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Efficient Optimization for Multi-Objective Decision-Making on Civil Systems Using Discrete Influence Diagram

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ABSTRACT: The breakdown of civil systems, e.g. bridge networks and water distribution networks, has a significant social and economic impact, highlighting the importance of optimal decision-making on such systems. Modeling and optimization of probabilistic decision-making problems for civil systems, can be facilitated by graphical methodologies such as influence diagram (ID). However, the converging structure in IDs representing civil systems, which relates the random variables standing for component events and that for system event, results in the exponential increase in the number of modeling parameters and variables to be optimized as that of component events increases. In order to address these challenges, in this paper, the recently proposed matrix-based Bayesian network (MBN) is employed to quantify the IDs. To facilitate the optimization process, a proxy objective function is also proposed. The proxy function not only significantly reduces the number of variables to be optimized, but also allows an efficient framework for multi-objective optimization in which the weighted sum of the objectives is optimized to obtain a set of non-dominated solutions. Three numerical examples demonstrate the performance of the proposed methodology.

1. INTRODUCTION

The breakdown of civil systems such as bridge networks and water distribution networks, has a significant social and economic impact. In order to minimize such risk, optimal decisions need to be made on the systems, generally in consideration of multiple objectives, e.g. cost of retrofits, and system performance. Influence diagram (ID), an extension of Bayesian network (BN) for the purpose of decision-making, facilitates the probabilistic modeling and inference of complex systems while discrete ID allows the development of general-purpose algorithms for inference. However, since real-world civil systems consist of a large number of components and the definition of system events is highly complex, the exact modeling and optimization of the IDs remain elusive. In particular, the converging structure between the random variables (r.v.'s) standing for component events and that for system event, may result in an exponential increase in both the number of parameters that quantify given probability distributions and the variables to be optimized as that of component events increases.

In order to address this issue, multiple methodologies have been developed, e.g. a method to exploit the regularity in the definition of system event (Poole 1996). Recently, as a generalization and expansion of such efforts, a matrix-based Bayesian network (MBN) has been developed (Byun et al. 2018) to provide an alternative data structure for the probability mass functions (PMFs) in BN. On the other hand, algorithms for optimization of discrete IDs have been continuously studied to develop structured procedures of optimization (Olmsted 1984; Diehl and Haimes 2004). However, these algorithms are limited to relatively small-size problems, and thus unable to handle large real-world civil systems.

In this paper, IDs are modeled by the MBN, and the use of a proxy objective function is proposed to facilitate their optimization. The proxy function decomposes the optimization problem into smaller ones, so that the size of optimization problem linearly increases as the number of decision variables increases. Moreover, by using the proposed proxy function, an efficient scheme for multi-objective optimization is developed, in which the weighted sum of objective values is optimized. Optimization with these weights leads to a set of non-dominated solutions.

In the formulations of this paper, upper and lower cases respectively denote a r.v. and the assigned value, e.g. X and x, while bold letters are used to denote a set of r.v.'s. For simplicity, x^k indicates the assignment of value k to r.v. X, i.e. $x = k$. The set of values that a r.v. X can take is denoted as $Val(X)$.

2. BACKGROUND

2.1. Influence diagram (ID) for civil systems

BN is a graphical representation of a joint probability distribution based on conditional independence between r.v.'s (Koller and Friedman 2009). In BN, circular nodes and directed arrows respectively represent the r.v.'s ($\boldsymbol{X}$) and their statistical dependence. ID is an extension of BN for the purpose of decision-making in which variables of decision ($\boldsymbol{D}$) and utility ($\boldsymbol{V}$) are additionally introduced to describe the design alternatives, and utility (or risk) quantified on the instances of interest. They are respectively visualized by rectangles and rhombuses in ID. In the followings, the terms *node* and *r.v.* are used synonymously.

Utility variables $V \in \boldsymbol{V}$ are deterministic function of their parent nodes, and in the following illustration, the symbol is also used when referring to the corresponding function, i.e. $V(Pa_V)$, where Pa_V denotes the parent nodes of V. For decision variables, only deterministic decision rules are considered in this paper, i.e. the probability distribution $P(D_n|Pa_{D_n})$ for $D_n \in \boldsymbol{D} = \{D_1, \cdots, D_N\}$ assigns nonzero probability to exactly one value of D_n.

