Modeling spatial variability of steel corrosion using Gumbel distribution

Mingyang Zhang
Graduate Student, Dept of Civil and Environmental Engineering, Waseda University, Tokyo, Japan

Sopokhem Lim
Assistant professor, Dept of Civil and Environmental Engineering, Waseda University, Tokyo, Japan

Mitsuyoshi Akiyama
Professor, Dept of Civil and Environmental Engineering, Waseda University, Tokyo, Japan

Dan M. Frangopol
Professor, Lehigh University, 117 ATLSS Drive, Bethlehem, PA, USA

ABSTRACT: Performance of corrosion-affected RC structures depends on the spatial variability of steel corrosion of reinforcing bars. The effect of spatial variability of steel corrosion on the remaining service life of concrete structures is significant. In this paper, an experimental study was conducted to investigate the effect of current density on the spatial variability of steel corrosion. This variability is modeled using Gumbel distribution derived from experiments and incorporated with FE method to estimate the yield loading capacity of RC beams. The results show that using Gumbel distribution parameters derived from the specimens with high current density may lead to an overestimation of load capacity of corroded RC beams.

1. INTRODUCTION

Steel corrosion is one of the most important factors causing performance degradation of RC structures exposed to marine environment. This has become an important issue over the past decades. There has been substantial evidence that the localized damages of reinforcements have detrimental effects on structural performance of corroded RC structures. Zhu and François (2014) reported that the capacity loss of corroded RC beams was affected by the steel cross-section loss at the failure location. Malumbela et al. (2010) showed that using average cross-section loss in the theoretical model resulted in an overestimation of the loading capacity of RC members. Stewart (2004) found that the failure probability considering the effect of spatial corrosion pits was twice larger than that of uniform corrosion for the reliability analysis for RC members in flexure. Therefore, it is important to model the spatial variability of steel corrosion in reliability analysis of corroded RC structures.

Gumbel distribution has been widely used for modeling spatial variability in steel corrosion. Stewart (2009) proposed a pitting factor $R_p = P_{max}/P_{av}$, where P_{max} and P_{av} are the maximum and average pitting depths, respectively. The pitting factor R_p was assumed as a time-invariant parameter while R_p was observed to decrease with time. Gu et al. (2018) proposed a spatial variability factor $R = A_{ave}/A_{min}$, where A_{ave} and A_{min} are average and minimum cross-section area, respectively. The Gumbel parameters of R shows a linear relationship with the increase of corrosion level. However, these parameters were derived using the data of steel corrosion from different corroded rebars with corrosion level lower than 5%. Lim et al. (2016) applied a non-destructive method using X-ray and imaging processing technique to quantify the spatial variability of steel weight loss. However, the relationship of

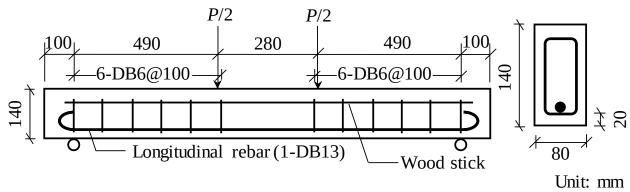

Figure 1: Front view and cross-section of RC beam

Gumbel distribution parameters for R_{swl} and mean steel weight loss was proposed with the experimental data without considering the effect of current density.

In this paper, Gumbel distribution parameters are derived from two RC beams corroded by two level of current densities (I_{corr}) 50 μA/cm^2 and 500 μA/cm^2 to model the steel weight loss distribution of corroded rebar. The effect of current density on the Gumbel distribution parameters for modeling the spatial variability of steel weight loss along the length of rebar is highlighted.

2. EXPERIMENTAL INVESTIGATION

2.1. Experimental program

Two RC beams with the dimensions shown in Figure 1 were cast for this study. The mixing proportions of concrete is listed in Table 1. The galvanostatic method using I_{corr} of 50 and 500 μA/cm^2 was applied to simulate corrosion-induced damage for specimens I50 and I500, respectively. During the corrosion process at various corrosion levels, the X-ray apparatus was used to take photograms of the non-corroded and corroded rebars from 8 different viewing angles as shown in Figure 2.

