TURUN KAUPPAKORKEAKOULU
TULEVAISUUDEN TUTKIMUSKESKUS

TURKU SCHOOL OF ECONOMICS AND BUSINESS ADMINISTRATION
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FUTU-publication 3/99

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DECOMPOSITION METHOD IN SUSTAINABILITY ANALYSIS

The purpose of the research programme *Citizenship and ecomodernization in the information society – the futures approach* – is to study the social and ecological dimensions of emerging information society. Particularly we aim at assessing social impacts of new informational structures that are impinged on citizens. We also focus on analyzing the ways application of information technology influences on targets and realization of sustainable development. The study programme comprises of ten individual research project organized around above sketched themes.

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ISBN	951-738-952-3
UDK	330.34
	504

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1. INTRODUCTION

Decomposition analysis has widely been utilised in energy efficiency analysis in different countries. In the recent two decades, numerous studies have been presented, where Divisia or Laspeyras (Paasche) energy indices are decomposed into factor contributions or, environmental pollution (CO_2 and others) figures are into contributions of relevant factors. These decomposition models reveal a quantitative relation between economic development and energy use, or relation between energy use and environmental pollution. They belong to the basic analytical tools of energy economics and energy and environmental policy.

In energy studies the total change of energy consumption from a base year is decomposed into contributions of economic activity, energy intensity development in different economic sectors, and structural shift of economy. The decomposition method applied has been either an approximate or exact one. The approximate decomposition method has a residual term, which is left unexplained, or the term is just named as a kind of a joint contribution not allocated to the factors used. In that sense its decomposition explanation is incomplete.

Different ways of decomposing the indexes has also been used. In the Factor Isolation Method the factorized contribution formulas are defined as a multiplication between the changes of variables and the values of the other variables in the base year. In the Combination Method the average values of the quantities in the base and the end year are used.

Decomposition, i.e. factorization into contribution effects, is carried out for variables expressed mathematically in the form of multiplication of two or more variables. The variables under considerations are empirically time series and the first year of the time series is often chosen as the base year. The empirical figures can be given either in absolute monetary or physical units or as indices with the index value 100 in the chosen base year. Or the values can be expressed as the dimensionless per unit figures assuming decimal values with the value 1.0 in the base year. As can be shown in the analysis the per unit value system offers some advantages over the others. In the exact methods of decomposition the changes of the decomposed variable is completely factorized into contributions by the specified factors without any unexplained residual left.

An exact decomposition approach was developed and applied by Malaska & Sun (1995), Sun (1996, 1998) and Sun & Malaska (1998) in a world energy efficiency study with a zero residual and complete allocation. Here we don’t present comprehensive literature review of the topics because one can find it in the above mentioned publications. In Sun's complete decomposition approach the principle 'jointly created, equally distributed' for allocation to the factor effects has been implemented, i.e. the residual has been divided equally to each factor contribution. In this article a more general allocation principle, which includes the complete decomposition calculus as a special case, is presented. A set of models of energy, material use, pollution, labour and capital are depicted for possible application of the general decomposition approach.

2. DECOMPOSITION

The principle for the allocation of the residual term to the factors in the generalized decomposition calculus is derived on the base of Figure 1 below.

With two explaining factors, *x* and *y* as in the equation (1) below, their additive contributions $(\Delta C_x, \Delta C_y)$ for the total change of C can be defined from the basic equations in (2) and (3) – (5).

BASIC DEFINITIONS FOR DECOMPOSITION

(1) $C = xy$

(2) $\Delta C = \Delta C_x + \Delta C_y$,

where ΔC_x is *x*'s contribution to ΔC and ΔC_y *y*'s respectively.

(3) $\Delta C_x = \Delta x y_0$

(4) $\Delta C_y = \Delta y x_0$

where subscript 0 refers to the base (year) situation. When a two factors is under investigation the complete expansion of ΔC is as follows

(5) $\Delta C = \Delta(xy) = \Delta x y_0 + \Delta y x_0 + \Delta x \Delta y$

The approximate decomposition would just omit the term $\Delta x \Delta y$, or call it the unexplained residual. The first term of the complete decomposition in (5) $\Delta x y_0$, is clearly a contribution of the variable *x* defined from the base situation, and similarly the term $\Delta y x_0$ is the contribution of *y*.