For instance, consider the system in Figure 1(a), which consists of three components X_1, X_2, and X_3 that can fail due to earthquake hazard. The system event is defined as the connectivity between the two nodes *s* and *t*. Decision on retrofit of each component (d_n^1 for retrofitting; d_n^0 for not) is considered, which may lower the system failure probability (V_S) at the expense of the related cost (V_n) as illustrated in the ID of Figure 1(b). In the figure, the r.v.'s H, X_n, and S respectively stand for the intensity of hazard (h^0 for insignificant; h^1 for significant), the state of *n*-th component, and that of system (x_n^0 and s^0 for failure; x_n^1 and s^1 for survival.)

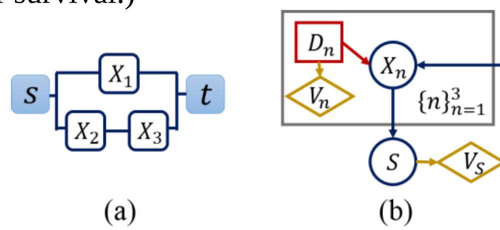

Figure 1. (a) Configuration and (b) ID of example system

The optimization of ID aims to obtain the optimal decision rule $\boldsymbol{d}^*$, i.e. an assignment over $\boldsymbol{D}$, that minimizes or maximizes the sum of expectations of utilities, i.e.

$$\min_{\boldsymbol{d} \in Val(\boldsymbol{D})} (\max) \sum_{V \in \boldsymbol{V}} E[V|\boldsymbol{d}] \qquad (1)$$

Regarding V_S in the example ID, the converging structure from $\boldsymbol{X} = \{X_1, \cdots, X_N\}$ to X_{N+1} makes the decision variables $\boldsymbol{D}$ dependent on each other in evaluating the expectation $E[V_S|\boldsymbol{d}]$. In other words, the decision variables cannot be optimized separately, requiring the optimization to consider their combinatorial states whose number exponentially increases as N increases. However, such structure inevitably takes place when modeling a system as it stands for the intrinsic characteristic of systems whose states are determined by the combinatorial states of their components. On the other hand, V_n, $n = 1, \cdots, N$, do not make the decision variables dependent to each other, implying that the optimization can be performed for each decision variable separately.

2.2. Extension of matrix-based BN (MBN) to ID

The converging structure increases not only the computational cost, but also the memory to store the parameters for quantifying the PMF $P(S|\boldsymbol{X})$ as $|Val(\boldsymbol{X})|$ exponentially increases to $|\boldsymbol{X}|$. In order to address this issue, MBN has been recently proposed as an alternative data structure for conditional PMFs in discrete BN (Byun et al. 2018).

In the MBN, the PMFs are quantified by conditional probability matrices (CPMs) $\mathcal{M} = \langle \mathbf{C}; \mathbf{p} \rangle$ that consist of two matrices, namely, event matrix $\mathbf{C}$ and probability vector $\mathbf{p}$. A CPM can be regarded as a set of rules $\mu = \langle \boldsymbol{c}; p \rangle$ each of which corresponds to an instance of interest with assignment $\boldsymbol{c}$ and probability p. $\mathbf{C}$ and $\mathbf{p}$ are also equivalent to a set of assignments and probabilities, respectively, while their data structure is matrix with each row corresponding to each instance. For example, the PMFs $P(H)$ and $P(X_n|D_n, H)$ can be quantified by CPM $\mathcal{M}_H = \langle \mathbf{C}_H; \mathbf{p}_H \rangle$ and $\mathcal{M}_{X_n} = \langle \mathbf{C}_{X_n}; \mathbf{p}_{X_n} \rangle$ as

$$\mathbf{C}_H = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{p}_H = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix},$$

$$\mathbf{C}_{X_n} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \mathbf{p}_{X_n} = \begin{bmatrix} 0.2 \\ 0.8 \\ 0.1 \\ 0.9 \\ 0.4 \\ 0.6 \\ 0.3 \\ 0.7 \end{bmatrix} \tag{2}$$

where $n = 1,2,3$. The subscript of a CPM denotes the r.v. whose conditional PMF on its parent nodes, is quantified by the CPM. The columns in $\mathbf{C}_{X_n}$ represent the assignments over X_n, D_n, and H in sequence. For instance, the first rows of $\mathbf{C}_{X_n}$ and $\mathbf{p}_{X_n}$ together indicate that $P(x_n^0|d_n^0, h^0) = 0.2$.

