Tobin's *q* of a Multi-Product Firm and an Endogenous Growth of a Firm

Shin-Haing Kim and Taegi Kim

This study considers the Tobin's *q* of a 'multi-product' firm with fixed capital goods. This modified version of Tobin's *q* includes a share of the fixed capital goods in a firm's investment. A firm in a developing economy, such as a South Korean *chaebol*, catches up the world frontier technology with its diverse products. The fixed capital investments of *chaebol*s are conducive in pursuing diversifications, thereby exhibiting high Tobin's *q*. Moreover, achieving an *Ak* technology enables *chaebol*s to reap their growth on the endogenous path. We observe a high disparity between the '*chaebol*-incumbent' and 'non-incumbent' firms in their growth performances in the previous half-century experience of the South Korean economy. We attribute this disparity to the endogenous growth of *chaebol*s.

Keywords: Tobin's *q*, Multi-product firm, Catch up, Fixed capital good, Endogenous growth, Korean *chaebol*, Gibrat's law

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I. Introduction

In a textbook version of macroeconomics, the rate of investment is explained through a comparison of the marginal efficiency of capital (MEC) with the market rate of interest. Investments occur as long

Shin-Haing Kim, Professor Emeritus, Department of Economics, Seoul National University, Seoul, South Korea. (Email): shk@snu.ac.kr, (Tel): +82-10-8139-6388; Taegi Kim, Corresponding Author, Professor, Department of Economics, Chonnam National University, Gwangju, South Korea. (Email): tgkim@jnu.ac.kr, (Tel): +82-62-530-1455, respectively.

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as MEC is higher than the rate of interest. Through Keynes' MEC schedule, a monetary sector of the economy is linked to the aggregate activity level of the real sector. Tobin's q bridges the gap between its value in the financial market and rate of investment in the real sector of the economy at a firm level. It refers to the ratio of a firm's market value in the financial market to its replacement cost (Tobin 1969, p. 21). If this ratio is above 1, then investments take place; otherwise, no investment occurs. Lucas (1967) adjustment cost theory of a firm paves the way to consider Tobin's q with respect to microeconomic theory of production function. Uzawa (1969) views the adjustment cost of investments in terms of the effective units of investment. Hayashi (1982) demonstrates the equality between the average and marginal Tobin's q, thereby leading to the empirical research on rates of investment related to Tobin's q.

We present a growth model of a firm based on Tobin's q to compare the growth performances of "*chaebol*-incumbents" and "non-incumbents," particularly given the experiences of the South Korean economy during its developmental stage in the last half century. We determine that "*chaebol*-incumbents" outperform "non-incumbents" in terms of growth, thereby disregarding "Gibrat's law." We attribute this non-proportionate growth pattern between the two to the capability of *chaebol*s to diversify multi-products.[1]

Chaebol investments occur across industries, which range from automobiles, constructions, ship-buildings, electronics, to semi-conductors, including wholesales. These investments contribute in the formation of a centralized group across industries. Amsden (1989, p. 151) views "the economy of scope" as one of the contributing factors for the emergence of the *chaebols* of the late industrializing countries such as Korea. A *chaebol*s' capacity to diversify provides them with the "economy of scope." Chandler's (1990) historical perspective on the emergence of "big corporations" inspires us to consider the "economy of scope" that arises from the "economies of scale" generated by investments in fixed capital goods.

Investments in fixed capital goods differ from those in working

[1] *Chaebol*s are often associated with their ownership. A small family owns stocks of firms and can control the management decisions of the group of firms. In this study, a "*chaebol*" is restricted to the production aspect and is separate from the issue of ownership.

capitals, such that reaping the final outputs from investments takes time. To compensate these costs, certain knowledge embodied in it should be shared with the forthcoming production of goods. We consider that knowledge on advanced economy embodied by imported fixed capital goods to small, open economies, such as South Korea, is transmitted to the latter. Knowledge is shared among adjacent succeeding industries in the production of goods. Accordingly, investments on fixed capital goods result in the emergence of the "economy of scope" to *chaebol*s.

We develop a Tobin's q in our model, which incorporates fixed capital goods in the investment decision of a firm. The difficulty arises after the inclusion of fixed capital goods in the typical Cobb–Douglas production function. This function is the problem of the "economy of scale" because of its indivisibility. We think that a "multi-production" firm can resolve this issue. Sharing of knowledge is embodied by the physical capital goods across the adjacent products of various industries. Hence, the indivisibility of the fixed capital goods become divisible by a linear combination of knowledge among them. The firm of our interest "*chaebol*s" can realize the "economy of scope" owing to the heavy investments in fixed capital goods at the initial period of development. We show that *chaebol*s' high Tobin's q, which incorporates investments in fixed capital goods, induces their high rates of investment. The outperformance of their growth is in their realization of "the economy of scope."

The remainder of this paper is organized as follows. Section II presents a model that incorporates fixed capital goods in the production function for which a technological frontier of *chaebol*s is introduced. Section III discusses the modified version of Uzawa-Hayashi's adjustment cost function. Section IV derives a modified Tobin's q for which a discussion on the role of fixed capital goods is included. Section V provides the empirical results, which compare the Tobin's q between the *chaebol* incumbents and non-incumbents. Section VI shows an endogenous growth path of a "multi-product firm." Lastly, Section VII provides the conclusion.