In Figure 1 we have a point (x_0, y_0) at the base year and another point (x_t, y_t) at time *t* which is apart from the previous by amounts $(\Delta x, \Delta y)$. Δy is divided into two separated pieces in Figure 2 so that

(6) $\Delta y^{(a)} + \Delta y^{(b)} = \Delta y$.

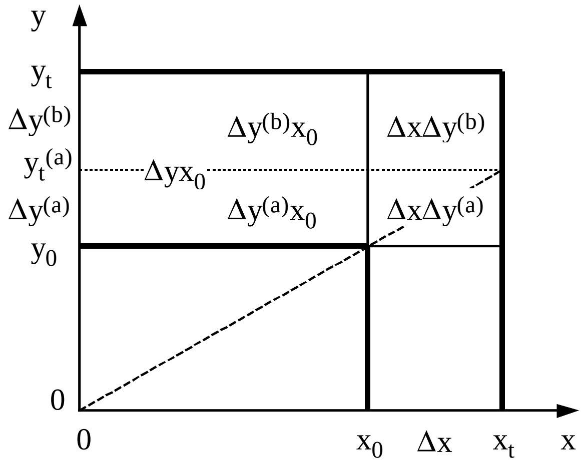

Figure 1. The contribution terms of generalized decomposition calculus.

If the point (x_t, $y_t^{(a)}$) corresponding to time *t* would lay on the line from origo (0,0) through the base year point (x_0, $y_0^{(a)}$) there would be no better principle to allocate the second order term ($\Delta x \Delta y$) to the factors than the principle 'jointly created, equally distributed'. In this case the exact, complete decomposition is

(7) $$\Delta C_x^{(a)} = \Delta x y_0 + \tfrac{1}{2}\Delta x \Delta y$$

(8) $$\Delta C_y^{(a)} = \Delta y \ x_0 + \tfrac{1}{2}\Delta x \Delta y \ .$$

This is just Sun's complete decomposition. This rule is the best possible decomposition for any pairs of (Δx, Δy) which lie on line defined by the equation

(9) $$\frac{\Delta x}{x_0} = \frac{\Delta y}{y_0} .$$

In the Figure 1 the Δy at the given Δx is however different from what the equation (9) assumes. In figure the value of the Δy from equation (9) Δy is marked by $\Delta y^{(a)}$, and it is less than real Δy. It is not at all evident by any rational reason that the amount of Δy exceeding the amount from equation 9 should be regarded as *x*'s contribution. It seems more evident that the excess should be regarded only as the variable *y*'s contribution because *x* has nothing more to contribute not already been taken into account by the contribution of equation (9).

By definition then the contribution share of residual allocated to ΔC_x in the case of Figure 2 equals the corresponding balanced share $\frac{1}{2}\Delta x \Delta y^{(a)}$ of the complete decomposition with the same Δx. Thus, the effects of explanatory factors of generalized decomposition calculus in Figure 1are

(10) $\Delta C_x = \Delta C_x^{(a)} = \Delta x y_0 + \frac{1}{2}\Delta x \Delta y^{(a)}$

(11) $\Delta C_y = \Delta C_y^{(a)} + \Delta y^{(b)}(x_0 + \Delta x) = \Delta y x_0 + \Delta x \Delta y - \frac{1}{2}\Delta x \Delta y^{(a)}$

In Equation 11 variable y's contribution of the residual term $\Delta x \Delta y$ is $\Delta x \Delta y - \frac{1}{2}\Delta x \Delta y^{(a)}$.

