Explaning the Correlation Between Output and Volatility in a Model of International Risk-Sharing and Limited Commitment

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Abstract

I study the constrained efficient allocations of a simple model of risk sharing and capital flows across countries assuming that each country cannot commit to fully repay its contract obligations. In the model, the degree of risk sharing and the amount of investment are interdependent. It is shown that, when individual rationality constraints are binding, the variance of consumption in any given country across states of nature (iid across countries) may be a non monotonic function of income: low in the early stage of development, high in an intermediate range and converging to zero as income converges to a high income level. A monotonically decreasing consumption variance can only obtain if the social welfare function assigns equal weights to all countries (equal treatment). The model also shows that a structure of competitive financial markets with appropriate borrowing constraints may not be sufficient to decentralize the constrained efficient allocation. A supernational authority forcing a specific redistribution of income within poorly capitalized countries may be necessary for decentralization. *JEL classification numbers*: A10, D80, G10, O17. *Keywords*: financial intermediation, moral hazard.

1 Introduction

Poor countries and emerging economies experience larger output volatility and more limited risk sharing than developed economies. This phenomenon has been documented in a variety of papers. As claimed by Aizenman and Pinto in a recent survey, "..cross country studies have consistently found that volatility exerts a significant negative impact on long-run (trend) growth, which is exacerbated in poorer countries [Aizenman and Pinto (2004), p. 3]". For example, Ramey and Ramey (1995) use a sample of 92 countries from 1962 to 1985 to show that higher standard deviation of output growth reduces mean

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growth by a significant amount. Acemoglu and Zilibotti (1997) find that, by plotting the logarithm of the standard deviation of a set of countries' GDP per capita growth rate over the period 1960-85 versus GDP per capita in 1960, the regression line has a negative and highly significant slope (a 1 percent increase in the initial GDP in 1960 is associated, on average, with a 0,25 percent decrease in the standard deviation of GDP). Wolf (2004) finds that, when we exclude volatility of output and inputs from the set of explanatory variable of the standard deviation of consumption growth across countries, per capita income seems to be the best predictor of a country's consumption volatility (with a negative effect).

This excess volatility is related to two celebrated puzzles that cannot be easily explained within a standard neoclassical model: (1) why there is so little risk sharing across countries and (2) why does so little capital flow from rich countries to poor. Evidence of the first puzzle is reported by Backus, Kehoe and Kydland (1992). Evidence of the second puzzle, known as the "Lucas paradox", is reported in Gertler and Rogoff (1989), Reinhart and Rogoff (2004) and Lane (2004), among many others. For instance, Reinhart and Rogoff claim that foreign lending "..rises more than percent for percent with per capita income among poor developing countries" [Reinhart and Rogoff (2004), p. 9], whreas neoclassical growth theory predicts that the amount of external debt financing investment in a given country should be inversly related with this country's capital-labor ratio (and per capita income).

A set of models that could resolve these puzzles are based on international capital market imperfections, mainly sovereign risk and limited contract enforcement. In these models, limits on credit and insurance may arise endogenously from the borrowers' incentives to default and their limited commitment to meet financial obligations. Strategic default is particularly important in studying international financial transactions and sovereign debt, due to the difficulty of enforcing contracts across borders and to the absence of supranational legal authorities. The seizure of a sovereign's foreign assets and the exclusion from future borrowing may be the only punishment strategies that are available to a lender in case of default. With limited enforcement, firms in low income countries face a larger cost of external finance because they have low collateral. This explains why these countries invest less in high return projects and have a larger marginal product of capital. Similarly, limited commitment may explain why low income countries cannot guarantee themself a "low" risk outcome by participating in a risk pooling agreement. Essentially, taking some risk may be the only way for an individual, or a country, to make *ex post* contingent repayments an incentive compatibile action (see Kocherlakota (1996)).

