Time-Dependent Pricing for Bandwidth Slicing under Information Asymmetry and Price Discrimination

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***Abstract*—Due to the bursty nature of Internet traffic, network service providers (NSPs) are forced to expand their network capacity in order to meet the ever-increasing peak-time traffic demand, which is however costly and inefficient. How to shift the traffic demand from peak time to off-peak time is a challenging task for NSPs. In this paper, we study the implementation of time-dependent pricing (TDP) for bandwidth slicing in software-defined cellular networks under information asymmetry and price discrimination. Congestion prices indicating real-time congestion levels of different links are used as a signal to motivate delay-tolerant users to defer their traffic demands. We formulate the joint pricing and bandwidth demand optimization problem as a two-stage Stackelberg leader-follower game. Then, we investigate how to derive the optimal solutions under the scenarios of both complete and incomplete information. We also extend the results from the simplified case of a single congested link to the more complicated case of multiple congested links, where price discrimination is employed to dynamically adjust the price of each congested link in accordance with its real-time congestion level. Simulation results demonstrate that the proposed pricing scheme achieves superior performance in increasing the NSP's revenue and reducing the peak-to-average traffic ratio (PATR).**

***Index Terms*—Time-dependent pricing, bandwidth slicing, price discrimination, information asymmetry, software-defined cellular networks.**

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I. Introduction

As pointed out by Cisco, global mobile Internet traffic has grown 17-fold over the past five years. In particular, with the proliferation of multimedia applications and ever-growing demands for multimedia data, peak-time traffic has increased by approximately 50 percent and will keep growing at high speed [1]. To accommodate the peak-time traffic, network service providers (NSPs) have to continuously expand their network capacity by investing in more infrastructure. Nevertheless, the pace of deploying new network infrastructure can hardly catch up with the growth of data traffic. When the network capacity is insufficient to meet the quality of service (QoS) requirements (e.g., bandwidth demand, delay, or jitter [2]) of all on-going traffic, coordination amongst different traffic flows is essential to guarantee reliable service provisioning. However, considering the bursty nature of multimedia data traffic and the corresponding high peak-to-average traffic ratio (PATR), it is not easy to achieve a network-wide coordination without a powerful centralized controlling unit and a scalable management framework.

Software-defined networking (SDN) which decouples the control plane from the data plane provides a flexible and programmable framework for implementing centralized traffic control and management in cellular networks [3]. Since all control functionalities are left to the control plane, the SDN controller with a network-wide view uses the southbound application programming interface (API) to apply different traffic control and management policies without incurring a significant modification to underlying routers or switches [4]. Moreover, by integrating SDN with network function virtualization (NFV) [5], network resources such as bandwidth, CPU, buffer, etc., can be virtualized, split into distinct slices, and allocated dynamically based on the overall network states. This provides a new paradigm for NSPs to cope with the peak-time traffic via on-demand resource slicing.

In this paper, we mainly focus on bandwidth slicing in software-defined cellular networks, while the derived results can be naturally extended to the slicing of other virtual resources. Towards implementing on-demand bandwidth slicing, bandwidth provisioning has to be adapted to the temporal-spatial variation of link condition and traffic demand. Pure-technical solutions mainly rely on dividing traffic into separate classes for applying different control policies via either parsing Ethernet, IP, or TCP/UDP header fields or using deep packet inspection technology [6]–[9]. However, these approaches require frequent signaling message exchange between the controller and the data plane. Moreover, the implementation complexity increases exponentially with the volume of data traffic. Thus, instead of relying on pure technical approaches, we explore an economic approach, i.e., pricing, which differentiates links with different congestion levels and enables autonomous user behavior adjustment. Specifically, effective peak-time traffic control can be realized by charging users not only based on the amount of required bandwidth, but also according to the time of usage. Such kind of pricing scheme is called time-dependent pricing (TDP) [10], [11]. Compared to other pricing schemes, TDP is more effective to flatten the fluctuation of the peak-time traffic demand over the whole network by taking both the spectral and temporal dimensions into consideration. For instance, some delay-tolerant users may choose to shift their bandwidth demands from the peak time to the non-peak time in order to reduce the high peak-time costs.

However, when implementing TDP for bandwidth slicing in software-defined cellular networks, there exist two major challenges that have to be addressed, i.e., information asymmetry and price discrimination. First, due to the heterogeneous nature of users, the delay sensitivity of each user is not identical. Namely, some users value the cost of delay more than others. Generally, users with low delay sensitivity are more prone to shift their demands from peak to non-peak time. Nevertheless, each user's delay sensitivity is generally private information, which is only known to the user itself and is unavailable to the NSP. This raises the so-called problem of information asymmetry. How to design a pricing scheme to motivate users to defer their traffic (or bandwidth demands equivalently) during traffic peak time while simultaneously maximizing the economic benefits of the NSP under with asymmetric information is still nontrivial.

Second, conventional TDP studies generally adopt a uniform pricing approach [12], [13], in which the congestion price per unit of bandwidth imposed on different congested links is uniform. Uniform pricing can be easily implemented since all the congested links are treated in the same way. However, the economic benefits of the NSP are severely degraded compared to the discriminatory pricing approach where the price of each congested link is dynamically adjusted in accordance with the real-time congestion level. On the other hand, price discrimination also increases the computation complexity, thus making the formulated optimization problem intractable.

The motivation of this work is to design a new TDP solution to motivate users to defer their traffic (or bandwidth demands equivalently) during traffic peak time while simultaneously maximizing the economic benefits of the NSP under information asymmetry and price discrimination. We formulate the pricing and bandwidth demand joint optimization problem by using a game-theoretical approach to capture the competitive interactions between the NSP and users. In particular, a two-stage Stackelberg leader-follower game is employed to model the dominant market position of the NSP over the users. The proposed scheme consists of two stages. In the first stage, the SDN controller issues congestion prices which indicate the current link congestion levels to users. The congestion price is used as a signal to incentivize delay-tolerant users to postpone their bandwidth demands. Then, each user individually determines whether to defer its demand or not by comparing the delay costs with the congestion penalty. In this way, the controller does not need to inspect the traffic of all the users, which can significantly reduce the overall signaling overhead. The proposed scheme can also be easily implemented in the more practical incomplete information scenario. Moreover, it is actually an economic approach which does not require any modification to the existing SDN architecture and preserves great consistency. The main contributions of this work are summarized as follows:

- **Multiple scenarios of information availability and cases of link congestion.** The complete information scenario where the NSP has the perfect knowledge of each user's delay sensitivity is firstly studied to serve as a benchmark. Then, the incomplete information scenario where the NSP possesses only the statistical information of users' delay sensitivity is investigated. Since the conventional deterministic approach cannot be directly applied, a stochastic modeling approach is employed to derive the optimal pricing strategy. Furthermore, we not only consider the simplified case of a single congested link but also investigate the more complicated case with multiple congested links with price discrimination for both the complete and incomplete information scenarios.
- **Joint pricing and bandwidth splicing optimization under information asymmetry and price discrimination.** We jointly optimize pricing and bandwidth slicing by using the backward induction approach under various practical constraints. We start from the second stage, and derive both the deterministic and stochastic expressions of users' best response strategies for the complete and incomplete information scenarios, respectively. Then, based on the derived users' best response strategies, the discriminatory pricing problem in the first stage is solved to maximize the NSP's utility. When considering both information asymmetry and price discrimination, the formulated optimization problem is NP-hard since the pricing variables of different congested links are coupled together. To provide a tractable solution, we transform the non-convex optimization problem into a variational inequality problem and then provide a gradient-based iterative pricing algorithm. We also provide rigorous theoretical analysis from the perspectives of convergence, computational complexity, and solution uniqueness.
- **Effective peak shaving.** Extensive simulations are

conducted to evaluate the performance of the proposed TDP scheme. Simulation results verify that the drastic fluctuation of bandwidth demand can be effectively flattened compared to other heuristic algorithms such as flat-rate pricing and uniform pricing. It is observed that the incorporation of price discrimination can significantly increase the profit of the NSP and reduce the network PATR.

The remainder of this paper is organized as follows. Section II provides a comprehensive literature review. Section III introduces the system model. Section IV describes the Stackelberg game formulation. Section V presents the optimal pricing design with complete information. Section VI presents the optimal pricing design with incomplete information. Section VI-B provides the numerical results. Finally, section VIII concludes this paper.

II. Related Works

SDN provides an open and programmable platform to implement policies of smart bandwidth slicing. Due to the promising features, numerous researchers have already studied SDN-enabled bandwidth slicing. In [14], Yiakoumis *et al.* proposed the concept of bandwidth slicing in software-defined wired networks, where bandwidth is virtually split into multiple slices for dedicated usage of different services. A similar idea was mentioned in [15], where Radhakrishnan *et al.* proposed *NetShare* to enable predictable bandwidth allocation for different services without changing the hardware of switches or routers. In [16], Ksentini *et al.* proposed a programmable framework to enable network slicing based on 3GPP dedicated core networks (DCNs) and a two-level medium access control (MAC) scheduler to facilitate physical resource abstraction and sharing. In [17], Li *et al.* proposed a SDN-based framework for machine-to-machine (M2M) communications. A feedback and control loop which dynamically adjusts resource allocation based on the performance gap was developed to address the random access problem. SDN-based bandwidth slicing was extended to 5G cellular networks [7], and a resource-slicing based architecture was proposed to enable mobility management, power control, and subchannel allocation.