One of the distinct features of the MBN is the use of "-1" state. The state accounts for the relationship of *context-specific independence*, i.e. dependent r.v.'s become statistically independent when a specific assignment is imposed on a subset of r.v.'s (Koller and Friedman 2009). Using the "-1" state, PMF $P(S|\boldsymbol{X})$ of the example system is efficiently represented by CPM $\mathcal{M}_S = \langle \mathbf{C}_S; \mathbf{p}_S \rangle$ with

$$\mathbf{C}_S = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \mathbf{p}_S = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{3}$$

where the columns of $\mathbf{C}_S$ respectively stand for the assignments over S and X_n, $n = 1,2,3$ in sequence. Specifically, the first rows of $\mathbf{C}_S$ and $\mathbf{p}_S$ indicate that

$$P\left(s^1|x_1^1, x_2^i, x_3^j\right) = 1, \forall i, j \in \{0,1\} \tag{4}$$

based on the fact that the nodes s and t are connected when X_1 survives, regardless of the states of X_2 and X_3. As noted in Eq. (3), the use of "-1" state allows an efficient description of the given PMF whereby 4 assignments are sufficient to define the event space that originally includes $|Val(\boldsymbol{X})| = 8$ instances.

Another distinctive feature of the MBN is that the CPMs do not need to include all exiting instances in an event space. Such condition is often required when quantifying probabilistic distributions, i.e. each possible assignment should be specified with a certain probability. In contrast, in the MBN, only the instances of interest are quantified while the unspecified instances are considered to have zero probability during the inference. For instance, in Eq. (3), as a counterpart to the first row, there is an instance

$$P\left(s^0|x_1^1, x_2^i, x_3^j\right) = 0, \forall i, j \in \{0,1\} \tag{5}$$

but they are not included in $\mathcal{M}_S$ as they have no effect on inference results because of their zero probability. This feature is advantageous especially for quantification of deterministic functions and approximate inference.

As the MBN can efficiently describe deterministic functions, its extension to ID is straightforward for utility variables are deterministic functions of their parent nodes. For instance, the utility variable V_S that quantifies the system failure probability can be defined as a deterministic function of the value of S, i.e.

$$V_s = \begin{cases} 1, & S = 0 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

This leads to the CPM $\mathcal{M}_{V_S} = \langle \mathbf{C}_{V_S}; \mathbf{p}_{V_S} \rangle$ as

$$\mathbf{C}_{V_S} = [1 \quad 0] \text{ and } \mathbf{p}_{V_S} = [1] \quad (7)$$

where the columns of $\mathbf{C}_{V_S}$ sequentially represent the assignments over V_S and S. It is noted that the instance associated with s^1 is not included in $\mathcal{M}_{V_S}$ as the associated utility value is zero, and the instance is not used during the inference. On the other hand, CPMs for decision variables do not need to be quantified in advance as they are the design variables determined by optimization.

3. PROXY OBJECTIVE FUNCTION FOR DEPEDNENT DECISION VARIABLES

In this section, a proxy objective function is proposed to alleviate the drastic increase in the complexity of optimization problem due to the presence of multiple dependent decision variables. Note that, regarding the set of assignments, $\mathbf{C}_{\boldsymbol{X}}$ over the r.v.'s $\boldsymbol{X}$, the expectation $E[V|\boldsymbol{d}]$ for a given rule $\boldsymbol{d}$ and basis rule $\bar{\boldsymbol{d}}$, is evaluated as

$$\begin{aligned} &E[V|\boldsymbol{d};\bar{\boldsymbol{d}}] \\ &= \sum_{\boldsymbol{c}\in\mathbf{C}_{\boldsymbol{X}}} \left\{ V(\boldsymbol{c}) P_{\mathcal{M}_{\boldsymbol{D}^-}}(\boldsymbol{c}) \prod_{n=1}^{N} P_{\mathcal{M}_{\boldsymbol{D}_n^+}[d_n]}(\boldsymbol{c}) \right\} \\ &= \sum_{\boldsymbol{c}\in\mathbf{C}_{\boldsymbol{X}}} \left\{ \left(V(\boldsymbol{c}) P_{\mathcal{M}_{\boldsymbol{D}^-}}(\boldsymbol{c}) \prod_{n=1}^{N} P_{\mathcal{M}_{\boldsymbol{D}_n^+}[\bar{d}_n]}(\boldsymbol{c}) \right) \cdot \prod_{n=1}^{N} \frac{P_{\mathcal{M}_{\boldsymbol{D}_n^+}[d_n]}(\boldsymbol{c})}{P_{\mathcal{M}_{\boldsymbol{D}_n^+}[\bar{d}_n]}(\boldsymbol{c})} \right\} \\ &= \sum_{\boldsymbol{c}\in\mathbf{C}_{\boldsymbol{X}}} \left\{ V(\boldsymbol{c}) P(\boldsymbol{c}|\bar{\boldsymbol{d}}) \cdot \prod_{n=1}^{N} (1+\Delta P_n) \right\} \\ &= \sum_{\boldsymbol{c}\in\mathbf{C}_{\boldsymbol{X}}} \left\{ V(\boldsymbol{c}) P(\boldsymbol{c}|\bar{\boldsymbol{d}}) \cdot \left(1 + \sum_{n=1}^{N} \Delta P_n + O(\Delta P_n^2) \right) \right\} \end{aligned} \quad (8)$$