Volatility and income: a closer inspection

Hence, the existence of an inverse relation between income and volatility is a theoretical possibility. I have already mentioned that the empirical correlation between these two variables in a cross section of countries appears to be negative. However, the significance level of the regression coefficients evaluated in a sub-sample of poor countries (whose income is, for example, up to 20% of the US level) is much smaller than those evaluated for the remaining sub-sample. This claim derives from the analysis of two different datasets available in the recent literature. All of them are a collection of two variables in a cross section of countries: the standard deviation of GDP growth (volatility) in a time interval and the average per capita GDP in the same interval. The first dataset is taken from Breen and Garcia-Peñalosa (1999) and consists of 73 country datas in the 1960-1990 time interval. The second dataset is taken from Jansen (2004)[1] and consists of 103 country datas in the 1980-2000 time interval. Within each dataset I have run three separate regressions of volatility on average p.c. GDP: one for the entire sample (All), one for the sub-sample of countries whose p.c. GDP is up to 20% of the US level (Poor) and one for the remaining sub-sample (Rich). The following table shows the estimated regression coefficients (slope of the regression line multiplied by 1,000), the t-statistics for the coefficients, the R^2 and the number of observations.

	1960-1990			**1980-2000**		
	All	Poor*	Rich	All	Poor*	Rich
Slope^o	-0,32	-0,73	-0,30	-0,09	+0,01	-0,08
t-stat.	-6,84	-2,35	-5,09	-6,18	0,06	-4,03
R^2	0,39	0,11	0,49	0,27	0,00	0,37
N. of obs.	73	44	29	103	74	29
* Average p.c. GDP over time interval up to 20% of US level						
^{o} Est. change in volatility due to a 1,000$ increase in GDP						

It is clear from the table that the negative effect on volatility of a rise in GDP is negative and highly significant for the entire sample of countries. However, the significance level as well as the estimated R^2 drops dramatically when we restrict the sample to the set of countries whose income falls below 20% of the US level. In the case of the 1980-2000 time interval, the slope of the linear regression in the poor countries sub-sample becomes positive, but not significantly different from zero. The graph in figure 1 may help to make this result more transparent. I have partitioned the 1980-2000 dataset into eight sub-samples consisting of countries whose per capita GDP is within eight consecutive ranges. These ranges are constructed so as to contain approximately the same number of countries (between 7 and 14), except for the sample in the first income range (less than $300 of per capita GDP), which contains 21 countries. The graph shows that volatility does not necessarily decrease (in some case it may actually go up) as a country is upgraded in terms of income range. Output volatility drops substantially as a consequence

[1] In my elaborations I have excluded from the original data set all countries whose population is less than 1.5 million and all countries of the former soviet block (whose volatility in the period is exceptionally high.

of a rise in income only when countries have a per capita GDP greater than $10.000.

Main contribution

In this paper I construct a very simple version of the limited commitment model for a world economy where countries differ in the level of initial income and consumers are risk averse. The main purpose of the paper is to make some predictions on the relation between risk (as represented by the variance of consumption across idiosyncratic states and countries) and the stage of development. A specific feature of my model is that, in each country, there are two type of heterogeneous agents: "consumers" and "entrepreneurs". The latter are endowed with a risky investment project affected by a two-state productivity shock i.i.d. across countries. I consider all possible redistributions within and across countries and across states compatible with the individuals' rationality constraints. The latter derive from an incentive to default due to the ability of agents to shield a constant fraction of their *ex post* income from liquidation. When these constraints are binding for a subset of countries, the latter are unable to insure completely against idiosyncratic states and their investment is a function of the degree of (incentive compatible) insurance (as represented by the marginal rate of substitution between bad and good state consumption).

One reason why we should expect idiosyncratic risk to be negatively correlated with income is that, within the limited commitment model, the degree of insurance across states (as well as investment) is increasing in collateralizable wealth. However, there are at least two reasons why this regularity may fail. First, because they can borrow more on the international financial market, countries with higher wealth may be induced to invest more in the risky project. Second, the lack of insurance can be compensated by within country redistributions of income, which may be carried out with the purpose of increasing the investors' collateral.

