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Reliability based optimization for semi-actively controlled seismic structures

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ABSTRACT: A stochastic semi-active control strategy for the MR damping controlled structure is provided in this paper. The integrated optimization of the weighting matrices pertaining to the active optimal control and the MR parameters pertaining to the semi-active optimal control is implemented. In order to reveal the advantages of this strategy, comparative studies on two aspects are involved, i.e. optimization schemes and probabilistic criteria. The comparison between the integrated and the separated schemes based on the probabilistic criteria in terms of statistical moments shows that the integrated scheme exhibits a better control effectiveness. While the comparison between the probabilistic criteria in terms of statistical moments and the reliability used for the integrated scheme shows that the probabilistic criteria in terms of reliability attains a more safe structure.

1. INTRODUCTION

Structural control has been developed into an effective method for mitigating the dynamic response and improving the safety and serviceability of structures since it was introduced into the civil engineering community (Yao, 1972; Housner et al, 1997). With the characteristics of low energy consumption and high efficiency, the semi-active control is regarded as an promising control strategy(Chu et al, 2005; Dan et al, 2015). The MR damper, for its excellent dynamic performance, is regarded as one of the most prospective semi-active control devices (Lozoya-Santos et al, 2012; Wang & Dyke, 2013). Since the randomness inherent in the structure system, a reliable stochastic semi-active control strategy is necessary for achieving satisfactory control effect of the structures with MR dampers.

According to the definition, the classical strategies of the semi-active control basically include the active optimal control and semi-active control. Firstly, although the LQG method has been widely adopted in the MR damping control of structures (Zhu et al, 2001; Rosół & Martynowicz, 2016), the introduced Gaussian white noise for the design of these strategies is far away from the characteristics of seismic ground motions. For this reason, Li et al developed a physically-based stochastic optimal control strategy (PSO), which is adaptable for the structures with any kinds of stochastic excitations (Li et al, 2010; Peng et al, 2013). On the other hand, the widely-used semi-active control schemes belong to the two-states control (Leitmann, 1994; Jansen & Dyke, 2000) and bounded Hrovat control (Hrovat et al, 1983). Peng et al further extended the PSO into the MR damping control of structures subjected to random seismic ground motion, and proved that the semi-active control strategy tracing the active optimal control gain can attain a satisfactory effectiveness (Peng et al, 2017).

Therefore, a reasonable semi-active control strategy for MR damping control of randomly excited structures shall be the combination between the physically-based stochastic optimal control and the bounded Hrovat control. However, a lots of researches indicated that the weighting matrices in the active optimal control have a serious influence on the control effect (Li et al, 2011; Shi et al, 2014). Although some researches have been made to optimize the weighting matrices, most of them adapt the trial-and-error method (Stengel et al, 1995; Shi et al, 2013), which are insufficient for searching out the global optimal weighting matrices. For solving this problem, Li et al. established the probabilistic criteria based on the structural performance of the optimal controlled structure (Li et al, 2011). However, how to optimize the weighting matrices corresponding to the optimal semi-active control still remains open.

This paper proposes a stochastic semi-active control strategy for the MR damping controlled structure. It involves the combination between the physically-based stochastic optimal control and the bounded Hrovat control and can achieve the integrated optimization of the weighting matrices and the MR parameters. In order to reveal the control effect of the proposed strategy, comparative studies optimization scheme and probabilistic criteria are carried out.

2. SEMI-ACTIVE OPTIMAL CONTROL STRATEGY

The proposed semi-active optimal control strategy includes the following contents: 1) generation of stochastic seismic samples based on the physical stochastic seismic model (Li & Ai, 2006); 2) calculation of the optimal control force corresponding to each sample; 3) analysis of the semi-active control gain; and 4) optimization of the weighting matrices and the MR parameters involved in the semi-active control algorithm.