where $\boldsymbol{D}^-$ denotes the set of r.v.'s that do not have decision variables as their parent nodes, that is, $\boldsymbol{D}^- = \{X \in \boldsymbol{X} | Scope[\mathcal{M}_X] \cap \boldsymbol{D} \neq \emptyset\}$ in which $Scope[\mathcal{M}]$ denotes the set of r.v.'s over which the CPM $\mathcal{M}$ is defined; $\boldsymbol{D}_n^+$ is the set of r.v.'s that have D_n as their parent nodes, i.e. $\boldsymbol{D}_n^+ = \{X \in \boldsymbol{X} | D_n \in Scope[\mathcal{M}_X]\}$; and $\mathcal{M}_X[x]$ denotes the CPM $\mathcal{M}_X$ conditioned on assignment x, i.e. the probabilities of instances that are not compatible with x are set as zero. For simplicity, some additional notations are introduced in Eq. (8) as

$$P_{\mathcal{M}_X}(\boldsymbol{c}) = \sum_{\langle \boldsymbol{c}'; p' \rangle \in \mathcal{M}_X[\boldsymbol{c}]} p' \quad (9)$$

and

$$\Delta P_n(\boldsymbol{c}|d_n;\bar{d}_n) = \frac{P_{\mathcal{M}_{\boldsymbol{D}_n^+}[d_n]}(\boldsymbol{c}) - P_{\mathcal{M}_{\boldsymbol{D}_n^+}[\bar{d}_n]}(\boldsymbol{c})}{P_{\mathcal{M}_{\boldsymbol{D}_n^+}[\bar{d}_n]}(\boldsymbol{c})} \quad (10)$$

The term $O(\Delta P_n^2)$ summarizes the products involving more than one ΔP_n term, i.e.

$$O(\Delta P_n^2) = \sum_{\substack{m,n\in\{1,\cdots,N\} \\ m<n}} \Delta P_m \Delta P_n + \cdots + \prod_{n=1}^{N} \Delta P_n \quad (11)$$

$\Delta P_n(\boldsymbol{c}|d_n;\bar{d}_n)$ in Eq. (10) is represented by ΔP_n in Eqs. (8) and (11) for concise representation. The probability $P(\boldsymbol{c}|\bar{\boldsymbol{d}})$ is evaluated as

$$P(\boldsymbol{c}|\bar{\boldsymbol{d}}) = P_{\mathcal{M}_{\boldsymbol{D}^-}}(\boldsymbol{c}) \prod_{n\in\mathcal{N}} P_{\mathcal{M}_{\boldsymbol{D}_n^+}[\bar{d}_n]}(\boldsymbol{c}) \quad (12)$$

In this study, the use of the following proxy measure $\tilde{E}[V|\boldsymbol{d};\bar{\boldsymbol{d}}]$ is proposed instead of $E[V|\boldsymbol{d};\bar{\boldsymbol{d}}]$ by ignoring $O(\Delta P_n^2)$ in Eq. (8):

$$\tilde{E}[V|\boldsymbol{d};\bar{\boldsymbol{d}}] = \sum_{\boldsymbol{c}\in\mathbf{C}_{\boldsymbol{X}}} \left\{ V(\boldsymbol{c}) P(\boldsymbol{c}|\bar{\boldsymbol{d}}) \cdot \left(1 + \sum_{n=1}^{N} \Delta P_n(\boldsymbol{c}|d_n;\bar{d}_n) \right) \right\} \quad (13)$$

