Stochastic Sampling for Efficient Seismic Risk Assessment of Transportation Network

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ABSTRACT: For accurate seismic risk assessment of transportation network under probabilistic seismic hazard, the uncertainty in the seismic hazard, the damage states of links/bridges in the network, and network performance need to be quantified. Stochastic simulation is well suited for this task. However, it typically requires large number of model evaluations, which entails significant computational effort, especially for large network. To address the above challenges, an efficient stochastic sampling-based approach is proposed. It relies on generating one set of samples for earthquake magnitude and carrying out analysis for the corresponding set of networks. This set of evaluations are used for seismic risk assessment under different risk measures, different probabilistic seismic hazards (e.g., with or without considering spatial correlation), and also for risk-based importance ranking of all bridges/links in the network for risk mitigation purpose. No additional evaluation of the network model is needed. The proposed approach is applied to seismic risk assessment and mitigation of the transportation network of Los Angeles and Orange countries. The impact of spatial correlation in seismic hazard on the seismic risk assessment and mitigation is investigated.

Keywords: Stochastic sampling; Stochastic simulation; Probabilistic seismic hazard; Transportation network; Spatial correlation

1. INTRODUCTION

Transportation network plays an important role in various social and economic activities. Disruption of transportation networks due to seismic events could cause significant economic losses. The evaluation of probabilistic performance (e.g., seismic risk) of transportation networks is critical for pre-event mitigation and post-event emergency responses and recovery activities (Kurtz et al. 2016). However, this evaluation requires quantification and propagation of high-dimensional uncertainties including uncertainties in intensity measures at each bridge site and uncertainties in the damage states of each bridge. Generally, stochastic simulation techniques, e.g., Monte Carlo simulation (MCS), need to be used for this evaluation (Taflanidis and Jia 2011). However, direct adoption of MCS would require large number of evaluations of the network model and entail significant computational challenges, which is further intensified considering the need to evaluate seismic risk of the network under different performance measures, hazard scenarios, and mitigation strategies.

This paper proposes an efficient sampling-based approach for seismic risk assessment of large-scale transportation network. This approach relies on only one set of simulations of the network model. Specially, it first generates one set of uniform samples for the earthquake moment magnitude, and then generates one set of corresponding samples for intensity measures at each bridge site and the damage states of each bridge (e.g., based on the fragility). Then network analysis is carried out for this set of samples. In the end, the information from this single set of analysis is used for seismic risk assessment under different risk performance measures, hazard characteristics, and mitigation strategies where the proposed approach only requires updating the corresponding risk measures or probability densities describing the hazard characteristics and/or mitigation strategies without the need to re-run any network analysis.

This paper is organized as follows. Section 2 introduces the seismic risk of transportation networks and challenges in evaluation of seismic risk. Section 3 presents the proposed sampling-based approach for efficient evaluation of seismic risk and risk mitigation of transportation networks. In Section 4, the proposed approach is applied to efficiently evaluate the seismic risk and different risk mitigation strategies for the transportation network of Los Angeles and Orange countries. The impact of spatial correlation in seismic hazard on seismic risk assessment and mitigation is investigated. The last section summarizes the research findings.

