Bayesian Updating of Embankment Settlement on Soft Soils with Finite Element Method

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ABSTRACT: Prediction of responses (e.g., embankment settlement) of geotechnical structures on soft soils is a challenging task due to their complex mechanical behaviors. In face with such complexity, the finite element method (FEM) combined with advanced soil constitutive models (e.g., soft soil creep (SSC) model) is frequently used to predict the short-term and long-term responses of geotechnical structures on soft soils, which involves a number of model parameters. Determination of these model parameters depends on knowledge obtained from site investigation data and/or monitoring information. This paper develops a Bayesian sequential updating (BSU) framework that incorporates monitoring information obtained at different construction stages to update FEM model parameters and their corresponding stochastic responses. To address the computational issues in Bayesian analysis, No-U-Turn Sampler (NUTS) Markov chain Monte Carlo (MCMC) algorithm is introduced to populate posterior samples, and multiple Hermite response surfaces are constructed for different monitoring phases to reduce the computational efforts costed by evaluating the likelihood function. The proposed method is illustrated by a settlement prediction example of Ballina trial embankment, New South Wales, Australia. Effects of different likelihood functions (namely with and without model bias factor (MBF)) on Bayesian updating of settlement predictions are investigated. Results showed that the proposed BSU framework improves the prediction accuracy of soft soil settlement compared with prior predictions. NUTS is much more efficient in generating posterior samples compared with Metropolis-Hastings (MH) algorithm as the number of model parameters is relatively large. When considering short-term settlement behaviors of soft soils, the likelihood function without MBF is preferred because the adopted SSC can properly characterize short-term behaviors of soft soils. On the other hand, the likelihood function with MBF is recommended because SSC is hard to represent long-term behaviors of soft soils.

1. INTRODUCTION

Accurate predictions of time-dependent settlement of embankment established on very soft ground are of great significance to achieve proper and economical design outcomes. However, it is a challenging task due to the complex mechanical behaviors of soft soils. To capture the real soft soil behaviors as accurately as possible, settlement predictions can be obtained from finite element method (FEM), along with the advanced soil constitutive models (e.g., soft soil creep (SSC) model), which may involve a number of model parameters. These parameters determined by laboratory tests often come with large uncertainty, and may lead to inaccurate predictions of the embankment settlement. The monitoring information obtained at different monitoring stages can be combined with the laboratory test data by Bayesian theory to update sequentially FEM model parameters and their corresponding stochastic responses (Kelly et al., 2018； Zheng et al., 2018), which is named as Bayesian sequential updating (BSU) herein.

There are two computational issues when performing BSU for soft soils. First, as the advanced soil constitutive model includes a large number of model parameters, the updating of the settlement prediction is a high-dimensional Bayesian updating problem. The commonly used Bayesian updating method, Metropolis-Hastings (MH) Markov chain Monte Carlo (MCMC) algorithm, is not suitable for this high-dimensional Bayesian updating problem (Betancourt, 2017). Second, FEMs provide a rigorous and versatile tool for settlement predictions, but FEM-based probabilistic simulation requires intensive computational power.

This paper develops a BSU framework to continuously integrate monitoring information that are newly obtained to improve the prediction of the settlement of embankment on soft soils. No-U-Turn Sampler (NUTS) (Hoffman and Gelman, 2014) is employed to effectively generate posterior samples for BSU. Multiple Hermite response surface models are constructed to reduce the computational efforts for evaluating the likelihood function. The paper starts with the introduction of BSU framework for embankment settlement on soft soils, followed by descriptions about posterior sampling algorithms based on NUTS. Then, the implementation procedure of the proposed approach is presented and illustrated through a settlement prediction example of Ballina trial embankment.