II. Model

We consider a "multi-product" firm, which can share variable factor and service inputs provided by fixed capital goods. The marginal physical productivity of the service inputs of fixed capital goods in each product can diminish separately. This case relates to the Cobb–Douglas production function in a firm's production theory. The law of diminishing returns can be resisted by the "multi-product" firm in sharing the services of two adjacent products' fixed capital goods. Panzar and Willig (1981) provide proof of the existence of a "multi-product" firm with the "economy of scope" in a competitive market. We consider the "economy of scope" of a multi-product firm as one of the possible methods to solve the "economy of scale" problem, which arises with the fixed capital goods in a production function.

We consider that a continuum of industries $\tau \in (0, 1)$ exists in the economy. A per capita output $y(\tau)$ is a Cobb–Douglas of the following form:

$$y(\tau)=\begin{cases}0 \\ \left[A(\tau)k(\tau)\right]^{\alpha}\end{cases} when \begin{cases}k_F(\tau)<k_F^*(\tau) \\ k(\tau)=k_\upsilon(\tau)+k_F(\tau)\geq k_F^*(\tau)\end{cases}, \tag{1}$$

where capital $k(\tau)$ comprises the variable factor input $k_\upsilon(\tau)$ and fixed capital good $k_F(\tau)$, which takes a share $0 < b(\tau) < 1$ of the total; $A(\tau)$ represents the knowledge embodied by the capital good τ; and the exponent $0 < \alpha < 1$ is a distributive share for capital.

Suppose that the world production function $\bar{A}k$ is determined by the world frontier technology. The price of good τ denoted by p_τ is given in the international market. Accordingly, we consider firm τ of a developing economy, the knowledge level $A(\tau)$ of which is at the level below the frontier level of $\bar{A}$: $A(\tau) < \bar{A}$.

Firm τ of our concern at the initial developmental stage of the economy intends to catch up the frontier through investment in the fixed capital goods of $k_F^*(\tau)$. This factor is obtained by the following equality condition of the marginal productivity of capital with its marginal productivity at the frontier. The output of good τ on the technological frontier is determined by the selection of its fixed capital stock $k_F^*(\tau)$, such that the following efficiency condition is satisfied:

$$\frac{1}{(k(\tau_j)-k_F^*(\tau_j))^{1-\alpha}}=\frac{1}{\left[(1-b^*(\tau_j))k(\tau_j)\right]^{1-\alpha}}=\frac{\bar{A}}{\alpha A(\tau_i)}.$$

Note that the marginal productivity of variable capital does not apply until the total amount of capital is no longer below its fixed amount of capital goods. Efficiency condition implies that the farther the distant of the firm of a developing economy situated from the frontier technology, the higher the amount of the fixed capital good required. We assume that the productivity level of the industry is higher in the order of the continuum of $\tau \in (0, 1)$, thereby increasing at a decreasing rate: $A'(\tau) > 0$, $A''(\tau) < 0$. This assumption suggests that when the productivity level of $A(\tau_i)$ is high, the forthcoming industry τ_j's share of fixed capital $b^*(\tau_j)$ decreases at an increasing rate: $b^{*\prime}(\tau) < 0$; $b^{*\prime\prime}(\tau) > 0$. In addition, we assume that the elasticity of the decrease of the investment share of fixed capital goods with respect to the increase in τ, as denoted by $\eta_{b(\tau)}$, satisfies the following elasticity condition:

$$0 < \eta_{b(\tau)} \equiv -\frac{b^{*''}(\tau)\tau}{b^{*'}(\tau)} < 2.$$

Elasticity condition is later discussed for the explanation of the endogenous growth path of *chaebol*s.

The technical frontier of "the multi-product" firm suggests that the required amount of investment in fixed capital must be the highest at the initial period of development. Thus, technological frontier determines a Rostovian "big-push" strategy. This unbalanced growth strategy argues that investments in heavy industries, such as steel, machineries, automobiles, and ships, provide breakthroughs for the growth of a developing economy. The growth of *chaebol*s in the past half century of the South Korean economy becomes a success story on this unbalanced growth strategy. Note that *chaebol*s' investment strategy was managed under international environments, in which prices of goods and inputs are given at international prices. The "big push" was also well incorporated with Park's regime of economic development policy, which was amenable to the market principle (Jwa 2018).

A "multi-product" firm has products with the interval of $\tau_{ij} \in (\tau_i, \tau_j)$ with capital stocks of $k(\tau_{ij}) \in (k(\tau_i), k(\tau_j))$. The firm shares the knowledge embodied by the fixed capital goods of the two goods, τ_i and τ_j. By sharing such knowledge, the "multi-product" firm can linearize the technological frontier it faces. A firm in small open economies faces the prices of goods τ_i and τ_j given at the level of $p(\tau_i)$ and $p(\tau_j)$, respectively, at the international market. A linear combination of the two prices is denoted by $p(\tau_{ij})$ with the weight of $0 < \xi < 1$. The weight is determined by the supply conditions of the two goods of small open economies. Moreover, a linear combination of the capitals used to produce the two goods yields $k(\tau_{ij})$.