2.1. Generation of samples of seismic ground motions

Based on the physical mechanism of seismic ground motions, Li and Ai established a physical stochastic seismic model, which can be expressed as (Li & Ai, 2006):

$$\ddot{X}_g(\boldsymbol{\Theta},\omega)=\frac{\Theta_{\omega_g}^2+2i\Theta_{\zeta_g}\Theta_{\omega_g}\omega}{\Theta_{\omega_g}^2-\omega^2+2i\Theta_{\zeta_g}\Theta_{\omega_g}\omega}\ddot{U}_b(\boldsymbol{\Theta}_b,\omega) \quad (1)$$

where $\ddot{X}_g(\boldsymbol{\Theta},\omega),\ddot{U}_b(\boldsymbol{\Theta}_b,\omega)$ denote the frequency domain expressions of seismic ground motions at the engineering site and the bedrock, respectively; $\boldsymbol{\Theta}=\{\Theta_{\omega_g},\Theta_{\zeta_g},\boldsymbol{\Theta}_b\}$ denotes the random vector characterizing the randomness involved in the ground motion at the surface of the engineering site. $\Theta_{\omega_g},\Theta_{\zeta_g}$ denote the random source of the site soil, i.e. the predominant frequency of the engineering site ω_g and the equivalent damping ratio ζ_g. $\boldsymbol{\Theta}_b=\{\Theta_{b,i}\}_{i=1}^{s_b}$ denote the random vector characterizing the randomness involved in the seismic ground motion inputted in the bedrock, s_b being the number of the random variables involved in this stage. ω denotes the circular frequency.

The time history of the stochastic seismic ground motion could then be obtained by the inverse Fourier transformation:

$$\ddot{x}_g(\boldsymbol{\Theta},t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\ddot{X}_g(\boldsymbol{\Theta},\omega)e^{i\omega t}d\omega \quad (2)$$

To describe the non-stationary of the seismic intensity, the following uniform modulation function is used:

$$f(t)=\begin{cases}t^2/4 & t\le t_a\\ 1 & t_a<t\le t_b\\ e^{-0.8(t-t_b)} & t_b<t\le T\end{cases} \quad (3)$$

where, t_a and t_a denote the starting and ending time point of the strong seismic intensity, respectively. T denotes the duration time of the seismic.

2.2. Calculation of optimal control force

With the stochastic samples of seismic ground motions, the active optimal control force corresponding to each sample can be calculated, which will be used as a reference for the bounded Hrovat semi-active control. Considering a structure controlled with MR dampers, the dynamic equation of the structure can be expressed as:

$$\mathbf{M}\ddot{\mathbf{X}}(t)+\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\mathbf{B}_s\mathbf{U}(t)+\mathbf{D}_s\mathbf{F}(\boldsymbol{\Theta},t) \quad (4)$$

where, $\mathbf{X}$ denotes a n-dimensional displacement vector; $\boldsymbol{\Theta}$ denotes the random source of the dynamic system; $\mathbf{U}$ denotes a r-dimensional control force vector; $\mathbf{F}$ denotes a p-dimensional random excitation vector; $\mathbf{M,C,K}$ denote the mass, damping and stiffness matrices, respectively; $\mathbf{B}_s$ denotes the $n\times r$ -dimensional location matrix of the control force; $\mathbf{D}_s$ denotes the $n\times p$ -dimensional location matrix of the excitation. For the sake of simplification, Eq.(4) can be simplified as the expression in state space:

$$\dot{\mathbf{Z}} = \mathbf{AZ} + \mathbf{BU} + \mathbf{DF} \quad (5)$$

where, $\mathbf{Z}$ denotes the 2n-dimensional state vector; $\mathbf{A}$ denotes the 2n×2n- dimensional system matrix; $\mathbf{B}$ denotes the 2n×r-dimensional location matrix of control force; $\mathbf{D}$ denotes the 2n×p-dimensional location matrix of excitation.

The linear quadratic cost function is defined according to the classical LQR control as (Li et al, 2011):

$$J_1(\mathbf{Z,U,\Theta}) = \frac{1}{2}\mathbf{Z}^T(t_f)\mathbf{S}(t_f)\mathbf{Z}(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\mathbf{Z}^T(t)\mathbf{Q}_Z\mathbf{Z}(t) + \mathbf{U}(t)\mathbf{R}_U\mathbf{U}(t)\mathrm{d}t \quad (6)$$

where, $\mathbf{S}(t_f)$, $\mathbf{Q}_Z$ denote 2n×2n-dominational positive semi-definite symmetric weighting matrices pertaining to the structural state; $\mathbf{R}_U$ denotes r×r-dimensional positive definite symmetric weighting matrix of control force. Without considering the influence of the cross terms, $\mathbf{S}(t_f)$, $\mathbf{Q}_Z$ and $\mathbf{R}_U$ are all the diagonal matrices.

