# Sensitivity Analysis of Interdependency Parameters Using Probabilistic System Models

Cynthia Lee
*Ph.D. Candidate, Dept. of Civil Engineering, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0355, USA*

Iris Tien
*Assistant Professor, School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0355, USA*

ABSTRACT: Comprehensive models of infrastructure networks feature many parameters characterizing the complex interdependencies that exist between systems. Most of these parameters are uncertain. Conducting sensitivity analyses is one way to characterize uncertainty in estimations of system-level performance based on component and interdependency parameters. Doing so provides an assessment of the importance of varying parameters and informs how to achieve targeted system outcomes through component- and system-level changes. To do this over interdependent infrastructure networks, we conduct inference over probabilistic Bayesian network-based models of these systems. We have developed a framework along with accompanying algorithms to conduct computationally tractable exact inference over the network model. Through a series of these analyses, we are able to analyze the impacts of changes in parameters on estimations of system-level performance. We apply the framework to a water distribution system including its dependencies with power and transportation networks. The results of the analyses show the effect of varying system parameters on probabilities of providing service across the network. We investigate the impacts on system performance of adding redundant power supplies, changing link configurations, and increased or reduced probabilities of component failures. The use of the sensitivity analysis results to support performance-based design based on system-level reliability measures is discussed.

Infrastructure systems are increasingly connected, with interdependencies between them often governing their performance and leading to increased vulnerabilities to cascading failures (Buldyrev et al. 2010). Previously, we defined three generalized, comprehensive infrastructure interdependency types (Johansen and Tien 2017), advancing upon previous work in interdependency analysis (e.g., Rinaldi et al. 2001) to include parameters affecting the recovery and resilience of infrastructure networks. Comprehensive models of infrastructure networks feature many parameters characterizing the complex interdependencies that exist between systems. Most of these parameters are uncertain. Conducting sensitivity analyses is one way to characterize uncertainty in estimations of system-level performance based on component and interdependency parameters. Doing so provides an assessment of the importance of varying parameters and informs how to achieve targeted system outcomes through component- and system-level changes. Prioritizing varying changes supports effective decisions to increase resilience of interdependent systems (Johansen et al. 2016, Ouyang 2016).

In this paper, we conduct these analyses for interdependent infrastructure networks by performing inference over probabilistic Bayesian network-based models of these systems. We utilize a previously proposed framework for probabilistic vulnerability of interdependent infrastructure systems and its accompanying algorithms to conduct computationally tractable exact inference over the network model (Applegate and Tien 2019). Through a series of these analyses, we are able to analyze the impacts of changes in parameters on estimations of system-level performance. We apply the framework to a water distribution system including its dependencies with electrical power and transportation networks to illustrate the approach. We investigate the impacts on system performance of adding redundant power supplies, changing link configurations, and increased or reduced probabilities of component failures. The results of the analyses show the effect of varying system parameters on probabilities of providing service across the network. We conclude with a discussion of the use of the sensitivity analysis results to support performance-based design based on system-level reliability measures.

## 1. PROBABILISTIC SYSTEM MODEL AND APPLICATION

To model the infrastructure network, we represent each component in the network as a node and the connections between them as links. With the connectivity and dependency relationships defined, we build the Bayesian network model of the interdependent system. The Bayesian network is a probabilistic graphical model that enables us to capture the uncertainties in the infrastructure system parameters, including uncertainties in the hazards a system is exposed to, individual component performance under hazards, and propagation to system-level responses.

We define and capture three interdependency types: service provision interdependencies, where the functioning of one component depends on a service provided by a component in another system; geographic interdependencies, where components are more likely to fail together under a hazard due to geographic proximity or physical similarity; and access for repair interdependencies, where the ability of a failed component to be repaired depends on physical or cyber access provided by a component in another system.

