Stochastic seismic response analysis of nonlinear structure with random parameters

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ABSTRACT: In the present paper, a dimension-reduction modeling method is proposed for a dual stochastic dynamic system of non-stationary ground motion stochastic processes and stochastic structures. In the proposed method, the random variables describing the stochastic ground motions and structural parameters are respectively represented by the functions of one elementary random variable, resulting in the entire stochastic dynamic system can be represented by merely two elementary random variables. Since the number of elementary random variables needed is extremely small, the set of representative points in regard to the elementary random variables can thus be selected by number theoretical method. Benefiting from the proposed method, it can be conveniently combined with the probability density evolution method to realize the dynamic response analysis and dynamic reliability evaluation of nonlinear stochastic structures. The seismic response analysis of an eight-storey reinforced concrete frame structure with random parameters subjected to non-stationary stochastic ground motions are investigated as case studies. Numerical results fully demonstrated the effectiveness of the proposed method.

In recent years, the dynamic response analysis of deterministic structures subjected to stochastic or deterministic excitations of dynamic disasters has been extensively investigated and well developed (Belytschko et al. 2000; Xu, et al. 2018). Nevertheless, the research on dynamic response analysis of structural system with random parameters, i.e., the stochastic structures, has been relatively lagged behind. In fact, the structural randomness, including the randomness of physical and geometric parameters upon a structural member caused by uncontrollable factors during the production process, would usually exist objectively and be unignorable. Practically, the external excitations acting on the engineering structures are commonly described as the stochastic processes. Therefore, it is reasonable and of great importance to study on the dynamic response of stochastic structures induced by the non-stationary stochastic excitations.

To this end, several stochastic approaches, such as the Monte Carlo simulation method (MCM) (Shinozuka, 1972) and its variants (Spanos & Zeldin 1998), the random perturbation method (Kleiber & Hien 1992) and the orthogonal polynomial expansion method (Ghanem & Spanos 1991, Li 1996), have been put forward in the past decades. Despite the progress being made, the application of these methods in large scale realistic problems is still limited, either due to prohibitive computational costs or due to their inability to deal with structures with complex behavior.

Lately, a valid method is added to the family of the stochastic methods, namely, the probability density evolution method (PDEM) (Li & Chen, 2004) which has a good development prospect. This method can obtain the instantaneous probability density function (PDF) as well as its response evolution of general stochastic dynamic systems, including linear and nonlinear stochastic structures. However, the main obstacle to the PDEM is that the convolution integral must be performed on the sample space of the elementary random variables, which limits the number of the elementary random variables that the method can handle. In particular, it is difficult to implement the modeling of the stochastic processes of the external excitations because of the large number of random variables generally retained in simulation function. On the other hand, theoretically, the efficiency of Monte Carlo simulation method is independent of the dimension of random variables. Even so, if too many random variables are involved, it will be very difficult to generate high-dimensional pseudo-random numbers, because there may be strong correlation between pseudo-random numbers of different dimensions (Glasserman 2013).

In recent years, remarkable progress has been made to effectively reduce the number of elementary random variables in the simulation of stochastic processes. A stochastic harmonic function representation was developed, which could express the stochastic processes with just a few dozens of random variables (Chen et al. 2013). Later, the dimension-reduction methods were proposed to realize the simulation of stochastic processes using just one or two elementary random variables (Liu et al. 2016; 2017).

In this paper, a dimension-reduction modeling method is developed for a dual stochastic dynamic structural system of non-stationary stochastic ground motion processes and stochastic structures, through which the probability density evolution analysis of the nonlinear structures can also obtained. Benefiting from this proposed scheme, the high-dimensional randomness degree is efficiently reduced to merely two. The remaining contents of this paper are arranged as follows. Section 1 introduces the dimension reduction simulation method for spectrum-compatible non-stationary ground motions. The dimension-reduction method of stochastic structural parameters is elaborated in Section 2. Section 3 shows the simulation results of stochastic ground motions and random structural parameters. In section 4, the responses of random structures under stochastic ground motions are analyzed. Conclusions drawn from this study are summarized in Section 5.