In my model, both consumers and entrepreneurs can borrow from abroad. This allows me to assume that the limited enforcement problem affects credit relations between international lenders and the two type of agents (consumers and entrepreneurs) in different degrees. Namely, I assume that the share of *ex post* income that foreign lenders can recover from a defaulting consumer is smaller than (or equal to) the share that they can recover from a defaulting entrepreneur. Other than being a more realistic assumption, this feature of the model implies some interesting predictions about international capital flows. In particular, by allowing for domestic transfers (between consumers and entrepreneurs) I find that output volatility may not be a monotonic function of a country's aggregate initial income. At an early stage of development, countries may run into a higher volatility of output as they increase their aggregate income.

How can we rationalize these findings within the simple limited commitment model that I use in this paper? In the paper I look at two specific constrained efficient allocations of the model: an *equal treatment allocation*, such that the Planner's social welfare function has equal weights for each country, and a *competitive allocation*, such that redistribution of resources across countries occurs through competitive financial markets (subject to zero profit conditions) and repayments are proportional to countries' liabilities. A constraint efficient allocation can be decentralized as a competitive equilibrium by means of financial markets where competitive financial intermediaries operating internationally offer a range of securities and agents in each country are subject to borrowing constraints[2] (which are always binding when full insurance is not feasible). Then, I study "observable consumption risk" or "volatility", i.e., the standard deviation of consumption across states, σ_{cw}, as a function of a country's individual income, w. I show that, whereas σ_{cw} is always monotonically decreasing with respect to w in the equal treatment allocation, it may be non monotonic in the competitive allocation. In particular, when the absolute degree of risk aversion is decreasing in consumption, consumption risk is increasing with a country's individual income in the early stage of development (low w) and decreasing in the final stage.

Notice that the degree of risk sharing and the level of output volatility in each country are not perfectly correlated. The first is naturally measured by the marginal rate of substitution between consumption (output) across states of nature or, equivalently, with a default premium, and the latter is measured by the standard deviation of output. I show that, while the degree of risk sharing in a country falls with its initial income, output volatility may increase.

The intuition can be explained as follows. Let MPK_w be the expected marginal product of capital, ρ a safe rate and w the country's initial income. We may call $\pi_w = (MPK_w - \rho)/\rho$ the *default premium* of this country. With decreasing marginal productivity of capital, π_w is decreasing in w, since, with partial risk sharing, ($\pi_w > 0$), a higher income implies a higher investment. Now assume that the marginal product of capital is hit by a stochastic shock with two possible realizations. A constrained efficient allocation equalizes the marginal rate of substitution between bad and good state consumption of each country, MRS_w, to a transformation of π_w. In particular, we have

$$MRS_w = 1 + \frac{\pi_w}{\beta - (1-\beta)\pi_w},$$

where $1 - \beta \in [0, 1]$ is the share of the borrower's *ex post* income that foreign lenders can recover in the good state in case of default. Taking a first order Taylor expansion of marginal utilities, one can show that

$$\sigma_{cw} \approx \left(\frac{1}{A_{bw}}\right)\left(\frac{\pi_w}{\beta - (1-\beta)\pi_w}\right),$$

where A_{bw} is the absolute degree of risk aversion at the bad state level of consumption. Since higher income countries enjoy a lower risk premium, they tend to have a smaller σ_{cw}. In the equal treatment allocation (when the Planner assigns equal weights to all countries), bad state consumption is equalized across countries and A_{bw} is invariant with respect to individual income, w. Hence, consumption volatility is increasing in the risk premium (and in w). Instead, the competitive allocation is such that bad state consumption is positively affected by individual income and, thus, A_{bw} goes up or down with w

[2]This is in the spirit of Alvarez and Jerman (2000).

according to whether absolute risk aversion is increasing or decreasing in consumption. This explains why the volatility of consumption across states, σ_{cw}, may not be decreasing with w. However, the model says more than this. It can be shown that, at a constrained efficient allocation, the degree of insurance (and the level of investment) in each country is bounded below. In other words, when a country is particularly poor, it is optimal to redistribute resources from consumers/lenders to entrepreneurs/borrowers (i.e., make the within country debt contracts cheaper) so as to keep the degree of insurance across states (and investment) from falling below a minimum. Hence, when a country is very poor, a rise in his income may leave π_w almost or completely unchanged. In this case, any change in the volatility of consumption would be caused by changes in the absolute degree of risk aversion.