The application network of interest is the City of Atlanta water distribution system located in the state of Georgia, USA, including its dependencies with power and transportation networks. Figure 1 shows the distribution system, where smaller solid circles indicate end-point distribution nodes and larger empty circles indicate water supply nodes. As the water supply components require electrical power to operate the treatment plants and pumping stations located at these nodes, the power supply components are also located at these nodes.

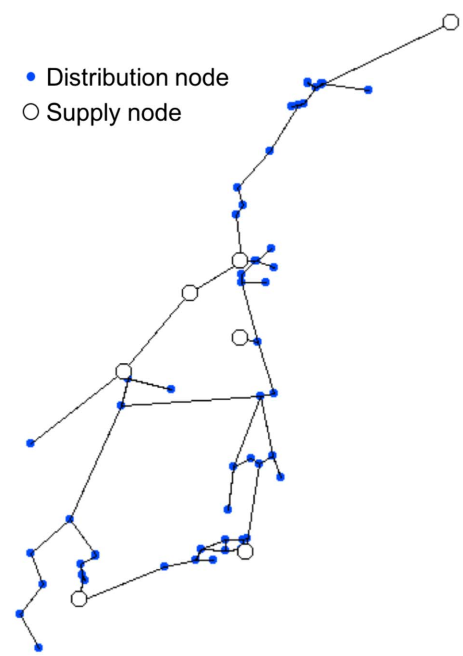



*Figure 1: Water distribution system and power supply dependencies for application*

A representation of the probabilistic Bayesian network model that is built for this system is shown in Figure 2. Hazards are included to capture the geographic interdependencies. The reliance of water supply components on power supply components to function captures the service provision interdependencies. In Figure 2, MLS indicates the minimum link sets for the system. These provide the minimum paths required to be functioning to transport the infrastructure resource, in this case water, from supply to distribution nodes. The Zone nodes indicate water services provided to specific zones in the network. As the network includes 112 components – seven of which are supply stations and 105 of which were transshipment or distribution nodes – and 244 links – or pipes – not all individual nodes in the network are shown.

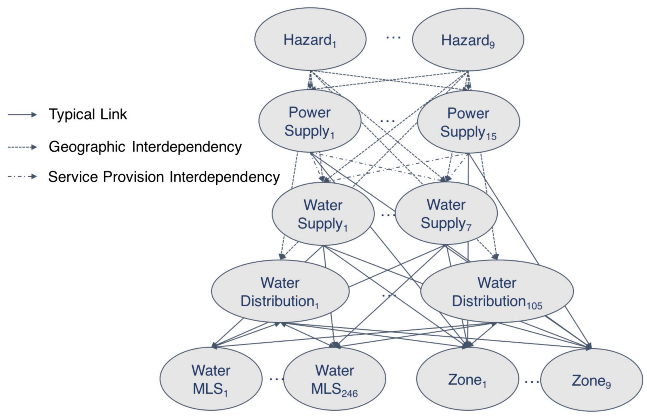



*Figure 2: Representative Bayesian network model of water and power distribution systems*

An example of an access for repair interdependency is shown in Figure 3. Here, access refers to the physical access provided by operational roads such that repair crews can access potentially failed water supply or distribution components. Two main roads, 17th Street NW and Northside Drive NW, are shown around the Supply 4 node.

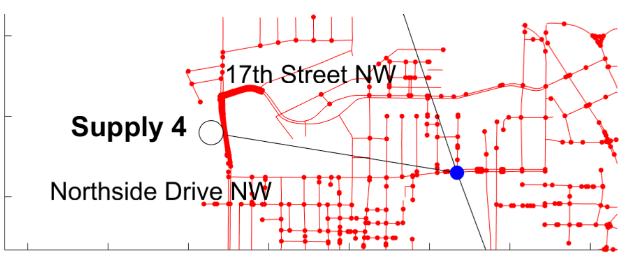



*Figure 3: Roads near water system supply node providing access for repair in the case of failure*