2. SEISMIC RISK ASSESSMENT OF TRANSPORTATION NETWORKS UNDER SPATIALLY CORRELATED SEISMIC INTENSITIES

2.1. Spatial correlation of seismic intensities

For a transportation network, typically bridges are the vulnerable links. The seismic performance of bridges will directly impact the overall network performance. Consider a network with a total of n bridges. A seismic event EQ can be defined by parameters such as magnitude M, the location $\mathbf{x}_{EQ}$ (or equivalently, vector of epicentral distances with respect to all bridge sites, i.e., $\mathbf{R}=[R_1,\ldots,R_i,\ldots R_n]$), depth etc. Let $\mathbf{IM}=[IM_1,\ldots,IM_i,\ldots IM_n]$ represent the vector of intensity measures at all bridge sites with IM_i the intensity measure at the i^{th} bridge. Let $p(\mathbf{IM}\,|\,M,\mathbf{x}_{EQ})$ represent the joint PDF for $\mathbf{IM}$ under given earthquake event defined by $(M,\mathbf{x}_{EQ})$. For spatially distributed lifeline infrastructure systems such as transportation networks, researches have shown that neglecting the uncertainties in ground motion intensities and the spatial correlations between multiple sites would result in significant errors in the seismic risk assessment of infrastructure systems (Jayaram and Baker 2010). The adoption of different correlation models will essentially impact $p(\mathbf{IM}\,|\,M,\mathbf{x}_{EQ})$. When intensity measures at different sites are assumed independent, $p(\mathbf{IM}\,|\,M,\mathbf{x}_{EQ})=\prod_{i=1}^{n} p(IM_i\,|\,M,\mathbf{x}_{EQ})$.

2.2. Seismic risk of transportation networks

For a network with a total of n bridges, and suppose all bridges are in service before a seismic event occurs; under a seismic event, each bridge has an associated probability of being in a certain damage state. The uncertainty in the damage states for all the bridges can be characterized by the vector of random variables $\boldsymbol{\theta}=[\theta_1,\ldots,\theta_i,\ldots,\theta_n]$ where θ_i represents the damage state for the i^{th} bridge. For given damage state, the capacity of the bridge will change accordingly. For a given realization of the damage states $\boldsymbol{\theta}$, the corresponding network response (e.g., total travel time, independent pathway) can be written as $y(\boldsymbol{\theta})$. For large-scale network, typically each evaluation of $y(\boldsymbol{\theta})$ requires significant computational effort (also depending on the adopted network model and performance measures). The performance of the network can be characterized through a performance function $h(\boldsymbol{\theta})$ (also called risk measure) related to the response $y(\boldsymbol{\theta})$, i.e., $h(y)=h(y(\boldsymbol{\theta}))=h(\boldsymbol{\theta})$. If this performance function defines failure, then $h(\boldsymbol{\theta})$ is simply the indicator function, i.e., $h(\boldsymbol{\theta})=I_F(\boldsymbol{\theta})$. If the failure of the network is defined as $y(\boldsymbol{\theta})\geq y_{thres}$ where

y_{thres} is some response threshold, then $I_F(\boldsymbol{\theta})=1$ if $y(\boldsymbol{\theta}) \geq y_{thres}$ and 0 otherwise.

Within the context of the above descriptions, propagating the uncertainties in the intensity measures, damage states and the seismic events, the seismic risk of a transportation network can be written as

$$H = \int h(\boldsymbol{\theta})\, p(\boldsymbol{\theta} \mid \mathbf{IM})\, p(\mathbf{IM} \mid M, \mathbf{x}_{EQ})\, p(M, \mathbf{x}_{EQ})\, d\boldsymbol{\theta}\, d\mathbf{IM}\, dM\, d\mathbf{x}_{EQ} \quad (1)$$

where $p(\boldsymbol{\theta} \mid \mathbf{IM})$ denotes the PDF for $\boldsymbol{\theta}$ conditional on given vector of intensity measures. For the i^{th} bridge we have $p(\theta_i \mid IM_i)$ for θ_i under given intensity measure IM_i, and when considering n_D possible damage states, we have $p(\theta_i \mid IM_i) = P(\theta_i = DS_j \mid IM_i)$ where $\theta_i = DS_j$ for any $j = 1, 2, \ldots, n_D$. In this case, $p(\theta_i \mid IM_i)$ can be established from the fragility curves for the i^{th} bridge.

