$$E(X^5\dot{X})=5E(X\dot{X})E(X^4)=0$$

(for stationary reaction, $E(X\dot{X})=0$)

After simplification, the equivalent parameters c_e and k_e can finally be expressed as two functions by the mean square value of reaction $E(X^2)$,

$$c_e=\frac{[-\varepsilon' E(\dot{X}^2)+\varepsilon' E(X^2)E(\dot{X}^2)]E(X^2)+2\varepsilon[-\varepsilon' E(\dot{X}^2)+\varepsilon' E(X^2)E(\dot{X}^2)]\cdot 3E^2(X^2)}{E(X^2)E(\dot{X}^2)+2\varepsilon E(\dot{X}^2)\cdot 3E^2(X^2)}$$
$$=\varepsilon'[-1+E(X^2)] \quad (15)$$

$$k_e=\frac{E(X^2)E(\dot{X}^2)+\varepsilon\cdot 3E^2(X^2)\cdot E(\dot{X}^2)}{E(X^2)E(\dot{X}^2)+2\varepsilon E(\dot{X}^2)\cdot 3E^2(X^2)}=\frac{1+3\varepsilon E(X^2)}{1+6\varepsilon E(X^2)} \quad (16)$$

4.2. Random reaction solution of equivalent system

The solutions of the equivalent nonlinear system (14) are shown in Table 1 according to FPK equation method.

Table 1: Analytic solution of equivalent system (14).

Joint probability density of displacement reaction and velocity reaction	$p_{X\dot{X}}(x,\dot{x}) = C\exp[-\frac{c_e}{\pi\varepsilon' S_0}(\frac{1}{2}\dot{x}^2+\frac{1}{2}k_e x^2+\frac{\varepsilon}{4}k_e x^4)]$
Edge distribution density of displacement reaction process $X(t)$	$p_X(x) = C\sqrt{2\pi}\sqrt{k_e}\sigma_{X_0}\exp[-\frac{1}{\sigma_{X_0}^2}(\frac{1}{2}x^2+\frac{\varepsilon}{4}x^4)]$
Edge distribution density of velocity reaction process $\dot{X}(t)$	$p_{\dot{X}}(\dot{x}) = \frac{1}{\sqrt{2\pi}\sigma_{\dot{X}_0}}\exp(-\frac{\dot{x}^2}{2\sigma_{\dot{X}_0}^2})$
Expectation	$E[X(t)]=0$, $E[\dot{X}(t)]=0$
Variance	$\sigma_X^2 \approx \sigma_{X_0}^2 - 3\varepsilon\sigma_{X_0}^4$, $\sigma_{\dot{X}}^2 = \frac{\pi\varepsilon' S_0}{c_e}$
Mean square value of displacement reaction	$E(X^2) = \frac{\pi\varepsilon' S_0}{c_e k_e} - 3\varepsilon\frac{\pi^2\varepsilon'^2 S_0^2}{c_e^2 k_e^2}$

In Table 1 C is a constant determined by normalization conditions, that is $C = \frac{1}{\sqrt{2\pi}\sqrt{k_e}\sigma_{X_0}}\{\int_{-\infty}^{\infty}\exp[-\frac{1}{\sigma_{X_0}^2}(\frac{1}{2}x^2+\frac{\varepsilon}{4}x^4)]dx\}^{-1}$; $\sigma_{X_0}^2$ and $\sigma_{\dot{X}_0}^2$ are respectively the stationary variance of the displacement reaction $X_0(t)$ and velocity reaction $\dot{X}_0(t)$, which can be obtained by linear random vibration analysis theory as follows,

$$\sigma_{X_0}^2 = \frac{\pi\varepsilon' S_0}{c_e k_e},\ \sigma_{\dot{X}_0}^2 = \frac{\pi\varepsilon' S_0}{c_e} \tag{17}$$