For a closed-loop control system with the consideration of state feedback, the control gain can be expressed as follows according to the Pontryagin maximum principle:

$$\mathbf{U}(\mathbf{\Theta},t) = -\mathbf{R}_U^{-1}\mathbf{B}^T\mathbf{PZ}(\mathbf{\Theta},t) = -\mathbf{G}_Z\mathbf{Z}(\mathbf{\Theta},t) \quad (7)$$

where $\mathbf{G}_Z = \mathbf{R}_U^{-1}\mathbf{B}^T\mathbf{P}$ denotes the control gain matrix; $\mathbf{P}$ can be obtained by solving the following equation:

$$\mathbf{PA} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}_U^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q}_Z = \mathbf{0} \quad (8)$$

The state vector $\mathbf{Z}$ and optimal control force $\mathbf{U}$ corresponding to each sample can be obtained through combining and solving Eq.(5) and Eq.(7), which will be used for calculating the semi-active control force. From Eq.(7), it can be seen that the selection of weighting matrices will directly affect $\mathbf{G}_Z$, which will further affect the control force and the control gain. Therefore, it is essentially to optimizing the weighting matrices $\mathbf{Q}_Z$ and $\mathbf{R}_U$ for achieving an optimal semi-active control.

2.3. Analysis of semi-active control force

According to the scheme of physically-based stochastic optimal control, the optimal control force corresponding to the given weighting matrices $\mathbf{Q}_Z$ and $\mathbf{R}_U$ can be obtained. Further, the semi-active control force can be calculated through tracing the stochastic optimal control force. In this paper, the bounded Hrovat semi-active control algorithm is used, which can be expressed as follow (Peng et al, 2017):

$$U_s(\mathbf{\Theta},t) = \begin{cases} C_d\dot{X}(\mathbf{\Theta},t) + U_{dc,\max}\,\mathrm{sgn}[\dot{X}(\mathbf{\Theta},t)], \\ \quad \text{Case A: } U_a\dot{X} < 0 \text{ and } |U_a| > U_{d,\max} \\ |U_a|\,\mathrm{sgn}[\dot{X}(\mathbf{\Theta},t)], \\ \quad \text{Case B: } U_a\dot{X} < 0 \text{ and } |U_a| < U_{d,\max} \\ C_d\dot{X}(\mathbf{\Theta},t) + U_{dc,\min}\,\mathrm{sgn}[\dot{X}(\mathbf{\Theta},t)], \\ \quad \text{Case C: } U_a\dot{X} > 0 \end{cases} \quad (9)$$

where $U_s(\mathbf{\Theta},t)$ denotes the semi-active control force offered by MR damper; $U_a(\mathbf{\Theta},t)$ denotes the reference optimal control force; $U_{d,\max}(\mathbf{\Theta},t)$ denotes the maximum damping force generated by MR damper; $\dot{X}(\mathbf{\Theta},t)$ denotes the damper velocity; C_d denotes the viscous coefficient; $U_{dc,\max}$ and $U_{dc,\min}$ denote the maximum and minimum Coulomb forces of MR damper, respectively. Generally, the Coulomb force equals to be zero if no current input to the MR damper. Therefore, the parameters C_d and $U_{dc,\max}$ need to be determined. In this paper, the parameters will be optimized together with the weighting matrices forming into an integrated scheme for parameter optimization.

With the definition of parameters, the semi-active control force can be obtained through Eq.(9). Then, substituting the achieved semi-active control force into Eq.(1), the semi-active control gain of the structure can be obtained.