2. BSU FRAMEWORK FOR EMBANKMENT SETTLEMENT ON SOFT SOILS

2.1. Prior analysis

Let $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_D]$ represent uncertain parameters of the soft soil constitutive model (e.g., the modified compression index λ^*, the modified swelling index κ^*, the modified creep index μ^* in SSC model), where D is the total number of uncertain parameters. y_j denotes the monitoring results of the embankment settlement at time T_j, j = 0, 1, 2, …, N_T, and j refers to different monitoring phase. $M_j(\boldsymbol{\theta})$ denotes the prediction of settlement Y_j calculated by FEM-based numerical analysis at time T_j. For a given embankment, its prior mean prediction for settlement $\overline{Y}_j$ based on FEM can be calculated as (Li et al., 2016)

$$\overline{Y}_j = E\left(Y_j\right) = \int \beta_j M_j\left(\boldsymbol{\theta}\right) f\left(\boldsymbol{\theta}\right) \mathrm{d}\boldsymbol{\theta} \qquad (1)$$

where $f(\boldsymbol{\theta})$ is the prior joint probability density function (PDF) and β_j is the model bias factor (MBF) to characterize the model error. (e.g., Cao et al. 2016(a)).

2.2. BSU with sequential monitoring information

The accuracy of prior predictions of the embankment settlement given by Eq. (1) depends on the model parameters. However, these parameters usually contain large epistemic uncertainty from prior knowledge, which may result in biased predictions, especially when confronted with soft soils whose accurate characterization is a challenge (Kelly et al., 2018). Monitoring information can be incorporated in Bayesian framework to provide a more accurate prediction of the settlement.

Under the framework of Bayesian updating, when monitoring data of embankment settlement y_j is available, the updated or posterior PDF of uncertain parameters $f(\boldsymbol{\theta}|y_1,\ldots,y_j)$ can be formulated as (e.g., Cao et al., 2016(b))

$$f(\boldsymbol{\theta}|y_1,\ldots,y_j) = K_j f(\boldsymbol{\theta}|y_1,\ldots,y_{j-1}) L_j(\boldsymbol{\theta}) \qquad (2)$$

where K_j is a normalized constant in j-th updating phase; $f(\boldsymbol{\theta}|y_1,\ldots,y_{j-1})$ is posterior distribution of $\boldsymbol{\theta}$

in updating phase $j-1$, and it is used as the prior distribution in updating phase j; $L_j(\boldsymbol{\theta})$ is the likelihood function when observing y_j. When considering measurement error and model error, settlement prediction result Y_j at T_j can be expressed as (e.g., Li et al., 2016)

$$Y_j = \beta_j M_j(\boldsymbol{\theta}) + \xi_j \tag{3}$$

where ξ_j are measurement errors and are assumed to follow a zero-mean Gaussian distribution. In addition, ξ_j are assumed to be independent from each other.

Then, the likelihood function $L_j(\boldsymbol{\theta})$ can be written as

$$L_j(\boldsymbol{\theta}) = \varphi_{\xi_j}\left(y_j - \beta_j M_j(\boldsymbol{\theta})\right) \tag{4}$$

where $\varphi_{\xi_j}(\cdot)$ denotes normal PDF of ξ_j.

With the distribution of uncertain parameters is updated as $f(\boldsymbol{\theta}|y_1,\ldots,y_j)$, the updated mean prediction $\overline{Y}_i^{new}$ ($j < i < N_T$) can be represented by

$$\overline{Y}_i^{new} = E\left(Y_i^{new}\right) = \int \beta_i M_i(\boldsymbol{\theta}) f\left(\boldsymbol{\theta}|y_1,\cdots,y_j\right) \mathrm{d}\boldsymbol{\theta} \tag{5}$$

Based on the above procedure, monitoring data of the embankment settlement can be sequentially incorporated to estimate the embankment settlement in the next stage.

3. POSTERIOR SAMPLINGS USING NUTS

As the analytic expression of posterior PDF $f(\boldsymbol{\theta}|y_1,\ldots,y_j)$ is usually not available and the numerical model $M_i(\boldsymbol{\theta})$ is complex in Eq. (5), it is difficult to derive the analytical solution of $\overline{Y}_i^{new}$. Therefore, simulation-based methods are applied for obtaining the estimation of the updated settlement $\overline{Y}_i^{new}$. It can be expressed as (Li et al., 2018)

$$\overline{Y}_i^{new} \approx \frac{1}{N}\sum_{k=1}^{N} \beta_i M_i(\boldsymbol{\theta}_k) \tag{6}$$

where $\boldsymbol{\theta}_k$ (k = 1, 2, …, N) are the samples distributed as the posterior PDF $f(\boldsymbol{\theta}|y_1, \ldots ,y_j)$.