These results are obtained for a static (two-period) economy. A natural development of this model is to introduce a dynamic structure and analyze the wealth accumulation process.

Related literature

Kehoe and Levine (1993), Kocherlakota (1995) and Alvarez and Jermann (2000) have studied the characteristics of efficient allocations when individual rationality constraints arising from limited commitment are explicity taken into account. They have shown that perfect diversification of consumption risks across individuals may not be optimal and portfolio allocations may be subject to solvency constraints prohibiting agents from holding large amounts of contingent debt. Using a "real business cycle model", Kehoe and Perri (2002) show that limited commitment with respect to international contracts can greatly reduce the degree of risk sharing across countries. My model is much more simple and *ad hoc* in the formulation of the endogenous borrowing constraints. However, this simplicity allows me to make specific and analytic predictions on output variance along with the distribution of capital and default risks.

As a natural extention of this paper one may embed the model in a dynamic setting. For instance, it is not difficult to generate a wealth accumulation process by assuming that countries are populated by overlapping generations of agents deriving utility from bequests. Under this extension, the model can be related to the growing literature on credit and capital accumulation and on income distribution with financial frictions. Greenwood and Jovanovich (1990), Bencivenga and Smith (1991), Saint-Paul (1992) and Acemoglu and Zilibotti (1997) show that fixed costs on investment or indivisibilities or non convexities imply that countries develop by exploiting better diversification opportunities, more insurance and a gradual allocation of funds to high return/risky investment projects. In turns, this process implies (as shown, in particular, by Greenwood and Jovanovich (1990) and Acemoglu and Zilibotti (1997)) that the variability of output growth decreases as a country develops. Acemoglu and Zilibotti (1997) show that the time path of the development process is crucially affected by the stochastic realizations of the risky projects and, hence, slow growth may be a consequence of misfortune. Similar predictions can be obtained in a dynamic extension of the present model. However, Acemoglu and Zilibotti study a one country model and assume non convexities, linear stochastic technologies and incomplete markets. Moreover, their model concentrates on inefficient competitive equilibria characterized by market incompleteness and a non internalized pecuniary externality. In contrast, I concentrate on the constraint efficient allocations in a multi-country economy based on a simple (although rather *ad hoc*) limited enforcement problem.

2 The Model

Agents and technologies

I consider a two-period economy characterized by a continuum of "countries", indexed by i, with $i \in I = [0,1]$. Countries are *ex ante* identical except for their initial (first period) "aggregate income", w_i, which takes one of a finite number of values in $W = \{w_1, .., w_N\}$. Then, for all $w \in W$, there exist a non empty set, $I_w \subset I$, such that $i \in I_w$ implies $w_i = w$, where $\{I_w : w \in W\}$ is a partition of I. I assume that I_w is an interval for all $w \in W$ and let $\mu(I_w) = \mu_w$ be the length of these intervals. Hence, $\mu = \{\mu_w : w \in W\}$ is the standard Lebesgue measure defining the probability distribution of initial wealth across countries and $\bar{w} = \sum_w w\mu_w$ is the average world income.

Each country is populated by two agents, a consumer and an entrepreneur, both consuming in the second period only. All agents in each country have access to a safe investment opportunity (or storage technology) such that any unit of consumption stored in period one yields a gross return $\rho > 0$ in all contingencies. In the first period of his life, the entrepreneur is endowed with a fraction $\gamma \in (0,1)$ of the aggregate initial income, w_i, and the consumer is endowed with the remaining amount. Entrepreneurs are risk neutral and able to activate an investment project. The latter is a technology transforming $k \geq 0$ units of the good in the first period into $\epsilon f(k) \geq 0$ units of the same good in the next period, where ϵ is a random variable with observable ex post realizations. I make the following assumptions.

