Probabilistic Assessment of Decentralized Decision-making for Interdependent Network Restoration

Hesam Talebiyan
Ph.D. Student, Dept. of Civil and Env. Engineering, Rice Univ., Houston, TX, USA

Leonardo Dueñas-Osorio
Associate Professor, Dept. of Civil and Env. Engineering, Rice Univ., Houston, TX, USA

ABSTRACT: This study introduces a statistical model that guides decentralized infrastructure restoration processes aligned with field practices. In particular, we make more analytically tractable the previously proposed Judgment Call method to simulate real-world decisions under time and resource constraints. The Judgment Call method explicitly models the largely ignored feature of decentralization in the restoration planning across interdependent networks. The method solves the Decentralized Interdependent Network Design Problem (D-INDP) while acknowledging the lack of proper communication among decision making agents, and hence, the lack of essential information. Here, we use a Bayesian Hierarchical Model (BHM) to simulate the agents' practical use of their field expertise and judgments to compensate for essential information shortage. We train the model using synthetic restoration plans that emphasize the local preferences of the agents. The method is applied to the interdependent infrastructure network of Shelby County, TN, and the results show that the performance of BHM-aided restoration plans is close to the conceptual upper bound.

Decisions for real-world interdependent networks are typically made by multiple distributed agents. Modeling such a decision-making environment must be decentralized and probabilistic. The former reflects the interaction of several potentially selfish agents, and the latter accounts for the uncertainties of real environments. However, most mathematical methods for decision-making in interdependent networks rely on centralized or deterministic formulations. In a previous study (Talebiyan and Duenas-Osorio, 2018), we proposed the Judgment Call method, which accounts for the decentralized nature of the realistic decision-making processes guiding the restoration of interdependent networks. Judgment Call recognizes the fact that human decision-makers use their field expertise and judgment to compensate for their bounded rationality (Sarma, 1994) as well as the lack of necessary information stemming from poor or no communication among agents. In this paper, we employ a Bayesian Hierarchical Model (BHM) to probabilistically study the intuitive decisions and utilization of expert opinion on the part of agents, and therefore, push the Judgment Call method closer to realism. As for the formulation of the network restoration problem, we adopt the Interdependent Network Design Problem (INDP), a family of centralized optimization problems concerned with the restoration of disrupted networked systems subject to budget and operational constraints, which serves as baseline for quantifying the effects of decentralization. Like the general problem of decentralized decision-making (Tsitsiklis, 1984), the lack of proper communications among agents is a crucial aspect of the new Decentralized-INDP (D-INDP) class. Judgment Call assumes that agents use their field expertise about the potential decisions of other agents to compensate for scant communications. To model such compensation, we explored several simple assumptions such as optimistic and pessimistic agents (Talebiyan and Duenas-Osorio, 2018). In this study, we replace these simple assumptions with a BHM, which aims to simulate the decision process of agents when they make judgments in a more realistic way. Also, using statistical models is relevant given the current trend of artificial intelligence tools for agents to make more informed decisions. In particular, we select BHM as our modeling framework because it is known to be an apt paradigm to model spatio-temporal data (Cressie and Wikle, 2015), such as restoration strategies. Moreover, BHMs accommodate the fact that agents update their mindset when they face discrepancy between their judgment and actual decisions by other agents. We employ our model to study the interdependent infrastructure networks in Shelby County, TN disrupted by hypothetical earthquakes.

In the next section, the relevant literature is reviewed. Then, we briefly explain INDP and the Judgment Call method as the basis of the proposed approach in this paper. Thereafter, we introduce BHM and particularly the model we use for restoration plans. Next, we apply the method to the restoration planning of infrastructure networks of Shelby County, TN. Finally, we present conclusions and ideas for future work.

