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A Novel Repair Sequence Scheduling Method for Post-Disaster Critical Infrastructure Systems

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ABSTRACT: This paper formulates the repair sequence scheduling problem for damaged component in post-disaster critical infrastructure systems (CISs) under limited repair resources in a general form and proposes a heuristic method to solve the problem. The proposed method are compared with typical existing solution methods in the literature in terms of the optimality gap and computational cost. All these methods are applied into post-earthquake damage scenarios for a real electric power system. Results show that the proposed method has better performance than existing methods and can be applied to the recovery of large-scale CISs with extensive disruptions.

1. INTRODUCTION

Critical infrastructure systems (CISs), including electric power, transportation, water supply and telecommunication systems, provide essential services to support the economy of a region as well as the well-being of its citizens. However, these CISs are subjected to types of disruptions, such as natural disasters and terrorist attacks, and the failures of these systems may cause severe societal and economic disruption (Hackl et al., 2015). Designing resilient CISs is the key to make a city or nation resilient and a resilience CIS requires absorptive capacity (absorb the negative effect of a disruption), adaptive capacity (adapt to the new conditions after a disruption) and restoration capacity (rapidly recover after a disruption). Many studies proposed various types of measures, such as protecting and reinforcing critical components and adding line switch in the electric power systems, to resist and absorb potential hazards (Salmeron et al., 2004; Zhao et al., 2013; Fang et al., 2016), while this paper addresses enhancing restoration capacity of CISs after large-scale disruptions, such as seismic hazards.

The restoration capacity of post-disaster CISs mainly depends on how to rapidly recover from disruption in the restoration process. However, the restoration processes of CISs can be very complicated in practice, which vary with different types of CISs, disruptions and objectives. A restoration process can be generally divided into three periods: response recovery period, which last from 1 to 7 days and some emergency actions are taken; short-term recovery period, which takes weeks to months and urgent components have been rehabilitated and repaired; long-term recovery period, which spends long times to totally recovery from disruptions or to improve the system better (Kaviani et al., 2018). In the restoration process of a post-disaster CIS, the most important phase is how to schedule the limited restoration resources to the damaged components. Before the repair phase, several firstphase preparations, such as initial inspection and damage assessment, have been made (Cagnan and Davidson, 2004). Hence, this paper focuses on

how to schedule the limited repair resources for the damaged components in post-disaster CISs.

In the literatures, scholars have proposed different methods and optimization formulations to assign the limited repair resources or schedule repair teams for post-disaster CISs. Some scholars evaluated the importance of those damaged components and the damaged components with high importance values have the high priority to be repaired. The importance value of a damaged components can quantified by betweenness (Ulusan and Ergun, 2018), degree (Sun and Zeng, 2017), and the ratio of the functionality increase to its required repair time if repairing that component (Nojima et al., 1992). Moreover, Sato and Ichii (1995) used the travel time based total un-restored ratio as the objective function for the post-earthquake road network in the Izu Penisula, and solved the optimum repair sequence by using a genetic algorithm. Similar methods solution techniques have been also used to schedule the repair resources to the damaged components (Ozdamar, et al., 1999; Xu et al., 2007).

scholars proposed optimization Some formulations which model the repair sequence as decision variable, and optimize an objective function that describes the efficiency of a repair sequence. Hentenryck et al. (2011) studied the repair sequence scheduling problem of postdisaster power systems with the consideration of the vehicle-routing constraints, and Coffrin et al. (2012) investigated the last-mile restoration for interdependent power and gas systems. The models in these two studies were solved by heuristic algorithms. Considering a single CIS, if its operation is described by a linear programming model, the problem for identifying the optimum repair sequence can be formulated as a mixed integer linear programming (MILP), which can be directly solved by commercial solvers, such as CPLEX. Nurre et al. (2012) proposed a time index based optimization method to schedule repair teams for CIS restoration, which divides the whole restoration period into several equal small time periods and uses the total demand loss over all time periods as the objective function. Similar

modelling approaches have been also used for post-earthquake interdependent power, water and gas systems in Shelby County, and interdependent power and telecommunications system in New York City (Cavdaroglu et al., 2013; Gonzalez et al., 2016). Instead of dividing the restoration period into equal small time period, recently Ouyang and Fang (2017) proposed a component index based optimization method, which repairs one damaged component at the beginning of each time period and uses the resilience loss as the objective function. However, this method was only applied for small-scale damage scenarios (a few components' failures under the worst-case attack), how efficient this method is under largescale damage scenarios has not been investigated.

