Wavelet Analysis of Round-Trip-Time of a Packet on the Internet

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Information is carried on the Internet in the form of packets. Each packet is handed from one server (called gateway) to another until it reaches the destination assigned in the packet. As the topology of the network becomes complex and the number of network users increases, the traffic of the network gets crowded with lots of packets, and a large system consisting of lots of servers and packets is expected to attain a statistically stationary and more or less universal state.

Recently, Leland et al. [1,2] reported a self-similarity in temporal fluctuation of packet density passing through a server on network, and proposed a new self-similar stochastic model. observational data. Statistical analysis has also been performed for a time-series of round-trip time (RTT) of a packet between two servers. Csabai (1994) found that Fourier spectrum of RTT series has 1/f specral form, and proposed an idea that the jamming of the Internet would yield 1/f-spectrum of time series of RTT[3], based on an observation that highway traffic also gives 1/f-spectrum. These ideas were examined in detail by Takayasu et al. [4]. They discussed the Fourier spectrum and one-point distribution function of time series of RTT of a packet obtained by using ping-command of the Unix operating system. They confirmed the 1/f-spectrum and reported a typical form of the one-point distribution of RTT had a high peak around at 100msec and a long tail up to several seconds, when the number of gateways was 17 to 20. They observed a distribution of time intervals between amplitudes higher than the threshold RTT(they gave 1023 ms)[5] when the number of gateways was 17 and 20. They suggested that the times series of RTT showed the critical behavior at the critical points[6].

In this paper we give an obserevational report that there exists a limiting distribution for small-scale fluctuation in the time series of RTT. As far as the analyzed cases are concerned, this distribution is independent of paths and machines relevant to the round-trip of packets, although no theoretical explanation has yet given to the limiting distribution. We sampled RTT between two machines every second with an accuracy of 10^{-3} second by using 'ping'- command. We can get the (effective) round-trip-time of the packet by measuring the time between the emission and the arrival of these packets. In the following we show results in seven cases where a typical time series consists of $262144 (= 2^{18})$ points

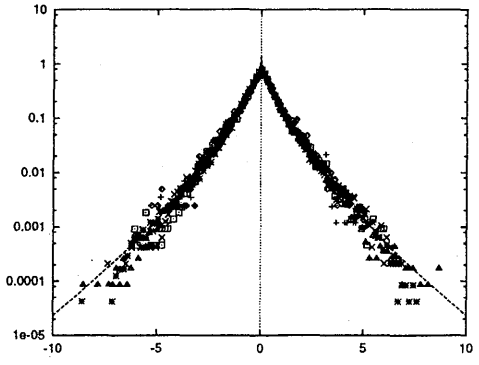

Fig.1: $12 \leq j \leq 17$ for 17 gateways.

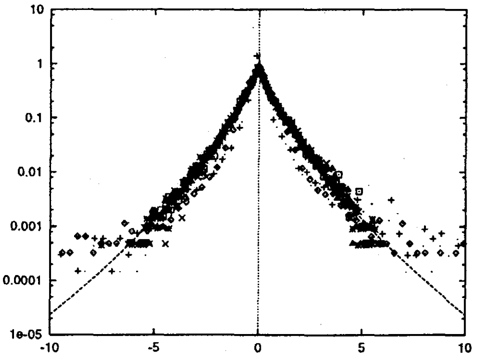

Fig.2: 11 to 17 gateways for $j = 14$.

($\sim$ 3 days). These time series were obtained by using paths containing 11 to 17 gateways between host and destination. Fourier spectrum of the time series in the case of 17 gateways shows 1/f-spectrum, consistently with the observation by Csabai[3] and Takayasu et al.[15]. The one-point distribution of RTT depends on the number of gateways and also on network conditions. Therefore clearly, no universal behaviour of the distribution function can be expected with respect to the magnitude of RTT itself. But these variations of distributions have long period, such as 12 hours or 7 days, and we may still expect that short-period components of fluctuations have some general properties.