1. Literature Review

The problem of decentralized decision-making has been tackled from a general and abstract perceptive via *Markov decision process* (MDP). In particular, the extension of MDP to the multi-agent setting is called the decentralized partially observable MDP (DEC-POMDP), which is known to be computationally hard; specifically, it is NEXP-complete (nondeterministic exponential time) even with just two agents (Bernstein et al., 2002). Also, approximation algorithms, which are tailored to the structure of specific subclasses of DEC-POMDP, can only solve small problems (Seuken and Zilberstein, 2008), and are not scalable to the size of practical instances.

Decentralized restoration planning can be thought as a decentralized (or distributed) optimization. The main body of work in this class of optimization is devoted to convex, continuous problems, as pioneered by Tsitsiklis (1984). A short review of these methods can be found elsewhere (Nedic et al., 2010). Few studies have addressed the decentralized optimization for discrete problems. Nemhauser et al. (1978) introduced the distributed maximization of submodular functions subject to cardinality constraints, and solution methods for this problem are proposed by Mirzasoleiman et al. (2013). Karabulut (2017) studied the integer programming problems in which agents are coupled by resource constraints. INDP differs from this problem mainly because it includes coupling interdependency constraints. In a more general setting, Feizollahi (2015) proposed decentralized mixed-integer programming for the unit-commitment problem in electric power systems. Also, Sharkey et al. (2015) and Singh and O'Keefe (2016) solved the decentralized scheduling problem. These studies assume an unbounded number of communications among agents during the process of decision-making, which is hardly possible in real-world settings of the network restoration. The authors propose a method that accounts for the more realistic case of delayed and limited communications among agents (Talebiyan and Duenas-Osorio, 2018). This heuristic method is improved in the current study by using BHM as a more realistic and analytically tractable model of the decision-making process for restoration of interdependent networks.

2. Judgment Call

The most critical issue in decentralized decision-making is communication (Tsitsiklis, 1984). In real-world decision-making environments, the communications among agents are usually noisy or delayed. However, decisions that these agents make are affected by other agents' because of the interdependency between different parts of the system. Due to communication problem, agents cannot solicit enough, timely information (pertinent to their decisions) from other agents. Therefore, as a practical human approach, agents use their expertise and judgment to compensate for lack of information (Sarma, 1994). Previously, the authors introduced the Judgment Call methodology to recognize and model this human approach to decision-making for the restoration of interdependent network.

The main goal of Judgment Call is to solve the Interdependent Network Design Problem (INDP) (González et al., 2016) in a decentralized fashion while taking the lack of communication into account. INDP, a family of Mixed Integer Programming (MIP) problems, finds the restoration strategies for disrupted networked systems with limited budgets and operational constraints. INDP minimizes the sum of four types of cost that the interdependent network incurs during the restoration process: 1) flow cost (of commodities), 2) reconstruction cost of arcs and nodes, 3) penalties due to the unbalance of supply and demand at nodes, and 4) and geographical co-location cost. INDP finds the minimized sum of the costs while satisfying five types of constraints:

C1. Flow balance constraints at nodes, which compare inflow, outflow, and demand/supply of commodities at each node, and record potential unbalance among them.

C2. Capacity constraints on commodity flow in arcs, which ensure that flows in functional arcs are not more than their capacities, and also, damaged arcs do not carry any flow.

C3. Resource constraints, which prevent employing more resources than the Resource cap, R_c, which is the number of available resources.

C4. Physical interdependency constraints, which make sure a node will not be functional if their *dependee nodes* are not functional. Dependee nodes are in other networks, and the node in question relies on them for its functionality. For example, when a power substation provides electricity to a water pump, the substation is a dependee node with respect to the pump.

C5. Co-location constraints, which ensure any given area has to be prepared only once even if several agents carry out reconstruction tasks inside the area.

For the detailed mathematical formulation of INDP, readers are referred to the original study (González et al., 2016). Here, we adopt the iterative INDP (iINDP) in which we solve INDP iteratively until all demands across the network are met. On each iteration (equivalent to a time step), the problem is solved to optimality to find the minimum-cost strategy. Figure 1 shows a schematic of the procedure of iINDP.

