Efficient Uncertainty Propagation through Inelastic Wind-Excited Structures Subject to Stochastic Excitation

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ABSTRACT: The growing interest in applying probabilistic performance-based design to wind excited structural systems has increased the need for models capable of efficiently estimating the inelastic responses of these systems. This paper outlines the development of such a model that combines the theory of dynamic shakedown with distributed plasticity and simulation methods, providing a framework for estimating any system-level probabilistic performance metric of interest. The potential of the proposed framework is illustrated on a full scale three dimensional building.

1. INTRODUCTION

Performance assessment in wind engineering is going through a period of significant change with performance-based design becoming ever more prevalent. Various system-level frameworks that estimate performance in terms of probabilistic measures that are consistent with the needs of decision-makers have been proposed (Ciampoli et al., 2011; Spence and Kareem, 2014; Chuang and Spence, 2017). These frameworks, however, were developed within the setting of elastic systems. Integration of methods that enable the inelastic response of the structural system to be considered in the performance estimations are lacking. This can be traced back to the significant computational and theoretical challenges inherent to modeling the long duration inelastic response of wind-excited systems for which complex failure mechanisms can occur (e.g. low-cycle fatigue failure in the across-wind direction, or ratcheting in the along-wind direction). The need to propagate uncertainties through the system in order to estimate probabilistic performance metrics further complicates the problem.

Recently, a framework for treating this problem based on recent advances in probabilistic dynamic shakedown theory was proposed, thereby enabling the inelastic response of the structural system to be considered in the performance estimations (Chuang and Spence, 2019a). Finite element models for the inelastic response can be divided into two categories: concentrated plasticity and distributed plasticity. In the framework outlined in (Chuang and Spence, 2019a), the inelastic response of the system is modeled under the assumption of concentrated plasticity through the theory of plasticity with the state of dynamic shakedown identified as a desirable performance objective. Indeed, this state ensures the safety of the system against both the aforementioned collapse mechanisms. Models are introduced for efficiently estimating not only the state of dynamic shakedown for a given wind load trace but also the plastic strains and deformations occurring at shakedown. This is achieved through the development of pathfollowing algorithms that only require the estimation of the steady-state purely elastic response of the system. This enables inelastic responses to be efficiently estimated for any given load trace therefore allowing simulation models to be directly applied for propagating uncertainty through the system and consequently estimating any system-level probabilistic performance metric of interest.

This work is focused on extending this framework to a distributed plasticity setting with the aim of overcoming the limitations associated with concentrated plasticity models (Spacone et al., 1996; Spacone and El-Tawil, 2004). In particular, the dynamic shakedown problem is reformulated within the setting of section-based distributed plasticity and then integrated into the space of probabilistic performance-based wind engineering. To illustrate the framework, a case study of a full scale 3D building subject to wind-tunnel informed stochastic loads is presented.

2. PROBLEM SETTING

The performance of a wind excited system can be expressed in terms of a set of probabilistic decision variables (DVs) of interest, e.g. repair time and repair cost, which consider both collapse and non-collapse scenarios, as follows (Chuang and Spence, 2017):

$$\begin{aligned} P(DV > dv|im) =& P(DV > dv|NC, im)P(NC|im) \\ &+ P(DV > dv|C, im)P(C|im) \end{aligned} \tag{1}$$

where dv is a decision variable threshold of interest; im is an intensity measure; $P(NC|im)$ and $P(C|im)$ are respectively the conditional probabilities of non-collapse (NC) and collapse (C) of the structural system given the intensity im; while $P(DV > dv|NC, im)$ and $P(DV > dv|C, im)$ are the corresponding conditional exceedance probabilities of dv. The estimation of the system-level performance therefore requires a full description of failure as well as an efficient approach for modeling the inelastic behavior of the system.

In general, a building can be identified as collapsed under the following failure scenarios: (1) low cycle fatigue, instantaneous plastic collapse, or ratcheting; (2) excessive plastic deformations. Recently, a strain-driven approach based on the dynamic shakedown theory has been proposed to rapidly identify failures due to both scenarios under the assumption of concentrated plasticity (Chuang and Spence, 2019a). To further account for the spread of plasticity along the element, this paper presents development of the approach within the context of distributed plasticity.

