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Innovation, Patent Races, and Endogenous Growth

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Abstract

This paper presents a model of innovations and economic growth, which departs from standard endogenous growth models by assuming that the set of potential projects for innovation in each period is limited. The model differs in a number of results from former endogenous growth models. First, it explains patent races, where many research teams search for the same potential innovation. Second, the rate of growth of the economy is bounded and does not rise too much with the scale of the economy. Namely, the model gives rise to a non-linear relationship between the size of the R&D sector and the rate of growth. Third, R&D is Pareto-inefficient, as there are too many research teams searching for the same breakthrough. This problem increases with scale. Fourth, concentration of R&D by monopolistic firms is explained in this model by risk aversion.

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Innovations, Patent Races and Endogenous Growth

1. Introduction

This paper presents a model of innovations and economic growth, which departs from standard R&D-based endogenous growth models in one single assumption.[1] It assumes that the set of potential projects for innovation in each period is limited rather than unlimited, as implicitly assumed in those models. Changing this assumption leads to a number of interesting results. First, this model generates patent races, where many researchers, or research teams, search for the same potential innovation. We examine what happens if they search in similar ways for an innovation, or by using different search strategies. Second, in this model the rate of growth of the economy is bounded when the scale of the economy increases. Namely economic growth does not present strong scale effects. Third, the model predicts a non-linear relation between the amount of R&D and the rate of growth, where increases in R&D have a diminishing effect on economic growth. Fourth, R&D is Pareto-inefficient, as there are too many research teams searching for the same breakthrough using the same research methods. This problem increases with the scale of the economy. Finally, the model shows that the concentration of much R&D by monopolistic firms is a result of risk aversion.

The assumption that limits the number of potential innovations in each period is a very realistic assumption and touches on the essence of the innovation process. This is a gradual process, in which innovations follows previous innovations and need for the new innovations emerges after some experience with the previous ones. Thus, the inventors of

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[1] The main R&D based endogenous growth models, to which I refer in the paper, are Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

the car could not invent the air bag immediately at the same time. Some years of experience with traffic problems and a growing need for safety led to it. Another example is the invention of talking movies, which came almost 30 years after the silent movies. The delay was not due to insufficient inventors, but due to gradual development of need and of know-how. We capture this gradual development of innovation in our assumption that in each period the number of potential innovations is limited.

The paper develops an aggregate model of economic growth through innovations that increase the productivity of workers. The model has overlapping generations, where each individual can choose whether to become a production worker or an innovator, when young. Each innovator (or each research team) searches for one of the potential innovations. If it is found, she sells its use in next period, when old. Production workers purchase the patent rights from the old innovators, who were successful last period. The size of the production sector and the size of the R&D sector are determined by equalization of expected lifetime utilities across sectors.

As the scale of the economy increases, the gains from each invention increase. It lures individuals to enter the R&D sector. If they could, they would search for a whole new potential innovation, in order to reap the full gains from innovation. This is what happens indeed in the standard endogenous growth models. But in this model they soon exhaust all potential innovations, which are limited. If the gains from successful innovations are very high, they might still want to search for them and enter a patent race. In such a race many search for the same potential innovation, and the first who finds, gets the patent rights. The expected gains from innovation are therefore reduced as more innovators enter the race, until equilibrium is reached, where the expected gains from innovation are equal to gains from production.[2] Thus, the first result of the model is an explanation of patent races.

Next we examine the effect of scale on economic growth. Scale increases the gains from innovation, which attracts more innovators. But the increase in R&D sector does not lead to more rapid economic growth, as the number of potential projects is bounded. This result is interesting for two reasons. First, it removes the strong scale effect of the initial R&D-based endogenous growth models, which has been in odds with the empirical evidence. Second, this result is related to the critique of Jones (1995) on R&D-based growth models, where he shows that increases in R&D in OECD countries did not lead to higher growth rates. This paper's explanation to this puzzle is that much of the increased R&D activity was due to more participants in each patent race, without an increase in the number of innovations.

The most interesting result of this paper is that if the amount of potential innovations is limited, the equilibrium is inefficient, as there is too much R&D, since too many innovation teams search for the same innovation. This holds in the case of a single research strategy, but it also holds in the case of multiple research strategies. In the first case the result is obvious, since all competing innovators follow the same strategy, so a single innovator is sufficient. In the second case competing innovators can pursue different search strategies and increase the probability of finding the innovation. But the paper shows that most innovators follow the most promising strategies, while only few follow the less promising ones. Hence there is still too much R&D activity.

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[2] In the initial endogenous growth models the equilibrating mechanism is entry to the R&D sector that reduces the size of the production sector, until the gains from innovation are sufficiently low.

Finally, the paper deals with the effect of risk aversion of patent races. Risk aversion should lead to cooperation between participants of a patent race, as they can share the gain from innovation and have the same expected income, but with less risk. While such cooperation is hard to achieve by agreement, it can emerge when a single firm employs all the teams that search for an innovation and it thus internalize the patent race. On the one hand such a firm wants to have fewer teams, in order to increase the gains per team, but on the other hand it needs to hire enough teams to deter potential competition. We show that as a result such R&D monopolies have more teams than under competition, so that they are less efficient.