If the equation (9) holds the generalised decomposition coincides with the Sun's complete decomposition. An imbalance between the generalized decomposition and complete decomposition occurs when

(12) $$\frac{\Delta x}{x_0} \neq \frac{\Delta y}{y_0}$$

Let the imbalance be measured through a coefficient *c* so that

(13) $$\frac{\Delta x}{x_0} = c\frac{\Delta y}{y_0}$$

When $c = 1$ we have the balanced complete decomposition calculus, and $c < 1$ means that Δy exceeds the balanced value and $c > 1$ that Δx exceeds its balanced value. From the previous results the contribution equations of the generalized exact decomposition calculus can be derived as follows

(14) $\Delta C_x = \Delta x y_0 + c\frac{1}{2}\Delta x \Delta y$

(15) $\Delta C_y = \Delta y x_0 + (2 - c)\frac{1}{2}\Delta x \Delta y$

The above demonstration of generalized decomposition analysis was based on a two-dimensional case, but a multi-dimensional treatment can be easily carried out.

3. A PRELIMINARY MODEL FRAMEWORK

The framework of a decomposition analysis of energy-economic-environment model is outlined in Figure 2. Energy use, labour force (employment), material use, environment pollution and capital formation are determined by the economy through (GDP)-relations. Furthermore, environment pollution is determined by emission coefficients from the energy relations. The submodels can reflect both quantitative and structural changes. Sustainability means certain standards or objective norms to the environmental effects and energy development, which may condition the economic growth quantitatively or structurally. These new elements are indicated in the framework figure.

There are five sub-structures in the framework. They are the pollution structure, and the P-model of environmental effects; the material flow structure, with the M-model; the labour force structure, and the L-model; the energy use structure, with the E-model and the capital structure, with the C-model.