3. DYNAMIC SHAKEDOWN FRAMEWORK

To model the distribution of plasticity along each element of a structure experiencing inelastic behavior, the members composing the structural system are modeled through a fiber-based framework. This section firstly presents the displacement-based (DB) fiber formulation used in this work and then the corresponding dynamic shakedown model based on the DB formulation.

3.1. *Mechanical model*

The formulation considered in this work is based on Euler-Bernoulli beam theory with three-dimensional (3D) displacement field described by:

$$\mathbf{v}_n(x) = \begin{bmatrix} u(x) & v(x) & w(x) \end{bmatrix}^T \tag{2}$$

where $u(x)$, $v(x)$ and $w(x)$ are respectively displacements in the x, y and z-direction for the nth element, as shown in Figure 1. The section deformation vector, which contains the axial strain $\varepsilon_x(x)$ and curvatures $\kappa_z(x)$ and $\kappa_y(x)$, is given by:

$$\begin{aligned} \mathbf{d}_n(x) &= \begin{bmatrix} \varepsilon_x(x) & \kappa_z(x) & \kappa_y(x) \end{bmatrix}^T \\ &= \begin{bmatrix} \dfrac{\partial u(x)}{\partial x} & \dfrac{\partial^2 v(x)}{\partial x^2} & -\dfrac{\partial^2 w(x)}{\partial x^2} \end{bmatrix}^T \end{aligned} \tag{3}$$

The corresponding section forces $\mathbf{D}_n(x) = \begin{bmatrix} N_x(x) & M_z(x) & M_y(x) \end{bmatrix}^T$ can be defined through section constitutive relation as follows:

$$\mathbf{D}_n(x) = \mathbf{k}_{s_n}(x)\mathbf{d}_n(x) \tag{4}$$

where $N_x(x)$, $M_z(x)$ and $M_y(x)$ are respectively the axial force and bending moments in local z and y directions; while $\mathbf{k}_{s_n}(x)$ is the section stiffness matrix, which can be derived from a fiber discretization of the cross section.

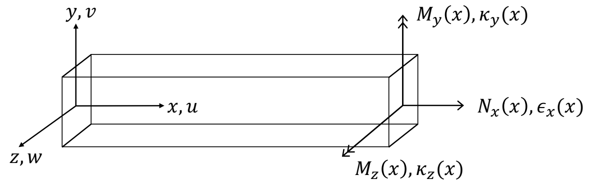

Figure 1: Displacements, section forces and deformations for a three-dimensional beam-column element.

The DB stiffness method follows the standard finite element approach, in which the displacement field of the element is expressed by the nodal displacements through appropriate interpolation functions. The most commonly used functions for beam-column elements are linear Lagrangian interpolation functions for the axial displacements and cubic Hermitian polynomials for the lateral translations and rotations. The degrees of freedom at each node are three displacements and two rotations for a 3D beam-column element. The response in torsion is assumed linear elastic and uncoupled from the axial and flexural response. The displacement field along the axis of the nth element, $\mathbf{v}_n(x)$, can then be related to nodal displacements through the following expression:

$$\mathbf{v}_n(x) = \mathbf{N}_n(x)\mathbf{q}_n \tag{5}$$

where $\mathbf{q}_n$ is the vector collecting nodal displacements and rotations at the element ends in the local coordinate system while $\mathbf{N}_n(x)$ is a matrix collecting the interpolation functions for all nodal degrees of freedom.

The section deformations $\mathbf{d}_n(x)$ are then related to the nodal displacements $\mathbf{q}_n$, as follows:

$$\mathbf{d}_n(x) = \mathbf{B}_n(x)\mathbf{q}_n \tag{6}$$

where $\mathbf{B}_n(x)$ is the strain-deformation matrix containing the first derivative of the axial displacement interpolating function and the second derivatives of the transverse displacement interpolating functions.