Solving INDP for the whole interdependent network is equivalent to considering a single agent deciding for the entire network. The more realistic assumption is to consider several interacting agents, each of them deciding for one layer of the network. The Judgment Call method embodies this assumption by letting each agent solve a subproblem of INDP that pertains to its respective layer. To this end, agents need information about the dependee nodes, but they usually lack such information. Therefore, they assign a *Restoration Probability*, P_r, to each dependee node in other networks. P_r captures the chance that the dependee node is going to be repaired during the current time step. Then, the agent carries out a Bernoulli experiment with the probability of success equal to P_r for each node. If the result is one, then the agent assumes that the dependee node is repaired and solves his/her INDP subproblem accordingly. The procedure of Judgment Call is depicted in Figure 2. In the figure, the double-lined arrows show the communication among agents.

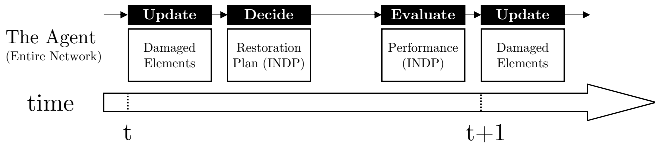

Figure 1: Procedure of iINDP (Talebiyan and Duenas-Osorio, 2018).

Agents may take different approaches to determine P_r. For example, the agents may be pessimistic and assume that other agents will not repair dependee nodes or $P_r = 0$. On the other hand, an optimistic approach is to assume that agents will repair all dependee nodes until the next time step or $P_r = 1$. These two can be thought as extreme schemes, and there can be different assumptions that fall in between (Talebiyan and Duenas-Osorio, 2018). In the next section, we propose using BHMs —trained on historical data and suitable for taking real-time evidence— to guide assigning P_r and making a judgment.

3. Bayesian Hierarchical Model

In this section, we introduce and employ the Bayesian Hierarchical Model (BHM) as a realistic model of the judging process by decentralized decision-makers. We assume that agents use BHM

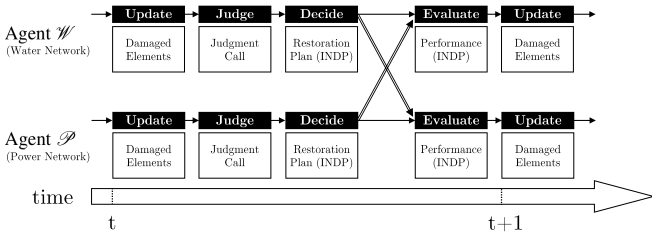

Figure 2: Procedure of Judgment Call (Talebiyan and Duenas-Osorio, 2018).

to represent their knowledge about the history of the restoration process. In particular, we train a BHM using restoration strategies devised during historical events or hypothetical disruption scenarios. The agents run the model whenever they want to make a judgment about other agents' decisions, and the output of the model is the judgment. In other words, we assume that the domain expertise and judgment of the agents is embedded in the BHMs. Here, we aim to fit the model to restoration plans, which are temporal data over spatially-distributed entities (networks), and BHM is known to be suitable to model spatio-temporal data (Cressie and Wikle, 2015).

BHM is a class of hierarchical statistical modeling, which is an approach to quantify uncertainties in a dataset using levels of conditional probabilities. In accordance with Berliner (1996), a hierarchical model consists of three levels: data, process, and parameter. At the top level, the data model describes the conditional probability of data given a hidden process. The middle level is the process model, which captures the true phenomenon of interest. The process model, in turn, may be conditioned on several parameter models, which are the distributions of parameters of the process model. Formally, a BHM is

$$Pr(Z,Y,\theta) = Pr(Z|Y,\theta) \times Pr(Y|\theta) \times Pr(\theta) \quad (1)$$

where $Z =$ data, $Y =$ the quantity or process of interest, and $\theta =$ parameters of the model.