This paper is related to the literature of endogenous growth, which began with Romer (1986) and Lucas (1988), and had a large contribution to understanding global economic growth. The main line in this literature has explicitly modeled the creation of technical progress within a macroeconomic framework of sustained growth, as in Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). These models have been very successful in using the increasing returns to scale of innovations to explain the impressive global economic growth over the recent two centuries.[3] But these models also faced criticism, mostly because the scale effect they use has come out too strong. Their prediction that the rate of growth increases unboundedly with the scale of the economy has been in contrast with empirical evidence. As a result, a number of papers have refined the endogenous growth models in order to eliminate this strong scale effect, mainly Kortum (1997), Young (1998), Segerstrom (1998) and Howitt (1999). In a way, these papers share a similar assumption, namely that as technology progresses and

[3] In the years 1820-1992 world GDP increased 40 fold, and GDP per capita increased 8 fold. US output per capital increased 17 fold. See Maddison (1995).

becomes more advanced, it becomes more difficult to create new innovations, and this change in the production function of innovations enables these models to have sustained growth without scale effects.

This paper follows a very different route, by focusing on the gradual process of innovations, which cannot be invented all at the same time, but have to follow previous innovations, one at a time. This assumption leads the paper to a more micro-oriented analysis of the innovation process, which studies how innovators compete with one another in finding the same potential innovation, and on how they might use similar or different methods in their search. As a result this paper has much wider results than just the elimination of scale effects. It explains how patent races form, how they run, what is their market structure, and it examines their efficiency.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibrium in the case of a single research strategy, while Section 4 extends the analysis to the case of multiple research strategies. Section 5 examines the introduction of risk aversion and how it leads to concentration of R&D by large monopolies that internalize patent races. Section 6 concludes.

2. The Model

Consider an economy with a single final good, which is produced by many intermediate goods, indexed on [0, 1]. The final good is produced according to the following Kobb-Douglas production function:

(1) $$\log y_t = \int_0^1 \log x_{j,t} dj .$$

The intermediate goods are produced by labor only. A worker, who uses all available technologies at time t, can produce an amount $a_{j,t}$ of intermediate good j. This productivity rises from one period to the other through innovations. Namely, production of each intermediate good goes through a process of technical change. Each innovation increases productivity of a worker by an amount, which is proportional to last period productivity, namely by $ba_{j,t}$, where $b > 0$.[4] Thus, if the current productivity in sector j is $a_{j,t}$ and if $i_{j,t}$ innovations are found in the sector in period t, the next period productivity will be $a_{j,t+1} = a_{j,t}(1 + i_{j,t}b)$.

We next introduce the main assumption of the paper, namely that potential innovations are limited. Formally, we assume that the number of potential innovations in each sector in each period of time is finite. For simplicity, we assume that the number of potential innovations in each sector in each period is 1, although the results carry over for higher numbers as well, as can be easily shown. As a result, each sector can have at most one innovation in each period. Formally, $i_{j,t}$ can be either 1, if the innovation is found, or 0 if it is not found.

We next turn to describe the search for innovations, which is conducted by innovation teams. For simplicity we normalize the size of each innovation team to 1. A potential innovation is searched by a number of teams, where each can find the innovation, but only one finds it first. This team gets the patent rights on this innovation and sells its use to the producers of the intermediate good. Note, that if the set of potential innovations were unbounded, each innovation team would choose a different innovation to search for in order to increase the probability of getting the patent rights. Patent races

[4] This proportionality assumption is common to all endogenous growth models. It reflects what is

of this type appear only when the set of innovations is limited. We next turn to describe how teams search for the potential innovation. One possible assumption is that there is only one way to search for the innovation, namely there is a single search strategy. In that case all the teams, which are searching for the innovation, follow the same search strategy, but only one of them gets there first. An alternative assumption is that there can be many different search strategies, where each has some probability of success, but only one is ultimately successful. The paper explores these two alternative assumptions on R&D and shows that they yield similar results. We call the first assumption the *single search strategy* case, and the second one the *multiple search strategies* case. We next present the assumptions on these two cases more formally.

Single Search Strategy

In this case all teams search similarly. The probability of finding the innovation in each sector is 1.[5] As assumed above, only one team finds the innovation first and gets the patent rights. The probability of finding it first is the same for all teams. Hence, if the number of teams that search for the innovation in sector j in period t is $n_{j,t}$ the probability of success for each team is:

(2) $$P_{j,t} = \begin{cases} 0 \text{ if } \; n_{j,t} = 0 \\ \dfrac{1}{n_{j,t}} \text{ if } \; n_{j,t} \geq 1. \end{cases}$$

Multiple Search Strategies

In this case there are T search strategies, $T \leq \infty$, for each innovation. The probability of finding the innovation while using strategy s is e_s. We order the strategies by decreasing

sometimes called the "spillover effect" of innovations: they increase productivity of future innovators.