2.3. *Computational challenges in evaluation of seismic risk*

To evaluate how spatial correlation in seismic intensities impact the seismic risk, we need to evaluate the seismic risk under many different combinations of (1) risk measure (e.g., different definition of risks), (2) spatial correlation models, i.e., different $p(\mathbf{IM} \mid M, \mathbf{x}_{EQ})$, (3) probabilistic hazard scenarios, i.e., different $p(M, \mathbf{x}_{EQ})$, and (4) mitigation strategies, i.e., different $p_s(\boldsymbol{\theta} \mid \mathbf{IM})$ (where $p_s(\boldsymbol{\theta} \mid \mathbf{IM})$ denotes the updated PDF for $\boldsymbol{\theta}$ resulting from retrofitting some of the bridges in the network). To evaluate risks under different combinations, direct use of MCS (e.g., repeating MCS for each combination) would create huge computational challenges, especially for large-scale network where each evaluation of network response requires huge computational effort and for rare events.

3. EVALUATION OF SEISMIC RISK BY EFFICIENT SIMULATION

To address the computational challenges in the evaluation of seismic risk, this paper proposes an efficient sampling-based approach to assess the seismic risk of the transportation networks and investigate how the spatial correlation in seismic intensities would impact the risk assessment under any of different combinations mentioned in Section 2.3. It relies only on one set of simulations of the network model and has significant efficiency improvement compared to using direct MCS.

3.1. *Evaluation of seismic risk integrals*

The risk integrals in Eq. (1) correspond to high-dimensional integrals, especially for large-scale networks. Stochastic Simulation (e.g., MCS) is the general approach to estimate such integrals. However, each estimation typically requires a large number of model evaluations. Using N samples $\{\boldsymbol{\theta}^k, \mathbf{IM}^k, M^k, \mathbf{x}^k_{EQ}\}, k = 1, \ldots, N$ from some proposal density $q(\boldsymbol{\theta}, \mathbf{IM}, M, \mathbf{x}_{EQ})$ for the uncertain parameters $[\boldsymbol{\theta}, \mathbf{IM}, M, \mathbf{x}_{EQ}]$, the risk integral in Eq. (1) can be estimated through

$$\hat{H} = \frac{1}{N}\sum_{k=1}^{N} \frac{h(\boldsymbol{\theta}^k)\, p(\boldsymbol{\theta}^k \mid \mathbf{IM}^k)\, p(\mathbf{IM}^k \mid M^k, \mathbf{x}^k_{EQ})\, p(M^k, \mathbf{x}^k_{EQ})}{q(\boldsymbol{\theta}^k, \mathbf{IM}^k, M^k, \mathbf{x}^k_{EQ})} \quad (2)$$

The accuracy of the estimate in Eq. (2) can be quantified by the c.o.v (coefficient of variation) of the estimate,

$$\delta \approx \frac{1}{\sqrt{N}} \sqrt{\frac{\frac{1}{N}\sum_{k=1}^{N}\left[h(\boldsymbol{\theta}^k)\, r^k_{IS}\right]^2}{\hat{H}^2} - 1} \quad (3)$$

where

$$r^k_{IS} = \frac{p(\boldsymbol{\theta}^k \mid \mathbf{IM}^k)\, p(\mathbf{IM}^k \mid M^k, \mathbf{x}^k_{EQ})\, p(M^k, \mathbf{x}^k_{EQ})}{q(\boldsymbol{\theta}^k, \mathbf{IM}^k, M^k, \mathbf{x}^k_{EQ})} \quad (4)$$

3.2. Steps of the proposed efficient simulation

The proposed approach starts by generating a set of N samples from a selected proposal density $q(\boldsymbol{\theta},\mathbf{IM},M,\mathbf{x}_{EQ})$ and evaluate the corresponding network responses $y(\boldsymbol{\theta})$. This evaluation is the most computationally demanding task in estimation of seismic risks. This is especially true when each run of the network model (e.g., for large-scale network) takes a lot of time. Then, instead of re-running MCS for risk assessment under all different combinations mentioned earlier, the proposed approach uses the same set of samples and for different combinations only updates the r_{IS}^k defined in Eq. (4). More specifically,