In Bayesian updating analysis, the Markov chain Monte Carlo (MCMC) simulation method has been widely applied for generating samples following the posterior PDF $f(\boldsymbol{\theta}|y_1, \ldots ,y_j)$. The Metropolis-Hastings (MH) algorithm is a commonly used sampling method in MCMC due to its conceptual simplicity and easy implementations. However, for the high-dimensional Bayesian updating analysis, where a large number of uncertain parameters are needed to infer, the random-walk behaviors of MH make it challenging to accept the candidate states (Betancourt, 2017). This leads to a large number of repeated samples. The No-U-Turn Sampler (NUTS) (Hoffman and Gelman, 2014) was proposed to deal with complex, high-dimensional problems with minimal user intervention. The basic idea of NUTS originates from Hamiltonian dynamics and it takes advantages of gradient information to make larger jumps away from the initial point into new and unexplored posterior regions, achieving much faster convergence than MH methods. Therefore, NUTS is applied in this study to the prediction of embankment settlement on soft soils with FEM, which includes a relatively large number of uncertain parameters. For the purpose of conciseness, details of NUTS, including the algorithm and implementation procedure, are not provided here. Interested readers can be referred to Hoffman and Gelman (2014).

4. ILLUSTRATIVE EXAMPLE

The proposed BSU framework is applied to analyzing the settlement of Ballina trial embankment, New South Wales, Australia (Kelly et al., 2018). Figure 1 displays the geometry of the representative cross-section of Ballina embankment. It covers a horizontal distance of 140 m and soil profile beneath the ground surface is divided into four layers, from top to bottom, including a 1.4 m alluvial clayey sandy silt layer, a layer of 9.4 m estuarine clay that is further divided into 4 sub-layers, a 3.3 m transition zone, and a 5 m sandy layer. To build the embankment, a working platform was constructed to about 0.6m thick initially, a 0.4 m drainage sand layer where wick drains began was replaced on top of it, and lastly main embankment construction with 2.0 m thickness was carried out. The ground water table is 1.2 m beneath the original terrain.

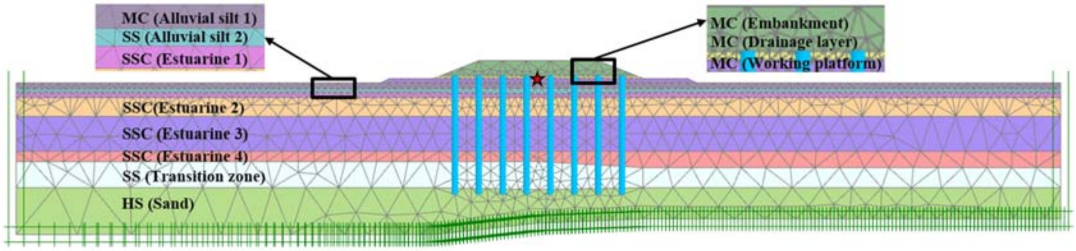


Figure 1: Geometry of finite-element model of Ballina trial embankment (After Jostad et al., (2018))

Table 2: Consolidation stages during the creep phase and their corresponding monitoring data

Phase ID, j	12	13	14	15	16	17	18	19	20	21	22	23
Time, T_j (Days)	63	81	97	122	157	196	220	248	258	297	322	389
Monitoring data (m)	0.44	0.58	0.68	0.78	0.85	0.94	0.97	1.03	1.06	1.12	1.17	1.19
Phase ID, j	24	25	26	27	28	29	30	31	32	33	34	35
Time, T_j (Days)	447	501	682	802	902	979	1090	1240	1390	1540	1690	1833
Monitoring data (m)	1.24	1.27	1.34	1.37	1.42	1.45	1.52	–	–	–	–	–