[5] We could model search with probability of success less than 1. The results are unchanged.

probability of success: $e_1 > e_2 > ... > e_T$. The search strategies are independent of one another, so that the probability of finding the innovation is $\sum_{s \in Q} e_s$ if Q is the set of strategies followed. If all strategies are followed the probability of success is 1. The probabilities of the different strategies can in principle vary over sectors and over time, but in order to simplify the presentation of the equilibrium at the steady state, we assume that these probabilities are the same for all sectors and for all times. Hence, we must further specify these probabilities to be: $e_s = e(1-e)^{s-1}$ for all s, $1 \leq s < \infty$, where $0 < e < 1$. Under this specification, if an innovation is not found in period t, after using strategies 1,…, S, innovators can use the remaining strategies from next period on and the conditional probabilities of success of these strategies are going to be exactly the same as the original probabilities: $e_s = e(1-e)^{s-1}$. Hence, the probabilities of the various strategies are the same, whether research on the innovation has just begun, or if it has been going on for some time. Let us denote the number of research teams in sector j in time t, which use strategy s by $n_{s,j,t}$. The success probability of such a team is:

(3) $$P_{s,j,t} = \begin{cases} 0 \text{ if } n_{s,j,t} = 0 \\ \dfrac{e_s}{n_{s,j,t}} \text{ if } n_{s,j,t} \geq 1. \end{cases}.$$

We next turn to describe individuals in the economy. Assume that this is an overlapping generations economy, where individuals live two periods each. There is no population growth and the size of each generation is L. Individuals are assumed to be risk neutral, so that utility from consumption is:

(4) $$u = c_1 + \frac{1}{1+\rho} c_2 ,$$

where c_1 is consumption when young, c_2 is consumption when old, and ρ is the subjective discount factor.[6] Individuals work in first period of life only. They work either in the production sectors or in the R&D sector. A worker, who produces an intermediate good in period t, sells it in the market to earn income. If the worker uses a new innovation in production of the intermediate good, he pays patent fees to the patent holder, who has innovated it in period t-1. A member of an innovation team receives income from patent fees. We assume that patent rights hold one period only. Hence a team, which finds an innovation in period t, earns the patent fees in period t+1 in their second period of life.

3. Equilibrium with a Single Search Strategy

We begin the analysis of equilibrium with the simpler case of a single search strategy for each innovation.

3.1 The Markets for Innovations

Denote the market prices of the intermediate goods by $p_{j,t}$, where the final good serves as the numeraire. Consider a sector j, in which an innovation has been found in period t-1. The team that has patent rights has monopoly over the innovation at t. Denote the patent fee paid in period t in sector j by $z_{j,t}$. Workers are willing to purchase the innovation as long as their net income is greater or equal to their income without the innovation:

(5) $$p_{j,t}a_{j,t} - z_{j,t} \geq p_{j,t}a_{j,t-1}\,.$$

Hence, the demand for the innovation is a step function that depends on the amount of workers in the sector, $l_{j,t}$:

[6] This assumption is changed in Section 5, where the effect of risk aversion is analyzed.

(6) $$q_{j,t}(z_{j,t}) = \begin{cases} l_{j,t} & \text{if } \ z_{j,t} \le p_{j,t}(a_{j,t} - a_{j,t-1}) \\ 0 & \text{if not.} \end{cases}$$

Hence, the monopoly patent fees are set at the maximum price and all workers purchase the innovation and use it. The patent fees are therefore equal to

(7) $$z_{j,t} = p_{j,t}a_{j,t} - p_{j,t}a_{j,t-1} = p_{j,t}a_{j,t-1}b.$$

3.2. Income, Prices and Employment

Due to free entry in first period of life the income of workers must be equal across sectors and that must be equal to the expected present value of income of innovators across sectors as well. Denote the equal income of workers across sectors in period t by w_t. Then for every sector j, which has a new technology, a worker's income is

(8) $$w_t = p_{j,t}a_{j,t} - z_{j,t}.$$

From (7) and (8) we get:

(9) $$w_t = p_{j,t}a_{j,t-1}.$$

This holds for all sectors j, both for those with new innovations and for those without. Note, that as a result, the patent fee is equal to: $z_{j,t} = w_t b$.

We next turn to determine the wage rate in each period. Profit maximization in production of the final good leads to:

(10) $$p_{j,t} = \frac{\partial y_t}{\partial x_{j,t}} = \frac{y_t}{x_{j,t}}.$$

Combining (1), (9) and (10) we get:

(11) $$\log w_t = \int_0^1 \log a_{j,t-1} dj.$$

Hence, if f_t is the amount of sectors, which have an innovation in period t, the rate of change of income is

(12) $$\frac{w_{t+1}}{w_t} = (1+b)^{f_{t-1}}.$$

We next show how the amount of workers in each sector is determined. From equations (9) and (10) we get:

(11) $$l_{j,t} = \frac{x_{j,t}}{a_{j,t}} = \frac{y_t}{w_t}\frac{a_{j,t-1}}{a_{j,t}}.$$

Hence, if there has been an innovation in t-1 in sector j, the number of workers is $l_{j,t} = y_t/(1+b)w_t$. Since this number does not depend on j, we denote it by l_t. If there has not been an innovation in sector j, the number of workers is $l_{j,t} = y_t / w_t = (1+b)l_t$. If we sum up all labor inputs in all sectors we get the overall number of workers, which is the total number of young individuals L minus the number of people who work in the R&D sector in period t, I_t:

(12) $$L - I_t = L - \int_0^1 n_{i,t}di = l_t f_{t-1} + (1+b)l_t(1-f_{t-1}) = l_t\left(1+b-bf_{t-1}\right).$$

This equation determines the size of l_t and it thus determines how many workers are in each sector of the economy.

