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Multifold Local Minimum Structure of Total Transport Cost in Logistic Distribution Allocation

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Abstract

We solved a logistic problem for a Group Company which was how they should allocate the transport distribution routes to minimize the total transport cost from their manufacturing companies to retailers via distribution centers [1]. We propose to solve the problem by a perturbation method, which is based on the assumption that the distribution center fees weakly affect the total transport cost. We point out that the analytical expression of the total transport cost has a multifold local minimum structure. In the case when the distribution center fees affect strongly the total transport cost, the perturbation method fails to allocate the optimum logistic distribution routes for the overall minimum of the total transport cost, while the method leads to a local minimum that is not the overall minimum of the total transport cost. Therefore whenever one has solved the logistic problem by the perturbation method, he must further examine if the total transport cost is reduced to be lower by allocating any transport distribution routes so that the distribution routes extend via a smaller number of distribution centers.

Key Words: logistic distribution allocation, total transport cost

1. Introduction

We solved a logistic problem for a Group Company which was how they should allocate the transport distribution routes to minimize the total cost for the transport from their manufacturing companies A_i to retailers B_k via distribution centers D_j. The Group Company is assumed to have r manufacturing companies and n retailers, and the logistic distribution network from manufacturing companies to retailers via m distribution centers. The total transport cost is expressed as follows:

$$z = \sum_{j=1}^{m} \Big(\sum_{i=1}^{r} c_{ij} X_{ij} + \sum_{k=1}^{n} d_{jk} Y_{jk} \Big) + \sum_{j=1}^{m} v_j W_j^{\theta} \qquad (1)$$

with $0 < \theta < 1$. In Eq. (1), c_{ij} stands for the cost of transport per unit from manufacturing company A_i to distribution center D_j, d_{jk} also for the cost of transport per unit from distribution center D_j to retailer B_k and v_j is a parameter for the fee of distribution center D_j. The

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values of these quantities are given. Each one of transport routes from a manufacturing company to a retailer extends through only one distribution center. We allocate the optimum logistic distribution routes, determining the variables X_{ij} for transport quantity from manufacturing company A_i to distribution center D_j and the variables Y_{jk} for transport quantity from distribution center D_j to retailer B_k so as to minimize the total transport cost z. The variable W_j is the throughput of distribution center D_j.

We solve the optimum logistic distribution allocation problem under the following conditions: The supply of manufacturing company A_i is given by

$$\sum_{j=1}^{m} X_{ij} = a_i \tag{2}$$

and the demand of retailer B_k is given by

$$\sum_{j=1}^{m} Y_{jk} = b_k. \tag{3}$$

The throughput of distribution center D_j is

$$W_j = \sum_{i=1}^{r} X_{ij} = \sum_{k=1}^{n} Y_{jk}. \tag{4}$$

In this paper we propose to solve the logistic problem by the perturbation method, which is based on the assumption that the distribution center fees weakly affect the total transport cost. We will show that the total transport cost z in Eq. (1) has a multifold local minimum structure. Especially in the case when the parameters v_j for distribution center fees are large and when each one of v_j almost degenerates with others, the perturbation method fails to allocate the optimum logistic distribution routes for the overall minimum of the total transport cost. The perturbation method in principle finds a minimum of the total transport cost z in the vicinity of the minimum determined in the initial solution. Therefore, the minimum obtained in the final solution of the perturbation method is not necessarily the overall minimum of the total transport cost. In a logistic distribution route allocation space, the overall minimum is separated from local minima by the barriers of total transport cost. To reach the overall minimum from a local minimum, we must cross the barrier, where the total transport cost is higher than that at the local minimum.

In Section 2, we explain the perturbation method. In Section 3, we discuss the multifold local minimum structure of the total transport cost z in Eq. (1), using simple examples of the logistic distribution network.

2. Perturbation Method

By the perturbation method, we start to solve the logistic distribution route allocation problem. The perturbation method proceeds in iteration procedures based on the assumption that the distribution center fees weakly affect the total transport cost. The following are the solution steps.