1. DIMENSION-REDUCTION OF NON-STATIONARY STOCHASTIC GROUND MOTIONS

According to Priestley's evolutionary spectral representation theory, a non-stationary stochastic ground motion process $\ddot{U}_{\mathrm{g}}(t)$ with zero-mean can be expressed as the following finite series form (Liu et al., 2016)

$$\ddot{u}_{\mathrm{g}}(t)=\sum_{n=1}^{N}\sqrt{G_{\ddot{U}_{\mathrm{g}}}(t,\omega_n)\Delta\omega}\left[\cos(\omega_n t)X_n+\sin(\omega_n t)Y_n\right] \tag{1}$$

where $\ddot{u}_{\mathrm{g}}(t)$ denotes the representative time histories of the ground motion process $\ddot{U}_{\mathrm{g}}(t)$. $G_{\ddot{U}_{\mathrm{g}}}(t,\omega)$ denotes the one-sided evolutionary power spectral density (EPSD) function of the non-stationary ground motion process $\ddot{U}_{\mathrm{g}}(t)$.

In Eq. (1), $\{X_n, Y_n\}$ $(n=1,2,\cdots,N)$ refers to a set of the standard orthogonal random variables, which must satisfy the following basic conditions (Liu et al. 2016)

$$E[X_n]=E[Y_n]=0,\ E[X_mY_n]=0,$$

$$E\left[X_m X_n\right] = E\left[Y_m Y_n\right] = \delta_{mn} \qquad (2)$$

where $E[\cdot]$ indicates the mathematical expectation, δ_{mn} denotes the Kronecker-Delta.

After adopting the idea of random functions, the set of the standard orthogonal random variables $\{X_n, Y_n\}$ $(n = 1, 2, \cdots, N)$ can be expressed as the following random function form (Liu et al. 2018)

$$X_n = \sqrt{2}\cos(\bar{n}\Theta_1 + \varphi_1),\ Y_n = \sqrt{2}\sin(\bar{n}\Theta_1 + \varphi_1),$$
$$(n, \bar{n} = 1, 2, \cdots, N) \qquad (3)$$

where Θ_1 is the elementary random variable following the uniform distribution over the range $[0, 2\pi)$; φ_1 is a constant in the range $[0, 2\pi)$ and is taken as $\pi/4$ in this paper; $\bar{n}$ is a unique one-to-one mapping of n. This mapping operation can be carried out by employing the Matlab Tool box functions $rand('state', 0)$ and $randperm(N)$. Obviously, Eq. (3) completely satisfies the basic conditions involved in Eq. (2).

Consequently, the fully non-stationary stochastic ground motions could be represented by just one single elementary random variable through the above treatment.

2. DIMENSION-REDUCTION OF STRUCTURE WITH RANDOM PARAMETERS

In general, the random field model cannot be directly used in the finite element method, which causes the random field has first to be replaced by N_r (a limited number) of random variables. In this paper, all input structural parameters are supposed to be deterministic except for Young's modulus (elastic modulus) and Poisson's ratio of the material which are treated as the random parameters. It is assumed that the elastic modulus $E_i (i = 1, 2, \cdots, N_r)$ and the Poisson's ratio $\nu_i (i = 1, 2, \cdots, N_r)$ obey normal distributions and they are irrelevant. For convenience, the standard normal random variables α_i and $\beta_i (i = 1, 2, \cdots, N_r)$ can be further introduced to respectively describe the random variables E_i and ν_i which and satisfy the following condition

$$E_i = \mu_{E_i}(1 + \delta_{E_i}\alpha_i),\ \nu_i = \mu_{\nu_i}(1 + \delta_{\nu_i}\beta_i) \qquad (4)$$

where μ_{E_i} and δ_{E_i} are the mean and coefficient of variation (COV) of E_i, respectively. μ_{ν_i} and δ_{ν_i} are the mean and COV of ν_i, respectively. Obviously, the stochastic structure degrades to the deterministic structure when all the COV are equal to zero.