<u>3.3. Determination of Innovative Activity</u>

We next turn to determine $n_{j,t}$, namely how many teams search for the innovation in each sector. While the income of a worker is w_t, the expected present value of the future income of an innovator in sector j is:

(13) $$\frac{z_{j,t+1} l_{j,t+1}}{1+\rho} \frac{1}{n_{j,t}}.$$

From the above analysis we can tell that the future patent fee is $z_{j,t+1} = w_{t+1} b$, and that the expected demand for the innovation if found is $l_{j,t+1} = l_{t+1}$. Hence the expected income of an innovator in period *t* is:

(14) $$\frac{w_{t+1} b l_{t+1}}{1+\rho} P_{j,t} = \frac{w_{t+1} b l_{t+1}}{1+\rho} \frac{1}{n_{j,t}}.$$

As long as this discounted income is higher than w_t, more teams are added and $n_{j,t}$ increases, until it equals w_t. If this discounted income is smaller than w_t, there are no innovators and no R&D sector. If the discounted income equals the wage when there is only one team, then it is indifferent between becoming a worker or an innovator, and can be either. Hence, the equilibrium number of innovating teams depends on the ratio of incomes in the R&D sector and the production sector when there is only one team searching. We denote this ratio by R_t, since it is identical across sectors:

(16) $$R_t = \frac{w_{t+1} b l_{t+1}}{w_t (1+\rho)}.$$

Hence, if $R_t > 1$, then $n_{j,t} = R_t$, if $R_t < 1$, $n_{j,t} = 0$, and if $R_t = 1$ then $n_{j,t}$ can be either 0 or 1. We can now calculate the overall size of the R&D sector $I_t = \int_0^1 n_{j,t} dj$. Note, that if R_t is greater than 1, the number of R&D teams is equal across sectors, so that $I_t = R_t$. If R_t is smaller than 1, there are no teams in any sector and $I_t = 0$. If $R_t = 1$, innovators enter only some of the sectors and $0 \le I_t \le 1$.

The amount of innovations across the economy in period t is therefore determined by the size of the R&D sector I_t. Clearly, $f_t = I_t$ if $I_t \leq 1$, since in this case there is at most one innovator in each sector. But if $I_t > 1$ and there are more innovators in each sector, $f_t = 1$. We next show that in equilibrium f_t can be only 0 or 1 and there cannot be equilibrium with innovations in some sectors only. To see this note that R_t is equal to:

$$R_t = \frac{(1+b)^{f_{t-1}} b}{1+\rho} \frac{L - I_{t+1}}{1+b-bf_t}. \tag{17}$$

Hence R_t is an increasing function of f_t. Assume for the contrary that $0 < I_t < 1$. If $R_t \geq 1$ more innovators enter, f_t increases and that further increases R_t. This will continue until $I_t = R_t > 1$. If $R_t \leq 1$ innovators leave, f_t declines and that further reduces R_t. This goes on until no innovators are left. Hence, the only stable equilibrium outcome is either $I_t = 1$ or $I_t = 0$. The following figure 1 demonstrates this result. The curve R represents equation (17), while the curve f describes how f_t depends on I_t. If the curve R intersects with the horizontal axis to the right of 1, there is a unique stable equilibrium at $f_t = 1$. If the curve R intersects with the curve f to the left of 1, there is a unique stable equilibrium at $f_t = 0$. If the curve R is in between, as is the case in Figure 1, there are two stable equilibria, one at $f_t = 0$ and the other at $f_t = 1$.

[Insert Figure 1 here]

3.4. Equilibrium Dynamics

As indicated by equation (17), the number of innovation teams in period t depends on R_t, which itself depends on the equilibrium in the past and in the future. Intuitively, it depends on past innovations, as they determine the anticipated rise in wages and it depends on the number of future R&D workers, as it affect the future number of production workers. These intertemporal links make the dynamic analysis of equilibrium somewhat complex. For the dynamic analysis we introduce the following parameter:

(18) $$h = \frac{b(1+b)}{1+\rho}.$$

We also introduce two threshold levels of scale:

(19) $$L_0 = \frac{1}{h},$$

and:

(20) $$L_1 = \frac{(1+\rho)(1+b)}{b}\frac{1}{1-h} = \frac{(1+b)^2}{1-h}L_0 > L_0.$$

From here on we assume that b is small enough, so that h is very small, and in particular we assume that $h < 1$.

Proposition 1: If $L < L_0$, then there is no R&D in the economy. If $L > L_1$, there is R&D activity and there are innovations in every sector. In this case the size of the R&D sector is $\frac{h}{1+h}L$. The amount of inventions is 1 and the rate of growth of the economy is b. If the size of the economy satisfies $L_0 \le L \le L_1$, the invention activity can fluctuate from 0 to 1 and back.