1) When we allocate the optimum transport distribution routes in the initial iteration procedure for the perturbation method, we ignore the distribution center fees $v_j W_j^{\theta}$ in the total transport cost z, Eq. (1). At the beginning, we calculate the transport cost per unit for each a transport route from a manufacturing company to a retailer via a distribution center. For each combination of a manufacturing company A_i with a retailer B_k, we select one optimum

transport route from A_i to B_k on which the transport cost per unit is lowest. Selecting lower transport cost routes, we allocate an optimum set of transport distribution routes at the lowest total transport cost z appropriate for the supplies a_i by manufacturing company A_i and the demands b_k by retailer B_k given in the conditions (2) and (3). Thus the optimum transport distribution routes for the initial solution of the perturbation method are allocated. We calculate the throughputs W_j of distribution center D_j proper to the optimum transport distribution route allocation for the initial solution. The total transport cost including the fees for the throughput W_j of distribution center D_j determined from the initial solution are calculated.

2) Proceed to a higher order of the iteration procedure for the perturbation method, where the fees $v_j W_j^{\theta}$ of distribution center D_j are taken into account. The fee per unit transport quantity for distribution center D_j is assumed to be the amount of $v_j W_j^{\theta} / W_j$, where the throughput W_j is determined in the previous iteration procedure. Calculate the transport cost per unit for each transport route from a manufacturing company to a retailer via a distribution center. For each combination of a manufacturing company A_i with a retailer B_k, select one optimum transport route from A_i to B_k on which the transport cost per unit is lowest. Allocate an optimum set of transport distribution routes at the lowest total transport cost z appropriate for the conditions (2) and (3). The optimum transport distribution routes and the total transport cost are determined as a solution in the higher order of the iteration procedure for the perturbation method.

3) Repeat the procedure 2) until a same solution is obtained as that in the previous iteration procedure. The final solution is the most accurate solution obtained in the perturbation method.

3. Multifold Local Minimum Structure of Total Transport Cost

In this section, we argue that the total transport cost z in Eq. (1) has several local minima. In order to develop this argument in a case of simplest examples of the logistic distribution network, we assume a Group Company to have two manufacturing companies A_i, two distribution centers D_j and two retailers B_k. In Table 1 we show the costs c_{ij} of transport per unit from manufacturing company A_i to distribution center D_j and the supplies a_i by A_i. In Table 2 we show the costs d_{jk} of transport per unit from distribution center D_j to retailer B_k and the demands b_k by B_k. In this example we take two cases for the distribution center fees, i.e., Cases 1 and 2. In Case 1 the distribution center fees are assumed to be $100\sqrt{W_1}$ for D_1 and also to be $100\sqrt{W_2}$ for D_2. In Case 2 the fees are $200\sqrt{W_1}$ for D_1 and $200\sqrt{W_2}$ for D_2. The distribution center fees in Eq. (1) with $\theta = 1/2$ are taken as shown in Table 3. We will show that in Case 1 the perturbation method successfully leads to the overall minimum of the total transport cost z in Eq. (1) but that in Case 2 the method fails to lead to the overall minimum of the total transport cost.

Table 1 The costs c_{ij} of transport per unit from manufacturing company A_i to distribution center D_j and the supplies a_i by A_i are shown.

	D_1	D_2	a_i
A_1	5	10	100
A_2	10	5	100

Table 2 The costs d_{jk} of transport per unit from distribution center D_j to retailer B_k and the demands b_k by B_k are shown.

	B_1	B_2
D_1	5	10
D_2	10	5
b_k	100	100

Table 3 The fees $v_j W_j^{\theta}$ for the throughput W_j of distribution center D_j with $\theta = 1/2$ in Cases 1 and 2 are shown.

Case	D_1	D_2
Case 1	$100\sqrt{W_1}$	$100\sqrt{W_2}$
Case 2	$200\sqrt{W_1}$	$200\sqrt{W_2}$

1) *Solutions in perturbation method*

(1) Case 1

Starting for the initial solution in the perturbation method, we select an optimum transport route from manufacturing company A_i to retailer B_k at the lowest transport cost by neglecting the distribution center fees $v_j\sqrt{W_j}$. The transport costs per unit on the optimum transport route from manufacturing company A_i to retailer B_k are shown in Table 4. The distribution centers D_j on the optimum transport route are also indicated in the Table. Selecting lower transport cost routes, we allocate the two routes for the optimum transport distribution for the initial solution appropriate for the conditions (2) and (3), i.e., the route $A_1 \to D_1 \to B_1$ for the transport of 100 units supplied by manufacturing company A_1 and the route $A_2 \to D_2 \to B_2$ for the transport of 100 units supplied by A_2 as shown in Table 5. The throughputs and the fees of distribution center D_j on the optimum transport distribution routes determined in the initial solution are shown in Table 6. The initial solution yields the transport fee to be 2,000, the distribution center fee to be 2,000 and the total transport cost $z = 4{,}000$.