A production function $y^m(\tau)$ of "multi-product" firm $\tau \in (\tau_i, \tau_j)$, which produces its nearby products τ_i and τ_j, is shown as Equation (2) as follows:

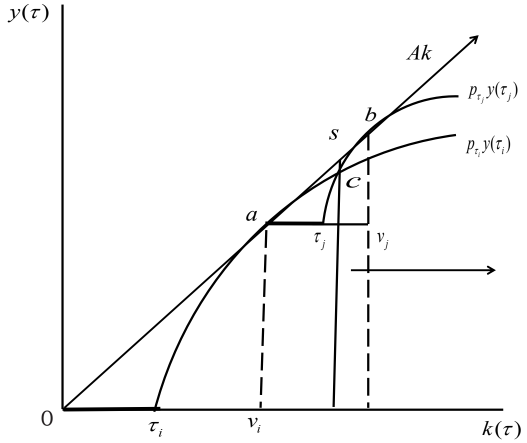
$$\begin{aligned} & y^m(\tau) = \int_{\tau_i}^{\tau_j} p(s)\left(A(\tau_{ij})k(\tau_{ij})\right)^{\alpha} ds \\ & ;\, A(\tau_{ij}) \geq \xi A(\tau_i) + (1-\xi)A(\tau_j) \\ & ;\, k(\tau_{ij}) = \xi k(\tau_i) + (1-\xi)k(\tau_j) \\ & ;\, \tau_{ij} \in (\tau_i, \tau_j) \\ & 0 < \xi < 1. \end{aligned} \tag{2}$$

Although the production functions of products i and j are of the Cobb–Douglas form separately, this multi-product firm shows the technology of $A(\tau_{ij})k(\tau_{ij})$. The concavity of the knowledge function in the input-sharing of knowledge implies that its productivity $A(\tau_{ij})$ on the second row of Equation (2) is no longer lower than any of the $A(\tau_i)$ and $A(\tau_j)$ of the non-multi-product-firm of products i and j. The input-sharing of the different production lines of a multi-product firm makes the intrinsic problem of "indivisibility" of fixed capital goods "divisible" by a linear combination of services provided by such goods of adjacent products.[2]

Figure 1 presents a technological frontier of a "multi-product" firm. Its horizontal axis represents the level of capital stock $k(\tau)$ associated with the production of goods $\tau \in (0, 1)$. *Ak* technology is exhibited by the line of tangential points of the per capital output of each industry.

The fixed capital goods of firm τ are represented in the units of variable capitals. The horizontal axis spans industries in a sequential order of the required fixed capital goods. The axis is also interpreted as the development level of industries as the knowledge level embodied by the fixed capital goods increases in the rightward direction. No final output of good τ_i is produced until after the investment in fixed capital

[2] These adjacent goods are presumably close substitutes for each other.



Distance "$0\tau_i$" on the horizontal $k(\tau)$ represents the amount of fixed capital goods in the units of the variable capital of the distance of "$\tau_i v_i$" by the multiple of "$x(\tau_i)$".

Figure 1
Technological Frontier of a Multi-Product Firm

good $k_F(\tau_i)$ for industry i is complete. It is represented as the "thick line" of "$0\tau_i$" on the horizontal axis. The final output τ_i is produced from the investments on its variable capital for the interval of "$\tau_i v_i$". Given that fixed capital good is a constant multiple $x(\tau_i)$ of variable capital, the share of the fixed capital of the total capital $b(\tau_i) = x(\tau_i)/1 + x(\tau_i)$ increases in its constant multiple of $x(\tau_i)$. This procedure is repeated over the forthcoming industries, in which the share of fixed capital goods in investment decreases.

At point a, the production possibility curve of industry i is tangential to frontier technology A, thereby fulfilling efficiency condition. Moreover, we superimpose the amount of the fixed capital of "$a\tau_j$" for the nearby industry j. We also determine the output at the tangential point of "b" to the frontier technology for the level of variable capital of "$\tau_j v_j$". Accordingly, *chaebol*s can combine the two adjacent techniques of i and j to form the linearized technology at point "s" on the frontier technology.[3] The linearized Ak technology of *chaebol*s deters the fall of

[3] Point "s" on line "ab" is determined by the relative price of the two adjacent goods, "i" and "j" on the international market.

the marginal productivity of capital in producing final good *i*, whereas such a technology increases the productivity of creating final good *j*. A triangular shape as shown by the area of "*abc*" indicates the efficiency benefited by the "multi-product" firm on its operation on the world frontier *Ak* technology.

The sequence of investments is followed by the strategy of a firm of a developing economy attempting to catch up the world frontier technology. Thus, the sequence is path dependent. The linearization of the techniques of a "multi-product" firm is not solely attributed to *chaebol*s. Any "multi-product" firm can also combine adjacent techniques and can linearize them. However, doing so should be at the lower level than that of the frontier. That is, $A < \bar{A}$. A heavy requirement for the investment in fixed capital goods at the initial developmental stage of the economy to access frontier technology is limited to a few. Nevertheless, *chaebol*s succeed in this situation.

An alternative strategy can be used to catch up the world frontier technology. Instead of exploring the scale effects of fixed capital goods, the strategy concentrates on the supply of parts and components generally from the side of small and medium-sized firms. This case is relevant to the growth experience of the Taiwanese economy. Grossman and Helpman's quality ladder can also be relevant to explain the steps that should be followed up to the frontier.

Our next agenda is to determine whether the "multi-product" of our concern is consistent with a competitive equilibrium. Panzar and Willig (1981) suggest that "the economy of scope" is a sufficient condition for the existence of the competitive equilibrium prices of multi-products. The multi-product firm, which shares the services of fixed capital goods at its disposal, has "the economy of scope" (Appendix provides its proof).