Assumption 1 (Production) $f : \mathbb{R}_+ \to \mathbb{R}_+$ *is smooth, strictly increasing, concave, bounded above, such that*

$$f(0) = 0, \qquad \lim_{k\to 0} f'(k) = +\infty, \qquad \lim_{k\to\infty} f'(k) = 0.$$

Assumption 2 (Risk) *The random variable ϵ is identically and independently distributed across entrepreneurs (i.e., countries) with support $\{\epsilon_b, \epsilon_g\} = \{0,1\}$ and probabilities $(p_b, p_g) \gg 0$.*

Consumers in each country i are unable to activate projects and are risk averse. Their welfare is evaluated through a standard expected utility, $\sum_{j=b,g} p_j u(x_{ji})$, defined over next period income, or consumption, $(x_{gi}, x_{bi}) \geq 0$, which is contingent on idiosyncratic states.

Assumption 3 (consumers) *The Bernoulli utility, $u : \mathbb{R}_+ \to \mathbb{R}$, is bounded, smooth, smoothly strictly increasing and smoothly strictly concave, with* $\lim_{x\to 0} u'(x) = \infty$.

Since countries are *ex ante* identical, I concentrate on allocations that assign the same amounts of investments and state contingent consumptions, to all countries with the same first period income. Hence, from now on, all variables referring to the resources assigned to a country, before idiosyncratic states are realized, will be indexed by w. For instance, k_w will denote the amount of risky investment assigned to a country with initial income w and (x_{gw}, x_{bw}) will denote the state contingent consumption assigned to the same country.

Feasible allocations

A consumer in each country has the option to make a direct transfer, b_{hw}, to the local entrepreneur in period one for investment in the stochastic technology, against the promise to get a repayment, R_{hw}, from the local entrepreneur in the next period if he has been successful. The array $\mathcal{H} = (b_{hw}, R_{hw}; w \in W)$ will be called the set of *home transfers.*

Moreover, I assume that there is an International Agency redistributing resources across countries. In particular, this agency collects d_w units of consumption in period one from consumers in each country. I assume that $b_{hw} + d_w$ can be greater (or smaller) than consumers' initial resources, $(1-\gamma)w$.

Once resources have been collected, the Agency allocates b_{fw} units of consumption to each country with initial income w for investment in the stochastic technology in period one against the promise to be delivered R_{fw} units of consumption by each successful entrepreneur[3] in period two. The remaining part of the collected resources,

$$\sum_w (d_w - b_{fw})\mu_w,$$

is stored and yields an aggregate output $\rho \sum_w (d_w - b_{fw})\mu_w$ in period two. Finally, the Agency uses this aggregate output to make a contingent transfer (z_{gw}, z_{gw}) to each country with initial income w. The array $\mathcal{R} = (z_{jw}, d_w, b_{fw}, R_{fw}; j = b, g, w \in W)$ will be called a *redistribution.*

Since the realizations of ϵ are i.i.d. across countries, by the law of large numbers the Agency makes choices under the following feasibility constraint:

$$\sum_w \left(\sum_{j=b,g} p_j z_{jw} \right) \mu_w \leq \sum_w (pR_{fw} + \rho(d_w - b_{fw}))\mu_w.$$

Now let y_w be the consumption of the entrepreneur in the good state in each country with initial income w. Since entrepreneurs are risk neutral, we only consider allocations such that they get no positive (and negative) transfers in the bed state. Thus, y_w is

[3] If an entrepreneur is not successful, he gets zero output and is unable to repay.

unambiguously a measure of the entrepreneur's welfare in these countries. By the above definitions,

$$k_w = \gamma w + b_{hw} + b_{fw}, \qquad y_w = f(k_w) - R_{hw} - R_{fw}.$$

Notice that, under this arrangement, contingent consumption across country types is a pair (x_{gw}, x_{bw}) such that

$$\begin{aligned} x_{gw} &= \rho((1-\gamma)w - b_{hw} - d_w) + z_{gw} + R_{hw}, \\ x_{bw} &= \rho((1-\gamma)w - b_{hw} - d_w) + z_{bw}. \end{aligned}$$