When studying SDN-enabled bandwidth slicing, the domain-specific challenges of SDN such as scalability and consistency problems have to be considered. Namely, as the number of flows increases in SDN, it takes a large amount of time for the controller to handle all the control functionalities. Consequently, the controller might become the bottleneck of the whole system. One way to alleviate the scalability problem is to reduce the load on the controller. For instance, in DevoFlow [18], only large flows are forwarded to the controller for coordination, while other short-lived flows are handled in the data plane locally. This way, the amount of requests forwarded to the controller could be significantly reduced. However, the main drawback of this approach is that it imposes modifications to the existing SDN architecture, i.e., it raises new concerns about consistency. Some research attempts try to alleviate both the scalability and consistency problems by distributing the control functionality across hierarchical controllers. In Kandoo [19], the control functionalities that require network-wide coordination are handled by a root controller, while the others are handled by local controllers. Therefore, Kandoo can preserve scalability without changing the design of SDN switches. Nevertheless, retaining network state consistency through frequent propagation of state updates will introduce unbearable signaling overhead between the controllers and switches. Another thread of research attempts focuses on using economic approaches such as pricing to control the bandwidth demands and regulate link access behaviors of users. For instance, given a well-designed congestion price, users can autonomously adjust their bandwidth demands to avoid the high congestion costs from the perspective of individual profit maximization. There already exist some studies on the design and optimization of pricing strategies [20], [21]. Flat-rate pricing has long been the prevailing pricing model for both wired and wireless networks due to its simplicity [22]. The drawback is that light users cannot be distinguished from heavy users, which implicitly forces light users to subsidize heavy users.

Since May 2011, worldwide NSPs, such as AT&T, Comcast, etc., have started to impose a data cap (i.e., an amount of data that can be used during a certain period of time) on users and charge a penalty as long as the usage exceeds a pre-defined limit [23]. This mechanism is named usage-based pricing (UBP). Furthermore, with the emergence of mobile virtual network operators (MVNOs), some hybrid UBP schemes start to play an important role in real-world network operations. For example, Karma [24] designed a UBP scheme by offering a fixed reimbursement to its subscribers when they share their Internet connectivity to other nonsubscribers in the vicinity. In [25], Iosifidis *et al.* extended the Karma model by adjusting the reimbursement according to the amount of traffic that a subscriber actually forwards. UBP is efficient in controlling the total data usage for a certain period of time, but inefficient in resolving the peak-time congestion unless the price is charged in accordance with the real-time network congestion level.

By incorporating the information of temporal dimension, TDP has emerged as a promising solution for peak-time traffic management [10], [11]. Specifically, TDP addresses the peak-time congestion problem by charging users according to both the amount of bandwidth required and the time of usage. It can effectively flatten out the peak-time traffic demand and improve the overall bandwidth utilization efficiency [26].

TDP has received intensive research interest from both academia and industry. Jiang et al. proposed a TDP scheme to optimize the NSP's revenue in a monopoly market, and proved that if the NSP has full information about the users' utilities, the revenue-optimizing TDP also results in social welfare optimization [27]. In [28], Zhang et al. extended the analysis of TDP from the monopoly market to an oligopoly market, and developed a game-theoretic approach to solve the revenue optimization problem. In [29], Wong et al. proposed a novel day-ahead pricing (DAP) scheme by considering the feasibility of creating time-dependent charging model for real-world deployment, where prices are determined based upon historical traffic load, and are offered to users on a day-ahead basis. The feasibility and benefits of adopting TDP and DAP were extensively discussed in [30]–[32]. In [33], Ding et al. demonstrated that a unified TDP used by a whole network achieves poor performance for specific locations, and hence proposed a TDP for large-scale mobile networks by combining both spatial and temporal traffic patterns. In [34], Ma et al. formulated the payoff optimization problem of both the NSP and users as a two-stage decision process, and then derived an optimal time and location aware pricing scheme by solving the optimization problem. Nevertheless, the more practical scenario of incomplete information is neglected in [34] and [33]. In [35], Ha et al. presented an end-to-end TDP system named TUBE, in which a pricing-based feedback control loop is created between NSP and users. In [36], Sen et al. developed a framework of dynamic day-ahead TDP based on NSP's costs as well as users' usage volumes and delay willingness. Two real-world trials demonstrate that TDP can effectively incentivize users to adjust their usage demands and increase NSP's revenue with multimedia-rich applications. Both [35] and [36] adopt the simplified uniform pricing approach without considering price discrimination.

Different from the work mentioned above, we jointly consider pricing and bandwidth demand optimization in software-defined cellular networks, under the scenarios of both complete and incomplete information. Particularly, we emphasize on how to derive the optimal pricing strategy by using only the statistical information of users' preferences. Furthermore, we extend the previous results from the simplified case of a single congested link to the more complicated case of multiple congested links, where the optimal price of a congested link depends on the prices imposed on other congested links. To provide a tractable solution, we explore the variational inequality theory and propose a gradient-based iterative pricing scheme which converges to the optimal strategy.

Our work is different from the standard Stackelberg game based pricing schemes [37]–[40] due to information asymmetry and price discrimination. The details are elaborated as follows. First, different from the standard Stackelberg game where the leader has the perfect knowledge of followers' information, we investigate the more practical incomplete information scenario where the leader possesses only the statistical information of followers. With information asymmetry, the best response strategies of users are not deterministic, and the optimal pricing has to be derived based on a stochastic modeling approach. Second, for the case of multiple congested links, the formulated optimization problems with price discrimination are NP-hard. We propose two iterative heuristic algorithms to find the optimal solution, and provide rigorous property analysis for convergence, computational complexity, and uniqueness.

III. SYSTEM MODEL

Fig. 1 shows the architecture of a software-defined cellular network. It consists of three layers: a control layer, a data forwarding layer, and a user layer. In the control layer, a

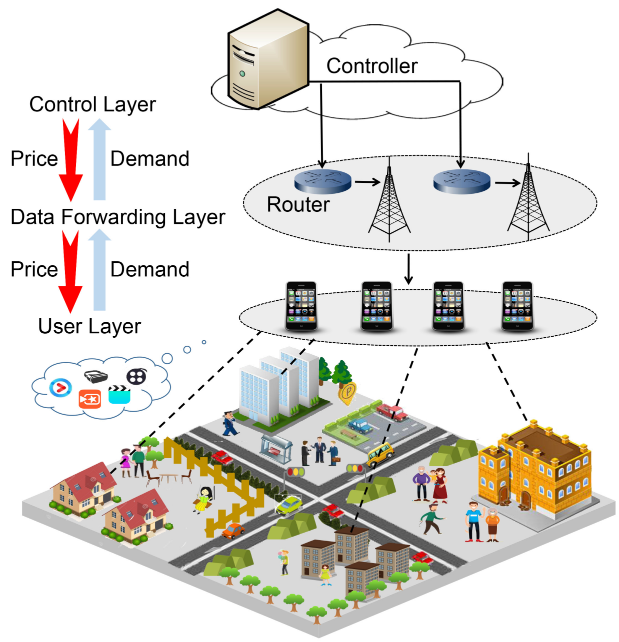

Fig. 1. Architecture of a software-defined cellular network.

centralized controller with a network-wide view takes charge of all control logic and enforces pricing policies by decoupling the network control functions from data delivery. The data forwarding layer contains various routers and base stations, which are responsible for forwarding multimedia data to users. In the user layer, there are multiple users who send (or fetch) a large amount of data to (or from) the multimedia servers. The path between each user and its destination server is composed of a set of logical links. That is, the data traffic of a user has to go through a set of logical links in order to reach its destination.

The overall negotiation and pricing procedure is composed of three stages. In the first stage, each user notifies its bandwidth requirement to the controller in an on-demand manner. In the second stage, the controller sets the congestion price of each logical link according to the bandwidth demand reflecting its congestion level. Note that the congestion price could be zero in the case that the link capacity is sufficient to meet the bandwidth demand (i.e., no congestion exists). Then, the congestion prices are broadcasted to users through the data forwarding layer. In the final stage, each user independently determines whether to defer its traffic or not upon receiving the corresponding congestion price. The users' decisions are fed back to the controller enabling it to dynamically adjust the bandwidth slicing based on the feedback. It is noted that in practical implementation, the users do not have to be involved in bandwidth demand submission and decision making. Alternatively, the proposed joint bandwidth slicing and pricing algorithm can be programmed as a software to autonomously collect bandwidth demands and make delay decisions. This way, the burden of decision making of users can be alleviated.

We assume that the network is composed of a set of logical

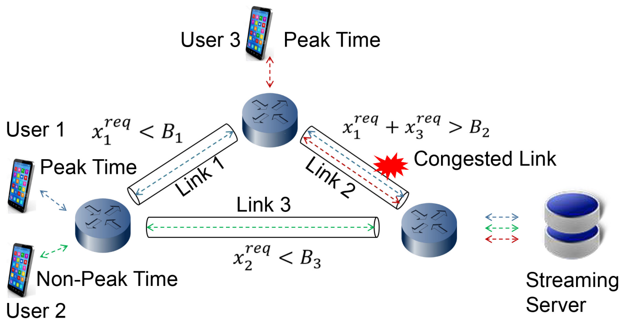

Fig. 2. Illustration of the congested link.

links (wired or wireless) denoted by $\boldsymbol{L}$, and a set of multimedia application users denoted by $\boldsymbol{S}$. A total amount of bandwidth B_l is reserved on each link $l \in \boldsymbol{L}$ for multimedia applications while the residual bandwidth is utilized by other types of applications. In this paper, we mainly focus on link congestion caused by multimedia applications. We henceforth refer to the multimedia application users as *users*, and the bandwidth reserved for multimedia applications as *available bandwidth* for simplicity.