Proof: Consider first the case that $L < L_0$. Note that $f_{t-1} \le 1$, $f_t \le 1$, and $I_{t+1} \ge 0$. Hence:

$$R_t \le \frac{(1+b)b}{1+\rho}L = hL < 1.$$

Hence, $n_{j,t} = 0$ for all j, and there is no R&D in any sector in the economy.

Next, note that we always have: $I_{t+1} \leq hL$, since:

$$I_{t+1} = \frac{(1+b)^{f_t} b}{1+\rho} \frac{L - I_{t+2}}{1+b-bf_{t+1}} \leq \frac{(1+b)b}{1+\rho} L = hL .$$

Consider next the case that $L > L_1$. Using the constraint on I_{t+1} we get:

$$R_t = \frac{(1+b)^{f_{t-1}} b}{1+\rho} \frac{L - I_{t+1}}{1+b-bf_t} \geq \frac{b}{1+\rho} \frac{(1-h)L}{1+b} = \frac{L}{L_1} > 1 .$$

Hence, there are innovators in all sectors in all periods. Hence, in every period we have $f_{t-1} = 1$, and the dynamic condition becomes:

$$I_t = h(L - I_{t+1}) .$$

Since $h < 1$, there is a unique stable rational expectations solution to this dynamic model, and it is the fixed saddle solution:

$$I_t = \frac{h}{1+h} L .$$

As for $L_0 \leq L \leq L_1$, we do not describe the full dynamics, but since the equilibrium can be either 0 or 1, the economy might fluctuate between periods of full innovation and periods of no innovation at all. QED.

3.5. R&D and Economic Growth

Note first, that if the economy is not large enough, namely if $L < L_0$, there is no R&D, there are no innovations, and there is also no economic growth. Consider next an economy, which is large enough to have R&D in all its sectors. In such an economy there is economic growth. The rate of growth of wages is equal to *b*. Output can be calculated as well and is equal to:

(21) $$\log y_t = \log l_t + \log(1+b) + \log w_t .$$

Since l_t is fixed over time the growth rate of output, which is also equal to the growth rate of total factor productivity, is equal to the growth rate of wages, namely to *b*. The growth rate is therefore fixed over time, and is independent of the size of the population.

Note, that the size of the R&D sector and the rate of growth are uncorrelated when the economy is growing. The size of the innovation sector increases with population, while the growth rate remains unchanged. Only if population is small enough, so that the R&D sector drops to 0, the growth rate is 0 as well. This non-linear relationship between growth and R&D fits well the findings of Jones (1995).

The results of this model with respect to economic growth and R&D are both similar and different from the results of earlier endogenous growth models. This model presents a limited scale effect, since if the economy is too small there is no innovation activity and no economic growth, because the scale of the economy is not large enough to provide incentive to innovators. As the population becomes large enough, innovations become beneficial, innovation activity begins and with it economic growth. But according to this model the rate of growth does not grow with the scale of the economy, but remains fixed instead, since the amount of potential projects is limited. The greater incentives to innovation, due to a larger scale, just lead to patent races. More and more innovators are trying to find a limited number of potential innovations. They are doing it because the benefits from finding an innovation are very large. Hence, a large population increases the rate of growth from 0 to *b*, but it then remains fixed at this level whatever the size of the population. This result differs significantly from the strong scale effect of the original endogenous growth models. It is next shown that this model differs also in its welfare implications.

3.6. Patent Races and Pareto Efficiency

An interesting question is whether the equilibrium described above is efficient. The fact that many innovation teams are looking for the same innovation, where only one team can find it, means that there is some misallocation of resources. To see this more formally consider a central planner, who allocates individuals between production and R&D (in a growing economy). This planner can assign only one team of innovators for each sector, and assign all others to work in production. The rate of growth will be the same, while the level of output will be larger. Hence, the equilibrium we observe is not optimal. It is not clear whether there is a simple policy that can Pareto-improve the allocation of resources in the economy, but it is obvious that subsidization of R&D works in the opposite direction.

Note that this result is drastically different from the initial endogenous growth models. In these models potential innovations are unbounded, hence subsidization can always increase R&D activity, and that raises the rate of growth and might increase welfare as well. This channel is blocked in our model, as the number of potential innovations is bounded.

4. Equilibrium with Multiple Search Strategies

In the previous section it is shown that patent races lead to significant Pareto-inefficiency, since too many innovators are trying to find the same innovation, using the same search strategy. This naturally raises the question, whether these inefficiencies are reduced when innovators can use different strategies in order to search for the innovation. This is what this section examines, as it describes the equilibrium of the economy in the case of multiple search strategies. It is shown that in this case as well there are significant inefficiencies due to overcrowded patent races. Innovators tend to pursue different strategies in searching for the innovation, but they still tend to crowd the strategies with the highest success probabilities.