We assume that the Markov assumption holds for the restoration process so that the state of a node is affected only by its state at the previous time step. We presume that it is difficult for an agent to keep track of the entire history given time and cognitive constraints. Also, the assumption follows the sequential nature of iINDP. To materialize this assumption, we define the levels of BHM as follows,

$$\begin{aligned} \text{Data model:} \quad & w_t^v \sim \text{Bernoulli}(p_t^v) \\ \text{Process model:} \quad & p_t^v = \mathscr{M}(p_{t-1}^v + \varepsilon^v) \\ & \mathscr{M}(x) = \frac{1}{1+e^{-x}} \\ \text{Parameter model:} \quad & p_0^v \sim \text{Beta}(\alpha = 2, \beta = 2) \\ & \varepsilon^v \sim \text{Gaussian}(0,1) \end{aligned} \quad (2)$$

The first level of the model describes the state transition of node v at time step t, w_t^v, as a Bernoulli distribution with success probability p_t^v. A state transition w_t^v is unity if a damaged node at time $t-1$ is repaired at time t, and zero otherwise. This definition is consistent with the Judgment Call method in which agents care and speculate about the state transition of damaged dependee nodes. In other words, they use their judgment to guess if the state of the damaged node will change or not during each step of decision-making. The process model embeds the Markov assumption by conditioning p_t^v on p_{t-1}^v and an stationary error term ε^v. We apply a logistic function, $\mathscr{M}(x)$, to confine the probabilities p_t^v in [0,1]. One can observe that, to find any p_t^v, we only need p_0^v and ε^v, which are the parameters of the model. We choose a beta prior for p_0^v to keep them in [0,1]. To model the non-informative prior knowledge, the initial shape parameters, α and β are chosen so that the prior becomes symmetric around the mode at 0.5. The non-informative prior of ε^v is a zero-mean Gaussian distribution with a relatively large standard deviation.

The goal of training a statistical model is to find the posterior distribution of parameters. To this end, one has to sample from the prior distributions, and likelihood functions; the later is a conditional probability. For this purpose, we use the Markov Chain Monte Carlo (MCMC) method (Metropolis et al., 1953) which efficiently generates samples from conditional probabilities. In particular, we use No-U-Turn Sampler (NUTS) (Hoffman and Gelman, 2011), which avoids random walk behavior, and needs minimal tuning on the user's part. In this context, the main issue of MCMC is convergence. We employ *Scale Reduction Factor* ($\hat{R}$) (Gelman and Rubin, 1992), which compares the variance between multiple chains to the variance within each chain. If these variances are identical or $\hat{R} \approx 1$, then MCMC has converged. Also, we use 1000 samples to tune the sampler, and then, draw 4000 samples to find the posterior distribution.

The model has to be trained on a dataset of historical restoration plans. To the best of our knowledge, such a dataset is not available for the infrastructure network in Shelby County, TN, which we study in the Application section. Therefore, we use INDP to build up the dataset in accordance with seismic scenarios by Wu (2017). In particular, we use these scenarios and INDP to find restoration plans for each layer of the infrastructure network while ignoring interdependencies among layers. In other words, we let agents separately solve the subproblem of INDP that is related to their respective layer. The decoupled nature of the solutions results in restoration plans that are purely based on the local preferences of each agent, which is of interest to other agents during the judgment process. Note that each agent speculates about other agents' preferences when he/she makes a judgment. Solving the subproblems, we obtain a host of restoration plans. In many seismic scenarios, no node is damaged, and therefore we select a subset of plans in which at least one node is damaged in the entire network. These are 508 damaging scenarios, which are 11% of a total of 4800 scenarios, whose corresponding restoration plans are employed to train the model and find posterior distributions of parameters.

The mean and standard deviation (sd) of posterior distributions of parameters are computed and presented in Table 1 for two nodes. Node 65 corresponds to a relatively high number of state transition samples (average 21 per time step), while the model for Node 83 is trained on a relatively small number of 10 samples per time step on average. The number of samples decreases with time because there is a smaller chance that a node is still damaged in later time steps. Table 1 also presents $\hat{R}$ of different estimated parameters which are fairly close to unity and indicate MCMC convergence. This observation holds for all estimated variables. The other piece of information in Table 1 is the Monte Carlo standard error (MCSE). If we assume that the difference between an estimated value and its exact value is distributed according to a zero-mean Gaussian distribution, MCSE shows the standard deviation of the distribution. To compute MCSE while accounting for non-independent samples, the sample pool is divided into batches, and the standard deviation of batch means are calculated. Small values of MCSE show that our model fits the input data.