Note that the standard normal random variables α_i and $\beta_i (i = 1, 2, \cdots, N_r)$ are also unrelated random variables and they must satisfy the following basic conditions

$$E\left[\alpha_i\right] = E\left[\beta_i\right] = 0,\ E\left[\alpha_i \beta_j\right] = 0,$$
$$E\left[\alpha_i \alpha_j\right] = E\left[\beta_i \beta_j\right] = \delta_{ij} \qquad (5)$$

As we can see from the above equations, it is interesting that Eqs. (2) and (5) have exactly the same form. As a result, the dimension-reduction representation embedded in the random function articulated in Section 1 can also be applied herein to further reduce the number of random variables upon the stochastic structures. In this paper, a class of random functions proposed by Liu et al. (2017) is used to construct the standard normal random variables α_i and β_i, which are given as follows

$$\alpha_i = \Phi^{-1}\left\{\frac{1}{2} + \frac{1}{\pi}\arcsin\left[\cos(\bar{i}\,\Theta_2 + \varphi_2)\right]\right\} \qquad (6a)$$

$$\beta_i = \Phi^{-1}\left\{\frac{1}{2} + \frac{1}{\pi}\arcsin\left[\sin(\bar{i}\,\Theta_2 + \varphi_2)\right]\right\} \qquad (6b)$$

where $\Phi^{-1}\{\cdot\}$ represents the inverse function of standard normal distribution function. $i, \bar{i} = 1, 2, \cdots, N_r$. $\bar{i}$ is a unique one-to-one mapping of i, and the implementation of this mapping operation is similar to that in Eq. (3).

Θ_2 is the elementary random variable uniformly distributed over the interval $[0, 2\pi)$. φ_2 is an constant and is taken as $\pi/6$.

Through the conversions of Eqs. (4) and (6), only one elementary random variable, i.e., Θ_2 is required to describe the structural parameters. Thus, the number of random variables describing the structural parameters is reduced from $2N_r$ to merely one, which realizes the dimension-reduction representation of structure with random parameters. Coupled with the previous one elementary random variable used to simulate the stochastic ground motions as described in Section 1, there are merely two elementary random variables, namely Θ_1 and Θ_2, involved in the whole stochastic dynamic system.

3. SIMULATION OF STOCHASTIC GROUND MOTIONS AND STOCHASTIC STRUCTURES

3.1. Simulation procedure

i) In accordance with the number theoretical method (Li and Chen, 2007), n_{sel} representative points of the basic random variables $\mathbf{\Theta} = \{\Theta_1, \Theta_2\}$ are selected at intervals $[0, 2\pi) \times [0, 2\pi)$, and the corresponding assigned probability P_l is determined.

ii) Using Eq. (3) to generate the orthogonal random variables $\{X_n, Y_n\}$, and then use Eq. (1) to generate n_{sel} representative time histories of non-stationary stochastic ground motions. Meanwhile, each representative time history has a assigned probability of P_l.

iii) Using Eq. (6) to generate the standard normal random variables $\{\alpha_i, \beta_i\}$, and then the samples of elastic modulus and Poisson's ratio can be generated using Eq. (4). For each samples of elastic modulus and Poisson's ratio, there is a given probability P_l.

In this way, a dual stochastic dynamic structural system of stochastic excitations and stochastic structures represented by two elementary random variables $\mathbf{\Theta} = \{\Theta_1, \Theta_2\}$ is constructed.

3.2. Simulation of stochastic ground motions

It is assumed that the frame structure is located at the site with the seismic fortification intensity of eight degree. Other parameters involved are given as follows: $N = 1600$, $\Delta\omega = 0.15\,\mathrm{rad/s}$, $\omega_{\mathrm{u}} = 240\,\mathrm{rad/s}$, $\Delta t = 0.01\mathrm{s}$, and the duration of the ground motion process is $T = 25\mathrm{s}$.

According to the simulation procedures mentioned in Section 3.1, 144 representative time histories of the non-stationary stochastic ground motions are generated. Fig. 1 shows one representative time histories of the ground motions. It can be seen that the sample has typical characteristics of non-stationary.