## 2. INTERDEPENDENCY PARAMETER SENSITIVITY ANALYSIS

With the probabilistic system model built, we use it to assess the impacts of varying interdependent system parameters on the probabilities of providing service across the network. We use the previously proposed analysis framework, along with the accompanying modeling and inference algorithms, to perform exact inference over the network. This results in probabilities of failure or survival of each distribution node under varying scenarios. We investigate the effect of three system parameters on probabilities of providing service at each distribution node: adding redundant power supplies at the water supply nodes, changing link configurations to add redundant paths to distribution nodes, and increasing or reducing the conditional probabilities of water supply failure given a hazard. Each inference scenario analysis takes on the order of one minute on a personal computer with 4 GB RAM and a 1.3 GHz Intel Core i5 processor using MATLAB 2017b.

### 2.1. Redundant power supplies

The main service provision interdependency in the network is the electrical power that is required at the water supply components. If power fails, the water supply component fails, propagating to failures at end-point distribution nodes. An example of such a cascading failure is shown in Figure 4.

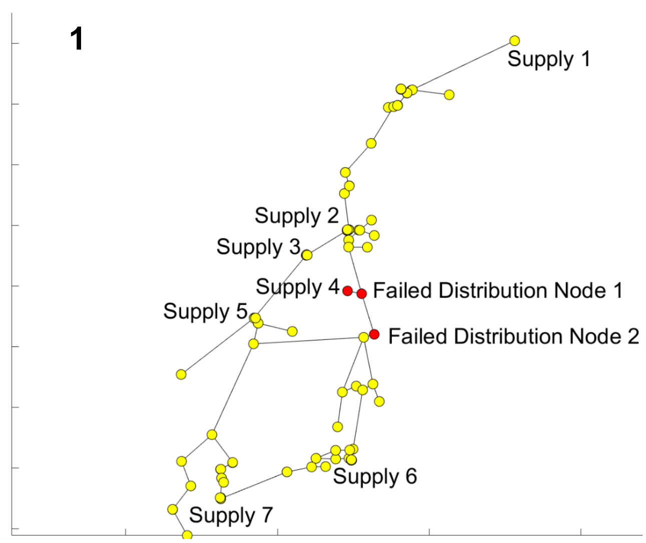



*Figure 4: Power supply failure at node Supply 4 propagating to failure of two distribution nodes*

To mitigate such potential failures, redundant power supplies can be added at the water supply components. This increases the probability that electrical power will continue to be able to be provided to the water supply component even if there is an outage in one power supply or the electrical feed between a power supply and water supply goes down.

Given a probability of failure for a power supply given a hazard of $10^{-2}$ and given no hazard of $10^{-4}$, the relative changes in distribution component probabilities of failure in adding a redundant power supply at the water supplies range between $-10^{-15}$, i.e., no change, and $-0.497$, i.e., a 49.7% decrease in probability of failure. As expected, adding redundant power supplies reduces component failure probabilities, protecting against single power supply failures. If a failure occurs for one power supply, the water supply node maintains functionality due to the added power backup. All prior probabilities of failure at the distribution nodes decrease with the addition of redundant power supplies.

*2.2. Changing link configurations*

In addition to power supply redundancies, link redundancies can increase the performance of a system. As the probability of providing a service at a distribution node depends on the ability to transport the resource from supply to distribution points, the configurations of the links in the network affects the number of possible paths along which the resource can be conveyed to end-point distribution nodes.

To investigate the effect of varying link configurations on system performance, we map the distribution nodes to specific service areas based on United States census blocks. This enables us to assess the impacts of infrastructure performance on populations. In the absence of specific block-level information, blocks are assigned to their closest nodes. Figure 5 shows the populations affected by failure in Supply 4 propagating to failures in the two distribution nodes, indicated as node A and node B.