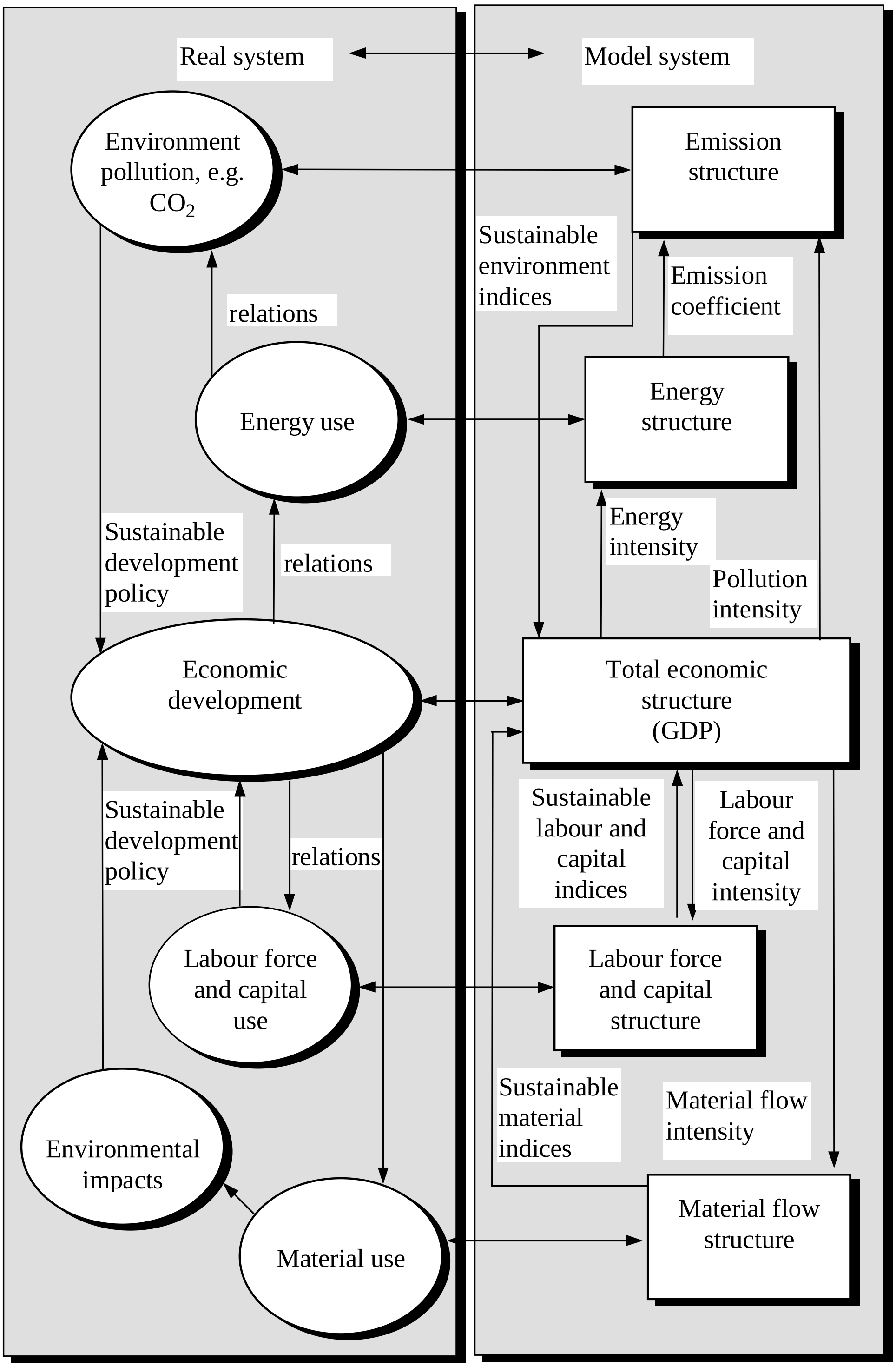

Figure 2. The framework of the structure and the models.

In the following, the mathematical principles of these models are outlined. The problem of sustainability as a tunnel is presented in Figure 3.

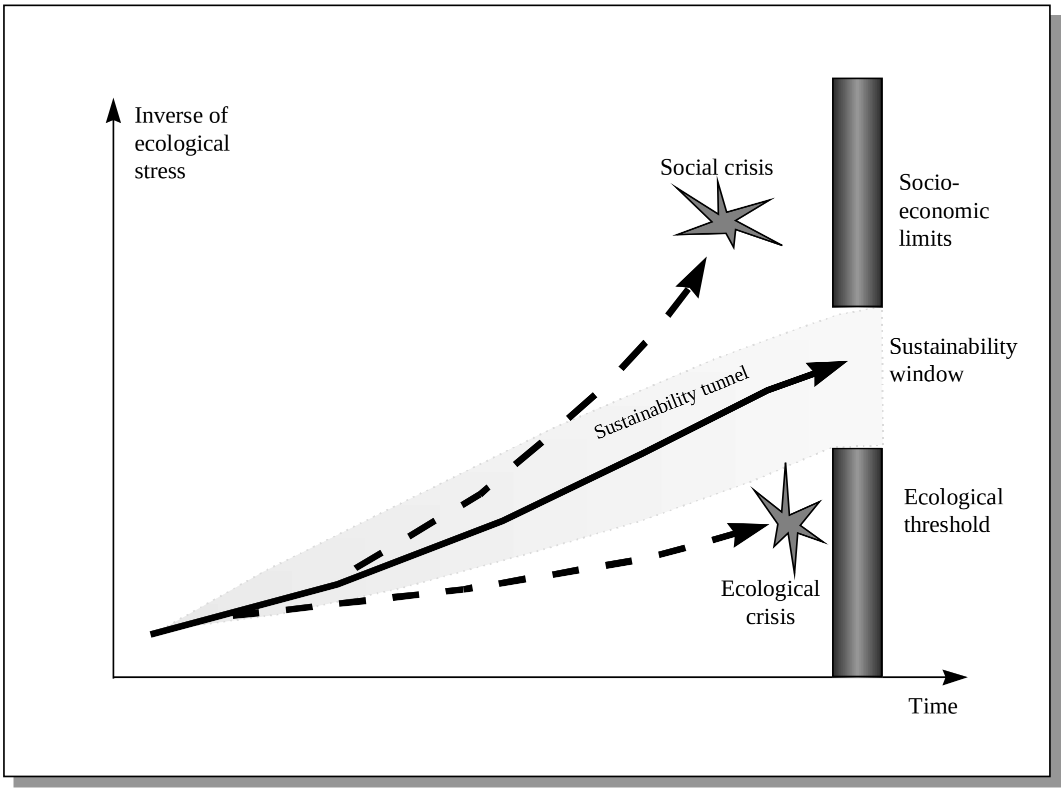

Figure 3. Sustainability window and tunnel.