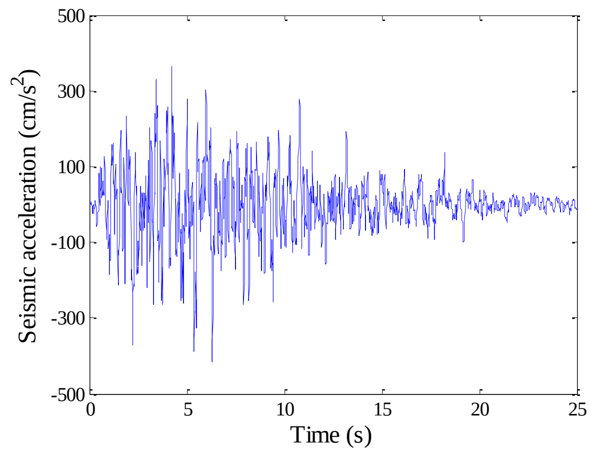


Figure 1: One representative time history

Fig. 2 shows the comparison of the mean and standard deviation upon its target value of the sample time-history set. It can be seen that the generated ground motion sample process is very close to the target value, which verifies the effectiveness of the method.

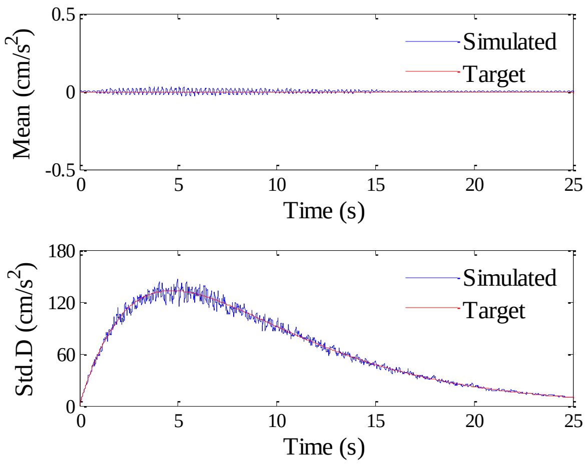


Figure 2: Comparison of samples mean and standard deviation of ground motion with target values

At present, the seismic design codes for various structures are based on the response spectrum as the ground motion input. Usually, the average response spectrum and the code response spectrum have certain fitting accuracy requirements.

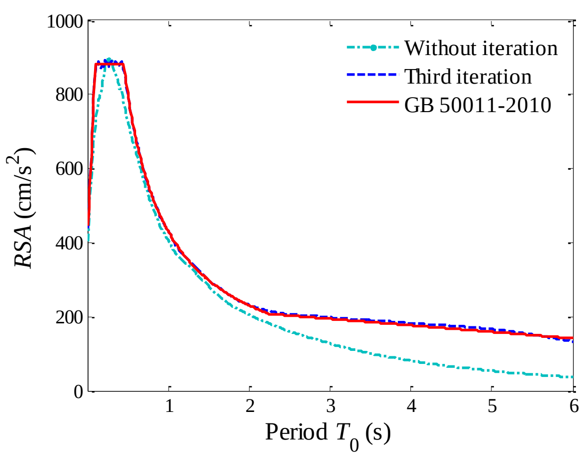


Figure 3: Response spectrum of the average sample processes and the Chinese code

Fig. 3 is a comparison chart of the obtained response spectrum, and it can be seen that the error between the two is large, especially in the long period, the difference is more obvious. For the seismic design time history of high-rise buildings or large-span structures, the long-period ground motion component plays an important role, so the fitting of the long-period partial response spectrum should have higher precision. To this end, Cacciola (2010) proposed a simple iterative correction method, the effect is also significant, this paper uses this method to correct. Fig. 3 also shows the comparison between the revised calculated response spectrum and the code spectrum (GB 50011-2010). It can be seen that the revised response spectrum and the target spectrum after iteration have a very good fitting effect. The average error and the maximum error are 2.1% and 6.03% respectively.

3.3. Simulation of structure with random parameters

An eight-storey reinforced concrete frame structure is studied within this section. The partial geometric sizes of the structure are listed as follows: the height of the first storey is 4.2m, and that of the rest, respectively, is 3m. The 3D model of the reinforced concrete frame structure is shown in Fig. 4. In order to reflect the nonlinearity of the structure, this study adopts the Modified Takeda Trilinear model as the hysteretic model for beams and columns. It is assumed that the bottom of the frame column is rigidly connected to the base.