Table 1: Estimated values of parameters of models for Nodes 65 and 83.

Parameter	Mean	sd	MCSE	$\hat{R}$
ε^{65}	-1.116	0.2310	0.0038	0.9999
p_0^{65}	0.4188	0.2063	0.0033	1.0003
ε^{83}	-1.2327	0.4024	0.0324	1.0146
p_0^{83}	0.4946	0.2412	0.0193	1.0129

To test the prediction quality of the model, we run a 10-fold cross-validation where the training set contains 85% of data (chosen randomly), and the test set has the remaining 15%. In each iteration of cross-validation, the trained model predicts the state of node v at different time steps t given the initial damages from the test dataset as well as the estimated p_0^v and ε^v. Then, the predicted values are compared to the exact values from test data. Figure 3 shows the mean value of the absolute error between predicted and exact values over all test scenarios for each iteration of cross-validation for Nodes 65 and 83. The figure shows a maximum cross-validation error of 0.065, which means that at most 6.5% of the predicted values do not match the exact value.

We construct BHM only when there are enough sample points for a node. In particular, some of the nodes are damaged in a small number of 508 damaging scenarios if any, and therefore, there are few samples of state transition for that node to train the model. For example, 53% of nodes in the power network show little or no damage. However, it also shows that the subset of nodes are not vulnerable, and therefore, we assume that $p_t^v = 1.0$ for all time steps. Thus, the agents do not need to run the model to judge the state of such nodes. Next, we apply above methodology to a real-world application.

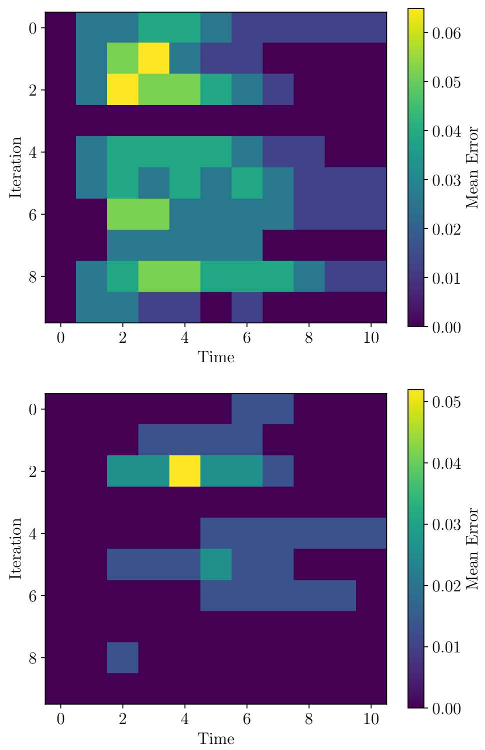

Figure 3: Mean absolute error of cross-validation iterations for Nodes 65 (top) and 83 (bottom).

4. APPLICATION

In this section, we employ our method to model the decentralized decision-making process for the restoration of interdependent infrastructure networks in Shelby County, TN, as disrupted by earthquake. The interdependent network comprises power, water, and gas networks, whose node and arc sets are presented in Table 2. This application features two types of interdependency: 1) physical interdependency between the water and the power network since water pumping stations depend on power stations to provide them with electricity, and 2) geographical interdependency which means water and gas networks share the site preparation cost if their elements are co-located. We generate the initial damage scenarios using the hazard analysis of Shelby County by Wu (2017). These hazard maps are computed based on a more comprehensive set of seismic scenarios (regarding magnitudes, rupture locations, etc.) compared to previous studies. They select a representative subset of scenarios, which captures the spatial variability of ground motion intensity (Miller and Baker, 2015), and is hazard-consistent and network-response-consistent. Based on each of these seismic scenarios, we compute initial damage to the network. Then, we find restoration plans for the network using centralized and decentralized methods and compare them.