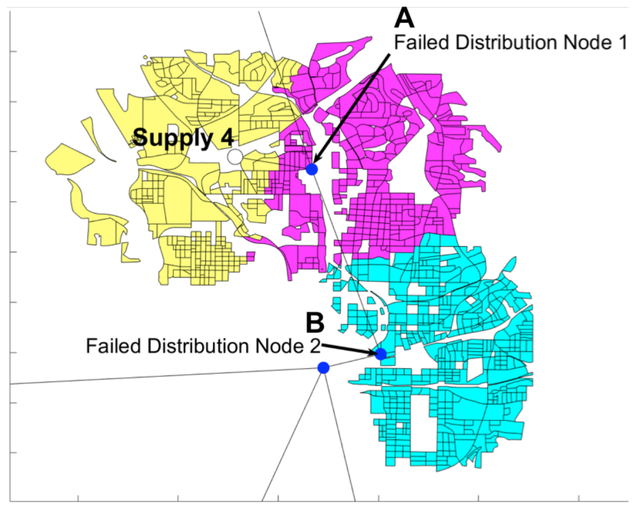



*Figure 5: Populations affected by failure in Supply 4 propagating to failures in distribution nodes A and B*

Considering the original configuration and failure event as shown in Figure 4 as Scenario 1, Figure 6 shows alternate Scenarios 2-5 where varying possible link configurations are presented to protect populations against potential failures. Link configurations are varied by adding a single link between a failed node and any other node. Added links are selected based on a search of potential links with nearby nodes. In Figure 6, darker red nodes indicate failed nodes; lighter yellow nodes indicate nodes with higher probabilities of survival.

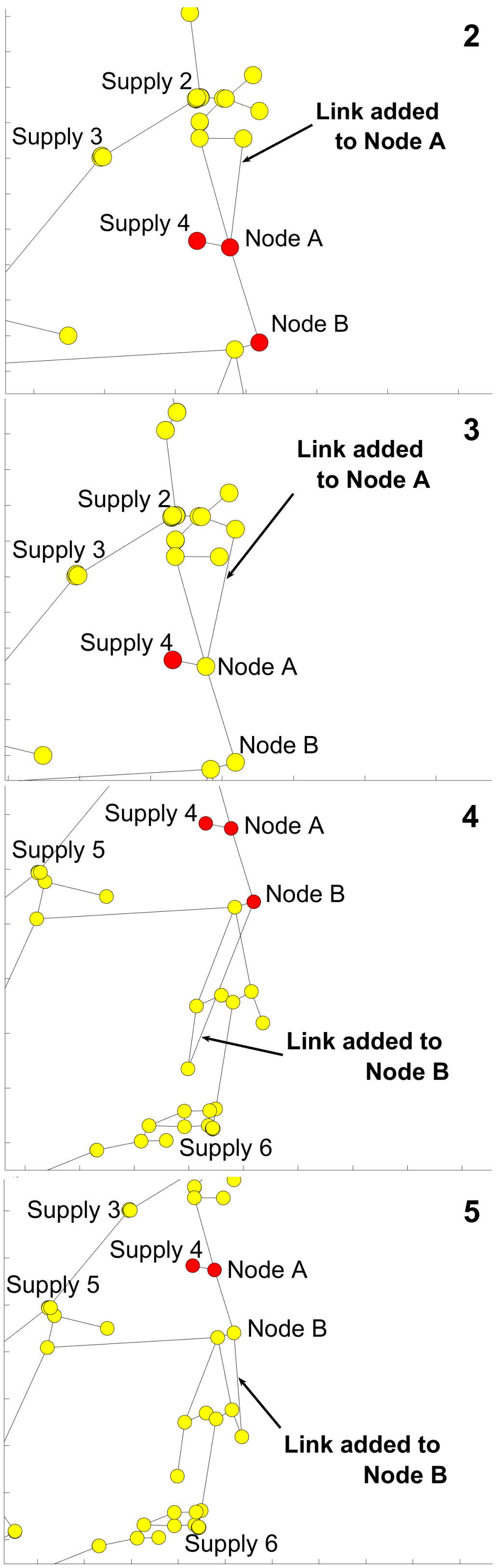



*Figure 6: Effect of varying link configurations (Scenarios 2-5) on probabilities of failure of distribution nodes*

Table 1 shows the differences in population, number of housing units, and number of critical facilities affected in the scenario where Supply 4 fails. This could be due to a loss of power at Supply 4 or failure of any other element of the supply node leading to Supply 4 failure. Values are based on service areas of failed components for corresponding census blocks.