As identifying the optimum repair sequence is a critical part for enhancing CIS resilience, adopting an efficient and accurate method is crucially important. Despite there are many methods for identifying the repair sequences for post-disaster CISs, how efficient each of those methods is has been seldom addressed in the literature. Hence, this paper first briefly introduces typical repair sequence scheduling methods in the literature and then proposes a novel heuristic algorithm. These methods are separately applied into post-earthquake damage scenarios for the electric power transmission system in Shelby County, USA, and are then compared in terms of the optimality gap and the computational cost, where resilience loss is taken as the metric to evaluate the solution. Note that if the exact solution cannot be obtained, the paper simple uses the minimal resilience loss among losses produced by all solution methods as the benchmark. The remainder of this paper is organized as follows. Section 2 formulates the problem, and section 3 briefly introduces or proposes different repair sequence scheduling methods. Taking the post-earthquake power transmission systems in Shelby County as an example, Section 4 compares different scheduling methods. Section 6 provides conclusions and future work.

2. PROBLEM FORMULATION

A CIS is represented by an undirected connected graph G(V, L), where V is the set of nodes and L denotes the set of lines. There is a set of supply nodes $V^s \subseteq V$, where each supply node $n \in V^s$ is associated with real supply $P_n^{S}(t)$ at time t and maximum supply \overline{P}_n^{S} , a set of demand nodes $V^D \subseteq V$, where each demand node $n \in V^D$ is associated with real supply $P_n^D(t)$ at time t and required demand \overline{P}_n^D . Each line $l \in L$ from an origination node o(l) to a destination node d(l) is characterized by its real flow $F_l(t)$ at time t and its capacity \overline{F}_l . Note that line damage can be equivalently modelled by adding a new node on each line, this paper only considers node damage. The set of damaged nodes under an event is denoted by V^A , and the state of node *n* at time *t* is denoted by binary variable $x_n(t)$, with its value 1 indicating normal operation, and 0 otherwise.

This paper formulate the repair sequence scheduling problem in a general form with the following assumptions:

(1) Repair resources are characterized by repair teams and the maximum amount of available repair resources is determined by an input parameter RR, and repair team are identical and share the same work efficiency;

(2) Each damaged component can only repaired by one repair team and the repair time for each damaged component n, which is determined by an input parameter τ_n to characterize the extent of the damage, is given and known beforehand;

(3) The travel time and routing for the repair teams are ignored;

(4) General network flow model and direct current power flow (DCPF) model are applied to simulate the operation of pipeline systems and electric power systems.

Based on the above assumptions and parameters, given a repair sequence of the set of damaged components under an event, V^A with K nodes damaged, the formulation of repair sequence scheduling problem can be written as following:

$$\min_{\substack{P_n^S(t_i), P_n^D(t_i), F_l(t_i)}} \sum_{i=1}^K \sum_{n \in V^D} w_n * \left(\overline{P}_n^D - P_n^D(t_i)\right) * T(i)$$
(1a)

Subject to:

$$T(i) = t_{i+1} - t_i, \forall i$$

$$P_n^S(t_i) - \sum \{l \in L | o(l) = n\} f_l(t_i) + \sum \{l \in L | d(l) = n\} f_l(t_i) - P_n^D(t_i) = 0, \forall n \in V, \forall t_i$$
(1c)

$$-\bar{P}_{l}^{L} x_{o(l)}(t_{i}) x_{d(l)}(t_{i}) \leq F_{l}(t_{i}) \leq \bar{P}_{l}^{L} x_{o(l)}(t_{i}) x_{d(l)}(t_{i}), \forall l \in L, \forall t_{i}$$
(1d)

$$0 \le P_n^S(t_i) \le x_n(t_i)\overline{P}_n^S, \forall n \in V^S, \forall t_i \quad (1e)$$

$$0 \le P_n^D(t_i) \le x_n(t_i)\overline{P}_n^D, \forall n \in V^D, \forall t_i \quad (1f)$$

where t_i is the exact finishing time of repair activity conducted on the (i-1)-th damaged components and t_1 is the initial time point of the planning horizon. The objective function for minimizing the resilience loss is described by (1a), where w_n denotes the weight of node n. Constraint (1b) presents the time interval between the current time point t_i and the next time point t_{i+1} at which one damaged component is repaired. Constraint (1c) ensures flow conservation. Constraint (1d) limits the flow capacity. Constraint (1e) states the maximum output. Constraint (1f) states the required demand level.