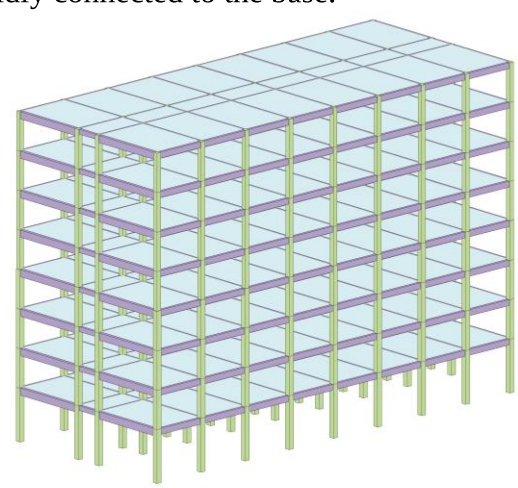


Figure 4: 3D model of the frame structure

For simplicity, the random parameters of the structure are divided by the number of storeys, namely $N_r = 8$. The mean and COV of the elastic modulus and Poisson's ratio of each layer are given as follows. $\mu_{E_1} = 3.15\times10^{10}\,\mathrm{N/m^2}$,

$\mu_{E_i} = 3.0\times10^{10}\,\text{N/m}^2$ for $i = 2,3,\cdots,N$, $\mu_{v_i} = 0.2$, and all of the COV is taken as 0.2. Based on the procedures **i)** and **iii)** mentioned in Section 3.1, 144 sets of $\{E_i, v_i\}(i = 1,2,\cdots,N_r)$ are generated for simulation purpose.

4. SEISMIC RESPONSE ANALYSIS OF STOCHASTIC NONLINEAR STRUCTURE

Under uniform earthquake excitations, the dynamic equation of a nonlinear multi-degree-of-freedom structural system using the finite element method can be written as

$$\boldsymbol{M}(\boldsymbol{\Theta})\ddot{\boldsymbol{U}} + \boldsymbol{C}(\boldsymbol{\Theta})\dot{\boldsymbol{U}} + \boldsymbol{G}(\boldsymbol{\Theta},\boldsymbol{U}) = -\boldsymbol{M}(\boldsymbol{\Theta})\boldsymbol{I}\ddot{U}_{\text{g}}(\boldsymbol{\Theta},t) \tag{7}$$

where $\ddot{U}_{\text{g}}(\boldsymbol{\Theta},t)$ is a stochastic seismic acceleration process generated by Eq. (1). $\boldsymbol{I}$ denotes column vector with element 1. $\ddot{\boldsymbol{U}}$, $\dot{\boldsymbol{U}}$ and $\boldsymbol{U}$ are the accelerations, velocities and displacements response vectors, respectively. $\boldsymbol{M}$ and $\boldsymbol{C}$ denote the mass and damping matrix of the structure; $\boldsymbol{G}$ denotes the nonlinear restoring force vector of the system.

The randomness of any physical responses (such as displacement etc.) of the dynamical system in Eq. (7) comes from stochastic seismic excitation and structural parameters. Therefore, arbitrary interested physical-response quantities involved in the system in Eq. (7) can be expressed as

$$Z = H(\boldsymbol{\Theta},t) \tag{8}$$

The velocity of the physical quantity Z is defined as $\dot{Z} = h(\boldsymbol{\Theta},t)$, where $h = \partial H/\partial t$.

Obviously, Eq. (8) can be regarded as a stochastic system by itself, in which the source randomness are completely described by $\boldsymbol{\Theta}$. Moreover, if we denote the joint PDF of $(Z,\boldsymbol{\Theta})$ as $p_{Z\Theta}(z,\theta,t)$, the generalized probability density evolution equation of the form can be obtained as follows (Li & Chen, 2009)

$$\frac{\partial p_{Z\Theta}(z,\boldsymbol{\theta},t)}{\partial t} + h(\theta,t)\frac{\partial p_{Z\Theta}(z,\boldsymbol{\theta},t)}{\partial z} = 0 \tag{9}$$

In order to assess the damage of the frame structure under the stochastic earthquake excitations, the inter-storey drift angle, i.e., $\Phi_j(t)$ $(j = 1,2,\cdots,N_r)$ is chosen as the interested physical-response. Again, the inter-storey drift angles of deterministic structure and stochastic structure are investigated in this paper for comparative purposes. By using the PDEM, the probability information of the inter-storey drift angle $\Phi_j(t)$ can be obtained. In this paper, only the probability information of stochastic response $\Phi_1(t)$ under stochastic earthquake excitations is given as an example.