In her book *Asia's Next Giant*, Amsden (1989, p. 151) shows that "an economy of scope" and "the capacity to diversify" are focal for the growth of "*chaebol*s" of late industrializing countries, such as South Korea. We consider Amsden's approach for the growth of South Korean "*chaebol*s," such that they can diversify and realize "the economy of scope."[4]

[4] *Chaebol*s are often associated with their ownership. A small family owns stocks of firms. They can control the management decisions of the group of firms. The "big firm" in this study refers to the production aspect separating itself from the issue of its ownership.

Amsden (1989) suggests that centralizing the knowledge and infrastructure of *chaebol*s at their disposal reduces the cost of entering a new industry. These advantageous situations for the investments of the *chaebol* incumbents compared with non-incumbents result in asymmetric growth among firms.

We consider a "catching-up" problem of a developing economy, in which the technology level embodied by physical capital goods is below the world technology level of A. High technology is embodied by the capital goods of an advanced economy. Thus, a trade structure of the imports of capital goods from advanced economies in exchange of the exports of consumer goods from developing economies explains how developing economies reach the world frontier.

The "multi-product" firm of small open economies is situated in an economic environment, in which the firm exports final goods to the international market and imports fixed capital goods. The technology embodied by its capital good for production comes from the imported ones from advanced economies. The firm strives for reaching the frontier technology. In exchange for the consumer goods produced by the firm, capital goods are imported from advanced economies on the frontier technology. Capital goods are vehicles through which technological knowledge is transmitted across economies.

In the development literature of the 1960s, two competing strategies are used for the development of underdeveloped economies. One is the balanced growth strategy, which provides externalities across domestic industries, as shown by the growth experiences of Taiwan. The other is the unbalanced growth strategy, which requires a "big push" of heavy industries, such as the steel industry, to break through the bottlenecks of fixed capital goods. A *chaebol*-led growth experience in the past half century of the South Korean economy becomes a success story of the "unbalanced growth strategy."

Figure 1 illustrates the process of reaching the world frontier Ak technology. This technology provides the following advantages to economies.

1. The technology is efficient because a linear combination of adjacent technologies yields several outputs.
2. The price level of each good in the world frontier technology is consistent with the international price.
3. Firms using the Ak technology increase at the growth rate of this

technology.

Figure 1 shows that world frontier technology Ak is drawn from the origin for a given constant output–capital ratio A. A tangential point of the slope of A to a production function of (1), $p_i y(\tau_i)$, presumably with the highest fixed capital requirements, is indicated as a.

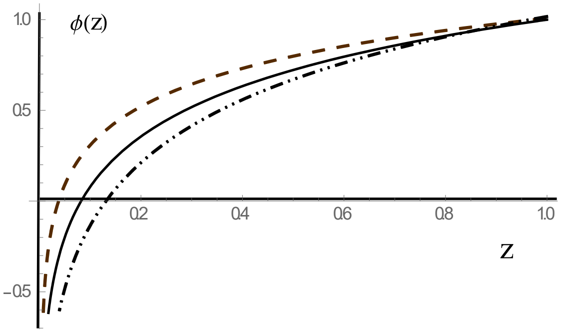
Investments in future industries unfold over the horizontal time axis of Figure 1. Each time a firm enters a new industry, it faces another "hurdle" of a fixed capital good to cross over. The height of the "hurdle" is likely high at the initial developmental period and may gradually decrease as the firm makes its investments along the horizontal axis. This scheme of a production technology of a "multi-product firm" is based on Rostow's "big-push" doctrine. Accordingly, breaking through the bottleneck of development is the required investment rate for economies. This rate should be higher than its critical level.

Economic environments at this early developmental era were favorable to *chaebol*s in this "hurdle race" of fixed-capital good investments in the following aspects.

1. Accesses to international markets for exports of the light-manufacturing consumer goods were favorable for small open economies, such as South Korea.
2. A trade pattern of importing capital goods, in which new technologies are embodied in exchange for exports, was favorable for technology transfer.
3. Government subsidies for the investments in fixed capital goods helped *chaebol*s win the race.

III. Uzawa–Hayashi's Adjustment Cost Function

The amount of outputs unconsumed in the economy is saved and invested for the output of the succeeding periods. A good model can show that consumption goods become de facto investment goods for the production in the next period. In Tobin's q, investment goods are distinguished from consumption goods with respect to their adjustment costs for production. These costs can emerge either on the administrative level of the overhead costs or the efficiency of the investment goods in production. For the former, adjustment costs directly enter the output function of final goods (Lucas 1967). The other



Effective units of investments vary based on the shared fixed capital. The solid curve for $b(\tau) = 0.4$, the dotted curve for $b(\tau) = 0.3$, and the double dotted curve for $b(\tau) = 0.5$. The critical rate $\hat{z}$ for each fixed capital share are $\hat{z}(0.4) = 0.082$, $\hat{z}(0.3) = 0.036$, and $\hat{z}(0.5) = 0.135$.

FIGURE 2
EFFECTIVE UNITS OF INVESTMENT[5]

considers adjustment costs in terms of the effective units of investments (Uzawa 1969, Hayashi 1982). Such costs are reflected in the installation of fixed capital goods, which we perceive as relevant to our purpose.