Steel weight loss (Rw) was estimated per 5 mm along the length of rebar using digital image processing technique of X-ray images which is defined as

$$Rw = \frac{1}{k}\sum \frac{W_{\theta n} - W'_{\theta n}}{W_{\theta n}} \tag{1}$$

where $W_{\theta n}$ and $W'_{\theta n}$ are the weights of the non-corroded and corroded bars per 5 mm, respectively; k = 8 is the number of viewing angles; and the subscript θ_n represent the viewing angle. More details are given in Lim et al. (2016).

Table 1: Mixing proportions (kg/m^3)

Water	*Cement*	*Sand*	*Stone*	*AE**
181	*362*	*754*	*961*	*2715*
**Air entrainment (mL/m^3)*				

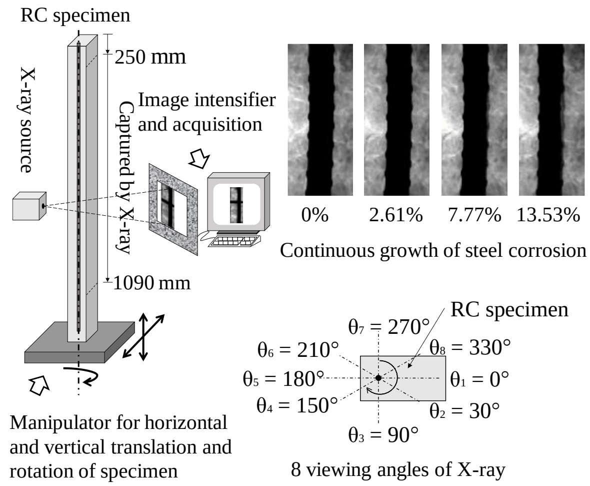

Figure 2: X-ray photography

At the end of the experimental procedure, a four-point bending test was conducted by applying monotonic loads at two points symmetrically as indicated in Figure 1. The applied load P and the mid-span deflection δ were recorded during the bending test.

2.2. Results and discussion

Figure 3 illustrates the Rw distributions for specimens I50 and I500 at various corrosion levels. In this study, comparison of Rw was conducted based on same mean steel weight loss (MRw). It can be seen that the I50 specimen shows a more non-uniform Rw distribution than the I500 specimen at three corrosion levels. A few large corrosion pits concentrating at three different regions (i.e. from 310 to 430 mm, 610 to 790 mm, and 1030 to 1090 mm) are found in specimen I50 at MRw = 10.96%. However, the Rw of specimen I500 distributes more evenly over the length of rebar. As a result, the standard deviation (SD) of Rw for specimen I50 is higher than that for specimen I500 at different corrosion levels.

These results suggest that the stochastic field of modeled spatial steel corrosion which is

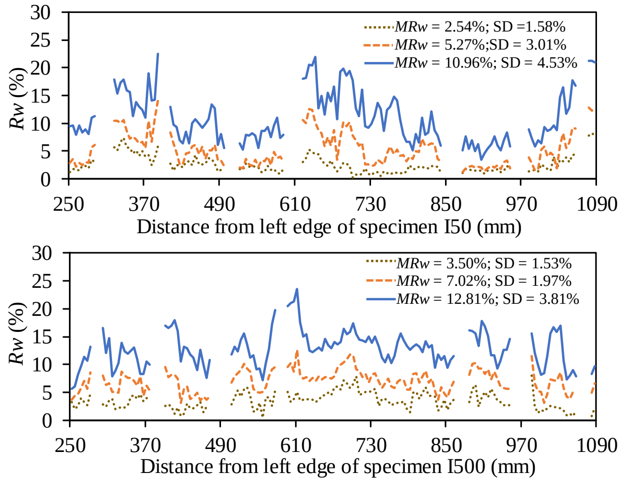

Figure 3: Spatial variability of the steel weight loss for specimens I50 and I500

generated using the statistical data of accelerated steel corrosion in the laboratory is significantly affected by the impressed current density.

3. PROBABILITY MODEL OF STEEL WEIGHT LOSS FOR CORRODED RC BEAM

3.1. Probability model

A probability model is established to estimate the spatial variability of the steel weight loss using the Gumbel distribution of the maximum steel weight loss ratio $R_{swl} = W_{max}/MRw$, where W_{max} is the local maximum steel weight loss at an interval of 50 mm. To represent the spatial variability of steel weight loss at different corrosion levels using Gumbel distribution, the location parameter μ and scale parameter σ can be determined via the expected value and variance of R_{swl} (i.e. $E(R_{swl})$ and $D(R_{swl})$).

$$E(R_{swl}) = \mu + \gamma\sigma \quad (2)$$

$$D(R_{swl}) = \pi^2\sigma^2/6 \quad (3)$$

where $\gamma = 0.5772$ is Euler's constant.