In order to achieve sustainable development the environmental stress must be lowered. This means lower energy and material requirements and less emissions. On the other hand, too rapid changes in the socio-economic system to achieve the environmental goals may cause socio-economic problems such as unemployment, slow growth, loss of competitiveness etc. To map the boundaries of the "tunnel of sustainable development" it is possible, by using the presented theory, to develop indicators of the socio-economic limits such as GDP growth, capital accumulation and unemployment and the environmental limits such as emissions and material use.

3.1. Sub-models

To analyse the energy use structure, the following E-model is defined:

E is energy use

G is gross domestic product (GDP),

Ie_i is energy intensity of industry i,

S_i is the share of value added of industry i in GDP

$$(16)\quad E = \sum_i^n E_i = \sum_i^n \frac{E_i}{G_i}\frac{G_i}{G}G = \sum_i^n Ie_i S_i G$$

To analyse the pollution structure, the following P-model can be used:

P is pollution (e.g. CO_2 emissions)

G is gross domestic product (GDP),

Ip_i is pollution (emission) intensity of industry i,

S_i is the share of value added of industry i in GDP

$$(17)\quad P = \sum_i^n P_i = \sum_i^n \frac{P_i}{G_i}\frac{G_i}{G}G = \sum_i^n Ip_i S_i G$$

To analyse the material flow structure, the following M-model can be used:

M is material flow

G is gross domestic product (GDP),

Im_i is material flow intensity of industry i,

S_i is the share of value added of industry i in GDP

$$(18)\quad M = \sum_i^n M_i = \sum_i^n \frac{M_i}{G_i}\frac{G_i}{G}G = \sum_i^n Im_i S_i G$$

To analyse the labour force structure, the following L-model can be used:

L is labour force

G is gross domestic product (GDP),

Il_i is labour force intensity of industry i,

S_i is the share of value added of industry i in GDP

$$(19)\quad L = \sum_i^n L_i = \sum_i^n \frac{L_i}{G_i}\frac{G_i}{G}G = \sum_i^n Il_i S_i G$$

To analyse the capital structure, the following C-model can be used:

C is capital input
G is gross domestic product (GDP),
Ic_i is capital input intensity of industry i,
S_i is the share of value added of industry i in GDP

(20)
$$C = \sum_i^n C_i = \sum_i^n \frac{C_i}{G_i}\frac{G_i}{G}G = \sum_i^n Ic_i S_i G$$

In the above equations (16-20) the share of value added of industry i is called the structural shift $S_i = G_i/G$. The activity level G is GDP and the sectoral energy, pollution, material, labour and capital intensities are:

(21)
$$Ie_i = \frac{E_i}{G_i} \qquad Ip_i = \frac{P_i}{G_i} \qquad Im_i = \frac{M_i}{G_i} \qquad Il_i = \frac{L_i}{G_i} \qquad Ic_i = \frac{C_i}{G_i}$$

The changes and corresponding explanatory factors or effects in the decompositions of energy use, pollution, material flow, labour force and capital are:

(22) $$\Delta E = EG_{effect} + EI_{effect} + ES_{effect}$$

(23) $$\Delta P = PG_{effect} + PI_{effect} + PS_{effect}$$

(24) $$\Delta M = MG_{effect} + MI_{effect} + MS_{effect}$$

$$\Delta L = LG_{effect} + LI_{effect} + LS_{effect}$$

$$\Delta C = CG_{effect} + CI_{effect} + CS_{effect}$$

Three-dimensional decomposition is needed for Equations 22-26. As an example, the decomposition for energy is presented using the complete decomposition calculus (superscript 0 refers to base year situation).