1) For different risk measures, we only need to update the $h(\boldsymbol{\theta}^k)$ value (i.e., based on value of $y(\boldsymbol{\theta}^k)$);
2) For different spatial correlation models (e.g., the level of correlation), we only need to update the values $p(\mathbf{IM}\,|\,M^k,\mathbf{x}_{EQ}^k)$;
3) For different definitions of probabilistic seismic hazard, e.g., different selection of $p(M,\mathbf{x}_{EQ})$, we only need to update PDF values for $p(M^k,\mathbf{x}_{EQ}^k)$;
4) For different mitigation strategies, we only need to update the PDF values $p(\boldsymbol{\theta}^k\,|\,\mathbf{IM}^k)$ to $p_s(\boldsymbol{\theta}^k\,|\,\mathbf{IM}^k)$ for the corresponding mitigation strategy. Take retrofitting the i^{th} bridge for example (with the updated PDF denoted as $p_{s,i}(\boldsymbol{\theta}\,|\,\mathbf{IM})$), the updated seismic risk of the network can be estimated as $\hat{H}_s(i)=\frac{1}{N}\sum_{k=1}^{N}h\left(\boldsymbol{\theta}^k\right)r_{IS,i}^k$ with

$$r_{IS,i}^k=\frac{p_{s,i}\left(\boldsymbol{\theta}^k\,|\,\mathbf{IM}^k\right)p\left(\mathbf{IM}^k\,|\,M^k,\mathbf{x}_{EQ}^k\right)p\left(M^k,\mathbf{x}_{EQ}^k\right)}{q\left(\boldsymbol{\theta}^k,\mathbf{IM}^k,M^k,\mathbf{x}_{EQ}^k\right)} \quad (5)$$

Therefore, the proposed approach facilitates efficient evaluation of seismic risk and risk mitigation relying only on one set of simulations of the network model, which leads to significant efficiency improvement compared to using direct MCS for each different definition of risk integrals. This approach is applicable to cases when the seismic risk needs to be evaluated for many different risk measures, different hazard models, different fragility models (e.g., mitigation) or combinations of them, and the system (e.g., large-scale network) model is computationally expensive to run.

3.3. Selection of proposal density

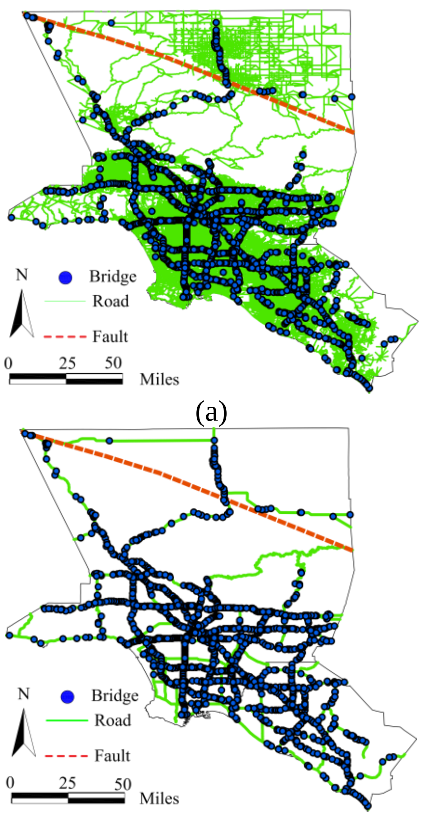
As one key element, the selection of proposal density will impact the risk estimation accuracies and the applicability of the proposal approach. Several considerations are taken into account in this selection. First, the support domain of $q(M)$ needs to be larger than the support domain $p(M)$ so that the density ratio $p(M)/q(M)$ is well-defined. Also, the proposal density needs to be applicable for a wide range of different combinations (as discussed earlier) including rare events. It is typically not efficient to establish proposal density for all the uncertain parameters $[\boldsymbol{\theta},\mathbf{IM},M,\mathbf{x}_{EQ}]$ due to the well-known intrinsic challenges in Importance Sampling for high-dimensional problems. Therefore, we focus on the more important parameters, and considering the importance of earthquake moment magnitude, we select a proposal density $q(M)$ for M while using prior distribution for the rest of the parameters. As for $q(M)$, the uniform distribution is a good candidate since it could generate samples for M that covers the entire range specified and also provide samples with large M (which helps with the simulation of rare events). Other selection of proposal densities (e.g., incorporate optimization) will be considered in future research.