We following Uzawa (1969) and represent the adjustment costs of investments in terms of the efficiency units of capital goods as follows:

$$\tilde{k}(\tau) = \phi(z(\tau))k(\tau);\ 0 < \phi(z(\tau)) < 1,$$

where $z(\tau)$ is our multi-product firm's investment rate for industry τ_{ij}.

$$0 < z(\tau) \equiv I(\tau) \,/\, k(\tau) < 1.$$

We denote the effective units of investment rate by $\phi(\tau)$ and expressed as follows:

$$\phi(z(\tau)) \equiv 1 + b(\tau)\log z(\tau);\ 0 < z(\tau) < 1.$$

This expression suggests that no adjustment costs occur when investment rate is equal to 1. The formula also trivially fulfills one

[5] This figure is drawn using the software Mathematica.

condition of Hayashi's homogeneous degree for the adjustment cost function.

Investments in fixed capitals differ from those in working capitals, such that reaping final outputs from production take time. The knowledge to be shared with the forthcoming production of goods compensate for such costs. We consider that the knowledge of advanced economy is embodied by imported fixed capital goods. Such knowledge is transmitted to small open developing economies. Knowledge on the production of goods in previous industries is shared with that of the production of goods in the succeeding adjacent industry production. Therefore, investments on fixed capital goods result in the "economy of scope" in *chaebol*s.

Figure 2 shows the effective units of investment $\phi(z)$ on the vertical axis with respect to the investment rate $z(\tau)$ on the horizontal axis. A relevant range of investments is indicated in the investments rates above the critical rate $\hat{z}$. That is, $\hat{z} < z < 1$. Sunk costs emerge for investments below the critical investment rate: $0 < z < \hat{z}$. The effective units of capital goods vary based on the shared investments of fixed-capital goods in the total amount of investment denoted by $b(\tau)$. The higher the shared investments of fixed capital goods, the lower the effective units and the higher the critical investment rates are. Figure 2 illustrates the critical investment rates for the three cases of the shared investments of fixed capital. This figure indicates that critical rates increase as shares increase from 0.3, 0.4, and 0.5.

No output is possible at an investment rate below the critical investment rate $\hat{z}$, in which

$$\varepsilon(\hat{z}(\tau)) \equiv \left(1 + b(\tau)\log \hat{z}(\tau) - b(\tau)\right) = 0.$$

We express the viability condition of investment for which $\varepsilon(z(\tau)) > \varepsilon(\hat{z}(\tau))$ as follows:

$$\varepsilon(z(\tau)) = \left(1 + b(\tau)\log z(\tau) - b(\tau)\right) > 0.$$

However, once the firm crosses over the critical investment rate $\hat{z}$, its effectiveness increases at a substantial rate with the increase of investment rate: $\varepsilon'(z(\tau)) > 0$.

IV. Role of Fixed Capital Goods in Tobin's *q*

Multi-product firm $\tau \in (\tau_i, \tau_j)$ produces its nearby products τ_i and τ_j. The firm employs labor in the amount of $L(\tau)$ at the wage rate of w. Intermediates for the amount of $v(\tau)$ are used at the price of $p_v(\tau)$ with fixed capital goods of $k_F(\tau)$. Such goods are presumably imported at the international price of $p_F(\tau)$. The rental service price of capital goods is r. The composite of the final goods $y(\tau)$ of the two nearby products are sold at prices $p(\tau_i)$ and $p(\tau_j)$. The profit of the firm is shown as Equation (3).

$$\pi(\tau) = y(\tau) - p_v(\tau)v(\tau) - wL(\tau) - rp_{k_F}(\tau)k_F(\tau). \tag{3}$$

Suppose that our "multi-product" has access to the loanable fund market for its investment at interest rate r.[6] Loans made by the firm are transformed into fixed capital goods by investments for the production of goods. This decision of the firm involves a trade-off between adjustment costs and their efficiency. These costs increase with the increase of investments. The increase in capital stock attained by the firm enhances the efficiency of the production by reducing adjustment costs. This problem can be solved best by the firm's optimization problem (P).

$$\begin{aligned}
&\max_{k(\tau),I(\tau)} \int_0^{\infty} \pi(\tau)\, e^{-\tilde{r}\tau} d\tau \\
&s.t. \\
&\dot{k}(\tau) = I(\tau) - \delta k(\tau) \\
&\tilde{r} = r - \theta - \gamma,
\end{aligned} \tag{P}$$

The constraint of this maximization problem indicates that capital stock decreases by its use at the rate of $0 < \delta < 1$. Therefore, the remaining investments are added to the previous one. Government subsidy denoted by θ influences favorably on the investment rate. A reduction of corporate tax rate γ has the same effect as government subsidy, such that the rate reduces investment cost. The effective rate of interest $\hat{r}$ is the rate of government subsidy. The rate of reduced

[6] We consider that the rental rate of capital goods is at the interest rate in the financial loanable fund market in perfect competition.

corporate taxes is deducted from the market rate of interest.