In analyzing the case of multiple search strategies note that most of the analysis of the equilibrium in the single search strategy case carries through to this case as well. The market for an innovation looks the same and the innovation price is the same as well. The same is true for the wage level and the employment figures as well, so that all the equations up to (12) hold here as well. The analysis differs when we turn to discuss the returns from innovation, since the production function of innovations is somewhat different. The returns to innovation from the *s* strategy in each sector, relative to the production wage rate, are equal to:

(22) $$\frac{w_{t+1} b l_{t+1}}{w_t (1+\rho)} \frac{e_s}{n_{s,j,t}}.$$

Let us use the following notation:

(23) $$N_t = \frac{w_{t+1} b l_{t+1}}{w_t (1+\rho)}.$$

Then, the strategies, which are adopted for search of innovation, are those that satisfy:

(24) $$n_{s,j,t} = N_t e_s \geq 1.$$

Clearly there exists a unique strategy S_t such that strategies 1,…, S_t are adopted and less promising strategies, for which $N_t e_s < 1$, are not. The amount of innovation teams declines with the probability of success, since $n_{s,j,t} = N_t e_s$. The more promising strategies draw a hot patent race, while the less promising strategies experience much smaller races. The total amount of innovation teams in each sector is:

(25) $$I_t = \sum_{s=1}^{S_t} N_t e_s = N_t \sum_{s=1}^{S_t} e_s = N_t f_t,$$

where the probability of success in each sector is denoted by f_t and this is also equal to the amount of sectors that have innovation in *t*. Note that N_t itself depends on the size of the R&D sector and on its probability of success in the following way:

(26) $$N_t = \frac{(1+b)^{f_{t-1}} b(L - I_{t+1})}{(1+\rho)(1+b-bf_t)}.$$

Together with (25) we get a complex difference equation that describes the dynamics of the economy. In what follows it is assumed that the economy is in a steady state and the analysis focuses on this steady state only.

At the steady state: $N_t = N_{t+1} = N$, $S_t = S_{t+1} = S$, and $f_t = f_{t-1} = f_{t+1} = f$. The level of *N* determines the last search strategy adopted *S*. Namely *S* is the integer, which satisfies: $Ne_S \geq 1$, while $Ne_{S+1} < 1$.[7] The index *S* determines the probability of finding the innovation:

(27) $$f = \sum_{s=1}^{S} e_s.$$

This probability is therefore an increasing step function of *N*, which is bounded by 1. It is described by the curve PROB in Figure 2. But *N* also depends on this probability *f*, since the returns from innovation depend on the amount of future and past innovations. This is reflected in the following condition, which is derived from (25) and (26):

(28) $$N = \frac{(1+b)^f b(L - fN)}{(1+\rho)(1+b-bf)}.$$

A simple manipulation leads to:

(29) $$N = L\frac{(1+b)^f b}{(1+\rho)(1+b-bf)+(1+b)^f bf}.$$

This relationship is described in the curve RET in Figure 2.

[Insert Figure 2 here]

The intersection of the two curves determines the steady state: how many strategies are adopted, and how big is the probability of success, but also how big is the R&D sector and what is the rate of growth of the economy. The equilibrium size of the R&D sector is given by:

(30) $$I = Nf = L\frac{(1+b)^f bf}{(1+\rho)(1+b-bf)+(1+b)^f bf}.$$

It is therefore proportional to the size of the economy. The rate of growth of the economy is equal to:

(31) $$\frac{y_t - y_{t-1}}{y_{t-1}} = (1+b)^f - 1.$$

We can now analyze the effect of the scale of the economy on innovation and growth. If L increases the innovations become more profitable and that shifts the RET curve to the right. As a result more strategies are followed, more innovation teams are operating and more innovations are found. The economy grows at a higher rate, but this gain in growth rate is diminishing with scale, since the rate of growth is bounded by b. Hence, the result of the initial endogenous growth models, of unbounded growth rates,

[7] Given our specification of the series e_s, S is the first integer for which logNe + S log(1-e) is negative.

does not apply in this case as well. Note that when L is very small, so that the RET curve is everywhere to the left of the PROB curve, there is no innovation and both the size of the R&D sector and the rate of growth are 0.

We next discuss the Pareto-Efficiency of equilibrium. Can we say, as in the single search strategy case, that there are too many R&D teams and workers? Here the results are more mixed. On the one hand there are too many teams working on the more promising strategies. But on the other hand the economy can benefit from putting these teams to work on the marginal search strategy, to increase the chances of finding innovations. The real efficiency issue is therefore how to reduce the number of teams working on strategies 1, …, S, without reducing, or even increasing S. Note, that an incentive to R&D, like a subsidy, might increase S, but might also increase the amount of people working on all other strategies, which is inefficient. The subsidy then increases the rate of growth, by increasing S, but also creates efficiency losses, by reducing the amount of available workers in production. Clearly, as the scale of the economy increases, the gain from increasing S is diminishing, as the increase in probability of finding the innovation e_{S+1} becomes quite small. At the same time the loss from increasing the R&D sector increases as the size of this sector increases as well. Hence, while it might make sense to subsidize R&D at some early stage of development, where S is rather low, its net benefit diminishes with the scale of the economy.