(27)
$$EG_{effect} = \Delta G \sum_i Ie_i^0 S_i^0 + \tfrac{1}{2}\Delta G \sum_i (Ie_i^0 \Delta S_i + S_i^0 \Delta Ie_i) + \tfrac{1}{3}\Delta G \sum_i \Delta Ie_i \Delta S_i$$
$$EI_{effect} = G^0 \sum_i S_i^0 \Delta Ie_i + \tfrac{1}{2}\sum_i \Delta Ie_i\,(S_i^0 \Delta G + G^0 \Delta S_i) + \tfrac{1}{3}\Delta G \sum_i \Delta Ie_i \Delta S_i$$
$$ES_{effect} = G^0 \sum_i Ie_i^0 \Delta S_i + \tfrac{1}{2}\sum_i \Delta S_i\,(Ie_i^0 \Delta G + G^0 \Delta Ie_i) + \tfrac{1}{3}\Delta G \sum_i \Delta Ie_i \Delta S_i$$

3.2. Sustainability

It is possible to operationalize the sustainable development concepts of dematerialization of production, immaterialization of consumption, and rebound effect using the developed concepts of activity, intensity and structural effects. The relationship between the ecological sustainability (*Ms*) to the sub-model quantities is presented in the following matrix form.

(28)
$$Ms = \begin{pmatrix} M_{De} \\ M_{Sa} \\ M_{Re} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & +1 & +1 \end{pmatrix} \begin{pmatrix} MI_{effect} \\ MS_{effect} \\ MG_{effect} \end{pmatrix}$$

where
M_{De} is dematerialization
M_{Sa} is immaterialization (material saving)
M_{Re} is material rebound effect

The other sustainability models can be formulated in the same way. With these models we should use concepts of de-energization, delaborization, decapitalization and depollutization and accordingly im-energization, imlaborization, imcapitalization and impollutization. By sustainability models we can also analyse rebound effects of different factors of production.

3.3. Energy-environment model

The major factors of the analysis in the energy-environment model are the emission coefficient of energy use and the total use of energy. The other factors in the balance equation of the total emission reflect the various structural elements of the relationship between environment and energy. The energy-environment structure, the Em-model, defined as an identity is in Equation 29. *Em* as for emission and *E* for total use of energy, and the *i*, *j*, subscripts refer to various contributing categories of decomposition.

(29)
$$\begin{aligned} Em = P &= \sum_{i}^{n}\sum_{k}^{p} Em_{ik} = \sum_{i}^{n}\sum_{k}^{p} \frac{Em_{ik}}{E_{ik}} \frac{E_{ik}}{E_i} \frac{E_i}{E} E \\ &= \sum_{i}^{n}\sum_{k}^{p} \lambda_{ik}\, \eta_{ik}\, \zeta_i\, E \end{aligned}$$

where the emission coefficients (λ_{ik}) and the various energy structural ratios are:

(30)
$$\lambda_{ik} = \frac{Em_{ik}}{E_{ik}}, \qquad \eta_{ik} = \frac{E_{ik}}{E_i}, \qquad \zeta_i = \frac{E_i}{E}.$$

When applied with country data subscript i means industries and subscript k refers to energy forms, if data available:

Em_{ik} is emission released by industry i from energy form (fuel) k,
E_{ik} is energy consumed by industry i from energy form (fuel) k,
λ_{ik} is emission coefficient of fuel k in industry i,
η_{ik} is the share of the energy use of energy form k in industry i,
ζ_i is the share of the energy use of industry i of the total industrial energy use.

Equation 29 defines an identity of the decomposition of the environmental pollution Em into its relevant factors by multiplication. Further manipulation of it is needed (as in Equations 22-26, but four-dimensional situation) in order to obtain the decomposition model of the contributions of the factors for analyses.

In the above equations of this chapter industrial sectors are dealt with in one level. If needed, it will also be possible to utilize a two level analysis in which e.g. three basic industries, primary, secondary and tertiary industry, are divided into industrial subsectors.

4. SUMMARY

Decomposition approach is a way to analyse a society and its structural changes. It can be applied in the analyses of energy, material, labour and capital use. Linking the decomposition analyses to sustainability remarkably widens the scope of efficiency analysis that has been the traditional way to utilise decomposition in the case of energy studies. The formulas given above can be utilised in sector or international analyses and comparisons of economies. The presented exact decomposition method together with the developed sustainability models are powerful tools to analyse the development of societies and their sustainability in quantitative terms.

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ABSTRACT

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