In the iteration procedure for the second solution of the perturbation method, we select an optimum transport route from manufacturing company A_i to retailer B_k at the lowest transport cost by adding the fees $v_j\sqrt{W_j}\,/\,W_j$ per unit throughput of distribution center D_j for which the throughputs W_j are determined by the optimum transport distribution route allocation in

Table 4 The transport costs per unit on the optimum transport route from manufacturing company A_i to retailer B_k determined for the initial solution by neglecting the distribution center fees $v_j\sqrt{W_j}$ are shown in a matrix expression. The distribution centers D_j on the optimum transport route are also indicated.

	B_1	B_2
A_1	D_1 10	D_1 D_2 15
A_2	D_1 D_2 15	D_2 10

Table 5 The transport quantities on the optimum transport distribution routes allocated in the initial solution are shown. The distribution centers D_j on the optimum transport distribution routes are also indicated.

	B_1	B_2	a_i
A_1	D_1 100		100
A_2		D_2 100	100
b_k	100	100	

Table 6 The throughputs W_j, fees $v_j\sqrt{W_j}$ and fees $v_j/\sqrt{W_j}$ per unit throughput of distribution center D_j determined in the optimum transport distribution route allocation for the initial solution are shown.

D_j	D_1	D_2
W_j	100	100
$v_j\sqrt{W_j}$	1000	1000
$v_j/\sqrt{W_j}$	10	10

Table 7 The transport costs per unit on the optimum transport route from manufacturing company A_i to retailer B_k determined for the second solution by adding the fees $v_j\sqrt{W_j}\,/\,W_j$ per unit throughput of distribution center D_j which are determined by the optimum transport distribution route allocation in the initial solution are shown. The distribution centers D_j on the optimum transport route are also indicated.

	B_1	B_2
A_1	D_1 20	D_1 D_2 25
A_2	D_1 D_2 25	D_2 20

the initial solution. The transport costs per unit on the optimum transport route from manufacturing company A_i to retailer B_k are shown in Table 7. The distribution centers D_j on the optimum transport route are also indicated in the Table. For the second solution we allocate the two transport distribution routes which are the same as those in the initial solution, i.e., the route $A_1 \to D_1 \to B_1$ for the transport of 100 units supplied by A_1 and the route $A_2 \to D_2 \to B_2$ for the transport of 100 units supplied by A_2. Further repetition of the iteration procedures allocates the same transport distribution routes. Thus, the optimum transport distribution routes are allocated for the final solution to the present problem in the perturbation method. The final solution yields the transport fee to be 2,000, the distribution center fee to be 2,000 and the total transport cost $z = 4{,}000$.

(2) Case 2

Now we proceed to Case 2, where the distribution center fees are modified from the values in Case 1 as shown in Table 3. The perturbation method for this case yields the initial solution in which we allocate, for the optimum transport distribution, two transport routes, i.e., the route $A_1 \to D_1 \to B_1$ for the transport of 100 units supplied by manufacturing company A_1 and the route $A_2 \to D_2 \to B_2$ for the transport of 100 units supplied by A_2 as in Case 1. The initial solution yields the transport fee to be 2,000, the distribution center fee to be 4,000 and the total transport cost $z = 6{,}000$.

In the iteration procedure for the second solution, we allocate the same two transport routes for the optimum transport distribution as those in the initial solution, i.e., the route $A_1 \to D_1 \to B_1$ for the transport of 100 units supplied by A_1 and the route $A_2 \to D_2 \to B_2$ for the transport of 100 units supplied by A_2. Further repetition of the iteration procedures allocates the same optimum transport distribution routes. Thus, the optimum transport distribution routes are allocated for the final solution in the perturbation method. The final solution yields the transport fee to be 2,000, the distribution center fee to be 4,000 and the total transport cost $z = 6{,}000$.