5. Risk Aversion and Concentration of R&D

Our paper shows that if the global economy is large enough, there will be patent races for all innovations and there many innovation teams will participate in these races. In reality though we observe many cases in which innovation are searched by a small number of competing large R&D teams and many cases where R&D is concentrate in a single monopoly. Interestingly our model can account for this phenomenon too, by attributing it to risk aversion. Assume that we change our original assumption that individual are risk neutral and assume instead that they are risk averse. The gains from innovation are very high, but the probability of success is low, due to large number of teams in each patent race. This creates a strong incentive for innovators to form a contract with others that they will share the gains from innovation if one of them finds it first. This way they can have the same expected income, but much smaller risk. Such arrangements cannot be created cooperatively, due to problems of free riding and contract enforcement, but they can be the outcome of a single firm, which hires many research teams to look for an innovation. By dividing the return from innovation between the teams, such a firm offers each team income above the alternative wage. This creates an incentive for this firm to have as few as possible teams on the one hand. But on the other hand, it has to have a sufficient number of teams to deter potential innovators from entering the race. Hence, even in this case, where the number of teams is smaller, there are still too many research teams, from a welfare consideration.

We next formalize these ideas by introducing a small change to the model. Let us assume that consumers are risk averse. For the sake of simplicity we assume that they work in second period of life only, so that utility is described by:

(32) $$u = \log c\,,$$

where c is utility in second period of life. We also assume that the physical good is storable without depreciation, so that the real interest rate in this economy is 0 (there are only lenders and no borrowers). In order to enable risk taking let us assume that in addition to working in first period of life (either in production or in R&D), consumers work in second period as well, whatever they did in first period. In the second period of life they are less productive, and hence they do not use the most recent technologies (patent fees are too expensive). Their income in second period of life, namely in period t+1, is therefore:

(33) $$\alpha w_t,$$

where α is much smaller than 1. For the sake of simplicity we describe the equilibrium under risk aversion for the case of single search strategy only.

The expected utility in this case of an R&D period t worker under a competitive patent race with n participants is:

(34) $$\left(1-\frac{1}{n}\right)\log(\alpha w_t)+\frac{1}{n}\log(\alpha w_t + R_t w_t)\,.$$

Note that R_t is the same as in Section 3 but with interest rate equal to 0. The expected utility of a production worker is:

(35) $$\log[(1+\alpha)w_t]\,.$$

Clearly, joining a firm that has m participants in it significantly increases utility of R&D workers, from (34) to:

(36) $$\left(1-\frac{m}{n}\right)\log(\alpha w_t)+\frac{m}{n}\log\left(\alpha w_t + \frac{R_t}{m} w_t\right).$$

We next turn to describe the creation of a monopoly in this market. Consider a leading firm, which has k innovating teams searching for the innovation. On the one hand this firm would like k to be as small as possible, in order to have larger gains per team. On the other hand it wants to deter potential competitor. Let us assume that a competing firm has m teams, so that the total amount of teams is $n = k + m$. The competing firm decides on its size m so as to maximize the expected utility (36):

$$\max\left[\left(1-\frac{m}{k+m}\right)\log(\alpha w_t)+\frac{m}{k+m}\log\left(\alpha w_t+\frac{R_t}{m}w_t\right)\right]. \tag{37}$$

The leading firm then chooses k such that the competing firm decides to stay out of the patent race, namely that its expected utility of its workers is smaller (or equal) than that of a production worker:

$$\left(1-\frac{m}{k+m}\right)\log(\alpha w_t)+\frac{m}{k+m}\log\left(\alpha w_t+\frac{R_t}{m}w_t\right)=\log[(1+\alpha)w_t]. \tag{38}$$

Under this condition the leading firm remains the single firm in the market and it can give its teams high certain income. The number of teams searching for each innovation is thus k, and it can be shown that it increases with the scale of the economy L as well.[8] Hence, in this case we also have too many teams searching for the innovation, but this is done in order to deter potential competitors. Interestingly, by enabling internalization of the patent race into one firm, the number of teams within a monopoly is greater than under competition. The intuitive reason is that the monopoly has to deter competitor firms, who diversify risk, and not only individual teams.[9] Hence, if firms are allowed to run a number of R&D teams together and become monopolies, instead of having competition between single innovation teams only, the R&D sector will become larger and at the same time more inefficient.

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[8] It can be shown that the FOC of maximization of (37) and condition (38) together with the definition of R in the steady state, which is $b(1+b)(L-k)$, yield two equations in the variables m/k and k/L. The solution is unique and thus the share of the R&D sector in each generation k/L is fixed. Thus the size of the R&D sector k rises with scale L.

[9] Formally, in competition the number of competing firms is given by condition (38) when m is restricted to be equal to 1. Clearly, when the LHS of (38) is not maximized, it is lower and hence k can be smaller. Namely, if only teams are allowed to compete in patent races, these races have less teams.

6. Summary and Conclusions

This paper departs from the initial R&D based endogenous growth models in one assumption, by assuming that the number of potential innovations in each period is limited. Bringing more innovators in a period of time cannot lead to more innovations necessarily, as some take time and take prior innovations to build on. It is shown that changing this assumption changes the results of the model quite significantly. Some of the changes lead to results that fit better the historical evidence of growth rates and the size of R&D activity. Thus, growth rates do not rise unboundedly with the size of the economy, and also the relation between the growth rate and the size of the R&D sector is not simplistically positive.