Time is divided into slots. The length of each time slot is defined as τ. In slot t, the set of links which are used to transmit the data traffic of user s is denoted as $\boldsymbol{L}_t(s)$, i.e., $\boldsymbol{L}_t(s) \subseteq \boldsymbol{L}$. Moreover, the set of users whose traffic traverses link l is denoted as $\boldsymbol{S}_t(l)$, $\boldsymbol{S}_t(l) \subseteq \boldsymbol{S}$. The bandwidth requirement of user s is defined as $x_s^{\text{req}}, \forall s \in \boldsymbol{S}$. In slot t, link l is marked as the congested link if the available bandwidth of link l is insufficient to satisfy the bandwidth demands of all users on it, i.e., $\sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}} > B_l$.

An example is shown in Fig. 2. Among the three links, link 2 is a congested link. Then, the corresponding slot t is marked as a peak-time slot for user 1 and user 3, whose traffic traverses link 2.

Since we have not made specific assumptions for the employed routing scheme, the proposed scheme is compatible with other dynamic routing schemes as long as $\boldsymbol{S}_t(l)$ remains constant during slot t. The joint optimization of routing, pricing, and bandwidth slicing is a new challenging topic. It is beyond the scope of this work and will be studied in the future.

The centralized controller monitors the bandwidth demand on each link, and notifies the users of a congestion price to be imposed on them as long as they stay connected during peak time. Note that the nonnegative congestion prices $\{p_{l,t}\}_{l\in\boldsymbol{L}}$ are the variables to be optimized by the NSP. For each user s, the overall penalty imposed on it in slot t is calculated as

$$P_{s,t} = \sum_{l\in\boldsymbol{L}_t(s)} p_{l,t} x_s^{\text{req}} \tau. \tag{1}$$

If the penalty is heavy enough, rational users may voluntarily defer their traffic (or bandwidth demand equivalently) in order to avoid the congestion cost in the peak time. As a result, peak-time congestion can be alleviated. Considering the contradictory interest between the NSP and users, the joint optimization problem of congestion pricing and bandwidth slicing can be formulated by using a game-theoretical approach. The details are elaborated in the next section.

A summary of the main notations used throughout the paper is given in TABLE I.

TABLE I
A SUMMARY OF THE MAIN NOTATIONS.

Notation	Description
$p_{l,t}$	Unit congestion price of link l in slot t
$P_{s,t}$	Additional payment of user s if he/she stays connected in slot t
$\boldsymbol{L}$	Set of logical links (wired or wireless)
$\lvert\boldsymbol{L}\rvert$	Number of links in $\boldsymbol{L}$
B_l	Maximum available bandwidth of link l
x_s^{req}	Bandwidth required by user s
τ	Slot length
d_s	Time delay of user s
$C_s(d_s)$	Delay cost of user s when the streaming service is delayed by d_s.
$\widetilde{C}_s(d_s,\tau)$	Marginal delay cost of user s the traffic of which is further deferred by one slot
θ_s	User-dependent factor indicating the scale of its utility function (i.e., delay sensitivity)
$b_{s,t}$	Indicator for the response of user s in slot t
$\boldsymbol{S}_t(l)$	Set of users on link l in slot t
$\boldsymbol{S}_t(l)\cap\boldsymbol{S}_t(n)$	Set of users, the traffic of which traverses both link l and link n in slot t
$B_{l,t}^{\text{demand}}(p_{l,t})$	Bandwidth demand on link l in slot t
$p_{l,t}^*$	Optimal congestion price of slot t

IV. STACKELBERG GAME FORMULATION

A. Game Formulation

In this section, a Stackelberg leader-follower game is formulated to model the competitive interaction between the NSP and users [10]. The Stackelberg game is a strategic game in which a leader player chooses its strategy first and then other followers move accordingly. In this work, the NSP is the leader that decides the congestion price of each link, i.e., $\{p_{l,t}\}_{l\in\boldsymbol{L}}$, and the users are the followers that determine whether to defer their traffic or not based on $\{p_{l,t}\}_{l\in\boldsymbol{L}}$.

We assume that both the NSP and the users are rational decision makers [41]. Let $b_{s,t}$ be the response indicator which is defined as follows: $b_{s,t} = 1$ if user s stays connected in slot t; or $b_{s,t} = 0$ if user s chooses to defer its traffic in slot t. Denote the response set of all users except user s as $\boldsymbol{b}_{-s,t}$. Namely, $\boldsymbol{b}_{-s,t} = \{b_{1,t}, ..., b_{s-1,t}, b_{s+1,t}, ..., b_{|\boldsymbol{S}_t|,t}\}$, where $|\boldsymbol{S}_t|$ is the number of users in set $\boldsymbol{S}_t$. The Stackelberg leader-follower game can be described by a tuple $\mathcal{G}$(Player, Strategy, Payoff), which is elaborated as follows:

- **Player**: The NSP is the leader and the multimedia application users are the followers.
- **Strategy**: For the NSP, the strategy is the selection of $p_{l,t}$ for each link l in time slot t. For each user s, the strategy is the decision on whether to defer its traffic or not given $p_{l,t}$, i.e., $b_{s,t}$.
- **Payoff**: For the NSP, the payoff is the revenue gain denoted by $\Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}}, \{b_{s,t}\}_{s\in\boldsymbol{S}})$; for a user s, the payoff is the net benefit denoted by $\Pi_s(b_{s,t}, \boldsymbol{b}_{-s,t}, \{p_{l,t}\}_{l\in\boldsymbol{L}_t(s)})$.

The optimal strategies of the NSP and users, i.e., ($\{p_{l,t}^*\}_{l\in\boldsymbol{L}}$, $\{b_{s,t}^*\}_{s\in\boldsymbol{S}}$), constitute a Stackelberg equilibrium, in which no player can improve its benefit by changing its own strategy unilaterally. In other words, each player chooses the best (locally optimal) response to the strategies adopted by other players [42].

The objective is to find the Stackelberg equilibrium, which is defined as follows:

Definition 1. *$\left(\{p_{l,t}^*\}_{l\in\boldsymbol{L}}, \{b_{s,t}^*\}_{s\in\boldsymbol{S}}\right)$ is a Stackelberg equilibrium if for any $\{p_{l,t}\}_{l\in\boldsymbol{L}}$ and $\{b_{s,t}\}_{s\in\boldsymbol{S}}$, we have*

$$\begin{aligned}&\Pi^{NSP}(\{p_{l,t}^*\}_{l\in\boldsymbol{L}}, \{b_{s,t}^*\}_{s\in\boldsymbol{S}}) \geq \Pi^{NSP}(\{p_{l,t}\}_{l\in\boldsymbol{L}}, \{b_{s,t}^*\}_{s\in\boldsymbol{S}})\\ &\Pi_s(b_{s,t}^*, \boldsymbol{b}_{-s,t}^*, \{p_{l,t}^*\}_{l\in\boldsymbol{L}_t(s)}) \geq \Pi_s(b_{s,t}, \boldsymbol{b}_{-s,t}^*, \{p_{l,t}^*\}_{l\in\boldsymbol{L}_t(s)}),\end{aligned} \tag{2}$$

where $\boldsymbol{b}_{-s,t}^*$ denotes the optimal value of $\boldsymbol{b}_{-s,t}$.

B. Users' Delay Sensitivities

The delay plays an important role in evaluating the users' satisfaction over the multimedia services. Therefore, we model the utility of a user as a function of delay. Specifically, the cost of user s when its traffic is deferred by a period of d_s is defined as $C_s(d_s)$, which is calculated as

$$C_s(d_s) = \theta_s f(d_s), \tag{3}$$

where θ_s is a user-dependent factor indicating the scale of its utility function, i.e., the delay sensitivities. Due to the heterogeneous delay sensitivities of users, the cost of the same delay period d_s perceived by different users may vary dramatically. Furthermore, $f(\cdot)$ is the delay cost function, which is assumed to be strictly increasing and convex [27], i.e.,

$$C_s(d_s^{'}) > C_s(d_s), \quad \forall d_s^{'} > d_s, \tag{4}$$

and

$$C_s(d_s^{'}+\tau) - C_s(d_s^{'}) > C_s(d_s+\tau) - C_s(d_s), \quad \forall d_s^{'} > d_s, \tag{5}$$

where τ is the slot length.

For the sake of simplicity, we adopt an exponential function as the delay cost function $f(.)$, which meets the requirements of strictly increasing and convex [43]. The derived result can also be extended to other delay functions which are strictly increasing and convex.

Thus, the delay cost function is rewritten as

$$C_s(d_s) = \theta_s \exp(d_s). \tag{6}$$

Let $\widetilde{C}_s(d_s, \tau)$ represent the marginal delay cost of user s when the traffic is further deferred by one slot, which is calculated as

$$\widetilde{C}_s(d_s, \tau) = \theta_s \left[\exp(d_s+\tau) - \exp(d_s)\right]. \tag{7}$$

Intuitively, user s will defer its traffic if the peak-time cost saving is sufficient to compensate for the marginal cost of the traffic delay, i.e., $\sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau \geq \widetilde{C}_s(d_s,\tau)$; or stay connected otherwise.