Soft Soil Creep (SSC) model is employed to model the shear deformation and compressibility of the estuarine clay and it can take into account the time-dependent stiffness and the creep behaviors of soft soils. For the alluvial silt layer between 0.75 and 1.4 m depth and the transition zone, Soft Soil (SS) model is adopted and Hardening Soil (HS) model is used to characterize the sandy layer beneath the transition zone. Other layers are modelled with Mohr-Coulomb (MC) model. Only the parameters of the main estuarine clay layer (i.e., the modified compression index λ^*, the modified swelling index κ^*, the modified creep index μ^*, the pre-overburden pressure POP, the friction angle ϕ', horizontal and vertical permeability k_x and k_y) are considered as uncertain variables in this study. All these random parameters are assumed to follow lognormal distributions and their prior statistics are shown in Table 1, which are consistent with those from Liu et al. (2018).

Although various monitoring equipment were installed in the adopted cross section (Kelly et al., 2018), the settlement of ground surface is chosen to illustrate the proposed approach because of its crucial role in assessing the safety level of embankments. Correspondingly, the

Table 1: Prior statistics of uncertain parameters of SSC in the estuarine clay layer (Liu et al., 2018)

Soil parameters	Mean	COV
λ^* (−)	0.216	0.26
κ^* (−)	0.032	0.25
μ^* (−)	0.006	0.33
POP (kPa)	24	0.23
ϕ' (°)	36	0.08
k_x (m/day)	0.00064	0.20
k_y (m/day)	0.0041	0.20

monitoring data of settlement plate2 (SP2) located near the cross-section will be incorporated to update FEM parameters and their corresponding stochastic responses. The monitoring point is denoted by pentagram in red color in Figure 1.

Numerical analysis is performed by finite element program PLAXIS 2D (www.plaxis.nl). Wick drains are modelled by vertical drain elements with a specific center distance of 3.2 m, starting from the sand layer above the ground to 14.9 m beneath the ground level. 15 node triangular elements with automatic meshing are selected, resulting in 2166 elements, which are similar to 2151 adopted by Jostad et al. (2018). For more details about PLAXIS finite element model and derivations of relevant parameters

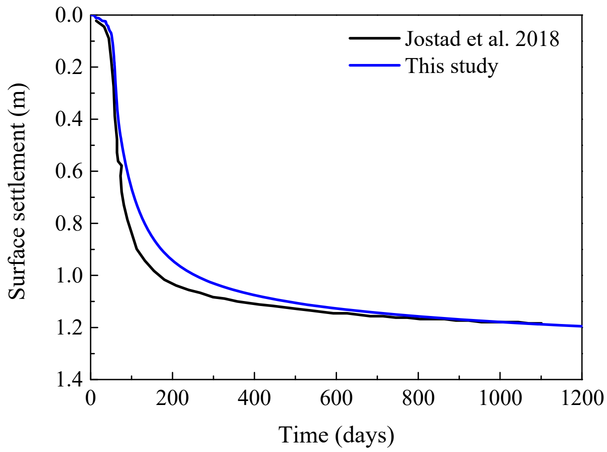


Figure 2: Settlement predictions at SP2 of Ballina embankment

corresponding to their constitutive models, please refer to Jostad et al. (2018) and Liu et al. (2018).

For validation, surface settlements from 0 to 1200 days at the centerline of Ballina embankment are evaluated using the FEM model. Figure 2 shows deterministic results by the blue line. It can be seen that the settlement increases with the increase of time. The settlement results from Jostad et al. (2018) are also plotted by black line, which is generally consistent with the results calculated by the model developed in this study.