To compare two restoration plans, we check their performance (fraction of the total demand that is met) and total cost over all time steps. We use areas corresponding to the performance curve and total cost curve as defined in Figure 4, which are related to the resilience of a networked system (Hosseini et al., 2016). We define the *Universal Relative Measure*, λ_U, as the average of *Relative Performance* and *Relative Total Cost*,

$$\lambda_U \triangleq \frac{1}{2}\left(\frac{A_c - A_d}{A_c} + \frac{B_c - B_d}{B_c}\right) \tag{3}$$

Theoretically, $\lambda_U \in (-\infty, 1]$, but we expect λ_U to be negative. In other words, we expect that $A_c \leq A_d$ and $B_c \leq B_d$ because the centralized plan is optimal and has to be superior to the decentralized one. A more negative λ_U indicates a worse performance of the decentralized plan. In the example shown in Figure 4, $\lambda_U = \frac{1}{2}(-0.58 - 0.63) = -0.605$.

We study the performance of decentralized plans when the damage to the network is severe by computing mean λ_U over the most damaging seismic scenarios. These are scenarios with more damaged elements than 95% of the whole set of damaging scenarios, which happens to be those with 18 damaged elements or more. There are 54 scenarios in the most damaging subset, which is the most stringent test to our method. Also, the average number of damaged elements in the subset is 30.8. Figure 5 shows mean λ_U for this subset along with confidence intervals for optimistic and pessimistic judg-

Table 2: Number of elements in the layers of the network. $|\mathscr{V}|$ and $|\mathscr{E}|$ denote the cardinality of the node and arc sets respectively.

Layer	Symbol	$\lvert\mathscr{V}\rvert$	$\lvert\mathscr{E}\rvert$	Total
Water	G_w	49	71	120
Gas	G_g	16	17	33
Power	G_p	60	76	136
Interconnections	$\mathscr{I}$		45	45
Whole network	G^*	125	209	334

* $G = G_w \cup G_g \cup G_p \cup \mathscr{I}$

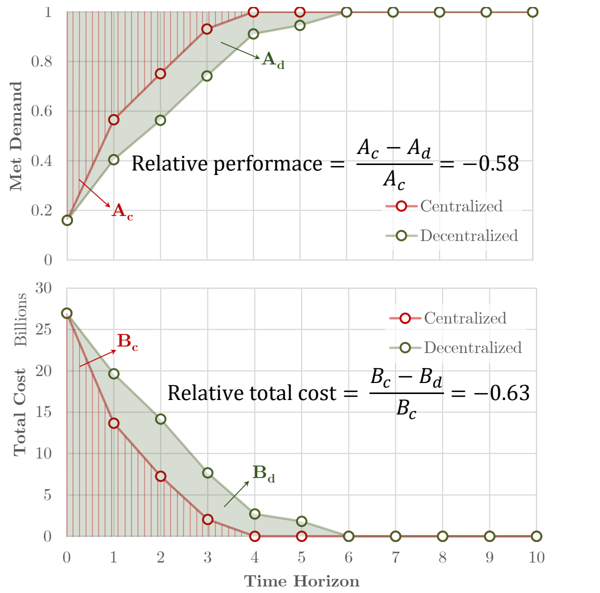

Figure 4: Measures of performance (top) and total cost (bottom) to compare centralized and decentralized plans. A_c (A_d) is the area above the performance curve of centralized (decentralized) plan. B_c (B_d) denotes the area under the total cost curve of centralized (decentralized) plan.

ments (Talebiyan and Duenas-Osorio, 2018) as well as the informed case and BHM-aided judgments. In the ideal informed case, agents have complete information about other agents' decisions. This case serves as a conceptual upper bound for the collective performance of decentralized agents. Note that our assumption here is that centralized agents can use resources only limited by their cap, while each of three decentralized agents gets a third of the resources as they are compelled to take action. The figure shows that with six units of resources (two for each agent) the performance of BHM-aided judgment is 32.4% worse than the centralized plan, which is close to the performance of the ideal informed case. The performance of BHM-aided Judgment Call is worse than the optimistic; however, BHM is a more realistic way of modeling the judgment process. The current BHM model can be thought of as a decision tool because its performance is close to the upper bound, and our future direction is to make it even closer by refining the model. The model may be refined by taking the spatial relation between nodes into account.