According to the results in the above table, we can obtain that,

$$\begin{cases} c_e = -\varepsilon' + \frac{\pi\varepsilon'^2 S_0}{c_e k_e} - 3\varepsilon\frac{\pi^2\varepsilon'^3 S_0^2}{c_e^2 k_e^2} \\ k_e = \dfrac{1+3\varepsilon\frac{\pi\varepsilon' S_0}{c_e k_e} - 9\varepsilon^2\frac{\pi^2\varepsilon'^2 S_0^2}{c_e^2 k_e^2}}{1+6\varepsilon\frac{\pi\varepsilon' S_0}{c_e k_e} - 18\varepsilon^2\frac{\pi^2\varepsilon'^2 S_0^2}{c_e^2 k_e^2}} \end{cases} \tag{18}$$

a two-element equation set of c_e and k_e can be obtained by the simultaneous of the upper equations, but it is difficult to solve, and its numerical solution can only be obtained by numerical method. After the determination of c_e and k_e, all the random reaction results can be obtained by taking c_e and k_e into equations in Table **Error! Reference source not found.**.

4.3. Analysis results and discussion

The approximate joint probability density function of van der Pol oscillator is given in the literature (Wang, Liu and Zhou 2010) as follows,

$$f_{X\dot{X}}(x,\dot{x}) = [\pi\sqrt{2\pi S_0}\,erfc(-\sqrt{\frac{2}{S_0}})]^{-1}\exp[-\frac{1}{8S_0}(x^2+\dot{x}^2-4)^2]$$

(19)

where $erfc(\cdot)$ is the residual error function and $erfc(x) = 1-erf(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-m^2)dm$. After the comparison of the numerical results, the result shows that ε' obtained by Eq. (19) is rather accurate when the parameters are small.

The results in this example are compared with the results of Eq. (19) and the results of equivalent linear analysis, then we find if the parameters $\varepsilon = 0$,the equivalent nonlinear system will degrade into a linear system and the results of equivalent linear analysis which has be transformed from the original nonlinear systems can be obtained.

From the comparison of the calculation results in Table 2, we can see that the calculation results given by the equivalent nonlinear method in this paper are quite close to the results in the literature (Wang, Liu and Zhou, 2010) and the results are reliable. In addition, the comparison with the results of equivalent linear analysis shows that the accuracy of the results given by the equivalent nonlinear analysis has some improvement than ever, and with the increase of the nonlinear parameter ε of the equivalent system, the accuracy of the calculation results is also improved. Therefore, the equivalent nonlinear analysis method in this paper is feasible. From the calculation results of Table 3, it can be seen that if the original nonlinear system is equivalent to a linear system to analyze its dynamic reliability, the error is indeed relatively large; the results in this paper shows great agreement with and the results in Monte-carlo numerical simulation (Liu and Yao, 2009), and the accuracy of the calculation results of the equivalent nonlinear method is obviously improved.

Table 2: Comparison of analysis results of random response.

ε'	0.05					0.2				
ε	0	0.2		0.5		0	0.2		0.5	
Method	Equivalent linearization	In literature (Wang 2010)	In this paper	In literature (Chen 2008)	In this paper	Equivalent linearization	In literature (Wang 2010)	In this paper	In literature (Chen 2008)	In this paper
$E(X)$	0	0	0	0	0	0	0	0	0	0
σ_X^2	0.2763	0.3496	0.3053	0.3496	0.3157	0.5645	0.6712	0.6143	0.6712	0.6547
$E(\dot{X})$	0	0	0	0	0	0	0	0	0	0
$\sigma_{\dot{X}}^2$	0.3516	0.6985	0.5684	0.6985	0.6047	0.6952	0.7265	0.7029	0.7265	0.7436

Table 3: Comparison of dynamic reliability calculation results

ε'	0.05				0.2			
ε	0	0.2	0.5	Monte-Carlo simulation	0	0.2	0.5	Monte-Carlo simulation
Method	Equivalent linearization	In this paper	In this paper		Equivalent linearization	In this paper	In this paper	
Structural dynamic reliability	0.8954	0.9398	0.9454	0.9715	0.8545	0.9465	0.9573	0.9788