2) ***Overall minimum of total transport cost***

In this subsection we argue that the total transport cost $z = 6{,}000$ obtained in the final solution in the perturbation method for Case 2 is not an overall minimum but a local minimum of the total transport cost z. The optimum transport distribution routes for an overall minimum of the total transport cost z are determined by bunching the transport distribution routes so that the transport distribution routes extend via a smaller number of distribution centers, only D_1 for example, so as $A_1 \to D_1 \to B_1$ for the transport of 100 units and $A_2 \to D_1 \to B_2$ for the transport of another 100 units. There is another way to allocate the optimum transport distribution routes: to bunch the transport distribution routes so that the distribution routes extend via only distribution center D_2 as $A_1 \to D_2 \to B_1$ for the transport of 100 units and $A_2 \to D_2 \to B_2$ for the transport of another 100 units. These two ways of the optimum transport distribution route allocation yield equally the transport fee to be 3,000, the distribution center fee to be 2,828 and the total transport cost $z = 5{,}828$. This value of the total transport cost for the optimum transport distribution route allocation by bunching the transport distribution routes is lower than the total transport cost $z = 6{,}000$ determined in the final solution of the perturbation method. Therefore the perturbation method fails to lead to the overall minimum of the total transport cost in Case 2.

Now we discuss the multifold local minimum structure of the total transport cost z in Eq. (1). The supplies a_i by manufacturing company A_i and the demands b_k by retailer B_k in Eq's. (2) and (3), respectively, in the present model are expressed as follows,

$$X_{11} + X_{12} = Y_{11} + Y_{21} = 100, \tag{5}$$

$$X_{21} + X_{22} = Y_{12} + Y_{22} = 100. \tag{6}$$

In order to demonstrate the multifold local minimum structure of the total transport cost z, we take a subspace of the transport distribution route allocation where the transport quantities are distributed symmetrically between manufacturing companies A_i and retailers B_k so that they are transported only on four routes, i.e., the route $A_1 \to D_1 \to B_1$ for the transport of $X_{11} = Y_{11}$ units, $A_1 \to D_2 \to B_1$ for the transport of $X_{12} = Y_{21}$ units, $A_2 \to D_1 \to B_2$ for the transport of $X_{21} = Y_{12}$ units and $A_2 \to D_2 \to B_2$ for the transport of $X_{22} = Y_{22}$ units. The transport distribution route allocation space is further limited so as

$$X_{11} = Y_{11} = 100, \quad \text{or} \quad X_{22} = Y_{22} = 100. \tag{7}$$

Then we define the asymmetric factor

$$W = W_1 - W_2 \tag{8}$$

for the throughputs W_1 and W_2 of distribution centers D_1 and D_2, respectively. Now the value of the total transport cost in Eq. (1) is expressed by

$$z(W) = 2000 + 5\,|W| + v\left(\sqrt{100 + \frac{W}{2}} + \sqrt{100 - \frac{W}{2}}\right) \tag{9}$$

as a function of the asymmetric factor W with a parameter $v = v_1 = v_2$ for the distribution center fees. The former two terms in Eq. (9) are for the transport fee and the last term is for the distribution center fee. The transport fee, distribution center fee and total transport cost calculated as a function of W are shown in Fig's. 1 and 2 for Cases 1 and 2, respectively. We see in both the Cases that the transport fee has a minimum at $W = 0$ but the distribution center fee has two minima at $W = -200$ and 200. The total transport cost z, i.e., the sum of the transport fee and the distribution center fee, has three minima. Only the minimum of the total transport cost at $W = 0$ is, however, obtained by the perturbation method based on the assumption that the distribution center fees weakly affect the total transport cost as shown in Subsection 1). The perturbation method fails to lead to the overall minimum of the total transport cost in the case that the distribution center fees dominate the total transport cost.