*Table 1: Disruption of service due to Supply 4 failure for varying link configuration scenarios*

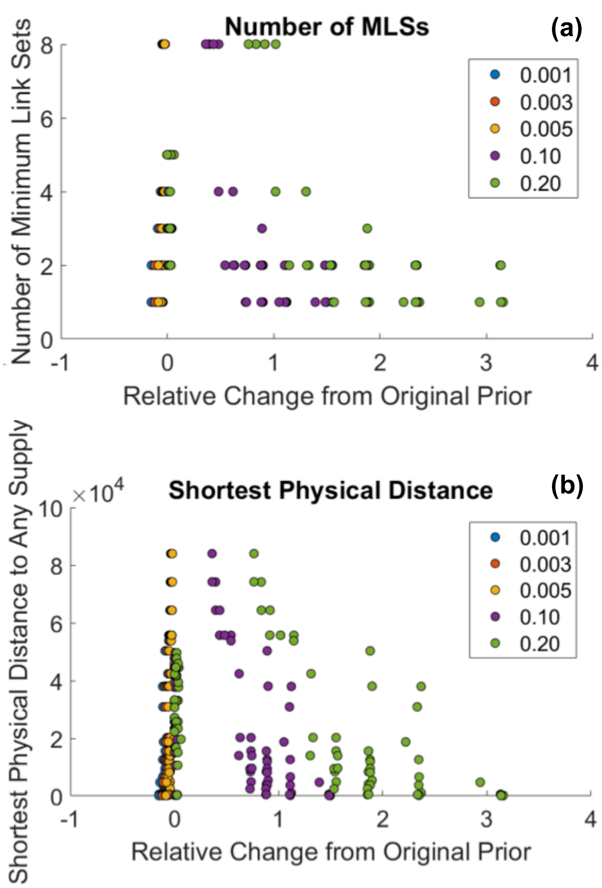
| Scenario | Population | Housing Units | Critical Facilities |
|---|---|---|---|
| 1 | 96,217 | 57,445 | 7 |
| 2 | 96,217 | 57,445 | 7 |
| 3 | 21,340 | 12,203 | 2 |
| 4 | 96,217 | 57,445 | 7 |
| 5 | 62,829 | 37,748 | 5 |

From Figure 6 and Table 1, Scenario 3 provides the link configuration solution that minimizes disruption to the population. In considering the varying link configurations, additional links protect against failures at distribution nodes due to supply failures if they are able to connect the node back to a different supply. Adding redundancies increases performance of the system; however, the redundancies need to be strategically placed to be effective in decreasing probabilities of failure under varying scenarios.

### *2.3. Varying component failure probabilities*

To assess the effect of varying component failure probabilities on the ability to provide service across the network, we vary the conditional probability of failure of the water supply components given a hazard. Decreasing the conditional probability of failure represents the case where the component is retrofitted or hardened to better withstand the hazard. Increasing the conditional probability of failure represents the case where damage may have occurred at a supply component and sufficient repairs were not made before a hazard may occur again. Varying component failure probabilities can also represent the case where new information is collected about a component to update one's belief in the state of the component, for example, as in Lee and Tien (2018).

The original probability of failure of the water supply component given hazard occurrence is set at 0.01. In this sensitivity analysis, the conditional probability of failure is varied from 0.001 to 0.20. To better characterize the change in probabilities of failure given these changes in water supply performance, the relative changes from the original values in terms of prior probabilities of failure are plotted. These changes are shown in Figure 7 as a function of the component characteristics: (a) number of minimum link sets, (b) shortest physical distance to a supply node, and (c) minimum number of hops (links) to a supply node. Each point indicates the relative change for a system distribution component. The relative change is given as the value and not percent, so a value of 2 indicates a 200% increase in probability of failure for that component.