The DCPF model can be formulated by adding the node phase angle constraint (1g) based on above equation (1a)-(1e).

$$f_l(t_i^j) = B_l(\theta_{o(l)}(t_i) - \theta_{d(l)}(t_i)), \forall l \in L, \forall t_i$$
(1g)

where B_l denotes the susceptance of line l and decision variable $\theta_n(t_i^j)$ denotes the phase angle of node n at time point t_i .

3. SOLUTION METHODS

The first type of methods is the component importance based methods which determine the repair sequence of a set of damaged components in terms of their importance values, and then schedule the available repair teams (initially available or available after repairing a damaged node) to damaged nodes according to this repair sequence. This paper selects degree based methods (DBM) for comparison, which provides the repair sequence by ranking the degree values of damaged nodes in a descending order (Sun and Zeng, 2017).

The second type of methods is the genetic algorithm based methods (GABM), which simulate an evolutionary process of the repair sequence that represent points in a search space. The GABM used in this paper firstly numbers the damaged components and expresses a repair sequence by a genotype, and secondly computes the fitness value of each genotype based on a fitness function. After that, this method uses the selection, crossover and mutation operators to produce the next-generation individuals, and then returns to the second step until the maximum generation is reached. The genotype in the final generation with the minimum resilience loss corresponds to the optimum repair sequence (Sato and Ichii, 1995).

The third type of approaches is programming based optimization methods which divide the whole restoration period into several small time periods and schedule the repair teams to repair damaged components during each time period for minimizing the accumulative system loss of all time periods. Depending on whether each time period is fixed (or set as input parameters) or not (as decision variables), this method can be further grouped into time index based optimization method (TIBOM) (Nurre et al., 2012) and component index based optimization method (CIBOM) (Ouyang and Fang, 2017). In these two methods, the time periods both start from 1 to T_p , where T_p is the numbers of all time periods, and t_s denotes the time point at the beginning of time period s. These two methods have some identical repair decision variables, including (1) binary variable $x_n(t_s)$ which represents the state of node n at the beginning of time period s, with 1 for normal operation and 0 otherwise; (2) binary variable $r_{kn}(t_s)$ which represents whether node n is repaired by repair team k at the beginning of

time period s, with 1 for repaired and 0 otherwise. In the TIBOM, each time period has the same interval T(s) which is the maximum recovery time T^{max} (to repair all damaged nodes) divided by T_p . There may exist several damaged components repaired by one repair team during the same time period, then this paper further ranks the repair sequence of those components in terms of their demands in a descending order. However, in the CIBOM, the number of time periods Tp is equal to the number of damaged components, and at most one damaged component is repaired at the beginning of each time period, and T(s) refers to the recovery time points for the two component repaired at the beginning of this time period s and at the beginning of the next time period s+1. Hence, TIBOM only provides an approximate optimal solution while CIBOM can provide the global optimum. Details of this two methods and formulations were introduced by Nurre et al. (2012) and Ouyang and Fang (2017).

For the above two optimization methods, the TIBOM can return the results quickly when the number of time periods is small, but it uses an upper bound estimation of the resilience loss as the objective function and then cannot ensure exact solutions; the CIBOM can ensure exact solutions, but it cannot solve the problem with large number of damaged components. To take the advantages and overcome the disadvantages of those two methods, this paper proposes a novel time index and component index combined optimization method (TI&CICOM). In each time step s, this method first decides the time duration T(s) of the next time period, and then uses the TIBOM with $T_p=1$ (for only the next one time period) to minimize system functionality loss at the end of the next time period and return the set of damaged components to be repaired at the next time period; after that this methods uses the CIBOM to further rank that set of damaged components by minimizing the resilience loss during the next time period. To ensure that the CIBOM can be effectively integrated, the proposed method needs to initially set an appropriate value N_{CIBESM}, which is the number of