Eq. (9) yields the following relations:

$$\begin{aligned} z(0) &= 2000 + 20v \\ z(200) = z(-200) &= 3000 + v\sqrt{200}. \end{aligned} \tag{10}$$

In Case 1 with the parameter $v = 100$ for the distribution center fees, the relation $z(0) < z(-200) = z(200)$ holds: The total transport cost z has an overall minimum at $W = 0$ with a dominant contribution of the transport fee to the total transport cost. In Case 2 with $v = 200$, however, the relation $z(0) > z(-200) = z(200)$ holds: The total transport cost z has two overall minima at $W = -200$ and 200 and a local minimum at $W = 0$ with the distribution center fee overcoming the transport fee. For the parameter $v > 170.8$ of the distribution center fees in the present simple model, the minima at $W = -200$ and 200 get lower than the minimum at $W = 0$: The perturbation method fails to lead to the overall minima at $W = -200$ and 200. When one has obtained the final solution of the perturbation method, he must further calculate the total transport costs by allocating any optimum transport

Total Transport Cost in Case 1

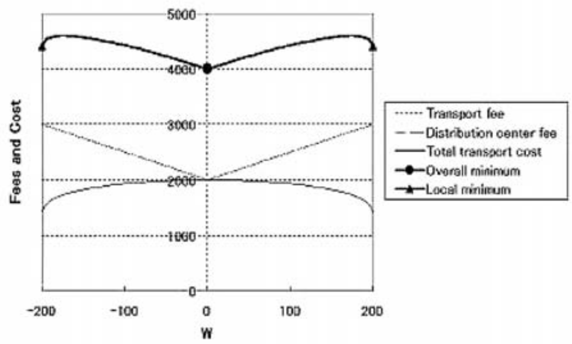

Figure 1 The transport fee, distribution center fee and total transport cost for Case 1 calculated as a function of the asymmetric factor $W = W_1 - W_2$ for the throughputs W_1 and W_2 of distribution centers D_1 and D_2, respectively. The total transport cost has one overall minimum at $W = 0$ and two local minima at $W = -200$ and 200.

Total Transport Cost in Case 2

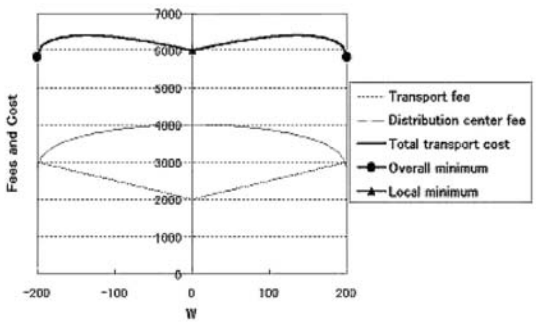

Figure 2 The transport fee, distribution center fee and total transport cost for Case 2 calculated as a function of the asymmetric factor $W = W_1 - W_2$. The total transport cost has two overall minima at $W = 200$ and -200 and one local minimum at $W = 0$.

distribution routes so that the distribution routes extend via a smaller number of distribution centers and to compare the calculated values of the total transport cost with the minimum of the total transport cost in the final solution of the perturbation method. Figures 1 and 2 visualize the multifold local minimum structure of the total transport cost z in Cases 1 and 2, respectively.

4. Discussions and Conclusion

We solved a logistic problem for a Group Company which was how they should allocate the transport distribution routes to minimize the total cost for the transport from their manufacturing companies to retailers via distribution centers. We proposed to solve the problem by a perturbation method, which is based on the assumption that the distribution center fees weakly affect the total transport cost. We have argued that the analytical expression of the total transport cost z in Eq. (1) has a multifold local minimum structure. In a case when the parameters v_j for distribution center fees are large and when each one of v_j almost degenerates with others, the perturbation method fails to allocate the optimum logistic distribution routes for the overall minimum of the total transport cost, while the method leads to a local minimum which is not the overall minimum of the total transport cost. Therefore when one has solved the logistic distribution route allocation problem by the perturbation method, he must further examine if the total transport cost is reduced to be lower by allocating any optimum transport distribution routes so that the distribution routes extend via a smaller number of distribution centers. The allocation of transport distribution routes via a smaller number of distribution centers is favored by the W_j^θ dependence of the distribution center fees $v_j W_j^\theta$ with $0 < \theta < 1$ on the throughput W_j of distribution center D_j. We will apply the present discussion to some real examples of the logistic distribution network in another context.

References

[1] Meilong Le and Yongchang Qian, Logistics Education and Research in China, Josai Journal of Business Administration, the present issue.