A modified Tobin's q^m of a "multi-product" firm is of the following expression:

$$\begin{aligned} q^m(\tau) &= \frac{\lambda(\tau)}{p_{K_F}(\tau)} \\ &= \frac{\hat{r}b(\tau)}{\varepsilon(\tau)}. \end{aligned}$$

Its numerator is the capital value, while the denominator is the replacement cost. The second row is determined by the first-order conditions of the firm's maximization problem with respect to investment rate $I(\tau)$ and capital stock $k(\tau)$. The numerator is the share of fixed capital goods in investments multiplied by the effective interest rate. Moreover, the denominator is the effective units of the investment rate for the production of good τ. We note that modified Tobin's q^m is a monotonically decreasing function of $z(\tau)$. Moreover, an equilibrium investment rate $z^*(\tau)$ exists, in which $q^m(\tau) = 1$ in the interval of $0 < \hat{r}b(\tau) < 1$.[7] Investment occurs if $q^m(\tau) > 1$. Furthermore, a decumulation of capital stock occurs if $q^m(\tau) < 1$. The modified Tobin's q^m fulfills a sufficient condition for a Tobin's q.[8]

A comparative statistics applied on the modified Tobin's q with respect to the technical coefficient of fixed capital good $b(\tau)$ implies that the higher the share of fixed capital goods, the higher the investment rate. Therefore, a Tobin's q of *chaebol*s, which succeeded to breakthrough high investments of fixed capital goods at the initial period of development, is also high as well as its investment rate. A high share of fixed capital implies high adjustment cost to a potential entrant to the industry.

Investments for the industry are also constrained by the amount of loans, in which interest rate $r(\tau)$ is paid. Constant δ is the rate of depreciation on capital, and the gross interest rate is the sum of the interest and depreciation rates of capital goods. That is, $r(\tau) + \delta$.

[7] This condition is fulfilled unless the interest rate is above 100%.

[8] We offer a derivation of the results on request.

Table 1
Average Value of the Selected Variables

	Number of observations	I	K	I/K	CF/K	Tobin's Q
Incumbents	2,205	484.5	4294.0	0.096	0.070	1.033
Non-incumbents	35,399	14.1	176.8	0.089	0.240	1.101
Total	37,604	56.4	519.9	0.090	0.226	1.095

Note: Numbers in the table are simple average values of the whole sample. I and K are in billion won (at 2010 constant). The nominal values of investments and K are converted into constant values by the price index.

V. Empirical Results

A. Data

All firm-level data are from the KIS Value database, except for firm investment data, which are obtained from the FnGuide database. Price indexes are from the Bank of Korea database. We select only manufacturing companies to analyze investment behavior under similar characteristics. The number of firms in our sample is 1,106, while the period is from 1982 to 2015. The total number of samples is 37,604.[9]

We classify all firms into two groups: firms affiliated with business groups (group firms) and firms that are not part of business groups (independent firms). We follow the classification of the KIS Value database, which reports whether a firm is included in a business group.[10]

Table 1 provides the average values of the selected variables for the incumbents and non-incumbents. The average investment is 484.5 billion won for incumbents and 14.1 billion won for non-incumbents. The average capital stock is 4,294 billion won for incumbents and 176.8 billion won for non-incumbents. Therefore, incumbents' capital stocks are 24 times higher than that of independents. Investment rate (investment [I] divided by capital stock [K]) is higher in incumbents (0.096) than in non-incumbents (0.089). Thus, incumbents invest more

[9] Data used in this study are the same as that in Kim and Kim (2018).

[10] The KIS Value reports that they classified the company group by the Korean Fair Trade Act.

than non-incumbents even with capital stocks. The ratio of cash flow to capital stock is higher in non-incumbents (0.240) than in incumbents (0.070), thereby indicating that non-incumbents hold relatively more cash than incumbents. The average value of Tobin's q is lower for incumbents than for non-incumbents (1.033 *vs.* 1.101, respectively). Therefore, the high replacement costs of capital stocks for incumbents in their conventional are the accounting measures of Tobin's q.[11]

B. Regression Results

The investment equation for a regression analysis for firm i is written as Equation (4):

$$\left(\frac{I}{K}\right)_{it} = cons + \beta_1 \left(\frac{I}{K}\right)_{i(t-1)} + \beta_2 Q_{it} + \beta_3 CFK_{it} + \varepsilon_{it}. \qquad (4)$$

We comprise a variable for a firm's cash constraint CFK_{it}, which determines the liquidity constraints that the firm may face. Given the asymmetric information between the managers of the firm and potential creditors, the firm occasionally determines that increasing external financing difficult. Thus, the availability of internal financing limits its investment.[12] Error term ε_{it} may contain company-specific effects α_i, time-specific effects α_t, and idiosyncratic shock υ_{it}. Variable Q is $q - 1$, where q is a Tobin's average q. The variable CFK divides cash flow by the beginning of period K.[13] Coefficient β_2 estimates the effect of a Tobin's q on its investment rate. In terms of our model, this effect is high for a firm with low adjustment cost. Therefore, the estimated coefficient for *chaebol* incumbent firms, with low adjustment costs for investment, is expected to show a high coefficient.

We use dummy variables to analyze the differences between incumbents and non-incumbents. The dummy variable DG is given

[11] The sum of equities and debts of the firm are divided by the replacement costs of capital stocks.

[12] Fazzari *et al.* (1988) argue that when financial markets are imperfect, firms rely on retained earnings to fund investment before they turn to external funds. Thus, investment increases with high cash flow or retained earnings.

[13] Abel and Eberly (2011) analytically show that investment is positively related to Tobin's q and cash flow even in the absence of adjustment costs or financing frictions.