damaged components that can be solved by the CIBOM within an acceptable computational time. Hence, this method needs to determine an appropriate T(t) during which at most N_{CIBOM} damaged components that can be repaired no matter how the repair teams are scheduled. An appropriate T(t) is determined by following steps: (1)find the minimum time interval $T^{upper}(t)$ during which at least $N_{\text{CIBOM}}+1$ damaged components can be repaired based on the undamaged components; (2) set time interval $T(t) = T^{upper}(t) - \varepsilon$, where ε is a small positive number. Note that there may exist spare time remained by each repair team in the last time period which should be considered in this methods. The minimum time interval $T^{upper}(t)$ can be determined by following formulation:

$$T^{upper}(t) = \min T(t) \tag{2a}$$

Subjtct to:

$$T(t) + rt_k(t-1) \ge \sum_{n \in V^N_CIBOM(t)} r_{nk}(t) * \tau_n , \forall k$$
(2b)

$$\sum_{k=1}^{RR} r_{nk}(t) = 1, \forall n \in V^{N_CIBOM}(t)$$
 (2c)

where variable $rt_k(t)$ represents the spare time of repair team k at time period t, and $V^{N_CIBOM}(t)$ denotes the $N_{\text{CIBOM}}+1$ damaged components can be repaired.

4. CASE STUDY

To demonstrate the efficiencies of the proposed existing solution methods method, those introduced in Section 3 and the proposed method are performed on the electric power transmission system in Shelby County, Tennessee (Shinozuka et al., 1998). Shown in Fig. 1, this system has eight gate stations, seventeen 23kv substations, twenty 12kv substations and fourteen transmission nodes, and they are connected by 73 transmission lines. Based on Adachi and Ellingwood (2010), this paper considers an earthquake scenario with seismic epicenter at 35.3° N and 90.3° W, including magnitudes within the range of $M_w \in [6.0, 9.0]$. For each seismic magnitude, 500 damage scenarios are generated. The computational experiments are performed on

a laptop with Intel i5 3210M quad-core @2.50 GHz and 4GB memory. Each optimization model in the solution methods is solved by MATLAB with CPLEX Toolbox.



Figure 1: Electric power transmission system in Shelby County, Tennessee, USA (Shinozuka et al., 1998).

Table 1 shows the average optimality gap and the average computational time over 500 damage scenarios under each earthquake magnitude when there is only one repair team (RR=1). Figure 2 further shows the cumulative distribution curves for the relative resilience loss error produced by each method when the seismic magnitude is 7.0 and 8.0, respectively. Note that if the optimal solution cannot be obtained, this paper simply use the minimal resilience loss achieved by all methods as the benchmark to measure the optimality gap. Hence, the relative resilience loss error is quantified as the difference between the resilience loss calculated from a particular method and the minimal resilience loss among all methods, normalized by the minimal resilience loss. From the table and figures, it can be found that the degree based method (DBM) shows the best performance in terms of computational cost, with the average computational time for each magnitude less than 0.5s, while its maximum and average gap are up to 106.50% and 1054%, respectively, which means this method may provide extremely bad solutions. The genetic algorithm based method (GABM) shows better performance in term of optimality gap, with the average optimal gap for GABM less than 1.34%, and it can produce the relative resilience loss error

Table 1: Average optimality gap and computational time for each repair sequence scheduling method over 500 component damage scenarios under seismic magnitude from 6.0 to 9.0 when there is only one repair team. N_{CIBOM} is set as 8. TIBOM1and TIBOM1means TIBOM with $T_p=5$ and $T_p=10$, respectively. Symbol '--' means that the results cannot be returned within one hour.

	$M_{w} = 6.0$		$M_{w} = 7.0$		$M_{w} = 8.0$		$M_{w} = 9.0$				
Methods	Gap(%)	CPU(s)	Gap(%)	CPU(s)	Gap(%)	CPU(s)	Gap(%)	CPU(s)			
DBM	0	0.1	53.28	0.1	106.50	0.3	59.22	0.4			
GABM	0	92.9	0.05	288.3	0.48	1,240.2	1.34	1,462.3			
TIBOM1	0	0.1	3.37	0.2	9.36	0.5	8.27	0.8			
TIBOM2	0	0.2	1.04	0.6	4.25	1.0	3.35	15.2			
CIBOM	0	0.1	0	0.6							
TI&CICOM	0	0.1	0.05	0.2	0.62	3.1	0.26	4.8			



Figure 2: Cumulative distribution curves for the relative resilience loss error produced by each method when the seismic magnitude is (a) 7.0 and (b) 8.0.

less than 1% for around 99.4% and 87.4% scenarios with $M_w = 7.0$ and $M_w = 8.0$, respectively.