4.1. Construction of multiple Hermite response surfaces

To address the computational cost for evaluating the likelihood function in the proposed BSU framework, the multiple Hermite response surface models (RSMs) (Li et al., 2011) are constructed for different monitoring phases, which can be written as

$$F_{p,j}(\mathbf{U}) = a_{0,j}\Gamma_0 + \sum_{i_1=1}^{D} a_{i_1,j}\Gamma_1(U_{i_1}) + \sum_{i_1=1}^{D}\sum_{i_2=1}^{i_1} a_{i_1i_2,j}\Gamma_2(U_{i_1},U_{i_2}) + \sum_{i_1=1}^{D}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2} a_{i_1i_2i_3,j}\Gamma_3(U_{i_1},U_{i_2},U_{i_3}) + \cdots + \sum_{i_1=1}^{D}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2}\cdots\sum_{i_D=1}^{i_{D-1}} a_{i_1i_2\cdots i_D,j}\Gamma_D(U_{i_1},U_{i_2},\cdots,U_{i_D}) \quad (7)$$

where $F_{p,j}(\bullet)$ is the stochastic responses of embankment settlement for the j-th phase with the order of p; $\Gamma_D(\bullet)$ is the multi-dimensional Hermite polynomials; $\mathbf{a}_j = \left(a_{0,j}, a_{i_1,j}, \ldots, a_{i_1i_2\cdots i_D,j}\right)$ are unknown coefficients needed to be determined; $\mathbf{U} = (U_1, U_2, \ldots, U_D)$ is a vector of independent standard normal variables, which can be transformed as

$$\mathbf{U} = \frac{\ln(\boldsymbol{\theta}) - \boldsymbol{\mu}_{\ln(\boldsymbol{\theta},\text{prior})}}{\boldsymbol{\sigma}_{\ln(\boldsymbol{\theta},\text{prior})}} \quad (8)$$

if model uncertain parameters $\boldsymbol{\theta}$ are lognormally distributed, where $\boldsymbol{\mu}_{\ln(\boldsymbol{\theta},\text{prior})}$ and $\boldsymbol{\sigma}_{\ln(\boldsymbol{\theta},\text{prior})}$ represent prior mean values and standard deviations of the logarithm (i.e., $\ln(\boldsymbol{\theta})$) of uncertain parameter $\boldsymbol{\theta}$, respectively.

To calculate unknown coefficients $\mathbf{a}_j$, the Latin hypercube sampling (LHS) based experimental design method is adopted. To demonstrate the proposed BSU framework, the final creep phase in Jostad et al. (2018) is further divided into different phases so that a monitoring settlement value is available at the end of each consolidation phase. Table 2 summarizes consolidation phases with actual measurements. It can be seen that there are 35 phases in total and only phase 12 to phase 30 have their corresponding settlement measurements and are sequentially updated in the following.

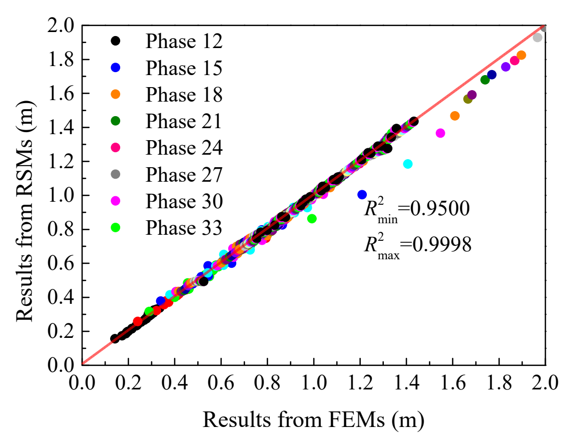


Figure 3: Comparisons between settlement prediction results calculated by FEMs and RSMs

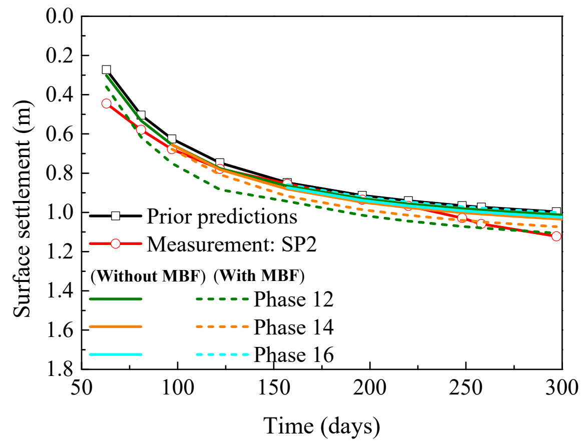
35 Hermite RSMs with the order of 3 are constructed with 120 LHS design points. To verify RSMs, additional 30 random samples are simulated. Settlement evaluation results for different phases from FEMs and RSMs are shown in Figure 3. The minimum R^2 that occurs in phase

12 is 0.9500, which indicates that the performance of RSMs is satisfactory as a whole.