4. ILLUSTRATIVE EXAMPLE

The proposed approach is used to evaluate the seismic risk and different risk mitigation strategies for the transportation network of Los Angeles and Orange countries (see Figure 1). The impact of spatial correlation in seismic hazard is also investigated.

4.1. Transportation network

The network data for Los Angeles and Orange are obtained from Southern California Association of Governments (SCAG 2012) (Southern California Association of Governments (SCAG) 2012). The database with bridge locations and physical characteristics of bridges is obtained from Caltrans Structure Maintenance and Investigations (SMI 2015) Database. The daily origin-destination (OD) data are based on the Southern California origin-destination survey of 2,950 traffic analysis zones (TAZ) in Los Angeles and Orange counties.



(b)

Figure 1: (a) The transportation network of Los Angeles and Orange countries ;(b) the corresponding aggregated network.

The transportation network data are aggregated through construction of Thiessen polygons for reducing the computational efforts. Only the freeway and state highway are considered in the aggregated network, which includes 155 nodes and 242 links. A node is defined by the location where two or more freeways/highways intersect, or the location where a freeway/highway passes through the boundary of the study area. A link is represented by a road segment between two adjacent nodes. The free flow speeds for the freeway and highway links are assumed to be 65 and 35 miles per hour, respectively. The practical capacity for a link on the freeway and highway is considered to be 2,500 and 1,000 passenger car units per hour, respectively. 2,600 bridges lie on the links of the aggregated network. The damage states of bridges due to specific intensity measures are estimated using the modified fragility curves based on the fragility function for each type of standard bridge in accordance with the HAZUS manual (DHS 2009). The bridge with the lowest mean capacity on one link is selected to represent all the bridges on the link for reducing the complexity of network analysis. And 217 bridges are selected as representatives of all bridges on the aggregated network. The daily OD data of 2,950 TAZs are aggregated to obtain the condensed daily OD data of 155 new TAZs.

4.1.1. Hazard model including spatial correlation of intensity measures

The seismicity information of Los Angeles and Orange counties is obtained from USGS and the Southern San Andreas Fault in the study region is considered (USGS 2015). $p(M)$ is selected as truncated exponential distribution in [5.5, 8.0] with regional seismicity factors $\beta = 0.9\log_e(10)$. The ground-motion model in Boore et al. (2013) is used to obtain the probabilistic distribution of intensity measure for each representative bridge in the network. And $S_a(T = 1.0\text{sec})$, which would be used in the fragility functions for bridges, is selected as the intensity measure. The probabilistic event location $\mathbf{x}_{EQ}$ is assumed to occur uniformly on the fault. To take into account the correlation between the ground-motion intensities at different sites, the spatial correlation models in Loth and Baker (2013) is used.

4.1.2. Network model

Based on bridge information from the National Bridge Inventory (NBI), fragility curves are established for all bridges using the HAZUS definition of fragility and damage states. Under given origin-destination (OD) matrix and given realization of damage states of all bridges, the network performance $y(\boldsymbol{\theta})$ (e.g., total travel time or TTT) can be evaluated through transportation network models. Here we use the combined distribution and assignment model in Bocchini and Frangopol (2011) to calculate TTT. For given realization of damage states for each bridge, the corresponding network characteristics such as capacity, trip production and attraction need to be updated. We assume the reduced capacities corresponding to the five different HAZUS damage states are 100% (none/slight), 75% (moderate), 50% (extensive), and 25% (complete). The non-zero capacity for the complete damage state is used considering the widely-used redundancies in transportation networks. The change in travel demands due to earthquake is not considered in this study, however this change can be easily incorporated when information on the OD patterns is available.