We analyze short-period fluctuations in the time series of RTT by employing an orthonormal wavelet expansion method. In the wavelet analysis we decompose the time series of RTT, $f(t)$, into components localized both in time and frequency domains, as follows

$$f(t) = \sum_{j,k} \alpha_{j,k}\psi_{j,k}(t), \quad \psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k) \quad (j,k \in \mathbf{Z})$$

where $\alpha_{j,k}$ are expansion coefficients. Note that the index j corresponds to frequency, and the index k corresponds to time. The expansion coefficient $\alpha_{j,k}$ represents the magnitude of component localized at $t \sim k/2^j$ and frequency $f \sim \text{Const}.2^j$. We can observe the temporal distribution of frequency components corresponding to the frequency index j by taking the distribution of the wavelet coefficients $\alpha_{j,k}(k \in \mathbf{Z})$[7,8]. Here we adopt the Meyer wavelet as the mother wavelet $\psi(x)$.

A naive conjecture may be that the gateways on the paths would be statistically independent and contribute equally to the distribution of RTT, which would lead to the normal Gaussian distribution through the central limit theorem. However, we see that the probability density distribution function is different from the Gaussian distribution;

it takes larger values than the Gaussian distribution both around the origin and at tail regions, and smaller values at intermidiate range of the abscissa, although the distribution is almost symmetric with respect to the origin (the skewness is 0.16). The kurtosis (8.63) of the distiribution function also show a clear deviation from the Gaussian distribution.

In order to see the variation of the distribution with the frequency index j, we plot in Fig.1 the normalized probability density distributions of wavelet coefficients for $12 \leq j \leq 17$ in the case of 17 gateways. Remarkably, as frequency (i.e. j) increases, the distribution function converges to a distribution function which is well approximated by

$$P(x) = Ae^{-\alpha|x|^{\beta}} \quad (A = 1.35, \alpha = 2.34, \beta = 0.67).$$

which is also depicted in Fig.1. In order to see the path-dependence of the distribution, we took the RTT data between many pairs of machines. In Fig.2 we show the normalized probability density distributions for 11 to 17 gateways in the case of $j = 14$. When the number of gateways is larger than 14, the distribution coincides with each other and with $P(x)$. This means that as the number of the gateways increases, the distributions converges to the above limiting distribution $P(x)$ independently of paths of packets. We remark that when the distribution is different ¿from $P(x)$, the Fourier spectrum does not show 1/f-form. This observation suggests that the limiting form $P(x)$ of the distribution function may be closely connected to the 1/f-spectrum.

We have observed that the distribution of wavelet coefficients of RTT converges to the distribution $P(x)$ as the frequency parameter j increases. The variation of the distribution with j is related to the multi-fractal property of the data series. According to the formulation of Parisi-Frisch [9], the Hausdorff dimension $h(\alpha)$ of a set of points $\{t\}$ where a small increment of data series $f(t+\tau) - f(t)$ has the order of $\tau^{\alpha}(\tau \longrightarrow 0)$, is connected by Legendre transformation to the scaling exponenent ζ_p of moments which is defined as $< (f(t+\tau) - f(t))^p >\sim \tau^{\zeta^p}$, where the bracket denotes an ensemble or spatial average. Noticing that the distribution of the increment is well approximated by the distribution of $\alpha_{j,k}$ for $t \sim k/2^j$ and $\tau \sim 1/2^j$, we conclude that the scaling exponent ζ_p is a linear function of p if the distribution of $\alpha_{j,k}$ is independent of j. This form of ζ_p indicates that the Hausdorff dimension $h(\alpha)$ is defined only at a single point of α, and therefore the data series has not a multi-fractal but a *mono-fractal* property. Note that this conclusion is a refinement of the observation that the RTT time series looks like fractal, i.e., invariant when appropriately scaled.

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