We do not assume that the bandwidth in the next time slot is sufficient. As shown in (6) and (7), the delay period d_s is accumulative over time. Furthermore, the potential delay in the future is taken into account in the sense that the marginal delay cost increases with the increasing of d_s, and a user with a larger marginal delay cost is more likely to defer its traffic given the same peak-time cost.

V. Complete Information Scenario

In this section, we investigate the scenario of complete information, i.e., the NSP has perfect knowledge of all the users' delay sensitivities ($\{\theta_s\}_{s\in\boldsymbol{S}}$). First, we study the simple case of a single congested link, and derive a closed-form expression of the optimal pricing strategy. Then, we extend the discussion to the more complicated case of multiple congested links, and propose a heuristic iterative algorithm by considering a price discrimination.

A. Single Congested Link

Let link l be the congested link. We adopt the backward induction approach and begin the analysis from the second stage. In the second stage, upon receiving $p_{l,t}$, user $s \in \boldsymbol{S}_t(l)$ chooses to stay connected if $p_{l,t}x_s^{\text{req}}\tau \leq \widetilde{C}_s(d_s,\tau)$, or to defer its traffic for a slot otherwise.

The optimization problem of user s is formulated as

$$\mathbf{P1}: \max_{b_{s,t}} \Pi_s(b_{s,t}, p_{l,t}) = b_{s,t}(\widetilde{C}_s(d_s,\tau) - p_{l,t}x_s^{\text{req}}\tau). \tag{8}$$

Formally, the best response of user s is described as

$$b_{s,t}^* = \begin{cases} 1, & \text{if } p_{l,t}x_s^{\text{req}}\tau \leq \widetilde{C}_s(d_s,\tau) \\ 0, & \text{otherwise} \end{cases}. \tag{9}$$

Then, the optimal net benefit of user s is given as

$$\Pi_s(b_{s,t}^*, p_{l,t}) = \begin{cases} \widetilde{C}_s(d_s,\tau) - p_{l,t}x_s^{\text{req}}\tau, & \text{if } b_{s,t}^* = 1 \\ 0, & \text{otherwise} \end{cases}. \tag{10}$$

Equation (10) can be rewritten as

$$\Pi_s(b_{s,t}^*, p_{l,t}) = \left(\widetilde{C}_s(d_s,\tau) - p_{l,t}x_s^{\text{req}}\tau\right)^+, \tag{11}$$

where $(\cdot)^+ = \max(\cdot, 0)$.

In the first stage, the pricing strategy of the NSP is to maximize the total revenue received from the set of users whose traffic traverses the congested link l, i.e., $\boldsymbol{S}_t(l)$. When all the users choose the best response, the total revenue of the NSP is given by

$$\Pi^{\text{NSP}}(p_{l,t}, \{b_{s,t}^*\}_{s\in\boldsymbol{S}_t(l)}) = \sum_{s\in\boldsymbol{S}_t(l)} p_{l,t}x_s^{\text{req}}\tau b_{s,t}^*. \tag{12}$$

The pricing problem of the NSP is formulated as

$$\begin{aligned}&\mathbf{P2}: \max_{p_{l,t}} \Pi^{\text{NSP}}(p_{l,t}, \{b_{s,t}^*\}),\\ &\text{s.t.} \quad C_1: \sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}} b_{s,t}^* \leq B_l,\\ &\qquad\; C_2: p_{l,t} \geq 0.\end{aligned} \tag{13}$$

Here, C_1 denotes the capacity constraint that the bandwidth slicing must be satisfied. C_2 implies that the congestion price must be nonnegative.

With the complete information about the users' delay sensitivities, the NSP can first sort the users in set $\boldsymbol{S}_t(l)$ in a descending order based on their net benefit values if they stay connected, i.e., $b_{s,t}=1, \forall s \in \boldsymbol{S}_t(l)$, as

$$\Pi_1(1,p_{l,t}) \geq \Pi_2(1,p_{l,t}) \geq ... \geq \Pi_{|\boldsymbol{S}_t(l)|}(1,p_{l,t}). \tag{14}$$

Suppose that the available bandwidth B_l is insufficient to simultaneously satisfy the bandwidth demands of all the users in set $\boldsymbol{S}_t(l)$ in slot t. Let $s^{'} \in \boldsymbol{S}_t(l)$ be a critical user such that

$$\sum_{s\in\{1,\cdots,s'\}} x_s^{\text{req}} \leq B_l \quad \text{and} \quad \sum_{s\in\{1,\cdots,s',s'+1\}} x_s^{\text{req}} > B_l. \tag{15}$$

The optimal pricing strategy $p_{l,t}^*$ of the NSP is to take away all the surplus of user $s^{'}$ while meeting the bandwidth constraint. Let $b_{s',t}=1$, we have

$$\begin{aligned}\Pi_{s'}(1,p_{l,t}^*) =&\widetilde{C}_{s'}(d_{s'},\tau) - p_{l,t}^* x_{s'}^{\text{req}}\tau \\ =&\theta_{s'}\left[\exp(d_{s'}+\tau)-\exp(d_{s'})\right] - p_{l,t}^* x_{s'}^{\text{req}}\tau \\ =&0.\end{aligned} \tag{16}$$

By solving (16), $p_{l,t}^*$ is derived as

$$p_{l,t}^* = \frac{\theta_{s'}\left[\exp(d_{s'}+\tau)-\exp(d_{s'})\right]}{x_{s'}^{\text{req}}\tau}. \tag{17}$$

B. Multiple Congested Links

In this subsection, we extend the above analysis to the case of multiple congested links. In the second stage, for user $s \in \boldsymbol{S}_t(l)$ whose traffic traverses a set of links $\boldsymbol{L}_t(s)$, the price penalty is defined in (1). The optimization problem of user s is formulated as

$$\begin{aligned}\mathbf{P3}: &\max_{b_{s,t}} \Pi_s(b_{s,t},\{p_{l,t}\}_{l\in\boldsymbol{L}_t(s)}) \\ &=b_{s,t}(\widetilde{C}_s(d_s,\tau) - \sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau).\end{aligned} \tag{18}$$

The best response of user s is described as

$$b_{s,t}^* = \begin{cases}1, \text{ if } \sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau < \widetilde{C}_s(d_s,\tau) \\ 0, \text{ otherwise}\end{cases}. \tag{19}$$

Then, the maximal net benefit of user s is given as

$$\begin{aligned}&\Pi_s(b_{s,t}^*,\{p_{l,t}\}_{l\in\boldsymbol{L}_t(s)}) = \\ &\begin{cases}\widetilde{C}_s(d_s,\tau) - \sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau, & \text{if } b_{s,t}=1 \\ 0, & \text{otherwise}\end{cases}.\end{aligned} \tag{20}$$

Equation (20) can be rewritten as

$$\Pi_s(b_{s,t}^*,\{p_{l,t}\}_{l\in\boldsymbol{L}_t(s)}) = \left(\widetilde{C}_s(d_s,\tau) - \sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau\right)^+. \tag{21}$$

In the first stage, the price of each congested link is determined based on its congestion levels by adopting a price discrimination. Given the second-stage best response of all users defined in (19), the revenue of the NSP is given as follows:

$$\Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{b_{s,t}^*\}_{s\in\boldsymbol{S}}) = \sum_{l\in\boldsymbol{L}}\sum_{s\in\boldsymbol{S}_t(l)} p_{l,t}x_s^{\text{req}}\tau b_{s,t}^*. \tag{22}$$

The optimal congestion price $\{p_{l,t}^*\}_{l\in\boldsymbol{L}}$ in slot t can be derived by solving the following optimization problem:

$$\begin{aligned}\mathbf{P4}: &\max_{\{p_{l,t}\}_{l\in\boldsymbol{L}}} \Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{b_{s,t}^*\}_{s\in\boldsymbol{S}}), \\ \text{s.t.} \quad &C_3: \sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}} b_{s,t}^* \leq B_l, \forall l \in \boldsymbol{L}, \\ &C_4: p_{l,t} \geq 0, \forall l \in \boldsymbol{L}.\end{aligned} \tag{23}$$

P4 is non-convex as it involves $b_{s,t}^*$, which is a non-continuous non-differentiable function of $\{p_{l,t}\}_{l\in\boldsymbol{L}}$ [44].

Therefore, we propose an iterative heuristic solution which is summarized in **Algorithm 1**. It consists of an initialization stage, followed by multiple iterations of price rising. In each iteration, the most congested link n can be found by comparing the gap between the total bandwidth demand and the available bandwidth. Then we increase the congestion price of link n by a small perturbation Δp, and update the traffic demand on link n accordingly. The algorithm terminates when no congested link exists.

Herein, we analyze the proposed algorithm in terms of its convergence and computational complexity.

Theorem 1. ***Algorithm 1*** *will eventually terminate.*

Proof. Theorem 1 can be proved by contradiction. In each iteration, the congestion price is increased by Δp. Suppose that **Algorithm 1** will not terminate, namely, there always exists a link l that $B_{l,t}^{\text{demand}} > B_l$, then the congestion price of link l will eventually increase to infinity, which forces $b_{s,t}^*=0, \forall s \in \boldsymbol{S}_t(l)$. As a result, $B_{l,t}^{\text{demand}}=0$, which contradicts with the initial hypothesis. ☐

Theorem 2. ***Algorithm 1*** *has a computational complexity of* $O(K\times|\boldsymbol{L}|\times|\boldsymbol{S}|)$.