4.1.3. Risk measures

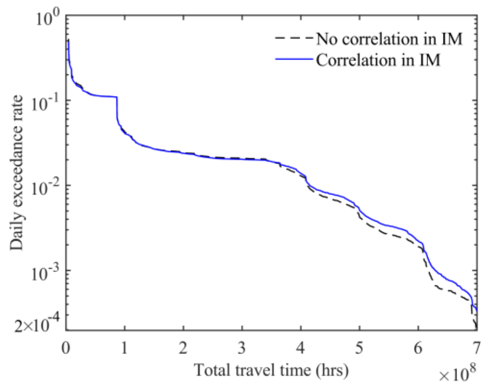
For seismic risk, here we consider the failure probability (or reliability) of the network) where failure is defined as TTT exceeding a certain threshold denoted y_{thres}. Other definitions of risk measures can be used as well depending on the interested network performance. Note that the proposed approach can easily incorporate different risk measures.

4.2. Implementation details

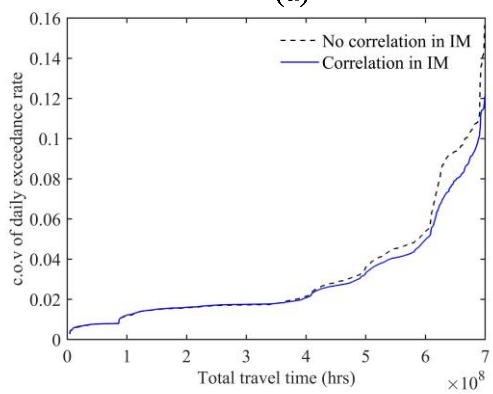
For the set of simulations, we use the uniform proposal density $q(M)$ to generate $N=100{,}000$ samples for *M*; then the prior distributions for intensity measures and damage states are used to generate realizations of damages states for each bridge. Then the network performance are evaluated for these samples. Based on this set of evaluations, we use the proposed approach to estimate the seismic risk of the transportation network under the different combinations mentioned in Section 2.3. For illustration, we will evaluate the TTT exceedance rate against y_{thres} (i.e., different risk measures), and estimate the failure probability of the network after retrofitting each bridge to establish risk-based rankings for evaluating the different risk mitigation strategies. For the seismic risk mitigation, we consider a definition of network failure with $y_{thres}=5y_0$ where y_0 is the TTT when all of the bridges are in service. The updated $p_{s,i}(\boldsymbol{\theta}\,|\,\mathbf{IM})$ is established by updating the fragility curve of retrofitted bridge through increasing the median value of the distribution of the fragility curves. In all these cases, intensity measures with and without spatial correlations are considered to explore the impact of correlation on seismic risk assessment and risk mitigation.

4.3. Results and discussions

Figure 2 (a) shows the TTT exceedance rate curve with and without considering spatial correlation in intensity measures. Figure 2 (b) shows the corresponding coefficient of variation (showing good accuracy over the different thresholds). All the information in Figure 2 is established using the same set of evaluations without additional evaluatiions of the network model. As seen from the figure, for a larger TTT threshold value, the exceedance rate obtained using the spatial correlation model is larger than that without spatial correlation considered. This shows the seismic risk assessment using correlation model would be more conservative compared to that when no correlation in intensity measures is considered. The results demonstrate the importance of taking into account the spatial correlation of ground-motion intensities at multiple sites when estimating seismic risk for spatially distributed network.



(a)

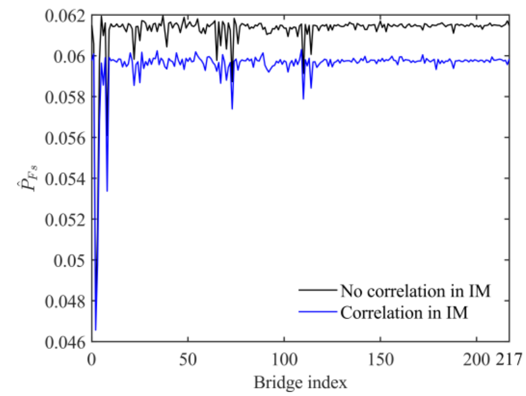


(b)