Since the displacement field is approximate, several displacement-based elements are required along the length of a member to represent the deformations. From the principle of virtual displacements, the element force vector $\mathbf{Q}_n$, i.e. nodal forces at element ends, can be expressed in the following form based on equilibrium:

$$\mathbf{Q}_n = \int_0^{L_n} \mathbf{B}_n^T(x)\mathbf{D}_n(x)dx \tag{7}$$

with L_n the length of the nth element. The corresponding element stiffness matrix $\mathbf{k}_{e_n}$, defined as the derivative of the element force with respect to the element displacement, can then be formulated in terms of the section stiffness, as follows:

$$\mathbf{k}_{e_n} = \frac{\partial \mathbf{Q}_n}{\partial \mathbf{q}_n} = \int_0^{L_n} \mathbf{B}_n^T(x)\mathbf{k}_{s_n}(x)\mathbf{B}_n(x)dx \tag{8}$$

Numerical quadrature, such as Gauss-Legendre quadrature, can then be adopted for the evaluation of the integrals involved in Eqs. (7) and (8). The elastic stiffness matrix $\mathbf{K}$ for the overall system can then be obtained by standard assembly over all n_b element:

$$\mathbf{K} = \sum_{n_b} \mathscr{A}\left(\mathbf{k}_{e_n}\right) \tag{9}$$

3.2. *Dynamic shakedown with distributed plasticity*

Dynamic shakedown is defined as a state in which an elastoplastic structure will eventually respond elastically after a finite amount of plastic deformation, which separates the safe state from collapses due to the aforementioned first failure scenario (Polizzotto et al., 1993). Under the assumption of a periodic and infinite duration dynamic load, a necessary and sufficient condition for an elastic perfectly plastic (EPP) structure to shakedown is that the sum of the steady state elastic stresses in $[0, T]$ (where T is the period of the load) and a time-independent self-equilibrated stress field lies within the elastic domain of the structure (Tabbuso et al., 2016; Chuang and Spence, 2017).

To account for the distributed plasticity along the element, this condition can be formulated in terms of the section forces along the element or, in a more sophisticated formulation, the fiber stresses at each section (Chuang and Spence, 2019b). In this paper, application to structural systems modeled with displacement-based beam-column elements was investigated with each section assumed to follow an EPP constitutive relation. Under this assumption, the structure will shakedown if the following condition holds for all sections of the structure:

$$\tilde{\mathbf{N}}_{ni}^{T}\left(\mathbf{D}_{ni}^{Es}(x,t)+\mathbf{D}_{ni}^{r}(x)\right)-\mathbf{R}_{ni}\leq\mathbf{0},\quad x\in[0,L_n] \tag{10}$$

where $\mathbf{D}_{ni}^{Es}(t)$ is the purely elastic steady state section forces of the ith section of the nth member due to external loads, while $\mathbf{D}_{ni}^{r}(x)$ is a time independent generalized self stress distribution; $\tilde{\mathbf{N}}_{ni}$ is the block diagonal matrix collecting the unit external normals to the piece-wise linear yield surfaces defining the yield function of the section; while $\mathbf{R}_{ni}$ is the plastic resistance vector. The shakedown problem then becomes the identification of the existence of $\mathbf{D}_{ni}^{r}(x)$. To this end, an amplification factor s of the external loads is introduced. Two critical values of s are of particular interest: (1) the elastic multiplier s_e, indicating the maximum amount the external loads can be amplified before at least one section yields; and (2) the dynamic shakedown multiplier s_p, identifying the maximum amplification that the external loads can undergo before dynamic shakedown no longer occurs, i.e. the conditions of Eq. (10) can no longer be satisfied.

Within this context, the aim is to not only identify the two critical amplification factors but also rapidly estimate the associated plastic deformation at shakedown. The first goal can be achieved through solving a linear programming problem (LPP) of the type otlined in Chuang and Spence (2019a). For structures that undergo plastic deformations but shakedown, i.e. $s_e<1$ and $s_p>1$, the inelastic responses will be estimated through an incremental strain-driven approach from $s=s_e$ to $s=1$.