Proof. **Algorithm 1** runs iteratively to obtain the desired congestion prices. It can be seen that there are at most K iterations. Within each iteration, the complexity is composed of two parts: (i) the subproblem of finding the most congested link with the complexity of $O(|\boldsymbol{L}|\log|\boldsymbol{L}|)$; and (ii) the traffic demand is updated with the complexity of $O(|\boldsymbol{L}|\times|\boldsymbol{S}|)$. Therefore, the overall complexity of each iteration is: $O(|\boldsymbol{L}|\log|\boldsymbol{L}|)+O(|\boldsymbol{L}|\times|\boldsymbol{S}|)$. We neglect the term $O(|\boldsymbol{L}|\log|\boldsymbol{L}|)$ as it is much smaller than $O(|\boldsymbol{L}|\times|\boldsymbol{S}|)$. As a result, the total complexity of the proposed algorithm is approximately $K\times O(|\boldsymbol{L}|\times|\boldsymbol{S}|)$, namely, $O(K\times|\boldsymbol{L}|\times|\boldsymbol{S}|)$. ☐

VI. Incomplete Information Scenario

In this section, we consider the incomplete information scenario, where θ_s is only known to user s, while the NSP only has the statistical information of θ_s via historical

Algorithm 1 The price-rising based heuristic solution.

```
Inputs:
    B_l, ∀l ∈ L; x_s^req, ∀s ∈ S_t(l); θ_s, ∀s ∈ S_t(l)
Outputs:
    b_{s,t}, ∀s ∈ S_t(l); p_{l,t}, ∀l ∈ L
Step 1: Initialization
for s = 1 to |S_t(l)| do
    b_{s,t} ⇐ 1
end for
for l = 1 to |L| do
    p_{l,t} ⇐ 0
    B_{l,t}^demand ⇐ Σ_{s∈S_t(l)} x_s^req
end for
Step 2: Congestion Price Update
while ∃l that B_{l,t}^demand > B_l do
    Find n such that B_{n,t}^demand − B_n ≥ B_{m,t}^demand − B_m, ∀m ≠ n;
    Update the congestion price: p_{n,t} ⇐ p_{n,t} + Δp
    while ∃s ∈ S_t(l) that user s voluntarily defers the traffic do
        b_{s,t} ⇐ 0
    end while
    Update traffic demand on link l: B_{l,t}^demand ⇐ Σ_{s∈S_t(l)} b_{s,t} x_s^req
end while
```

observation. Thus, the information of θ_s is asymmetric. We will demonstrate how to derive the optimal pricing strategy for the NSP under information asymmetry.

A. Single congested link

Let link l be the single congested link. Given the price $p_{l,t}$, the best response of any user $s \in \boldsymbol{S}_t(l)$ is exactly the same as (9). Without the complete information of the delay sensitivity θ_s of user s, the NSP cannot directly derive the best response $b^*_{s,t}$. Instead, it has to infer the probability that user s stays connected in slot t, i.e., $P(b^*_{s,t}=1)$, which is given by

$$\begin{aligned} P(b^*_{s,t}=1) &= P(p_{l,t}x_s^{\text{req}}\tau \leq \widetilde{C}_s(d_s,\tau)) \\ &= P\left(p_{l,t}x_s^{\text{req}}\tau \leq \theta_s\left[\exp(d_s+\tau)-\exp(d_s)\right]\right) \\ &= P\left(\theta_s \geq \frac{p_{l,t}x_s^{\text{req}}\tau}{\exp(d_s+\tau)-\exp(d_s)}\right). \end{aligned} \tag{24}$$

Without loss of generality, we assume that the delay sensitivity $\theta_s, \forall s \in \boldsymbol{S}_t(l)$ is independently and uniformly distributed in the range of [0, $\theta^{\max}$]. We assume that $\theta^{\max}$ satisfies the following inequality:

$$\theta^{\max} \geq \frac{p_{l,t}x_s^{\text{req}}\tau}{\exp(d_s+\tau)-\exp(d_s)}, \forall s \in \boldsymbol{S}_t(l). \tag{25}$$

Otherwise, the probability $P(b^*_{s,t}=1)$ might become a negative value, which has no practical meaning.

Hence, $P(b^*_{s,t}=1)$ can be rewritten as

$$P(b^*_{s,t}=1) = 1-\frac{p_{l,t}x_s^{\text{req}}\tau}{\theta^{\max}\left(\exp(d_s+\tau)-\exp(d_s)\right)}. \tag{26}$$

The above equation indicates that $P(b^*_{s,t}=1)$ is negatively related to $p_{l,t}$, x_s^{req}, and d_s.

The expected revenue of the NSP can be given as follows:

$$\begin{aligned} &\Pi^{\text{NSP}}(p_{l,t},\{P(b^*_{s,t}=1)\}_{s\in\boldsymbol{S}_t(l)}) \\ &= \sum_{s\in\boldsymbol{S}_t(l)} p_{l,t}x_s^{\text{req}}\tau P(b^*_{s,t}=1). \end{aligned} \tag{27}$$

The optimal congestion price $p^*_{l,t}$ in slot t is derived by solving the following problem:

$$\begin{aligned} \mathbf{P5}: &\max_{p_{l,t}} \Pi^{\text{NSP}}(p_{l,t},\{P(b^*_{s,t}=1)\}_{s\in\boldsymbol{S}_t(l)}), \\ \text{s.t.}\quad & C_2: p_{l,t} \geq 0, \\ & C_5: \sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}}P(b^*_{s,t}=1) \leq B_l, \end{aligned} \tag{28}$$

where C_5 implies that the constraint is the expected bandwidth demand.

Taking (26) into C_5, we can derive the upper bound of $p_{l,t}$, which is given by

$$p_{l,t} \geq \frac{\sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}} - B_l}{\sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}}\sigma_s} = p^{\min}_{l,t}, \tag{29}$$

where

$$\sigma_s = \frac{x_s^{\text{req}}\tau}{\theta^{\max}\left(\exp(d_s+\tau)-\exp(d_s)\right)}. \tag{30}$$

Eventually, **P5** can be transformed to a quadratic programming problem, which is given by

$$\begin{aligned} \mathbf{P6}: &\max_{p_{l,t}} \sum_{s\in\boldsymbol{S}_t(l)} p_{l,t}x_s^{\text{req}}\tau\left(1-\sigma_s p_{l,t}\right), \\ \text{s.t.}\quad & C_2: p_{l,t} \geq 0, \\ & C_6: p_{l,t} \geq p^{\min}_{l,t}. \end{aligned} \tag{31}$$

Since the objective function is quadratic and the constraints are affine, $p^*_{l,t}$ can be derived as

$$p^*_{l,t} = \begin{cases} \frac{|\boldsymbol{S}_t(l)|}{2\sum_{s\in\boldsymbol{S}_t(l)}\sigma_s}, & \text{if } \frac{|\boldsymbol{S}_t(l)|}{2\sum_{s\in\boldsymbol{S}_t(l)}\sigma_s} > p^{\min}_{l,t} \\ p^{\min}_{l,t}, & \text{otherwise} \end{cases} \tag{32}$$

B. Multiple congested links

In the case of multiple congested links, the best response and the maximum net benefit of any user $s \in \boldsymbol{S}$ are the same as what have been derived in (19) and (20).

In the fist stage, the NSP infers $P(b^*_{s,t}=1)$ as

$$\begin{aligned} &P(b^*_{s,t}=1) \\ =&P(\sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau \leq \widetilde{C}_s(d_s,\tau)) \\ =&P\left(\sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau \leq \theta_s\left[\exp(d_s+\tau)-\exp(d_s)\right]\right) \\ =&P\left(\theta_s \geq \frac{\sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau}{\exp(d_s+\tau)-\exp(d_s)}\right). \end{aligned} \tag{33}$$

$$\frac{\partial \Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})}{\partial p_{l,t}} = \sum_{s\in\boldsymbol{S}_t(l)} x_s^{req}\tau - \sum_{s\in\boldsymbol{S}_t(l)} \frac{(x_s^{req}\tau)^2}{\exp(d_s-\tau)-\exp(d_s)}(\sum_{m\in\boldsymbol{L}_t(s)} p_{m,t}+p_{l,t}) - \sum_{n\in\boldsymbol{L},n\neq l}\sum_{s\in\boldsymbol{S}_t(l)\cap\boldsymbol{S}_t(n)} \frac{(x_s^{req}\tau)^2}{\exp(d_s-\tau)-\exp(d_s)} p_{n,t}, \tag{34}$$

$$\frac{\partial^2 \Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})}{\partial^2 p_{l,t}} = -\sum_{s\in\boldsymbol{S}_t(l)} \frac{2(x_s^{req}\tau)^2}{\exp(d_s-\tau)-\exp(d_s)} < 0. \tag{35}$$

Following the same assumption of $\theta_s, \forall s\in\boldsymbol{S}_t(l)$ as shown in the previous subsection, $P(b_{s,t}^*=1)$ can be rewritten as

$$P(b_{s,t}^*=1) = 1-\frac{\sum_{l\in\boldsymbol{L}_t(s)} p_{l,t}x_s^{\text{req}}\tau}{\theta^{\max}\left(\exp(d_s+\tau)-\exp(d_s)\right)}. \tag{36}$$

The expected revenue of the NSP can be given as

$$\begin{aligned}&\Pi^{\text{NSP}}\left(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}}\right)\\ =&\sum_{l\in\boldsymbol{L}}\sum_{s\in\boldsymbol{S}_t(l)} p_{l,t}x_s^{\text{req}}\tau P(b_{s,t}^*=1).\end{aligned} \tag{37}$$

Then the optimal congestion price $p_{l,t}^*$ in slot t is derived by solving the following problem:

$$\begin{aligned}\mathbf{P7}: &\max_{\{p_{l,t}\}_{l\in\boldsymbol{L}}} \Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}}),\\ \text{s.t.}\quad & C_4: p_{l,t}\geq 0, \forall l\in\boldsymbol{L},\\ & C_7: \sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}}P(b_{s,t}^*=1)\leq B_l, \forall l\in\boldsymbol{L}.\end{aligned} \tag{38}$$

By analyzing the objective function of $\mathbf{P7}$, we have

Theorem 3. *$\Pi^{NSP}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})$ is concave with regards to any $p_{l,t}, \forall l\in\boldsymbol{L}$.*

Proof. The first-order derivative and the second-order derivative of the objective function $\Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})$ with respect to $p_{l,t}$ are shown in (34) and (35), respectively. Since the second-order derivative is negative, $\Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})$ is concave with regards to $p_{l,t}$. □

By observing (34), we conclude that $\mathbf{P7}$ is NP-hard since the congestion prices of different links are coupled together. In the following, we analyze how to find the optimal pricing strategy for the NSP.