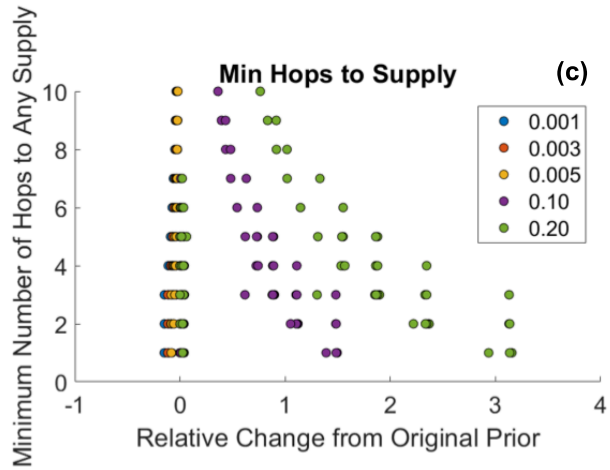




*Figure 7: Relative change in component prior probabilities of failure with changing water supply failure probabilities as a function of the (a) number of minimum link sets, (b) shortest physical distance to a supply, and (c) minimum number of hops (links) to a supply*

There are several observations from Figure 7. First, in evaluating the effect of decreasing the conditional probabilities of failure compared to increasing them, there is larger effect on the components when probabilities of failure increase, i.e., the increase in probabilities of failure when supply failure probabilities increase from 0.10 to 0.20 is greater than the increase from 0.01, or even 0.001, to 0.10.

Second, the effect on network performance is larger for components with fewer minimum link sets, shorter physical distance to a supply, and a fewer number of hops to a supply. This suggests that components that are closer to supplies are more directly affected by changes in the performance of the supply components. While it could be argued that farther components are more remote in the network and therefore less resilient to changes in the network, the results show that increases in distance actually correspond with more resilient components as there are more redundant paths to the component and the relative impact of the water supply on distribution component performance decreases.

Third, the effect of water supply performance on network performance as measured by probabilities of providing service at distribution nodes appears to follow an exponential trend. This is particularly apparent in Figure 7(c). As the number of hops between a distribution component and a supply increases, the effect of changing the water supply failure probability decreases exponentially.

## 3. USE OF SENSITIVITY ANALYSIS FOR PERFORMANCE-BASED DESIGN

The sensitivity results shown in Figure 7 provide a basis for component-level performance-based design decisions according to network-level reliability measures. The larger effect of increasing probabilities of failure compared to decreasing them suggests that it is more important to repair slightly damaged components, indicated by having slightly higher conditional probabilities of failure, compared to retrofitting components to have exceedingly low probabilities of failure.

The effect of increases in conditional probabilities of failure is magnified at higher failure probabilities. The relationship between the relative changes in distribution component probabilities of failure and the component characteristics informs those components that will have the largest changes in response to improvements or deterioration of supply components. This result supports repair or retrofit decisions based on the components that would most benefit from these actions for supply nodes.

Finally, the exponential trend in effect of supply node changes on distribution component performance indicates a point of diminishing returns in terms of investments to improve or increase performance of supply components in the system. All of these recommendations are based on inference results showing the ability under varying scenarios to meet service provision performance targets at distribution nodes throughout the network.

## 4. CONCLUSIONS

Given the number of parameters affecting interdependent infrastructure system performance and the uncertainty associated with the problem, conducting sensitivity analyses provides a way to quantify the effects on system performance due to variations in the system parameters. The parameters studied in this paper are selected based on interests in investigating practical changes that can be implemented for interdependent infrastructure networks. Comparing effects across parameters enables prioritization of different investment strategies to increase overall infrastructure performance from component- and system-level changes.

## 5. REFERENCES

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