Starting from the elastic limit state, the incremental approach estimates the inelastic responses by assuming an increment in load factor Δs and the residual displacement $\Delta\mathbf{u}_r$. From $\Delta\mathbf{u}_r$ the residual section deformations $\mathbf{d}_{ni}^{r}(x_j)$ of the jth section of the nth element can be obtained through the following geometric transformation:

$$\mathbf{d}_{ni}^{r}(x_j)=\mathbf{B}_n(x_j)\mathbf{T}_n\Delta\mathbf{u}_r \tag{11}$$

where $\mathbf{T}_n$ is the global to local coordinate transformation matrix of element n. The associated increment in section forces can be determined through the following return mapping scheme:

$$\begin{aligned}\Delta\mathbf{D}_{ni}^{r}(x_j)&=\Delta\mathbf{D}_{ni}^{E}(x_j)+\Delta\mathbf{D}_{ni}^{p}(x_j)\\&=\Delta\mathbf{D}_{ni}^{E}(x_j)-\mathbf{k}_{s_{ni}}\mathbf{d}_{ni}^{p}(x_j)\end{aligned} \tag{12}$$

where $\Delta\mathbf{D}_{ni}^{E}(x_j)=\mathbf{k}_{s_{ni}}\mathbf{d}_{ni}^{r}(x_j)$ is the elastic predictor of $\Delta\mathbf{D}_{ni}^{r}(x_j)$; while $\mathbf{d}_{ni}^{p}(x_j)$ is the plastic part of the deformation increment $\mathbf{d}_{ni}^{r}(x_j)$, which must satisfy the associated flow rule as well as the loading-unloading and consistency conditions. The return mapping scheme of Eq. (12) can be efficiently solved through the resolution of the following quadratic optimization problem:

$$\begin{aligned}&\min_{\Delta\mathbf{D}_{ni}^{p}(x_j)}\ \frac{1}{2}\Delta\mathbf{D}_{ni}^{p}(x_j)^{T}\mathbf{k}_{s_{ni}}^{-1}\Delta\mathbf{D}_{ni}^{p}(x_j)\\&\text{subject to}\\&\bar{\mathbf{D}}_{ni}(x_j)^{s}=\max_{0\leqslant t\leqslant T}\tilde{\mathbf{N}}_{ni}^{T}\mathbf{D}_{ni}^{Es}(x_j,t)\\&s\bar{\mathbf{D}}_{ni}(x_j)^{s}+\tilde{\mathbf{N}}_{ni}^{T}\Delta\mathbf{D}_{ni}^{r}(x_j)-\mathbf{R}_{ni}\leqslant\mathbf{0}\end{aligned} \tag{13}$$

This optimization problem ensures the shakedown feasibility condition of Eq. (10). To further satisfy the self-equilibrated condition, the section generalized self stresses $\Delta\mathbf{D}_{ni}^{r}(x_j)$ are updated iteratively until the resultant internal forces at each degree of freedom of the system, which can be estimated following the DB element assumption, are equal to zero, i.e.

$$\mathbf{S}^{r}(\Delta s,\Delta\mathbf{u}_r)=\mathbf{0} \tag{14}$$

Details on the solution scheme of the iterative process can be found in Chuang and Spence (2019a). The process is carried out until $s=1$, i.e. unamplified loads, is reached. At shakedown, in addition to the generalized self stresses $\mathbf{D}_{ni}^{r}(x_j)$ and residual displacements $\mathbf{u}_r$, section plastic deformations can also be evaluated as follows:

$$\mathbf{d}_{ni}^{p}(x_j)=\mathbf{B}_n(x_j)\mathbf{T}_n\mathbf{u}_r-\mathbf{k}_{s_{ni}}^{-1}\mathbf{D}_{ni}^{r}(x_j) \tag{15}$$

3.3. *Estimation of the collapse probability*

The efficiency of the strain-driven dynamic shakedown approach, which requires only the estimation of the peak elastic responses in $[0,T]$, allows simulation models to be directly applied for propagating uncertainty through the system and consequently estimating any system-level probabilistic performance metric of interest.

Within this context, a Monte-Carlo scheme is adopted to estimate the system-level collapse probability $P(C|im)$ with respect to the two failure scenarios defined in Section 2. For a wind storm of intensity $im = \bar{v}_y$, the collapse probability $P(C|\bar{v}_y)$ is therefore estimated through the following expression:

$$P(C|\bar{v}_y) = \frac{1}{N_s} \sum_{i=1}^{N_s} I_C^{(i)}(\bar{v}_y) \tag{16}$$

where N_s is the total number of samples used in the simulation while $I_C^{(i)}$ is an indicator function defining system-level failure within the context of dynamic shakedown or excessive plastic deformations estimated from the strain-driven shakedown algorithm.