2.4. Optimization of MR parameters and weighting matrices

As mentioned in the previous sections, the control effectiveness is highly relevant to the weighting matrices and MR parameters. In order to achieve the best control effectiveness, it is necessary to optimize them based on suitable criteria.

Since the reliability is an important index, it is reasonable to establish a criterion in terms of the reliability of structural state and control force. In this paper, the following index is established.

$$J_{R-Semi} = R^2_{\bar{X}-Semi} + R^2_{\ddot{\bar{X}}-Semi} + R^2_{\bar{U}} \quad (10)$$

where, $R_{X\text{-}Semi}$ $R_{\ddot{X}\text{-}Semi}$ denote the global reliabilities of displacement and acceleration of the structure; R_U denotes the global reliability of the semi-active control device. According to the existing literatures, the probability density functions of displacement, acceleration and the semi-active control force all are governed by the generalized probability density evolution equations (GDEEs) (Li et al, 2007; Chen & Li, 2007; Li & Chen, 2010; Li & Chen, 2008, whereby the global reliability in Eq.(10) can be readily solved.

In order to verify the benefit the reliability-based criterion shown in Eq.(10), another index based on the statistical moment is given by

$$J_{E-Semi} = \frac{E\left(\bar{X}_{Semi}\right)+\beta\sigma\left(\bar{X}_{Semi}\right)}{X_{S-Thr}} + \frac{E\left(\ddot{\bar{X}}_{Semi}\right)+\beta\sigma\left(\ddot{\bar{X}}_{Semi}\right)}{\ddot{X}_{S-Thr}} \quad (11)$$
$$\frac{E\left(\bar{U}_{Semi,i}\right)+\beta\sigma\left(\bar{U}_{Semi,i}\right)}{\bar{U}_{S-Thr,i}}$$

where $E(\bullet)$ and $\sigma(\bullet)$ denote the mean and standard deviation; β denotes the confidence level coefficient; $\bar{X}, \ddot{\bar{X}}$ denote the equivalent extreme values of $X, \ddot{X}$; X_{S-Thr}, $\ddot{X}_{S-Thr}$ and $U_{S-Thr,i}$ denote the thresholds of the corresponding variables.

3. CASE STUDY

In order to illustrate the effectiveness of the proposed control strategy, a SDOF structure system subjected to random seismic ground motions is investigated. The parameters of the system are: structural mass $m = 1\times10^5\text{kg}$; natural circular frequency $\omega_0 = 11.22\text{rad/s}$; damping ratio $\xi = 0.05$; maximum damping force of MR damper $U_{MR,\max} = 150\text{kN}$. The physical stochastic ground motion model is employed here. The random variable $\omega_g \sim N(12\text{rad/s}, 0.42)$ and $\zeta_g \sim N(0.1, 0.35)$. The peak value of seismic ground motions is 0.1g. Phase angle used in Eq. (2) is defined by $\phi_0 \sim N(\pi, 0.1)$. According to the strategy of probability-assigned partition via tangent spheres (Chen & Li, 2008), 221 representative samples of seismic ground motion are generated, the frequency and duration of which are 50Hz and 20.48s, respectively. The parameters t_a and t_b within uniform modulation function Eq.(3) are 2s and 16s. The threshold values of displacement and acceleration are 10mm, 1500mm/s^2, respectively. The threshold value of the control force equals to the damping force upper limit value of the MR damper, i.e. $U_{MR,\max} = 150\text{kN}$. The weighting matrices is defined as the form $\mathbf{Q}_Z = q\cdot\text{I}$ and $\mathbf{R}_U = r\cdot\text{I}$, which means that the influence of the weighting matrices to the control effect mainly depends on the value of q/r. With the assumption of q=100, the optimal weighting matrices could be obtained. The GA toolbox within MATLAB is used. The range of the variables are $r\in[10^{-20},10^{-5}]$, $C_d\in[0.2,2]$, $U_{dc,\max}\in[50,150]$. In order to reveal the difference of control effectiveness between the proposed integrated scheme in this paper and the separated scheme in the previous investigation (Peng et al, 2017), a comparative study is carried out. The separated scheme refers to as that the weighting matrices are optimized merely based on the active optimal control, then the MR parameters are designed based on the semi-active control algorithm. The optimized results of weighting matrices and MR parameters corresponding to different criteria and difference schemes are shown in Table 1, where $J_{E\text{-}Opt}$ corresponds to the separated strategy based on the moment criteria.