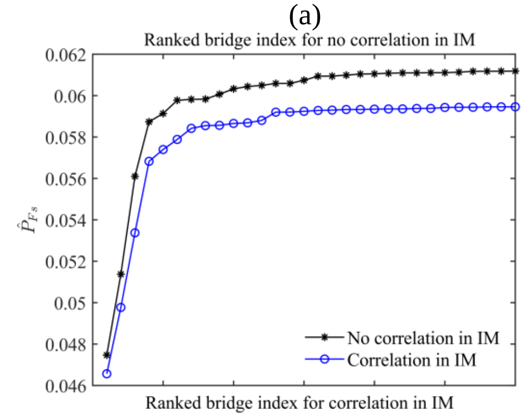
Figure 2: (a) Daily exceedance rate vs TTT;(b) coefficient of variation of the estimate for daily exceedance rate.

Figure 3 (a) shows the failure probability of the transportation network after retrofitting each bridge individually where x-axis corresponds to the bridge index from 1 to 217. Again, all information in Figure 3 are established using the same set of evaluations. Based on the amount of reduction in failure probability, we can rank the bridges in terms of their importance in seismic risk mitigation. The results are shown in Figure 3 (b) where the x-axis corresponds to the ranked bridge index. Note that here we evaluated the updated failure when only one bridge is retrofitted, the proposed approach can be easily extended to considering combination of several bridges to identify the optimal retrofit strategy (Wang and Jia 2018).

The impact of spatial correlation in intensity measures on the ranking of bridges can be seen in Table 1 below, where the top 10 ranked bridges are listed and $\hat{P}_{Fs}$ represents the failure probability of the network.



(a)



(b)

Figure 3: (a) Failure probability of the network after retrofitting each bridge individually; (b) Ranked failure probability of the network after retrofitting each corresponding bridge.

Table 1: Results of seismic risk mitigation for the transportation network.

No correlation in IM		*Correlation in IM*	
$\hat{P}_{Fs}$ *(%)*	*Bridge index*	$\hat{P}_{Fs}$ *(%)*	*Bridge index*
4.75	*2*	*4.66*	*2*
5.14	*3*	*4.98*	*3*
5.61	*8*	*5.34*	*8*
5.87	*73*	*5.68*	*4*
5.91	*110*	*5.74*	*73*
5.98	*65*	*5.79*	*110*
5.98	*4*	*5.84*	*114*
5.98	*22*	*5.86*	*22*
6.01	*114*	*5.86*	*6*
6.03	*70*	*5.87*	*67*

As can be seen, the spatial correlation in intensity measures impacts the ranking of the bridges, e.g., only the top three bridges in the ranking are the same whether considering the correlation or not while the rest of the top 10 ranked bridges are different for the case of considering correlation or not. The results in Figure 3 and Table 1 demonstrate the importance of considering spatial correlation in intensity measures when evaluating different seismic risk mitigation strategies.

5. CONCLUSIONS

This paper proposed an efficient sampling-based approach for seismic risk assessment and risk mitigation of transportation networks. The proposed approach requires only one set of simulations of the network model, which can be used to efficiently evaluate the seismic risk and risk mitigation for any combinations of risk measures, hazard models (e.g., with or without correlation in intensity measures), and mitigation strategies (e.g., change fragility models for bridges). The evaluation only requires updating the corresponding quantities in the sample-based estimation of seismic risk and no additional simulations of the network model are required. The proposed approach has great computational efficiency and is especially useful for seismic risk assessment and mitigation of large-scale transportation networks. The illustrative example for the transportation network of Los Angeles and Orange countries demonstrated the high efficiency of the proposed approach, and the results showed the importance of incorporating the spatial correlations in seismic hazard in seismic risk assessment and mitigation. Future work will investigate the application of the proposed approach for optimization of seismic mitigation strategies where groups of bridges need to be retrofitted (corresponding to challenging combinatorial optimization where large number of mitigation strategies need to be evaluated).

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