4. CASE STUDY

4.1. *Description*

The simulation-based methodology presented above is here applied to a full scale 3D case study. Illustration of the 59-floor building of interest and the lateral load resisting system are shown in Figure 2. The lateral load resisting system consists of a concrete core and an outrigger truss connected at floors 38-40 that engages six outrigger columns. The concrete core walls are connected by coupling beams at the floor levels, while the outrigger columns extend from the foundation to the outrigger truss. The concrete core system is composed of three cells from the foundation to Level 18, two cells from Level 18 to 40, and one cell from Level 40 to the roof. Each floor was considered to act as a rigid floor diaphragm for horizontal movements. Therefore, the floors could move freely in the X- and Y-directions and rotate about the Z-axis (indicated with u_X, u_Y and θ_Z respectively), giving the building a total of 177 degrees of freedom.

The material strengths considered in estimating the steady state elastic response of the system are summarized in Table 1. The Young's modulus of the concrete was calculated according to $E_c = 57{,}000\sqrt{f_c'}$, while the shear modulus was calculated using basic mechanics of materials with the assumption that the Poisson ratio of the concrete was 0.15.


(a) (b)

Figure 2: Rainier Square Tower: (a) Architectural and structural system rendering of the building (Hilburg, 2018); (b) OpenSees finite element model.

Table 1: Summary of material properties.

Material	Strength
Structural steel (wide flange)	$F_y = 50$ ksi
Concrete (shear walls and mega-columns)	$f_c' = 8$ ksi
Reinforcing steel	$f_y = 60$ ksi

To implement the distributed dynamic shakedown framework, the structure was modeled using DB beam-column elements. Gauss-Legendre quadrature was adopted for all elements with five integration points. To maintain continuity across all elements along the height of the building, adjacent wall elements were connected at each floor using two-node rigid link connections. All coupling beams were also connected to adjacent wall elements using two node rigid link connections.

In addition to the mass of all elements and the framing system, which consists of 2.5 inches of normal-weight (145 pounds per cubic foot) concrete over 3-inch ribbed steel decking (490 pounds per cubic foot), additional lumped mass equal to the superimposed dead loads, summarized in Table 2, was applied at the mass nodes, taken to be located at the geometric centers of each floor.

Table 2: Summary of the superimposed dead loading.

Use	Superimposed Dead Loading
Corridors and Stairs (within core)	15 psf
Level 2 to 38 and Level 59	10 psf
Level 39 to 58	30 psf
Level 60	25 psf

Wind load histories of total length of $T = 3600$ s with wind direction randomly selected from $\alpha = \{10°, 20°, \ldots, 350°, 360°\}$ following a uniform distribution were simulated from a wind tunnel driven stochastic model (Chuang and Spence, 2019a) for a 3-s gust wind speed, $\bar{v}_3$, at 33 ft height of 91 mph (corresponding to a MRI of 300 years for Seattle). In addition to the wind loads, gravity loads, including the self-weight of the structure and the superimposed dead loads, were also considered in the analyses. To estimate the steady state elastic response of the system, the first five modes were considered in the modal analysis with damping ratios of 5%.

To evaluate the inelastic response of the structure, EPP constitutive relationships were assumed for each section. The associated yield domain was approximated by piecewise linear yield surfaces. In particular, the 26-surface yield domain proposed in Malena and Casciaro (2008) was adopted for all reinforced concrete sections while the AISC yield surface (AISC 360-16, 2016) was used for the steel sections. A general description of collapse, considering both non-shakedown and failure due to excessive plastic deformations, was defined for estimating the conditional collapse probability $P(C|\bar{v}_3)$ as follows:

1. the inability of the structure to reach the state of dynamic shakedown;
2. peak interstory drift $\tilde{\tilde{u}}_r \geq h/100$;
3. permanent set $\tilde{u}_r \geq h/500$;

where h is the interstory height between each floor of the structure while the peak interstory drift $\tilde{\tilde{u}}_r$ at shakedown can be directly estimated as:

$$\tilde{\tilde{u}}_r = \max_{0 \leq t \leq T} \left[|\tilde{u}(t) + \tilde{u}_r| \right] \tag{17}$$