The probability density function is expressed as

$$f(x) = \tfrac{1}{\sigma}\exp[-\tfrac{1}{\sigma}(x-\mu)]\exp[\exp(-\tfrac{1}{\sigma}(x-\mu))] \quad (4)$$

The relationships between Gumbel distribution parameters (i.e. μ and σ) and MRw for

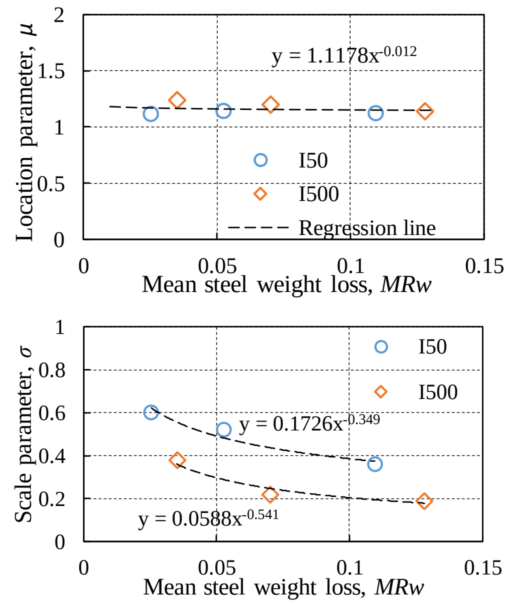

Figure 4: Relationship between Gumbel distribution parameter μ, σ and mean steel weight loss

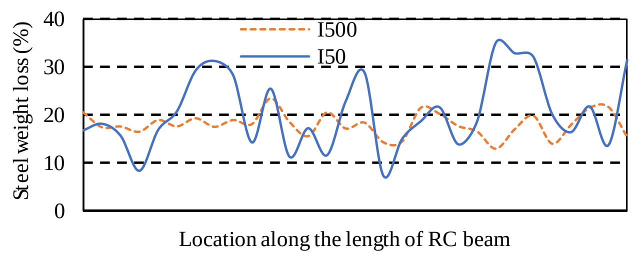

Figure 5: Example of generated steel weight loss

two specimens are illustrated in Figure 4. It can be seen that the scale parameter decreases with the increase of MRw. It is apparent that the Gumbel distribution scale parameter σ derived from the specimen using lower current density is higher than that derived from the specimen with higher current density.

3.2. Representation of Gumbel distributed variables

Normal distributed variables are commonly used as standard random variables because of mathematical tractability. When the random variables are independent, the uncertainty in the i-th variable x_i (i.e. R_{swl}) can be expressed as a function of the i-th standard normal distribution variable U_i.

To represent Gumbel distributed variables (i.e. R_{swl}), the transformation function

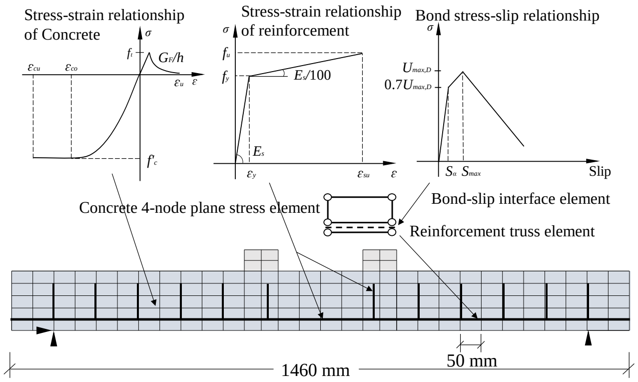

Figure 6: Constitutive model and mesh discretization of RC beam in FE analysis

$$x = \mu - \sigma \ln[\ln(\Phi(U))] \tag{5}$$

is used where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Examples of generated steel weight loss using two scale parameters are shown in Figure 5.