Since $\Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})$ is concave with respect to $p_{l,t}$, we can transform problem $\mathbf{P7}$ into a variational inequality problem, from which the uniqueness of the optimal pricing strategy is guaranteed.

Defining the set $\boldsymbol{K} = \{\{p_{l,t}\}_{l\in\boldsymbol{L}} \mid \sum_{s\in\boldsymbol{S}_t(l)} x_s^{\text{req}}P(b_{s,t}^*=1)\leq B_l, p_{l,t}\geq 0, \forall l\in\boldsymbol{L}\}$, then the equivalent problem of $\mathbf{P7}$ is given by

$$\begin{aligned}\mathbf{P8}: &\min_{\{p_{l,t}\}_{l\in\boldsymbol{L}}} -\Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}}),\\ \text{s.t.}\quad & C_8: p_{l,t}\in\boldsymbol{K}, \forall l\in\boldsymbol{L}.\end{aligned} \tag{39}$$

According to the variational inequality theory [45], solving $\mathbf{P8}$ is equivalent to finding a set of prices $\{p_{l,t}^*\}_{l\in\boldsymbol{L}}$ which satisfies

$$\left(\{p_{l,t}\}_{l\in\boldsymbol{L}}-\{p_{l,t}^*\}_{l\in\boldsymbol{L}}\right)F\left(\{p_{l,t}\}_{l\in\boldsymbol{L}}\right)\geq 0, \forall\{p_{l,t}\}_{l\in\boldsymbol{L}}, \tag{40}$$

where

$$\begin{aligned}&F(\{p_{l,t}\}_{l\in\boldsymbol{L}})\\ =&\nabla(-\Pi^{\text{CSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}}))\\ =&-\{\nabla_{p_{l,t}}\Pi^{\text{CSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})\}_{l\in\boldsymbol{L}}^{\top},\end{aligned} \tag{41}$$

Here, $\Big(\{p_{l,t}\}_{l\in\boldsymbol{L}}-\{p_{l,t}^*\}_{l\in\boldsymbol{L}}\Big)F\left(\{p_{l,t}\}_{l\in\boldsymbol{L}}\right)$ represents the element-wise difference between $\{p_{l,t}\}_{l\in\boldsymbol{L}}$ and $\{p_{l,t}^*\}_{l\in\boldsymbol{L}}$.

The problem defined in (40) falls into the category of variational inequality problem, and can be abbreviated as VI$(\boldsymbol{K},F)$. Based on [45], VI$(\boldsymbol{K},F)$ has the following property:

Theorem 4. *If $\boldsymbol{K}$ is a convex closed set, and the continuous function F is strictly monotone on $\boldsymbol{K}$, then VI$(\boldsymbol{K},F)$ admits at most one solution.*

Proof. The proof is omitted here due to the space limitation. A similar proof can be found in [45]. □

Theorem 5. *For the variation inequality problem VI$(\boldsymbol{K},F)$, F is strictly monotone on $\boldsymbol{K}$.*

Proof. To prove that F is strictly monotone on $\boldsymbol{K}$, we choose two solution sets from set $\boldsymbol{K}$, i.e., $\{p_{l,t}^{'}\}_{l\in\boldsymbol{L}}, \{p_{l,t}^{''}\}_{l\in\boldsymbol{L}}\in\boldsymbol{K}$, $\{p_{l,t}^{'}\}_{l\in\boldsymbol{L}}\neq\{p_{l,t}^{''}\}_{l\in\boldsymbol{L}}$, and check the positivity of $(\{p_{l,t}^{'}\}_{l\in\boldsymbol{L}}-\{p_{l,t}^{''}\}_{l\in\boldsymbol{L}})^{\top}(F(\{p_{l,t}^{'}\}_{l\in\boldsymbol{L}})-F(\{p_{l,t}^{''}\}_{l\in\boldsymbol{L}}))$, which is given by

$$\begin{aligned}&(\{p_{l,t}^{'}\}_{l\in\boldsymbol{L}}-\{p_{l,t}^{''}\}_{l\in\boldsymbol{L}})^{\top}(F(\{p_{l,t}^{'}\}_{l\in\boldsymbol{L}})-F(\{p_{l,t}^{''}\}_{l\in\boldsymbol{L}}))=\\ &\sum_{l\in\boldsymbol{L}}\left(\left(p_{l,t}^{'}-p_{l,t}^{''}\right)\left(-\nabla_{p_{l,t}}\Pi^{\text{NSP}}_{p_{l,t}=p_{l,t}^{'}}+\nabla_{p_{l,t}}\Pi^{\text{NSP}}_{p_{l,t}=p_{l,t}^{''}}\right)\right).\end{aligned} \tag{42}$$

Since the second-order derivative of the objective function $\Pi^{\text{NSP}}(\{p_{l,t}\}_{l\in\boldsymbol{L}},\{P(b_{s,t}^*=1)\}_{s\in\boldsymbol{S}})$ with respect to $p_{l,t}$ is negative according to (35), we can draw the conclusion that $\nabla_{p_{l,t}}\Pi^{\text{NSP}}$ is a monotonically decreasing function of $p_{l,t}$, while $-\nabla_{p_{l,t}}\Pi^{\text{NSP}}$ is a monotonically increasing function of $p_{l,t}$.

Thus, we have

$$-\nabla_{p_{l,t}}\Pi^{\text{NSP}}_{p_{l,t}=p_{l,t}^{'}}+\nabla_{p_{l,t}}\Pi^{\text{NSP}}_{p_{l,t}=p_{l,t}^{''}}\begin{cases}\geq 0, p_{l,t}^{'}\geq p_{l,t}^{''}\\ \leq 0, p_{l,t}^{'}\leq p_{l,t}^{''}\end{cases} \tag{43}$$

Algorithm 2 Gradient-based iterative pricing algorithm.

1: **Inputs:**
  $B_l, \forall l \in \boldsymbol{L}; x_s^{\text{req}}, \forall s \in \boldsymbol{S}_t(l); \theta_s, \forall s \in \boldsymbol{S}_t(l), \boldsymbol{K}$
2: **Outputs:**
  $z_{s,t}, \forall s \in \boldsymbol{S}_t(l); p_{l,t}, \forall l \in \boldsymbol{L}$
3: **Step 1: Initialization**
4: **for** $s = 1$ to $|\boldsymbol{S}_t(l)|$ **do**
5:   $z_{s,t} \Leftarrow 1$
6: **end for**
7: **for** $l = 1$ to $|\boldsymbol{L}|$ **do**
8:   $p_{l,t} \Leftarrow 0$
9:   $B_{l,t}^{\text{demand}} \Leftarrow \sum_{s \in \boldsymbol{S}_t(l)} x_s^{\text{req}}$
10: **end for**
11: **Step 2: Congestion Price Update**
12: **while** $\exists l$ that $B_{l,t}^{\text{demand}} > B_l$ **do**
13:   Find n such that $B_{n,t}^{\text{demand}} - B_n \geq B_{m,t}^{\text{demand}} - B_m, \forall m \neq n$;
14:   Set $i \Leftarrow 1$, precision threshold φ
15:   **while** $\frac{\|p^{[i]} - p^{[i-1]}\|_1}{\|p^{[i-1]}\|_1} \geq \varphi$ **do**
16:     Update the congestion price by a gradient assisted searching algorithm: $p_{l,t} \Leftarrow p_{l,t} + \lambda \nabla \Pi^{\text{NSP}}(p_{l,t})$
17: where λ is the step size of the price update.
18:     $i \Leftarrow i + 1$
19:   **end while**
20:   **while** $\exists s \in \boldsymbol{S}_t(l)$ that user s voluntarily defers the traffic **do**
21:     $z_{s,t} \Leftarrow 0$
22:   **end while**
23:   Update traffic demand on link l: $B_{l,t}^{\text{demand}} \Leftarrow \sum_{s \in \boldsymbol{S}_t(l)} z_{s,t} x_s^{\text{req}}$
24: **end while**

For any link $l \in \boldsymbol{L}$, we can prove that

$$\sum_{l \in \boldsymbol{L}} \left(\left(p_{l,t}^{'} - p_{l,t}^{''} \right) \left(-\nabla_{p_{l,t}} \Pi^{\text{NSP}}_{p_{l,t}=p_{l,t}^{'}} + \nabla_{p_{l,t}} \Pi^{\text{NSP}}_{p_{l,t}=p_{l,t}^{''}} \right) \right) \geq 0. \tag{44}$$

This proves that F is strictly monotone on $\boldsymbol{K}$. $\square$

Thus, by combining Theorem 4 and Theorem 5, we can prove that VI$(\boldsymbol{K}, F)$ admits at most one solution. Thus, the equivalent optimization problem $\mathbf{P8}$ also admits at most one solution. The uniqueness of the optimal solution is hence validated.