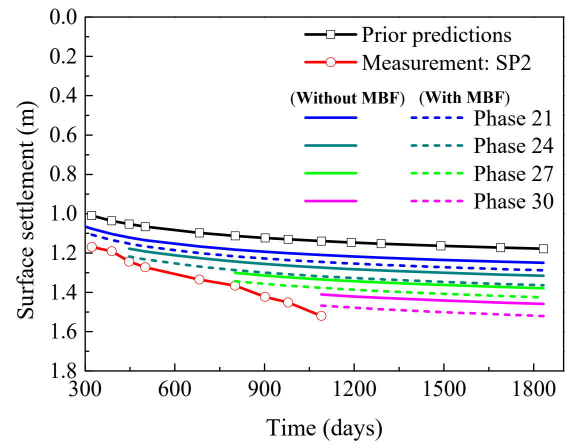
4.2. Sequential prediction results

Based on the previously proposed BSU framework, NUTS sampling algorithm is employed to estimate the sequential posterior predictions based on the multiple Hermite RSMs established in subsection 4.1. To investigate the effect of the MBF in the prediction of Ballina settlement, two situations are considered as follows: (1) MBF β is treated as a constant value 1 (i.e., predictions without MBF); (2) MBF β is treated as a random variable (i.e., predictions with MBF) following a lognormal distribution with a mean value of 1 and a coefficient of variation (COV) of 0.20 (Liu et al., 2018).

For convenience, the distribution type of the posterior distribution of all parameters in different updating phases is assumed as lognormal distribution in this study (Li et al., 2016). Measurement error ξ is assumed to follow a normal distribution, $\xi \sim N(0,0.10)$ (Zheng et al., 2018). 2 independent MCMC chains with 1000 warm-up iterations and 10000 post-warm-up draws per chain for all parameters are simulated. Figure 4 shows the results of sequential posterior predictions. The solid lines in different color denote updating results without MBF for different updating phases, and the dashed lines represent results with MBF. For comparisons, real monitoring values and prior mean predictions are also plotted by the lines with circles and squares, respectively. When considering short-term behaviors of soft soils, results for prior predictions are consistent with the real settlement measurements, as shown in Figure 4 (a). The results without MBF are more accurate than those with MBF. For example, after measurement data is incorporated in phase 12, prediction results in olive solid line (without MBF) is closer to real measurements than the olive dash line (with MBF). As for long-term responses of soft soils in Figure 4(b), the proposed BSU framework improves the prediction accuracy for embankment settlement compared with prior predictions. The analysis considering MBF provides a better prediction of the settlement than that without considering MBF, which is opposite to the observation on short-term behaviors of embankment settlement.



(a) Short-term prediction results



(b) Long-term prediction results

Figure 4: Sequential mean prediction results with without MRF

However, note that sequential prediction results deviate from the long-term monitoring data to some degree. A possible explanation might be that SSC model adopted in the estuarine clay layer cannot properly capture real soft soil behaviors because settlement calculated by SSC is more stable and slowly increasing compared with dramatic changes of real measurements shown in Figure 4 (b).

4.3. Comparisons of MH and NUTS

This section compares the performance of MH and NUTS. As shown in Figure 5, the straight lines with lower triangles and the dashed lines with upper triangles represent sequential posterior predictions considering MBF calculated by MH and NUTS, respectively. It can be seen that results from different methods are consistent. This validates the results obtained from NUTS.