4. FINITE ELEMENT MODELING OF CORRODED RC BEAM

4.1. FE model

Figure 6 illustrates a two-dimensional finite element model used to simulate the structural performance of a corroded RC beam. The concrete was modeled using a four-node plane stress element with 50 mm length. A two-node truss element was used to model reinforcement. The bond between reinforcement and concrete was modeled as a line-plane interface element. The concrete compression behavior of concrete was modeled based on Japan Society of Civil Engineers (JSCE) (2007) and softening curve of Hordijk, as described in DIANA10.2 User's manual (2017), was used for modeling tensile behavior. The bond-slip model used in Lim et al. (2016) was also applied herein to model the deteriorated bond between reinforcement and concrete. The data of R_{swl} was used as the input for modeling the reduced cross-section area of reinforcement (from 250 mm to 1090 mm from the left end of beam).

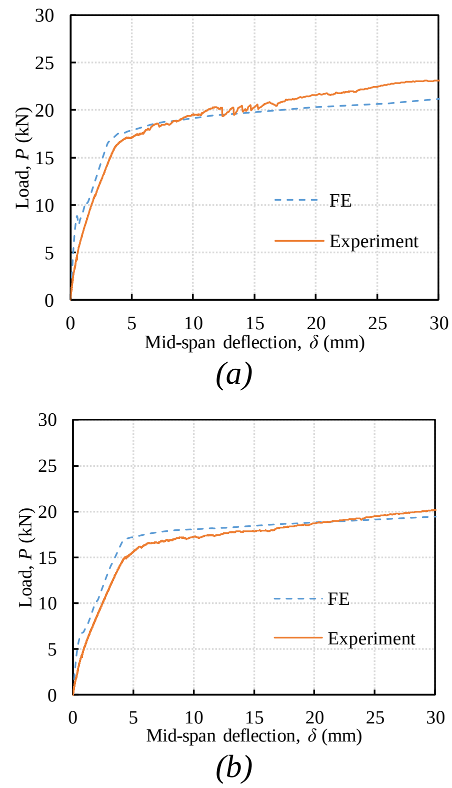

Figure 7: Experimental versus FE results for the flexural response of (a) beam I50 and (b)I500

4.2. Validation of FE model

The experimental and simulated load-deflection curves for specimens I50 and I500 are shown in Figure 7. Numerical results show a good agreement with the experiments. Therefore, the load capacity can be predicted with reasonable accuracy.

The FE method incorporated with the maximum steel weight loss at the interval of 50 mm can give the flexure responses which are compared to those of the test beams. This comparison provides a good accuracy. Therefore, FE method is applied herein to reflect the effects of current density on the Gumbel distribution parameters.

4.3. Modeling R_{swl} using Gumbel distribution

Ten samples of spatial distribution of steel weight loss with an assumption of *MRw* = 15% were generated based on Gumbel distribution parameters μ and σ calculated by two regression

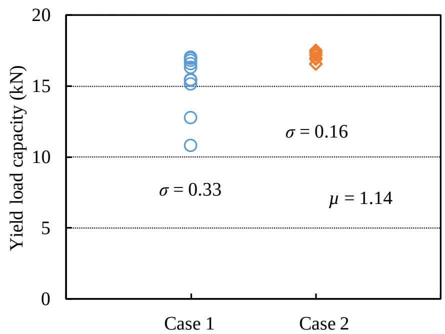

Figure 8: Yield load capacity distribution for two cases

equations shown in Figure 4 for two cases. The generated R_{swl} were used to calculate cross-section area of reinforcement as the input for the FE model to obtain the yield load capacity.

Figure 8 shows that the yield load capacity results obtained for two cases with different scale parameters. It can be seen that the yield load capacity in case 1 is lower and more scatter than that in case 2. This comparison suggests that the Gumbel distribution parameter is significantly affected by the impressed current density. Also, using Gumbel distribution parameter derived from the specimen with higher current density may underestimate the non-uniformity of steel weight loss distribution. This can lead to an overestimation of load capacity of a corroded RC structure.

5. CONCLUSIONS

1. Low current density can induce a more non-uniform steel weight loss distribution than using higher current density. This result indicates that the stochastic field of modeled spatial steel corrosion which is generated using the statistical data of specimens using galvanostatic method in the laboratory is significantly affected by the impressed current density.
2. Based on the FE modeling results, the corroded rebar samples generated by the Gumbel distribution parameters derived from the specimen with lower current density present a more scatter and lower yield load capacity, compared with that from a specimen with higher current density. Hence, caution should be taken when applying the galvanostatic method to simulate the corrosion of rebar to obtain the Gumbel distribution parameters. If these parameters are derived from specimens using higher current density, the non-uniformity of steel corrosion may be underestimated.

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