Similar to **Algorithm 1** developed in Section V, we develop an iterative price-rising algorithm to find the optimal solution. Since the objective function of $\mathbf{P7}$ is concave with regards to $p_{l,t}$, we can augment the price-rising part with a gradient-based searching algorithm to improve the convergence speed. The gradient-based iterative pricing algorithm is summarized in **Algorithm 2**.

Herein, we analyze the proposed algorithm in terms of its computational complexity.

Theorem 6. ***Algorithm 2** has a computational complexity of* $O(C + |\boldsymbol{L}| \times |\boldsymbol{S}|)$.

TABLE II

A SUMMARY OF THE SIMULATION PARAMETERS.

Average session duration	1 hour
Delay sensitivity (θ_s)	$[0, 1]$
Bandwidth requirement of user s (x_s^{req})	$[90, 320]$ Mbps
Available bandwidth of link l (B_l)	12 Gbps
Slot length (τ)	0.05 hour
Number of users	$100/10000$
User-dependent factor (θ_s)	$(0, 1)$
Price perturbation (Δp)	0.0005
Threshold (φ)	0.001
Step size (λ)	0.001

Proof. **Algorithm 2** runs in a gradient descent manner to obtain the desired congestion prices. Because we have proved that problem $\mathbf{P7}$ has at most one solution in Theorem 4, there will be at most C descent processes to obtain an optimal solution. Within each time slot, the complexity is composed of three parts: (i) the subproblem of finding the most congested link with the complexity of $O(|\boldsymbol{L}| \log |\boldsymbol{L}|)$; (ii) the subproblem of calculating the congestion prices of a specific congested link with the complexity of $O(C)$; and (iii) the traffic demand is updated with the complexity of $O(|\boldsymbol{L}| \times |\boldsymbol{S}|)$. Therefore, the overall complexity is $O(|\boldsymbol{L}| \log |\boldsymbol{L}|) + O(C) + O(|\boldsymbol{L}| \times |\boldsymbol{S}|)$. We neglect the term $O(|\boldsymbol{L}| \log |\boldsymbol{L}|)$ as it is much smaller than $O(|\boldsymbol{L}| \times |\boldsymbol{S}|)$. As a result, the total complexity of the proposed algorithm is approximately $C + |\boldsymbol{L}| \times |\boldsymbol{S}|$, namely, $O(C + |\boldsymbol{L}| \times |\boldsymbol{S}|)$. $\square$

It is noted that the optimization is carried out from the perspective of "expectation" due to information asymmetry, where the hard constraint of bandwidth capacity cannot be reliably guaranteed. To address this challenge, we modify **Algorithm 2** to satisfy the hard constraint of bandwidth capacity. Specifically, when the total bandwidth demand exceeds the maximum available bandwidth, **Algorithm 2** will continue to increase the congestion price until the bandwidth demand is satisfied based on Line 12. Therefore, **Algorithm 2** is a robust suboptimal pricing strategy, which provides a flexible tradeoff between optimality and robustness. Furthermore, the core concept of price rising enables the users with the least delay cost to defer their bandwidth demands. We will also demonstrate that the performance loss compared with the optimal algorithm is negligible via simulation results.

The proposed scheme is implemented in a slot-by-slot fashion, and can be easily implemented online since it does not require any noncausal knowledge of future information. We do not consider the joint bandwidth slicing and pricing optimization due to the following reasons. First, the noncausal knowledge of future information is required for decision making. For instance, in [35], a set of future day-ahead prices is given as a priori information. This is different from our work since we assume that the future information is unavailable. Second, some previous works of long-period decision making [46] only consider either bandwidth slicing or pricing. They cannot be directly applied in our model. Due to the bursty network traffic and uncertain real-time prices, not only the bandwidth slicing strategies and pricing strategies of the same slot are coupled, but also the bandwidth slicing strategies or pricing strategies of different slots are coupled. How to solve such a complicated problem is still nontrivial. Last but not least, conventional research attempts [47], [48] rely on some assumptions that the uncertain parameters follow some well-known probability such as Markov and Poisson distributions. They may suffer from severe performance loss if the practical probability distributions of uncertain factors disagree from the presumed statistical models. In comparison, we have not made any preassumption on the statistical model of traffic arrival or link congestion state.

VII. Numerical Analysis

We search the "Information and Communication Statistics Database" issued by Japanese government [49] and perform simulations based on the statistics usage data of Internet. More details can be found in [49]. We assume that session durations follow an exponential distribution with an average value of 1 hour. Detailed simulation parameters are summarized in Table II.

For comparison purposes, we implement and compare: (i) the uniform pricing where the NSP charges a uniform congestion price for all the congested links during a certain time slot, while the uniform congestion price may vary from one slot to another [12]; (ii) the dynamic TDP (DTDP) scheme where the prices are calculated according to the type of session through a Monte Carlo Method-based algorithm [33]; (iii) the flat rate pricing scheme where the NSP charges a fixed congestion price for all the congested links during the entire peak time [50]. In particular, we exhaustively try all feasible prices and find that the optimal flat-rate price for the NSP is 0.45, i.e., $p_{l,t} = 0.45, \forall l \in \boldsymbol{L}, \forall t$; and (iv) the proposed pricing scheme.

Fig. 3 depicts the traffic demand versus the time slot with multiple congested links with 10^4 users. It can be observed that the proposed schemes can largely reduce the traffic demand during the typical peak time compared to other schemes under both complete and incomplete information scenarios. Compared to the original bandwidth demand, the peak-time traffic demand is reduced by 20.66% and 27.39% under complete and incomplete information, respectively. The rational behind is that the NSP is able to determine an appropriate congestion price based on the congestion level as well as users' preference, which improves the utilization of the bandwidth while guaranteeing the QoS requirements of users. The peak shaving effect is achieved by shifting the traffic demand from peak time to off-peak time, which is evidenced by the fact that the traffic demand of the proposed scheme is lower than that of other schemes during the typical peak time, but higher during the typical off-peak time.

Fig. 4 shows the revenue versus the time slot with multiple congested links with 10^4 users. The proposed algorithm outperforms uniform pricing by 115.75% under incomplete information, and outperforms uniform pricing, optimal flat rate pricing and DTDP by 43.95%, 230.79%, and 22.77%, respectively. It can be easily observed that: (i) the NSP's revenue increases with the time slot; and (ii) there is a large gap between the revenue achieved under complete information and that under incomplete information. The reasons are as follows: (i) the increase in demand during peak time increases the congestion price, hence resulting in a higher revenue of the NSP; and (ii) with the prefect knowledge about users' preference, the NSP can improve its revenue by setting congestion prices that exactly take away all the surplus of users.

Fig. 5 shows the net benefit of users versus the time slot with multiple congested links with 10^2 users. It can be observed that the user net benefit under incomplete information outperforms that under complete information. The rationale behind is that the NSP can take away all the surplus with the prefect knowledge about users' preference (e.g., θ_s). Hence, information asymmetry actually increases users' net benefit and protects them from being over exploited. Although uniform pricing achieves the best performance under incomplete information, its performance of cumulative NSP revenue is much worse than that of the proposed scheme. Details will be explained in the next figure.

Fig. 6 depicts the revenue versus the number of users with multiple congested links with 10^2 users. Simulation results demonstrate that the NSP's revenue increases with the number of users. The reasons are as follows: (i) the competition for bandwidth gets more intensive as the number of users increase. As a result, a user has to bear a higher congestion price in order to remain connected during the peak time, which leads to a higher revenue of the NSP; (ii) given the fixed congestion price, the revenue of the NSP is also positively correlated with the number of users. Hence, the more users participating in the game, the more revenue the NSP can get. Compared to uniform pricing, optimal flat rate pricing, and DTDP under complete information, the performance improvements are 36.07%, 324.06% and 18.70%, respectively. Compared to uniform pricing under incomplete information, the performance improvement is 135.51%. In uniform pricing, the economic benefits of the NSP are severely degraded because the price cannot effectively reveal the real-time congestion level.

Fig. 7 shows the convergence performance of the proposed algorithms with 10^2 users. The optimality gap is calculated by minusing the performance achieved by **Algorithm 1** or **Algorithm 2** from the optimal performance. It can be evidently observed that the optimality gap of **Algorithm 2** decreases much faster than that of **Algorithm 1**, which demonstrates that **Algorithm 2** has a better convergence performance. The reason is the step size of **Algorithm 2** is dynamically adjusted to improve the convergence speed, while **Algorithm 1** adopts a fixed price perturbation. Simulation results also demonstrate that **Algorithm 2** has a smaller optimality gap. Compared with the optimal strategy, the performance loss of **Algorithm 1** is 5.25% after 600 iterations, while that of **Algorithm 2** is only 1.94%.