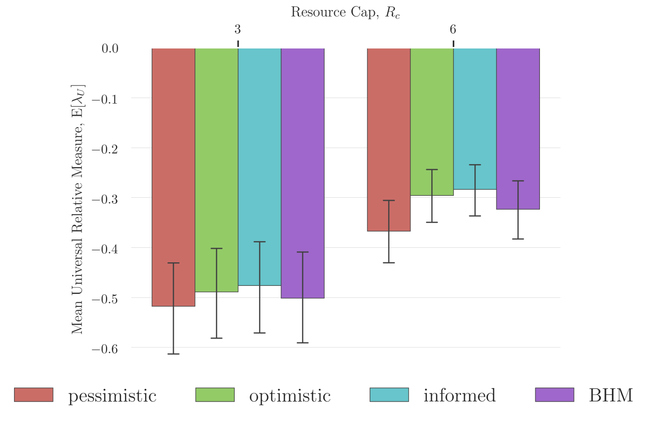

Figure 5: Mean λ_U for scenarios with more damaged elements than 95% of the whole set of damaged scenarios. Error bars show 95% confidence interval of mean values.

5. CONCLUSIONS

This paper puts forward a Bayesian hierarchical model to realistically capture the practical use of field know-how and expert judgment in the restoration decision-making of interdependent networks. Proposing the Judgment Call method, we first break the barrier of centralized models in resilience engineering and enter into the decentralized arena. Then, here, the proposed model enhances the realism of the Judgment Call method, which is designed to consider the poor communication among agents in real-world decentralized decision-making environments. Judgment Call assumes that agents use their intuition and expert judgment to compensate for the lack of communication, and solves INDP in a decentralized way based on this assumption. BHM adds a tool to Judgment Call that imitates the decision processes of agents more rigorously than previous studies. In particular, the Bayesian aspect of BHM imitates the accumulation of experience which constantly corrects and updates a person's mindset and shapes his/her intuition and expertise. Also, BHM is a rigorous way of modeling spatio-temporal data, which decomposes the uncertainty and helps us quantify different sources of it. Furthermore, as artificial intelligence and data-driven tools become prevalent in real-world decision-making environments, the proposed method will show even more resemblance to reality. The results show that guiding Judgment Call by the BHM-aided judgment leads to restoration strategies with realistic and acceptable performance.

Our next step is to improve the model form to include the spatial relations between state transitions of nodes. In particular, we can relate the restoration of nodes based on the connectivity or demand/supply values. Also, we will compare the model to other methods of approximating dynamics in layered networks (Alemzadeh and Mesbahi, 2018). Moreover, in the decentralized restoration planning, several agents are competing or cooperating to maximize their own utility and cost, which offers an opportunity to utilize game theoretic methods. In particular, we aim to improve previous studies on network recovery games (Smith et al., 2017) based on insights and tools from the Judgment Call method.

6. ACKNOWLEDGMENT

The authors gratefully acknowledge the support by the U.S. Department of Defense (Grant W911NF-13-1- 0340) and the U.S. National Science Foundation (Grant CMMI-1541033).

7. REFERENCES

Alemzadeh, S. and Mesbahi, M. (2018). "Influence Models on Layered Uncertain Networks: A Guaranteed-Cost Design Perspective." *arXiv preprint arXiv:1807.06612*.

Berliner, L. M. (1996). "Hierarchical Bayesian Time Series Models." *Maximum Entropy and Bayesian Methods*, Springer Netherlands, Dordrecht, 15–22.