Table 2
Regression Results

	(1)	(2)	(3)	(4)
$(I/K)t-1$	0.3362***	0.3097***	0.3369***	0.2988***
	(9.39)	(8.89)	(9.47)	(9.55)
Q	0.0274***	0.0259***	0.0266***	0.0236***
	(13.11)	(11.71)	(12.78)	(10.46)
CFK		0.1334***		0.1343***
		(3.61)		(3.85)
*Q*DG*			0.0427**	0.0466**
			(2.12)	(2.09)
*CFK*DG*				−0.0147
				(-0.14)
Hansen Test	677.47	750.50	624.82	571.03

Note: (1) The numbers in parentheses are the *z*-values. (2) *, **, and *** denote that the explanatory variable is statistically significant at the 10%, 5%, and 1% levels, respectively. (3) The Hansen test shows that the instrument set is valid. (4) AR (1) is statistically significant but AR (2) is not, thereby indicating that the error term of this model does not show autocorrelation.

"1" for incumbents and "0" for non-incumbents. Thus, if the coefficient estimate of the dummy variable is positive (+), then coefficient β_2 for incumbents is high as expected.

We use a generalized method of moments (GMM) estimation. The GMM estimator provides consistent estimates of parameters, regardless if a lagged dependent variable and other endogenous regressors are introduced into the model. In addition, a valid set of instrument variables should be used. We use the dynamic panel estimator method in two steps with the no-level option, while the lag values of explanatory variables are used as instrument variables with a maximum time lag of seven years.[14]

Table 2 shows the regression results. The sample contains 1,106 firms for 34 years from 1982 to 2015. Columns (1) and (2) are the results from the basic investment function, while (3) and (4) are the results using dummy variables to determine the difference between incumbents

[14] We use the estimation procedures of the STATA statistical software.

and non-incumbents. The estimated coefficient β_2 on Q is positive and statistically significant in the 0.0236–0.0274 range. Even if we add cash flow as an explanatory variable, the coefficient and its significance are still almost similar to the results without the variable. Coefficient β_3 for cash flow (CFK) is positive and significant, thereby indicating that the more cash holdings companies have, the more investments they make. However, the coefficient of CFK*DG is negative and insignificant.[15]

Columns (3) and (4) reveal the differences between incumbents and non-incumbents. The dummy variable of *DG* is "1" for incumbents and "0" for non-incumbents. Thus, if the coefficient of *Q*DG* is positive, then coefficient β_2 for incumbents is higher than that for non-incumbents. Columns (3) and (4) show that the coefficients for non-incumbents are 0.0236 or 0.0266, while those for incumbents are larger than those for non-incumbents, which are 0.0427 or 0.0466. Thus, the mean values of coefficients for non-incumbents are 0.0251 and 0.0697 for incumbents. This difference suggests that the adjustment cost of incumbents is 1.77 times low for non-incumbents.

VI. Endogenous Growth of *Chaebol*s

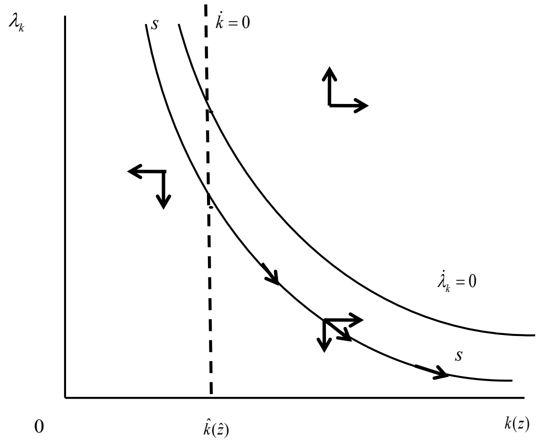
One often comes across a metaphor on the growth of the South Korean *chaebol* in the newspaper saying that "it is like riding a bicycle." If the *chaebol* stops growing, then it falls. The present model implies that *chaebol*s' view on investments is to maintain the accumulation rate from decreasing. Therefore, an endogenous growth path consistent with *chaebol*s' perception on the accumulation of capital exists.

*Chaebol*s' aim for capital accumulation modifies its constraint of the previous maximization problem (P) as follows:

$$\dot{k}(\tau) = \phi\left(z^*(\tau); k(\tau)\right) k(\tau) - \delta k(\tau).$$

This constraint partly relates to *chaebol*s' remaining ownership control on physical capital goods. The solution for the modified model of (P) is related to *chaebol*s' finding of fixed capital stock related to this aim. This solution is a plausible proposition for a firm situated at the

[15] Hoshi *et al.* (1991) show that liquidity had a great impact on investment in independent firms using Japanese firm data.



*Chaebol*s accumulate capital for $k(z) > \hat{k}(\hat{z})$. A shadow value of capital good λ_k for *chaebol*s decreases on the time path of ss, for which $\dot{\lambda}_k = 0$. *Chaebol*s with a capital stock below its critical level $\hat{k}$ shrink to the zero level of its capital stock.

FIGURE 3
PHASE DIAGRAM OF THE ENDOGENOUS GROWTH PATH OF *CHAEBOLS*

initial developmental stage of the economy, in which the existence of capital goods is nearly negligible. The shadow value of capital stock denoted by λ_k, which differs from the shadow value of investment λ for the preceding maximization problem, is expressed as follows:

$$\dot{\lambda}_k = -\frac{\partial H}{\partial k} = -(A - rp_{k_F}) + \lambda_k \left[\phi^{'}(k)k + (\phi(k) - \delta)\right].$$

Note that *chaebol*s' capital accumulation, which associates with the expansion of industries along the horizontal axis in Figure 1, reduces the required rate of fixed capital for the forthcoming industry. Therefore, the effective units of capital increase, thereby implying that $\partial\phi(\cdot) / \partial k = \left(\partial b(\tau) / \partial k\right) \cdot \log z(\cdot) > 0$.[16]

Figure 3 presents the solution of this problem in terms of a phase diagram of *chaebol*s' growth path.