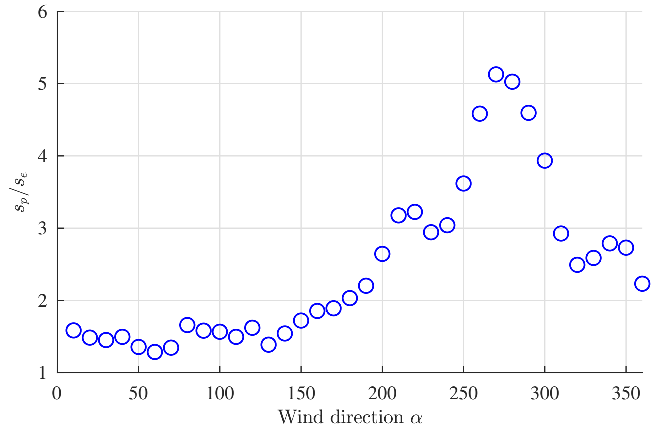

Figure 3: Mean values of the ratios s_p/s_e for all wind directions under 300 MRI wind loads.

with $\tilde{u}(t)$ the purely elastic interstory drift response at shakedown while $\tilde{u}_r$ are the residual interstory drifts estimated from the strain-driven shakedown algorithm.

4.2. Results

Analyses were first carried out to estimate the plastic reserve of the system in terms of the ratio between the plastic and elastic multipliers, i.e. s_p/s_e, over all wind directions. As can be seen from Figure 3, the plastic reserve of the system has a mean value larger than 1.5 for most wind directions, suggesting that the structure will still shakedown even under wind loads that are multiplied by 1.5. For wind loads coming from angles α between 190° and 290°, the plastic reserves are even higher with a maximum ratio of 5.1.

The Monte Carlo algorithm was then run for $N_s = 5000$ realizations. Under 300 MRI wind loads, 46.9% of samples remained elastic while none of the 5000 samples collapsed. From a traditional design standpoint, the structure is under designed with over a 50% chance of inelastic behavior. However, with respect to collapse as defined in this work, the system is safe, i.e. the structure can reach the safe state of dynamic shakedown without experiencing excessive permanent deformations for all 5000 realizations.

In addition, this simulation-based framework provides not only the system-level collapse probability but also the probability distributions of plastic deformations and residual displacements, which are

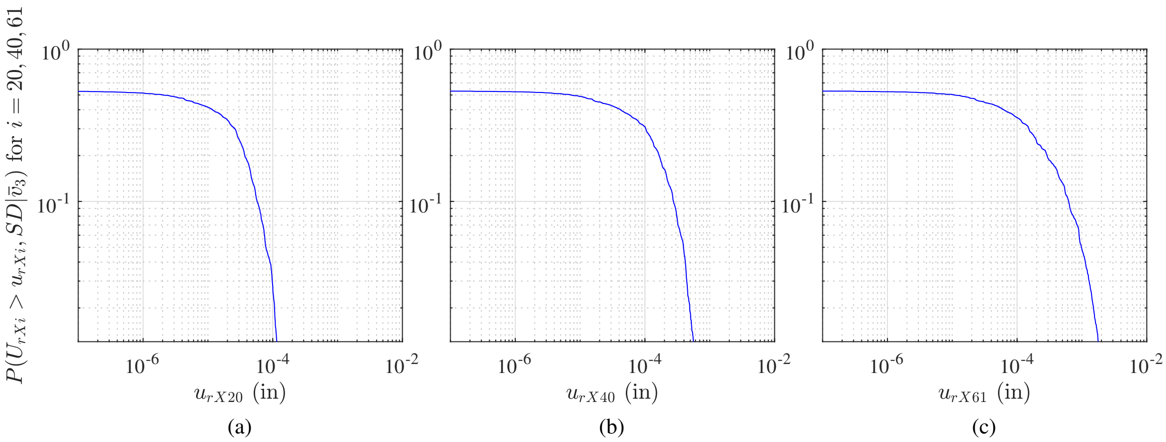

Figure 4: Exceedance probability of the residual displacements at the master nodes in the global X-direction at: (a) Level 20; (b) Level 40; (c) roof.

useful for estimating the non-collapse performance. Within this context, Figures 4 and 5 report the exceedance probability distributions associated with the residual displacements in the global X- and Y-directions at shakedown at three different floor levels. The response at the master node of each floor was chosen for representation. It can be seen that residual displacements in the Y-direction are larger than those in the X-direction, even though both are within the deformation limits.