We use the peak-to-average traffic ratio (PATR) to measure the degree to which the traffic demand is balanced over a day. A higher PATR implies that the traffic is more unbalanced and the NSP has more idle network capacity over-provisioned for the peak traffic demand. Fig. 8 shows that the proposed scheme allows the NSP to reduce the PATR by up to 15.00%

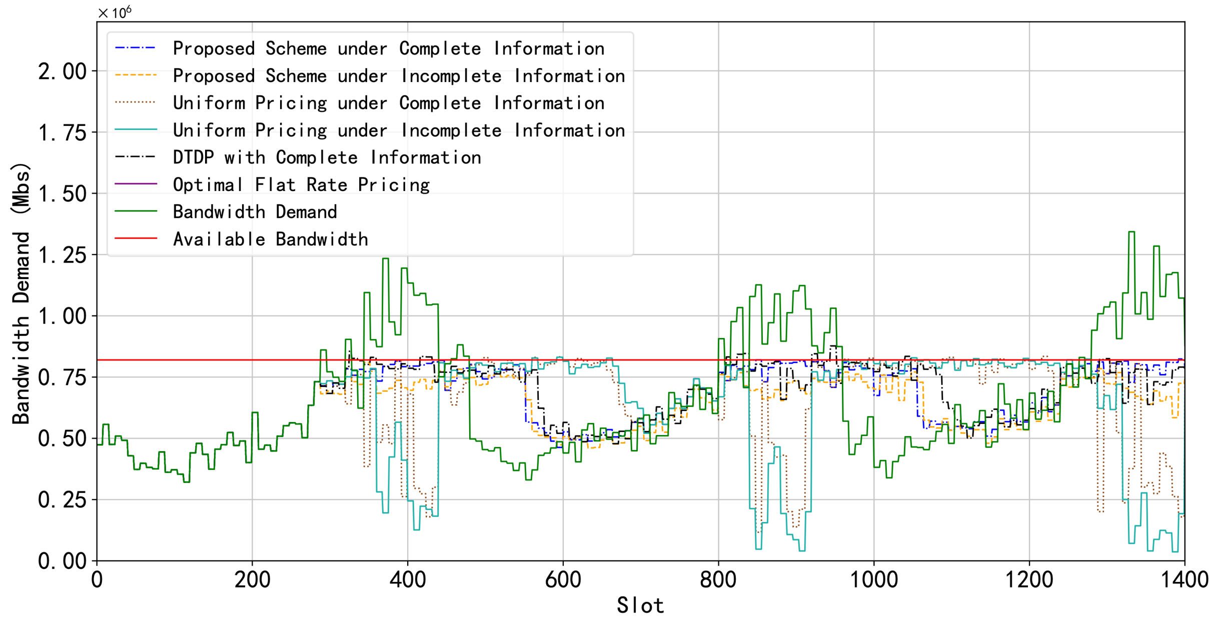

Fig. 3. Bandwidth allocation versus time slot with multiple congested links.

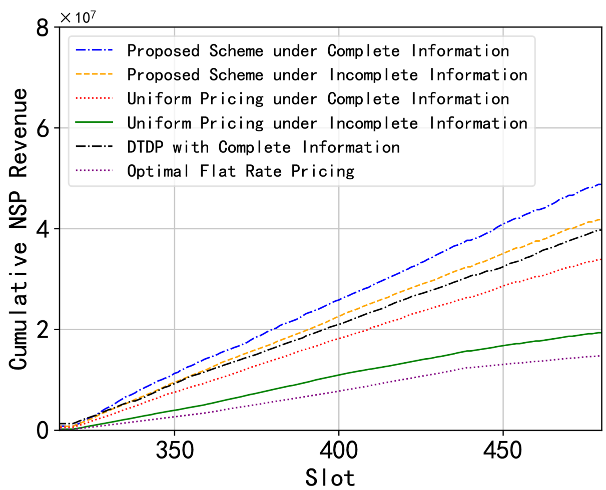

Fig. 4. Cumulative revenue of the NSP versus time slot with multiple congested links.

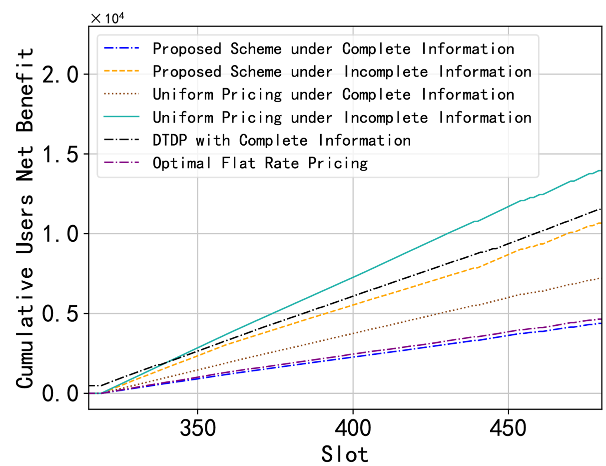

Fig. 5. Cumulative net benefit of users versus time slot with multiple congested links.

and 19.35% compared to the uniform pricing and the optimal flat rate pricing under incomplete information.

VIII. Conclusion and Future Work

This paper studies joint pricing and bandwidth allocation for QoS-guaranteed multimedia applications in software-defined cellular networks. Congestion prices are sent to multimedia application users as a signal indicating the real-time traffic load. With our proposed scheme, delay-tolerant users can make their decisions on whether to defer their traffic and vacate bandwidth resources for other delay-sensitive users based on the price signals perceived. The efficiency of our proposed scheme is demonstrated by means of simulation. Results show that the traffic demand can be efficiently shifted from peak time to off-peak time, and the revenue of NSP and net benefit of users are significantly improved.

Several issues are left to be further investigated, such as users are uncertain about how many slots they should wait until the next off-peak slot comes; and burden of decision making on whether to keep connecting or not in each peak slot. We believe that these problems are inevitable as long as the price is set to reflect the real-time traffic load. However, these problems can be alleviated, through letting users set their budgets as well as the delay tolerance in advance. If the users are not intended to monitor the time-varying congestion prices, an artificial intelligence could be introduced to assist the users for decision-making (i.e., to stay connected or not). Furthermore, the deferred online streaming contents could be downloaded offline within the budget and delay tolerance specified by the users. We will further investigate these issues

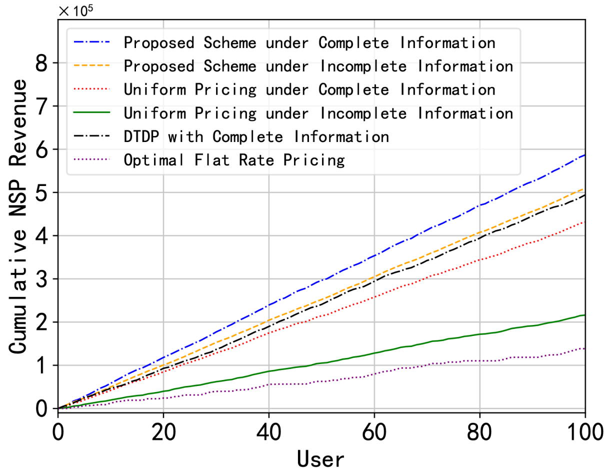

Fig. 6. Cumulative revenue of the NSP versus the number of user with multiple congested links.

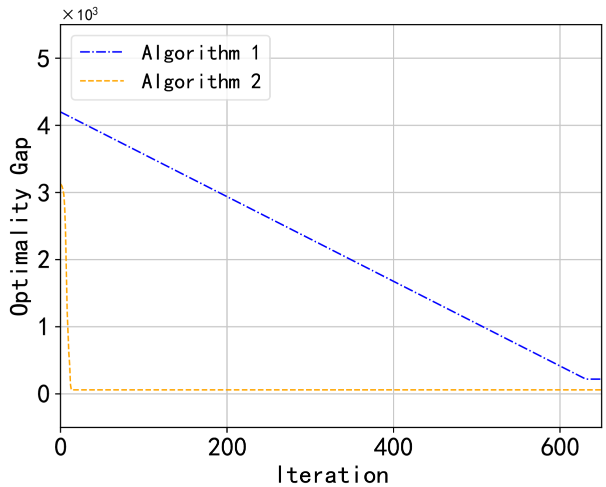

Fig. 7. Convergence performance.

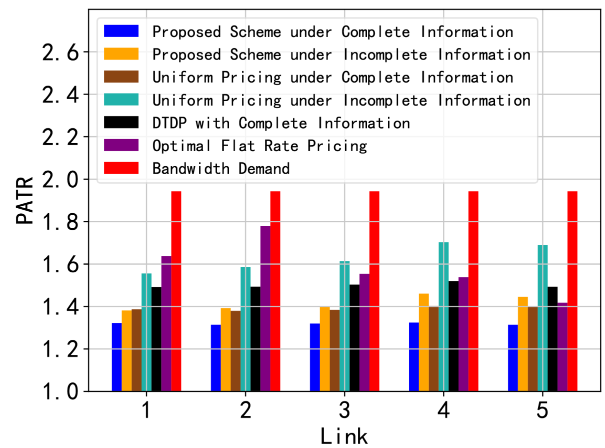

Fig. 8. Peak-to-average traffic ratio versus the links.

in our future work.

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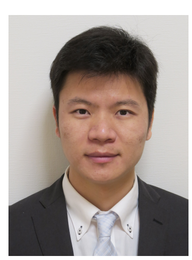

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