The optimization of $E[V|\mathbf{d}]$ is equivalent to optimizing $E[V|\mathbf{d}] - E[V|\bar{\mathbf{d}}]$ as $E[V|\bar{\mathbf{d}}]$ is a constant, and if $E[V|\mathbf{d}]$ is replaced by $\tilde{E}[V|\mathbf{d}]$, the optimization problem (for minimization) becomes

$$\min_{\boldsymbol{d}} \Delta\tilde{E}[V|\boldsymbol{d};\bar{\boldsymbol{d}}] = \sum_{n=1}^{N}\left(\min_{d_n} \Delta\tilde{E}[V_S|d_n;\bar{\boldsymbol{d}}]\right) \quad (14)$$

where $\Delta\tilde{E}[V|\boldsymbol{d};\bar{\boldsymbol{d}}] = \tilde{E}[V|\boldsymbol{d};\bar{\boldsymbol{d}}] - E[V|\bar{\mathbf{d}}]$ and

$$\begin{aligned}&\Delta\tilde{E}[V_S|d_n;\bar{\boldsymbol{d}}]\\ &= \sum_{\boldsymbol{c}\in\mathbf{C}_X} V(\boldsymbol{c})P(\boldsymbol{c}|\bar{\boldsymbol{d}})\Delta P_n(\boldsymbol{c}|d_n;\bar{d}_n)\end{aligned} \quad (15)$$

The subtraction of $E[V|\bar{\boldsymbol{d}}]$ eliminates the constant 1 in Eq. (13). The same procedure can be applied to maximization problems as well. It is noted that since summations are exchangeable, the optimization problem can be decomposed into each decision variable D_n whereby the number of variables to be optimized increases linearly in regards to N and $|Val(D_n)|$.

4. MULTI-OBJECTIVE DECISION-MAKING FOR CIVIL SYSTEMS

Decision-making processes for civil systems often consider more than one objective, e.g. cost ($\sum_n V_n$) and system performance (V_S), as illustrated in Figure 1(b). In such multi-objective optimization, a set of non-dominated solutions are usually of interest rather than a single solution since the optimal solutions can be changed depending on the relative importance of each objective which is often ambiguous.

One way to handle multiple objectives, is optimizing their weighted sum. For the example ID in Figure 1(b), the objective function of Eq. (1) can be alternatively formulated as

$$\sum_n E[V_n|d_n] + \lambda E[V_S|\boldsymbol{d};\bar{\boldsymbol{d}}], \qquad \lambda > 0 \quad (16)$$

Since D_n do not become dependent through V_n as implied in Eq. (16), the proxy measure proposed in Section 3, is introduced only for $E[V_S|\boldsymbol{d}]$. Thereby, the multi-objective minimization problem of cost and system failure probability can be approximated as

$$\sum_{n=1}^{N}\left\{\min_{d_n}\left(\Delta E[V_n|d_n;\bar{d}_n] + \lambda\Delta\tilde{E}[V_S|d_n;\bar{\boldsymbol{d}}]\right)\right\} \quad (17)$$

where $\Delta E[V_n|d_n;\bar{d}_n] = E[V_n|d_n] - E[V_n|\bar{d}_n]$.

A set of multiple optimal decision rules with different λ values, can be analytically obtained from Eq. (17). Specifically, consider the example in Figure 2 in which $Val(D_n) = \{d_n^1, \cdots, d_n^6\}$ and $\bar{d}_n = d_n^3$ (gray square). The blue squares and orange triangles denote respectively the optimal and non-optimal solutions in terms of the weighted sum formulation. The slopes of the lines connecting the blue squares correspond to the λ values, which change the optimal rule. Therefore, by collecting these values for each D_n and identifying the corresponding optimal decision rules, all optimal solutions in regards to Eq. (17) can be identified.

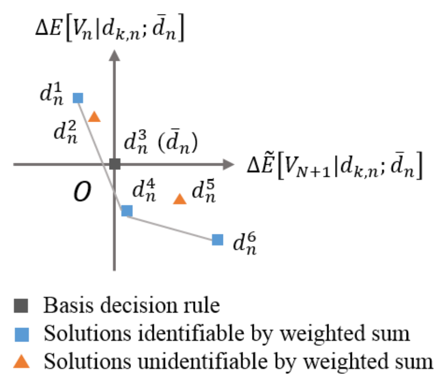

Figure 2. Example space of proxy objective function

To identify such λ values, one can start from the solution with the smallest $\Delta E[V_n|d_n;\bar{d}_n]$, i.e. d_n^1 in Figure 2, for it is always one of the non-dominated solutions. Then, from the current $d_n^{k_1}$, the solution with the most negative slope with respect to $d_n^{k_1}$ is selected among the solutions with greater $\Delta E[V_n|d_n;\bar{d}_n]$, as the next solution $d_n^{k_2}$. The absolute values of the slopes connecting those solutions are the λ values that change the preference between the optimal solutions. This identification process is terminated either when there is no more solutions with greater $\Delta E\left[V_n|d_n;\bar{d}_n\right]$ than the current one, or when there are no solutions with negative slopes.