[16] Recall the efficiency condition and Figure 2. The effective units of capital $\phi(\tau)$ increase for an industry, in which $b^{*}(\tau)$ is low for a given rate of investment $z^{*}(\tau)$.

This phase diagram shows that the horizontal axis is capital stock k, while its vertical axis is its shadow price λ_k. Line $\dot{\lambda}_k = 0$ slopes downwards by the following inequality condition:

$$\left.\frac{\partial\lambda}{\partial k}\right|_{\dot{\lambda}_k=0} = -\lambda_k \frac{\left[\phi^{''}(k)k + 2\phi^{'}(k)\right]}{\phi^{'}(k)k + \phi(k) - \delta} = -\lambda_k \frac{\phi^{'}(k)\left[\frac{\phi^{''}(k)k}{\phi^{'}(k)} + 2\right]}{\phi^{'}(k)k + \phi(k) - \delta} < 0. \tag{5}$$

New industries unfold along *chaebol*s' capital accumulation path on the horizontal axis in Figure 1. Therefore, the effective units of capital $\phi(k)$ are replaced as a function of $\phi(\tau)$ such that

$$\begin{cases} \phi^{'}(\tau) = b^{'}(\tau)\log z^{*}(\tau) > 0; \\ \phi^{''}(\tau) = b^{''}(\tau)\log z^{*}(\tau) < 0. \end{cases}$$

The expression $\frac{\phi^{''}(k)k}{\phi^{'}(k)}$ on the second row of Equation (4) can be expressed in the elasticity form of required fixed capital goods on the frontier technology with respect to the entrance on new industry τ, $\eta_{b(\tau)}$:

$$-2 < \eta_{b(\tau)} = \frac{\phi^{''}(\tau)\tau}{\phi^{'}(\tau)} = \frac{\phi^{''}(k)k}{\phi^{'}(k)} < 0.$$

In the elasticity condition on $\eta_{b(\tau)}$, the sign of Equation (5) is negative and it justifies the downward-sloping endogenous growth path of *chaebol*s denoted as curve "ss" in Figure 3.

Line $\dot{k} = 0$ is vertical at the critical capital stock level of $\hat{k}$. Capital accumulates continuously on its right side, whereas it decreases on its left side. The shadow value of capital goods for *chaebol*s increases above the steady path of $\dot{\lambda}_k = 0$ owing to elasticity condition, whereas the value decreases below the steady path.

This endogenous growth path is expected from *chaebol*s' linear technology as the path holds in an endogenous growth model. The growth rate is high for a firm, in which the share of fixed capital goods in investment is high. Thus, their effective units are also high. Once it is on the endogenous growth path, it maintains growing along the path similar to riding a bicycle as long as it crosses over the hurdles of fixed capital goods. The following is the result.

Result

*Chaebol*s with a capital stock above their critical level of $\hat{k}$ grows along the growth path of *ss* in Figure 3 at a growth rate of $(\tilde{\phi}(z^*) - \delta) \forall 0 < z^* < 1$.

As the catch-up effect of *chaebol*s is exhausted and as the effect approaches the frontier technology, the required amount of fixed capital goods gradually declines. In addition, its growth rate falls.

VII. Concluding Remarks

This study views the growth of South Korean *chaebol*s in the past half century from the viewpoint of their investment related to a modified Tobin's *q*. A high share of fixed capital goods in their investments provides *chaebol*s low adjustment cost advantages. A firm grows at an effective accumulation rate. The effective rate of capital accumulation is high for *chaebol*-incumbents. Moreover, they increase at a higher rate than non-incumbents, thereby contradicting Gibrat's law.

The growth of *chaebol*s enables them to play their role as a conduit for a structural change of the Korean economy in this respect. As a late industrializing country reaching its mature stage, we expect that fixed physical capital goods can give way to human capital.

Appendix

Let $c(y(\tau_i))$ denote the cost of producing good $y(\tau_i)$. From the Cobb-Douglas function of products τ_i and τ_j, we have the following expression for a unit cost function of product i for a given rental rate r and wage rate w.

$$c(y(\tau_i)) = A(\tau_i)^{-\alpha} \kappa r^{\alpha} w^{1-\alpha}$$

Similarly, the cost function for good $y(\tau_j)$ is as follows:

$$c(y(\tau_j)) = A(\tau_j)^{-\alpha} \kappa r^{\alpha} w^{1-\alpha},$$

where $0 < \kappa$ is a given constant in term of α. $A(\tau_{ij})$ is not lower than the technology terms of $A(\tau_i)$ and $A(\tau_j)$. Thus, the following inequality is provided:

$$A(\tau_{ij})^{-\alpha} c\left(\xi y(\tau_i) + (1-\xi) y(\tau_j)\right) = A(\tau_{ij})^{-\alpha} \xi c(y(\tau_i)) + A(\tau_{ij})^{-\alpha}(1-\xi) c(y(\tau_j))$$
$$< \left(A(\tau_i)^{-\alpha} \xi c(y(\tau_i)) + A(\tau_j)^{-\alpha}(1-\xi) c(y(\tau_j))\right).$$

Furthermore, "economy of scope" exists.

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