Bernstein, D. S., Givan, R., Immerman, N., and Zilberstein, S. (2002). "The Complexity of Decentralized Control of Markov Decision Processes." *Mathematics of Operations Research*, 27(4), 819–840.

Cressie, N. and Wikle, C. K. (2015). *Statistics for spatio-temporal data*. John Wiley & Sons.

Feizollahi, M. J. (2015). "Large-scale unit commitment: decentralized mixed integer programming approaches." Ph.d. dissertation, Georgia Institute of Technology, Atlanta, GA.

Gelman, A. and Rubin, D. B. (1992). "Inference from Iterative Simulation Using Multiple Sequences." *Statistical Science*, 7(4), 457–472.

González, A. D., Dueñas-Osorio, L., Sánchez-Silva, M., and Medaglia, A. L. (2016). "The Interdependent Network Design Problem for Optimal Infrastructure System Restoration." *Computer-Aided Civil and Infrastructure Engineering*, 31(5), 334–350.

Hoffman, M. D. and Gelman, A. (2011). "The No-U-Turn Sampler: Adaptively Setting Path Lengths in Hamiltonian Monte Carlo." *arXiv preprint arXiv:1111.4246*.

Hosseini, S., Barker, K., and Ramirez-Marquez, J. E. (2016). "A review of definitions and measures of system resilience." *Reliability Engineering and System Safety*, 145, 47–61.

Karabulut, E. (2017). "Distributed Integer Programming." Ph.d. dissertation, Georgia Institute of Technology, Atlanta, GA.

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A., and Teller, E. (1953). "Equation state calculations by fast computing machines." *Journal of Chemical Physics*, 21(6), 1087–1092.

Miller, M. and Baker, J. (2015). "Ground-motion intensity and damage map selection for probabilistic infrastructure network risk assessment using optimization." *Earthquake Engineering & Structural Dynamics*, 44(7), 1139–1156.

Mirzasoleiman, B., Sarkar, R., Karbasi, A., and Krause, A. (2013). "Distributed Submodular Maximization: Identifying Representative Elements in Massive Data." *Advances in Neural Information Processing Systems*, Vol. 26, 2049—-2057.

Nedic, A., Ozdaglar, A., and Parrilo, P. (2010). "Constrained Consensus and Optimization in Multi-Agent Networks." *IEEE Transactions on Automatic Control*, 55(4), 922–938.

Nemhauser, G. L., Wolsey, L. A., and Fisher, M. L. (1978). "An analysis of approximations for maximizing submodular set functions-I." *Mathematical Programming*, 14(1), 265–294.

Sarma, V. V. S. (1994). "Decision making in complex systems." *Systems Practice*, 7(4), 399–407.

Seuken, S. and Zilberstein, S. (2008). "Formal models and algorithms for decentralized decision making under uncertainty." *Autonomous Agents and Multi-Agent Systems*, 17(2), 190–250.

Sharkey, T. C., Cavdaroglu, B., Nguyen, H., Holman, J., Mitchell, J. E., and Wallace, W. A. (2015). "Interdependent network restoration: On the value of information-sharing." *European Journal of Operational Research*, 244(1), 309–321.

Singh, G. and O'Keefe, C. M. (2016). "Decentralised scheduling with confidentiality protection." *Operations Research Letters*, 44(4), 514–519.

Smith, A. M., González, A. D., Dueñas-Osorio, L., and D'Souza, R. M. (2017). "Interdependent Network Recovery Games." *Risk Analysis*.

Talebiyan, H. and Duenas-Osorio, L. (2018). "Decentralized Decision-making for the Restoration of Interdependent Networks." *ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering (Under review)*.

Tsitsiklis, J. N. (1984). "Problems in Decentralized Decision making and Computation.." Ph.d. dissertation, Massachusetts Institute of Technology, Cambridge, MA.

Wu, J. (2017). "End-to-end seismic risk analysis framework for the identification of infrastructure network retrofits." Ph.d. dissertation, Stanford University, Stanford, California.