Due to the errors by the approximation of $E[V_S|\boldsymbol{d}]$, some of the obtained solutions may not be optimal. One can sort out such solutions afterwards through the comparison between the obtained solutions. In addition, multiple basis decision rules $\bar{\boldsymbol{d}}$ can be utilized to obtain a sufficient number of non-dominated solutions as a single evaluation is unlikely to identify all of them due to the approximation errors.

5. NUMERICAL EXAMPLES

5.1. Illustrative example: a system with three components

Consider the example system and its ID in Figure 1. In the following analysis, CPM $\mathcal{M}_S$ is reduced to

$$\mathbf{C}_S = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{p}_S = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{18}$$

as $V_S = 0$ for the first and last rows in Eq. (3). Then, the set of all assignments of interest is

$$\mathbf{C}_{\boldsymbol{X}} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{19}$$

where the columns of $\mathbf{C}_{\boldsymbol{X}}$ sequentially denote the assignments over S, X_1, X_2, X_3, and H. The costs V_n for retrofitting X_n are set as 100, 60, and 50 for $n = 1,2,3$, respectively, leading to the CPMs $\mathcal{M}_{V_n}$ with

$$\begin{aligned} \mathbf{C}_{V_1} &= \begin{bmatrix} 0 & 0 \\ 100 & 1 \end{bmatrix}, \ \mathbf{p}_{V_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ \mathbf{C}_{V_2} &= \begin{bmatrix} 0 & 0 \\ 60 & 1 \end{bmatrix}, \ \mathbf{p}_{V_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ and} \\ \mathbf{C}_{V_3} &= \begin{bmatrix} 0 & 0 \\ 50 & 1 \end{bmatrix}, \ \mathbf{p}_{V_3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned} \tag{20}$$

where the first and second columns of $\mathbf{C}_{V_n}$ respectively stand for the assignments over V_n and D_n. Regarding the assignments $\boldsymbol{c}_1 = (0,0,0,-1,0)$ in the first row of $\mathbf{C}_{\boldsymbol{X}}$, the probability $P\left(\boldsymbol{c}_1|\bar{\boldsymbol{d}}\right)$ in Eq. (12) for the basis decision rule $\bar{\boldsymbol{d}} = (0,0,0)$ is evaluated as

$$P_J\left(\boldsymbol{c}_1|\bar{\boldsymbol{d}}\right) = (0.8 \cdot 1) \cdot (0.2 \cdot 0.2) = 0.032 \tag{21}$$

where the first parenthesis corresponds to $P_{\mathcal{M}_H}(\boldsymbol{c}_1) \cdot P_{\mathcal{M}_S}(\boldsymbol{c}_1)$; and the terms in the second are $P_{\mathcal{M}_{X_1}}(\boldsymbol{c}_1)$ and $P_{\mathcal{M}_{X_2}}(\boldsymbol{c}_1)$. It is noted that $P_{\mathcal{M}_{X_3}}(\boldsymbol{c}_1)$ is not considered as X_3 has the "-1" state in $\boldsymbol{c}_1$. The probabilities for other assignments $\boldsymbol{c}_k$, $k = 2,3,4$ can be evaluated by the same procedure and the result is summarized in Table 1. It is noted that $V_S(\boldsymbol{c}_k) = 1$ for all k.

Table 1: $V_S(\boldsymbol{c}_k)P_J\left(\boldsymbol{c}_k|\bar{\boldsymbol{d}}\right)$, $k = 1,\cdots,4$.

$\boldsymbol{c}_1$	$\boldsymbol{c}_2$	$\boldsymbol{c}_3$	$\boldsymbol{c}_4$
0.032	*0.032*	*0.0256*	*0.0192*

Table 2: Values of $\Delta\tilde{E}[V_S|d_n^1]$, $\Delta E[V_n|d_n^1]$, and λ, $n =$ 1,2,3 for basis decision rule $\bar{\boldsymbol{d}} = (0,0,0)$.

n	*1*	*2*	*3*
$\Delta\tilde{E}\left[V_S|d_n^1;\bar{\boldsymbol{d}}\right]$	*-0.0416*	*-0.0176*	*-0.0176*
$\Delta E\left[V_n|d_n^1;\bar{d}_n\right]$	*100*	*60*	*50*
λ	*240*	*3,410*	*2,840*