The average optimality gap provided by time index based optimization method (TIBOM) is less than 10% and 5% with $T_p=5$ and $T_p=10$, respectively. Increasing the number of T_p can

improve the solution quality in terms of the optimality gap, but it also increase the computational cost, with the average computation time increasing from less than 1s for $T_p=5$ to 15.2s for $T_p=10$, and the maximum computational time increasing from 3s for $T_p=5$ to 570s for $T_p=10$. The component index based method (CIBOM) can provide the exact solution for damage scenarios with $M_w \leq 7.0$ whose maximum number of damaged components is less than 8, and the average computational time is less than 1s. However, this method cannot provide the results within one hour for large scale of disruption. In the TI&CICOM, the number of N_{CIBOM} is set to 8, and the TI&CICOM have significantly better performance than existing methods, with the average optimality gap less than 0.62% and the average computational time less than 4.8s. Moreover, this proposed method can produce the relative resilience loss error less than 1% for around 99.4% and 81.0% scenarios with $M_w =$ 7.0 and $M_w = 8.0$, respectively.

The previous results are concluded from the case that there is only one repair team, while there may multiple repair teams working in parallel in practice. Hence, this section will further compare the solution methods when there are several repair teams (RR>1). Table 2 shows the average optimality gap and computational time when $M_w = 8.0$ and RR=2 to 5. Figure 3 show shows the cumulative distribution curves for the relative resilience loss error produced by each method when RR=3. From the table and figure, it can be still found that degree based method holds the best

Table 2: Average optimality gap and computational time for each repair sequence scheduling method over 500 component damage scenarios under seismic magnitude $M_w = 8.0$ when RR=2 to5.

	RR=2		RR=3		RR=4		RR=5				
Methods	Gap(%)	CPU(s)	Gap(%)	CPU(s)	Gap(%)	CPU(s)	Gap(%)	CPU(s)			
DBM	84.62	0.26	66.98	0.26	52.13	0.25	40.19	0.25			
GABM	0.05	1,093.8	< 0.01	1,091.0	0.02	1010.57	0	1104.0			
TIBOM	15.96	1.09	19.01	3.23	18.73	8.97	17.68	15.22			
TI&CICOM	3.12	3.16	6.12	2.73	9.46	1.95	14.19	1.69			



Figure 3: Cumulative distribution curves for the relative resilience loss error produced by each method when $M_w = 8.0$ and RR=2.

performance in term of computational cost, and GABM performs the best in terms of the optimality gap when $M_w = 8.0$ and RR from 2 to 5. Moreover, the average computational time of GABM is much larger than others and could be up to 1100s, which is hundreds to thousands times more than that for any other method. The optimality gap provided by the TIBOM and the TI&CIBOM is less than 20% and 15%, respectively. However, the TI&CIBOM provides the relative resilience loss error less than 10% for around 87.4% scenarios while only 19% for TIBOM when *RR*=2. Note that the computational time for the DBM and the GABM does not depend on RR, but the computational time for each of those methods with optimization models increases exponentially for larger RR.

5. CONCLUSION

This paper propose a novel heuristic method to solve the repair sequence scheduling problem for post-disaster CISs. The proposed method is compared with typical existing solution methods, including a degree based method, a genetic algorithm based method, a time index based method and a component index based method in terms of optimality gap and computational cost. All these methods are applied into postearthquake damage scenarios for the electric power transmission system in Shelby County, Tennessee, USA. Results show that the proposed methods better performance than existing method can be applied to the recovery of large-scale CISs with extensive disruptions.

This paper still has several issues remained and can be improved in the future work. First, improve the solution quality in terms of optimality gap for the proposed method when there are multiple repair teams working in parallel. Second, model the restoration process with more complexities, such as the routing for the repair team.

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