Table 1: Weighting matrices and MR parameters

Performance index	r	C_d (kN·s/mm)	$U_{\text{dc,max}}$ (kN)

$J_{R\text{-}Semi}$	$10^{-15.31}$	0.2060	110.258
$J_{E\text{-}Semi}$	$10^{-17.61}$	0.4830	105.832
$J_{E\text{-}Opt}$	$10^{-11.90}$	0.6119	82.280

Figures 1 and 2 show the displacement and acceleration in the sense of root-mean-square values. It is seen that both the integrated and separated schemes based on reliability and moment can achieve excellent control effectiveness.

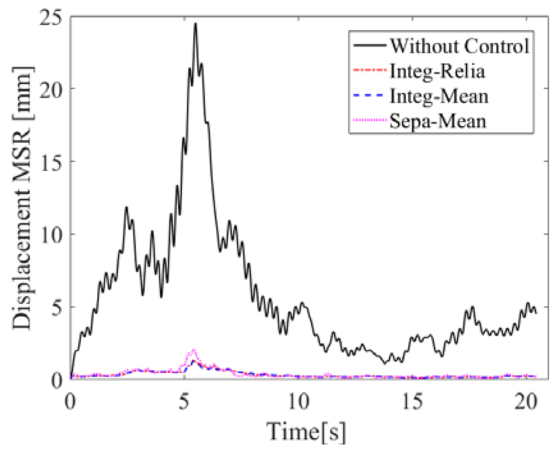

Figure 1: Comparison of root-mean-square displacement

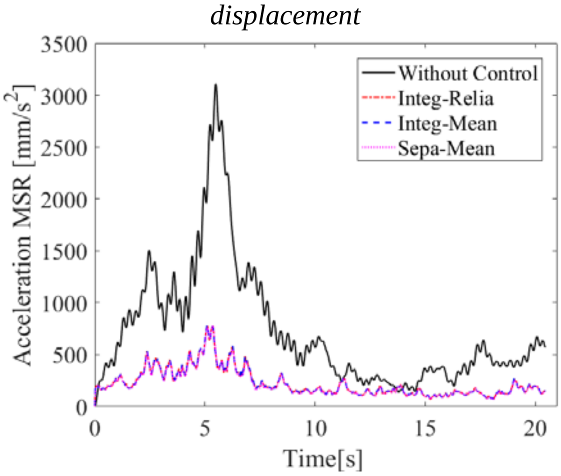

Figure 2: Comparison of root-mean-square acceleration

Reliabilities of arguments of the semi-active control based on different schemes and different criteria are shown in Table 2. It is revealed that the integrated scheme can achieve a better trade-off among the arguments of the controlled structure than the separated scheme. On the other hand, the integrated scheme with the criterion in terms of the reliability attains a better trade-off and larger reliabilities of arguments than the criterion in terms of the statistical moments.

Table 2: Reliabilities of arguments of semi-active control

Performance index	X (mm)	$\ddot{X}$ (mm/s^2)	U_{Semi} (kN)
$J_{R\text{-}Semi}$	0.9997	0.9803	1.0000
$J_{E\text{-}Semi}$	1.0000	0.9678	0.9982
$J_{E\text{-}Opt}$	0.9998	0.9046	0.9988

4. CONCLUSIONS

This paper proposes a reliability based optimization scheme for semi-actively controlled seismic structures with MR dampers. The integrated optimization of the weighting matrices pertaining to the active optimal control and the MR parameters pertaining to the semi-active optimal control is implemented. In order to reveal the control effect of the proposed strategy, comparative studies optimization scheme and probabilistic criteria are carried out. Numerical results show that: (i) the integrated scheme can achieve a better trade-off among the arguments of the controlled structure than the separated scheme; (ii) the integrated scheme with the criterion in terms of the reliability attains a better trade-off and larger reliabilities of arguments than the criterion in terms of the statistical moments.

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