Regarding the given basis rule, the remaining alternative for D_1 is d_1^1. Following Eq. (10), $\Delta P_1\left(\boldsymbol{c}_k|d_1^1;\bar{d}_1\right)$, $k = 1,\cdots,4$ are evaluated as

$$\begin{gathered} \Delta P_1\left(\boldsymbol{c}_k|d_1^1;\bar{d}_1\right) = (0.1-0.2)/0.2 = \\ -0.5, \text{ for } k = 1,3; \text{ and} \\ \Delta P_1\left(\boldsymbol{c}_k|d_1^1;\bar{d}_1\right) = (0.3-0.4)/0.4 = \\ -0.25, \text{ otherwise} \end{gathered} \tag{22}$$

$\Delta\tilde{E}\left[V_S|d_1^1;\bar{\boldsymbol{d}}\right]$ is then evaluated from the quantities in Table 1 and Eq. (22) as

$$\begin{aligned} \Delta\tilde{E}[V_S|d_1^1] &= (0.032 + 0.0256) \cdot (-0.5) \\ &+ (0.032 + 0.0192) \cdot (-0.25) = -0.0416 \end{aligned} \tag{23}$$

Also, $\Delta E\left[V_1|d_1^1;\bar{d}_1\right]$ is calculated as

$$\Delta E\left[V_1|d_1^1;\bar{d}_1\right] = V_1(d_1^1) - V_1(d_1^0) = 100 \tag{24}$$

Since there are only two rules for each D_n, the λ values that change the optimality, are evaluated from the slopes connecting them, i.e. for D_1,

$$\lambda = |(100-0)/(-0.0416-0)| = 240 \quad (25)$$

The same procedure applies to the other decision variables, and Table 2 summarizes the quantities.

The obtained λ values lead to the optimal decision rules $\boldsymbol{d}^*$ as:

$$\begin{aligned} \boldsymbol{d}^* &= (0,0,0) \quad \text{for } 0 < \lambda < 240; \\ \boldsymbol{d}^* &= (1,0,0) \quad \text{for } 240 < \lambda < 2{,}840; \\ \boldsymbol{d}^* &= (1,0,1) \quad \text{for } 2{,}840 < \lambda < 3{,}410; \text{ and} \\ \boldsymbol{d}^* &= (1,1,1) \quad \text{for } \lambda > 3{,}410 \end{aligned} \quad (26)$$

For each decision rule $\boldsymbol{d}$ among the total of $|Val(\boldsymbol{D})| = 8$, Figure 3 summarizes the total cost $\sum_n E[V_n|d_n]$ and the system failure probability $E[V_S|\boldsymbol{d}]$ (blue squares.) There are five non-dominated solutions (red asterisks), and four solutions can be identified as optimal using the weighted sum in Eq. (16) (dotted yellow line). The weighted sum of objective values cannot identify the non-dominated solutions that show a concave shape with their adjacent solutions. It is noted that the four decision rules identified in Eq. (26) correspond to the solutions that can be identified by the exact formulation in Eq. (16).

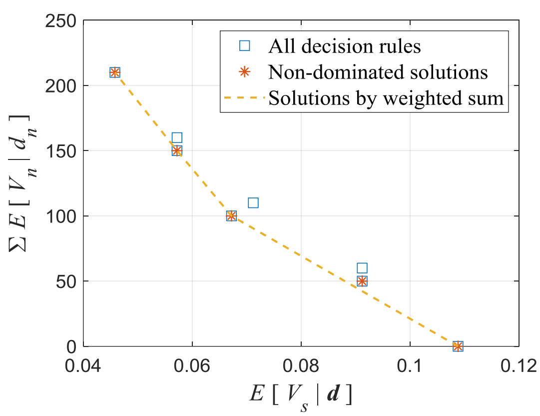

Figure 3: All and the optimal decision rules for example system.

5.2. Example reliability block diagram (RBD)

The proposed methodology is applied to an example reliability block diagram (RBD) consisting of 8 components in Figure 4(a). In the corresponding ID of Figure 4(b), the state of system event (S) is defined as the connectivity between the nodes s and t, which is determined by the states of components (X_n, $n = 1, \cdots, 8$). Decision variables D_n are associated with r.v.'s X_n, and have 3 decision alternatives that determine the reliability of n-th component. The cost V_n of decision rule on D_n, and the utility of system performance, V_S are modeled in the same way with Section 5.1.

There are a total of 3^8 decision rules as described in Figure 5 (blue dots), and all of the 27 non-dominated solutions (red squares) can be identified by the proposed method (yellow asterisks) by using 10 basis decision rules. It is noted that the proposed methodology can even identify the solutions that show concave shape with their adjacent solutions due to the errors caused by the approximation while the exact formulation cannot identify them.