5. CONCLUSION

For a nonlinear system, its accuracy of random reaction analysis determines the accuracy of dynamic reliability analysis. In this paper an equivalent nonlinear method based on equivalent Duffing system is proposed which transforms a general nonlinear system into a nonlinear

Duffing system which has linear damping and nonlinear stiffness, and for this kind of nonlinear systems, their accurate steady-state joint probability density function can be obtained by FPK equation. The example has demonstrated that, the method proposed in this paper has higher accuracy than the equivalent linear method in the results, and the former improves the accuracy of the dynamic reliability of nonlinear systems. In addition, because the equivalent nonlinear system adopted has the parameter ε controlling the intension of nonlinearization, it is easy to obtain the analysis results when the original nonlinear system is equivalent to different nonlinear systems with strong or weak linearization , by changing the value of ε; and in particular, when ε is zero, the analysis results of equivalent linearization can be obtained, which is quite useful to research the issue. The question of the optimal value of ε, when ε can get the most accurate calculation results of nonlinear system reliability, still requires further research work.

Finally, it should be pointed out that in order to simplify the calculation in the analysis process, the high order terms of small parameters are omitted, and some approximate hypotheses are used in the normal reduced order method used to deal with the high order reaction moment, which has a certain influence on the accuracy of the calculation results.

6. REFERENCES

A H-S ANG, W H TANG. (2007). *Probability Concepts in Engineering Emphasis on Applications to Civil and Environmental Engineering.* John Wiley & Sons Ltd.

Caughey T. K. (1986). On the response of non-linear oscillators to stochastic excitation. *Probabilistic Engineering Mechanics*, 1(1), 2-4.

Chen Yanmao and Liu Jike. (2008). Improved Equivalent Linearization Method. *Chinese Journal of Applied Mechanics*, 25(2), 296-298.

Guyader A C and Iwan D. (2006). Determining Equivalent Linear Parameters for Use in a Capacity Spectrum Method of Analysis. *Journal of Structural Engineering*, 132(1), 59-67.

Li Jie, Chen Jianbing. (2009). New Development of Random Vibration Method and Application Structures. *Shanghai: Tongji University Press*.

Lin Y Y and Miranda E. (2008). Noniterative Equivalent Linear Method for Evaluation of Existing Structures. *Journal of Structural Engineering*, 134(11), 1685-1695.

Liu Pei and Yao Qianfeng. (2009). Dynamic Reliability Calculation Based on Importance Sampling Method. *Chinese Journal of Computational Mechanics*, 29(6), 951-855.

Su Liang and Wang Yi. (2011). New Optimization Algorithm for Determining System Parameters of Equivalent Linearization Method. *Engineering Mechanics*, 28(9), 23-29.

Wang Wei, Liu Bideng and Zhou Zhenghua. (2010). Equivalent Linear Method Considering Frequency Dependent Stiffness and Damping. *Rock and Soil Mechanics*, 31(12), 3928-3933.

Yang Weijun, Zhang Zhenhao, and Lin Li. (2010). Research on Seismic Reliability of Structure Based on Boundary Value of Damage Index. *Journal of Earthquake Engineering and Engineering Vibration*, 30(1): 77-83.

Yang Wei-jun and Zhang Zhen-hao. (2011). Structural Dynamic Reliability Study Based on the Probability Analysis of First-passage Time of Continuous Markov Process. *Engineering Mechanics*, 28(7), 124-129.

Zhu Weiqiu and Yu Jinshou. (1989). Equivalent Nonlinear System Method for Predicting Response of Nonlinear Systems to Random Excitations. *Acta Mechanica Solida Sinica*, 10(1), 34-44.

Zhu Weiqiu. (2003).Nonlinear Stochastic Dynamics and Control. *Beijing: Science Press*.

Zhu WQ, Huang ZL and Suzuki Y. (2010). Equivalent Nonlinear System Method for Stochastically Excited and Dissipated Partially Integrable Hamiltonian Systems. *International Journal of Non-Linear Mechanics*, 36(5), 773-786.