Fig. 5 shows the mean and the standard deviation of the inter-storey drift angle $\Phi_1(t)$. As can be seen in Fig. 5, there is a great difference between the deterministic structure and the stochastic structure in the changing trend of the mean figure. Fig. 5 shows that the amplitudes of the standard deviation upon the stochastic structure are slightly larger than that of the deterministic one.

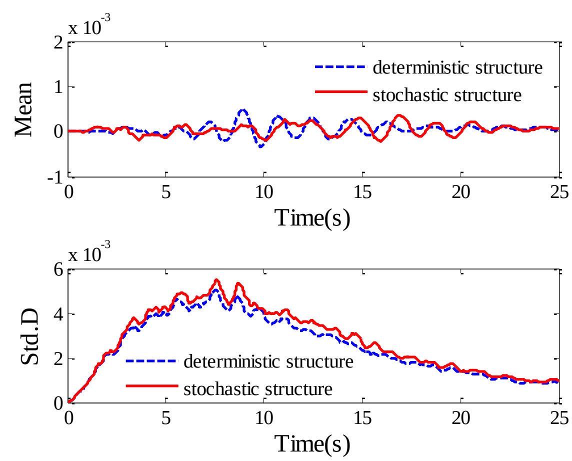
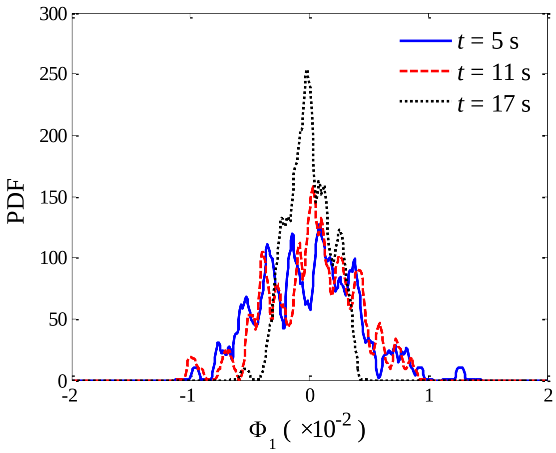
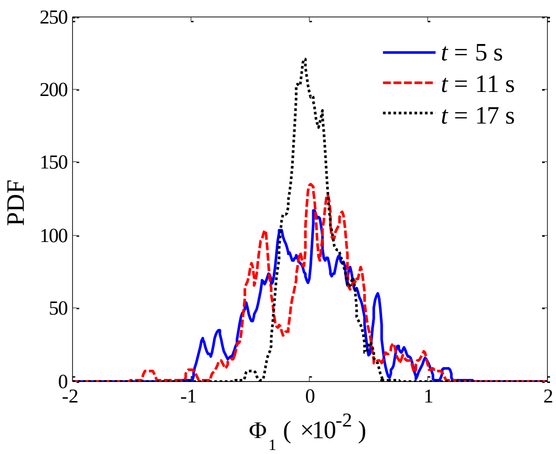


Figure 5: Mean and standard deviation of $\Phi_1(t)$

Fig. 6 shows the PDFs at three typical instants of $\Phi_1(t)$ with respect to the deterministic structure and the stochastic structure. It can be found that there is a remarkable difference of the PDFs at different instants upon both the deterministic and the stochastic structures. The PDFs of the response at the same time instant would change considerably when the randomness of the structural parameters are considered.



(a) Deterministic structure



(b) Stochastic structure

Figure 6. PDF at certain instants of $\Phi_1(t)$

5. CONCLUSIONS

In this study, a dimension-reduction modeling method is proposed for a dual stochastic dynamic structural system of the non-stationary stochastic ground motion processes and stochastic structures. By means of applying the random function to model the seismic excitations and structural random parameters, the entire stochastic dynamic system can thus be readily represented by just two elementary random variables. Due to a small number of the elementary variables required by the proposed method, the PDEM can be efficiently used for dynamic response analysis and dynamic reliability assessment of complex nonlinear structures with numerous random parameters. Numerical investigations adequately reveal the availability of the proposed scheme in engineering practices.

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