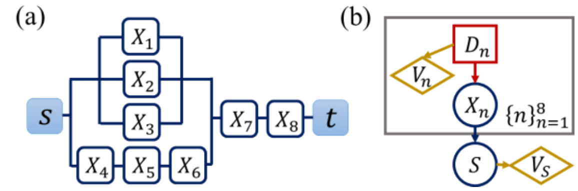

Figure 4: (a) Example RBD and (b) its ID.

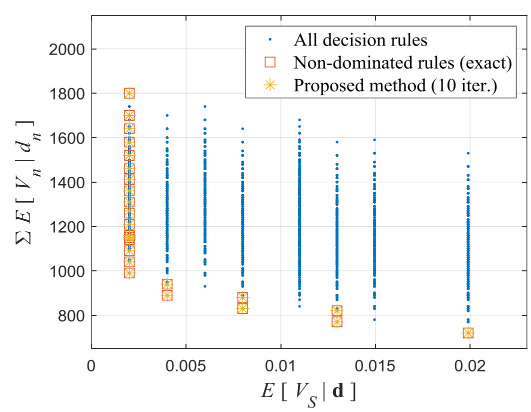

Figure 5: All and the optimal decision rules for example RBD.

5.3. Sioux Falls benchmark network

Consider the Sioux Falls benchmark network under a hypothetical earthquake hazard in Figure 6(a) consisting of 76 reinforced concrete (RC) bridges whose failure events are defined as in terms of RC columns. In the corresponding ID in Figure 6(b), the r.v.'s M and L respectively stand for the magnitude and location of earthquakes. The decision variable D_n, $n = 1, \cdots, 76$, has the 4 decision alternatives of the cross sectional area of n-th bridge's RC column whose state is represented by X_n. The utility variables V_n, $n = 1, \cdots, N$, and U_S respectively quantify the cost of D_n and system failure probability.

Figure 7 compares the results obtained by the proposed method with 50 basis decision rules and genetic algorithm (GA) after 10,000 generations, each with 50 populations. It is noted that the proposed methodology is able to obtain superior solutions while requiring the computational cost equivalent to evaluating one generation in GA.

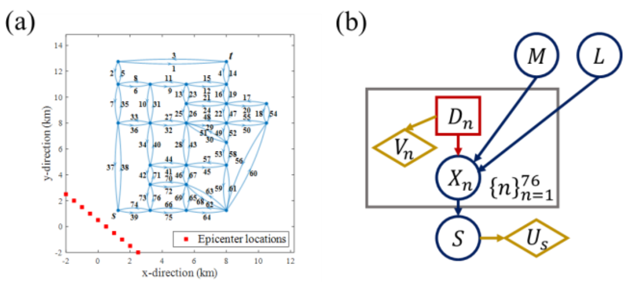

Figure 6: (a) Sioux Falls benchmark network and (b) its ID.

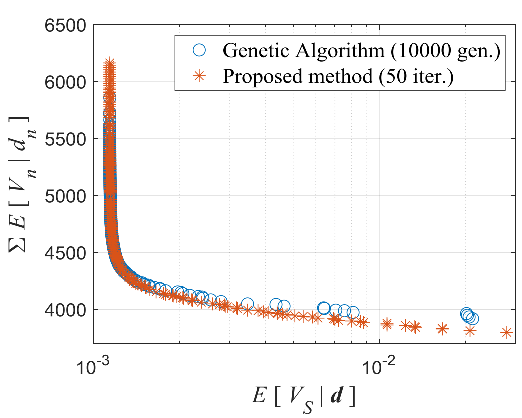

Figure 7. Optimal decision rules identified by proposed methodology and GA.

6. CONCLUSIONS

Influence diagram (ID) facilitates the operation of modeling and decision-making for civil systems. However, the large size and complex definition of system events of real-world civil systems, make such operation challenging. In particular, the converging structure between the random variables quantifying component events and that for system event, requires an exponentially increasing number of parameters, which leads to the exponentially increasing number of variables to be optimized as well. The application of matrix-based Bayesian network (MBN) allows the modeling of ID for large and complex civil systems. In order to optimize the IDs by MBN, a proxy objective function is proposed so that the computational complexity can be reduced from exponential order to polynomial. The errors caused by the approximation can be compensated by iterating the proposed scheme.

Another issue in decision-making for civil systems is the presence of multiple conflicting objectives. For such multi-objective optimization, the objective function is set as their weighted sum. The proposed proxy measure provides an efficient way for evaluating the weights that control the optimal decision rule. Three numerical examples successfully demonstrate the accuracy and efficiency of the proposed methodology.

7. ACKNOWLEDGEMENT

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