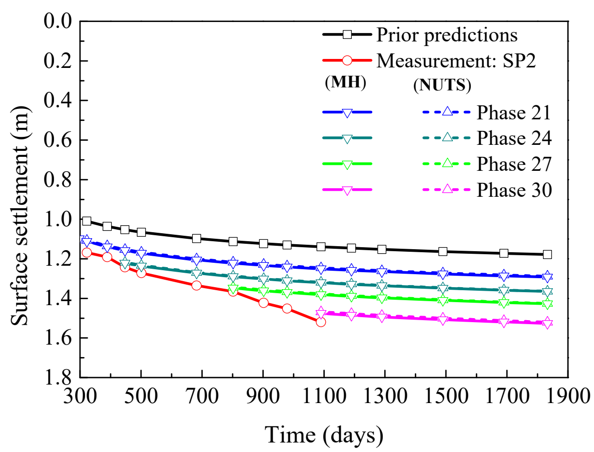


Figure 5: Sequential mean prediction results with MRF calculated by NUTS and MH

To further explore the advantageous performances of NUTS, effective sample size (ESS) is applied to measuring the efficiency of each MCMC algorithm for its convenience and simplicity. ESS is the number of independent samples needed to obtain a Monte Carlo estimate of the mean of a function with equal variance to the MCMC estimate of the mean of the function, (Hoffman and Gelman, 2014). ESS of NUTS and MH for all uncertain parameters in different phases are calculated and shown in Figure 6. Solid symbols with different colors denote ESS of different variables computed for MH, whereas, ESS of various parameters calculated for NUTS are shown in open symbols. It is obvious that NUTS performs much better than MH throughout all monitoring phases of interest. Such a high ESS indicates small autocorrelation among posterior samples. In this example, the minimum value of the proportion of non-repeated posterior samples for NUTS is 0.99. However, for MH, the value is significantly lower, with the mean value of 0.21 and COV of 0.26 for all concerned updating phases. However, it should be pointed out that NUTS requires gradient evaluation at each sample, which might imply a higher computational cost than MH if the derivatives are directly evaluated based on runs of the FE software. Further investigations are beyond the scope of this study.

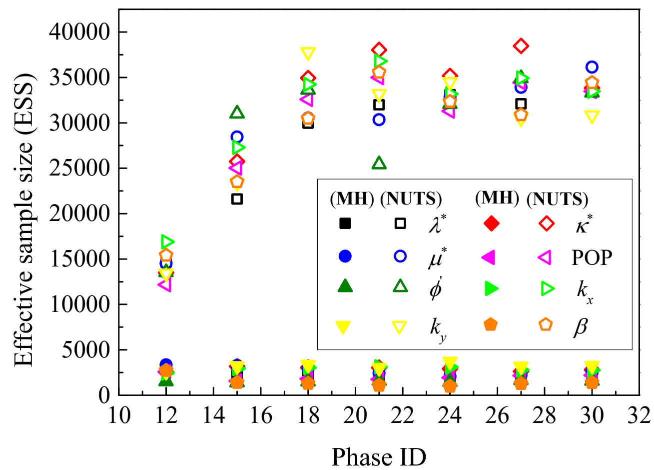


Figure 6: The results of ESS calculated from MH and NUTS for different monitoring phases

5. CONCLUSIONS

This paper developed a Bayesian sequential updating (BSU) approach for embankment settlement predictions on soft soils based on finite element models (FEM). The No-U-Turn Sampler (NUTS) algorithm is employed to efficiently produce posterior samples to estimate the mean prediction of the embankment settlement. To overcome the computational difficulty, multiple Hermite response surfaces for different updating phases were also established. An example of surface settlement predictions of Ballina trial embankment is investigated to demonstrate the efficiency and applicability of the proposed method. Results showed that the accuracy of settlement predictions of Ballina embankment on soft soils can be improved by incorporating monitoring information. The effective sample size (ESS) of NUTS is significantly higher than that of MH. NUTS is more efficient than MH for Bayesian model updating with high-dimensional uncertain parameters. It was also found that, when considering short-term settlement behaviors of soft soils, the likelihood function without MBF is preferred because the adopted SSC model can properly characterize short-term behaviors of soft soils. However, the likelihood function with MBF is recommended because SSC is hard to represent long-term behaviors of soft soils.

6. ACKNOWLEDGEMENTS

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