To illustrate the distributed plasticity, Figure 6 shows the plastic curvature χ_{pz} distributed along a representative element together with the locations of the five integration points marked by dashed lines. Based on the assumption of linear curvature along the element for a DB element, plastic deformations between integration points can be evaluated. For the selected element, plasticity (colored in red) occurred from the two ends of the element to around half the distance to the midpoint of the element.

Finally, it should be observed that the simulation-based approach provided the solutions discussed above in less than 72 hours while running the analysis on a typical dual processor desktop machine. If a similar analysis was carried out by direct integration for each of the $N_s = 5000$ windstorms of duration $T = 3600$ s, the estimated run time would be in the order of months.

5. CONCLUSIONS

This paper presented an approach for characterizing the inelastic response of structural systems within the setting of probabilistic performance-based wind engineering and distributed plasticity. The efficiency of the proposed dynamic shakedown based approach enables the integration of simulation methods for uncertainty propagation, thereby providing a probabilistic description of any performance metric of interest. With the knowledge of the wind climate at the site of interest, the proposed probabilistic framework can readily be extended for system-level structural reliability estimation by identifying all uncertainties involved and simulating over all intensity levels. The proposed approaches have the potential to open the door to the inelastic design of the structural systems during severe wind events.

6. ACKNOWLEDGMENTS

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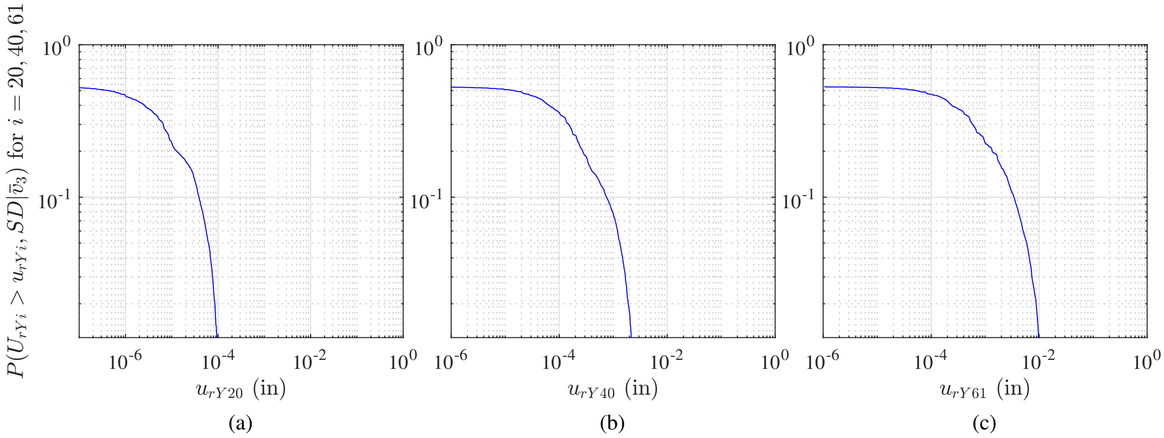

Figure 5: Exceedance probability of the residual displacements at the master nodes in the global Y-direction at (a) Level 20; (b) Level 40; (c) Core roof.

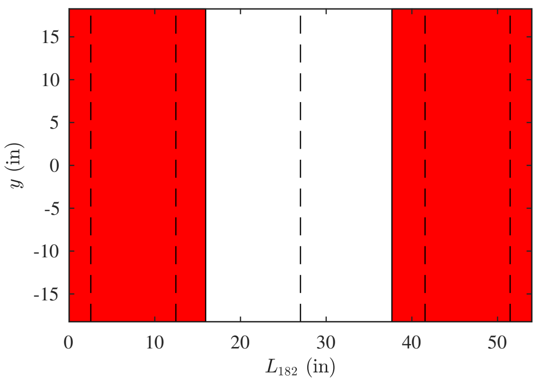

Figure 6: Plasticity distributed along an element of the structure for a representative sample.

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