But the main deviation of the paper from the initial R&D based endogenous growth literature is that it leads to patent races, both between innovating firms and within innovating firms. The limited number of potential projects leads innovators to join others, who already search for an innovation, and creates a patent race. This leads to some waste in resources and is clearly sub-optimal. The paper shows that this is true even if there are many different strategies to search for the same potential innovation. Even then searchers use different strategies, but they tend to crowd more the promising strategies, which offer a higher probability of success. Thus, there is still some inefficiency in this case as well.

We stop short of offering policy recommendations. Mostly because any general measure with respect to innovation activity, like subsidy, has a number of effects on growth. On the one hand a subsidy might increase the probability of finding innovations and can increase the rate of growth, though with diminishing success, if the scale of the economy is large. On the other hand, it increases incentives to innovators to join patent races, which already have too many participants, and that increase inefficiency. The ideal policy could be to support those innovators who travel the less frequented ways, namely those who try the strategies with the lowest probabilities of success. Hence, this model suggests that research incentives should be given to those who deviate from the crowd and who are doing less standard and more risky research.

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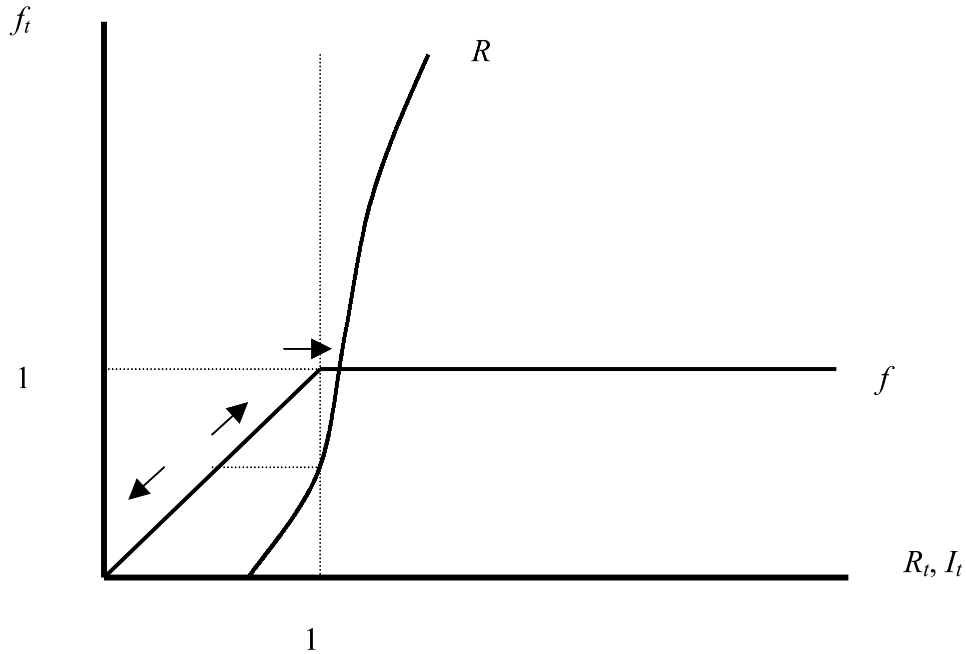
Figures



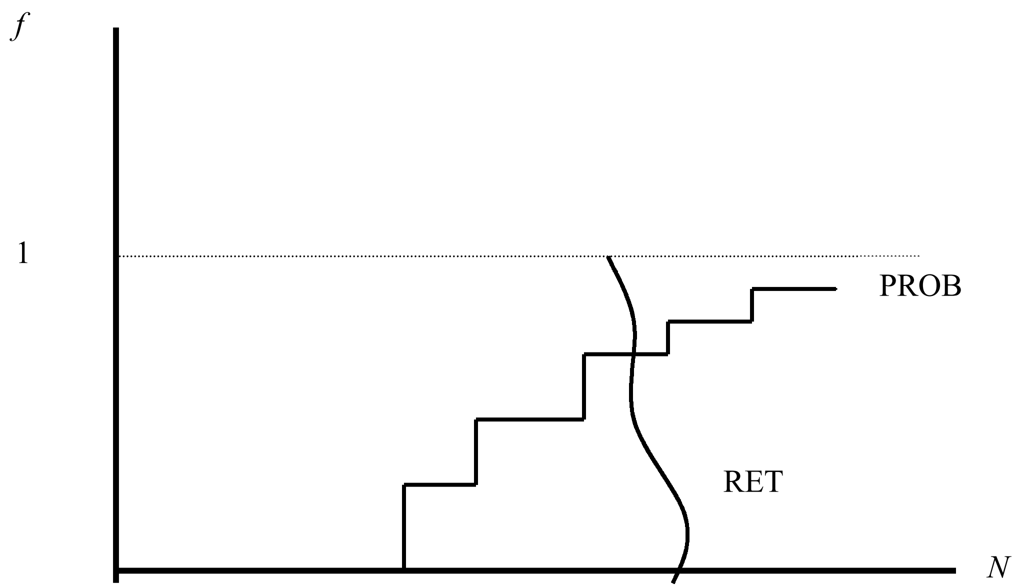


Figure 2