Hence, we can rewrite the feasibility constraint as

$$\sum_w \left(\sum_j p_j x_{jw} + p_g y_w - p_g f(k_w) - \rho(w - k_w) \right) \mu_w \leq 0. \tag{1}$$

Any set $(\mathcal{H}, \mathcal{R})$ of home transfers and redistributions across countries generates an assignment (or *allocation*), $\mathcal{A} = (x_{jw}, y_w, k_w; j = b, g, w \in W)$. The allocation is said to be *feasible* if it satisfies equation (1).

Limited commitment

Agents in each country cannot commit to repay the promised amounts, after uncertainty is revealed. I take it as an institutional constraint a rule similar to Chapter 7 in the U.S. bankruptcy law. Under this rule, all period 2 assets above a minimum threshold are liquidated by the courts and used to pay off the Agency. This implies that borrowers have an incentive to default (even in good states of nature) and redistributions across countries must satisfy a constraint implying that default is not individually rational. Since each country has two type of potential borrowers, consumers and entrepreneurs, there are two different constraints.

(a) Consumers. For any pair $(\mathcal{H}, \mathcal{R})$, let me define a country's *internally generated income*, as

$$x^I_{jw} = \max\{\rho((1-\gamma)w - b_{hw} - d_w), 0\} + \epsilon_j R_{hw},$$

where $j = b, g$ and, as I said earlier, $\epsilon_b = 0$, $\epsilon_g = 1$. Consumers' limited commitment problem arises from the assumption that any one of them can keep a fraction $\beta \in (0, 1)$ of his internally generated income when he does not fulfill his obligations with respect to the International Agency. Then, a consumer's individual rationality constraint reads

$$x_{jw} \geq \beta x^I_{jw}, \qquad j = b, g. \tag{2}$$

(b) Entrepreneurs. A similar type of constraint holds with respect to the (potential) debts contracted by entrepreneurs. Evidently, an entrepreneur's internally generated income is always zero in the bad state and it is equal to $f(k_w)$ in the good state. I make the crucial assumption that the entrepreneur's income which can be (partly) liquidated in case of default is net of home repayments. Namely, partial liquidation implies that the entrepreneur can get away with $\alpha(f(k_w) - R_{hw})$, where $\alpha \in [0, 1]$. Hence, entrepreneurs can fully commit with respect to debt repayments contracted with local lenders (consumers). It follows that an entrepreneur's individual rationality constraint reads

$$y_w \geq \alpha(f(k_w) - R_{hw}). \tag{3}$$

From the above assumptions, it is clear that any given allocation, $\mathcal{A}$, generated by the pair $(\mathcal{H}, \mathcal{R})$ of home transfers and redistributions, satisfies the individual rationality constraint if and only if

$$x_{gw} - \beta \max\{\rho((1-\gamma)w - b_{hw} - d_w), 0\} \geq \beta R_{hw} \geq \beta(f(k_w) - y_w/\alpha) \quad \text{if} \quad \epsilon = 1,$$

$$x_{bw} - \beta \max\{\rho((1-\gamma)w - b_{hw} - d_w), 0\} \geq 0 \quad \text{if} \quad \epsilon = 0.$$

Since (b_{hw}, d_w, R_{hw}) are unrestricted values, the above constraints imply the following definition.

Definition 1 (Individual rationality) *An allocation $\mathcal{A}$ is said to be individually rational if it satisfies*

$$x_{gw} \geq \beta(f(k_w) - y_w/\alpha). \tag{4}$$

The parameters α and β represent the threshold for the fraction of period two assets above which liquidation occurs. I will make the following key assumption:

Assumption 4 $\beta \geq \max\{p_g, \alpha\}$.

i.e., liquidation of consumers' assets in case of default is sufficiently difficult and liquidation of consumers' assets is no less difficult than liquidation of entrepreneurs' assets. The last assumption generates a richer set of optimal allocations that may be potentially interesting.

3 Efficient allocations

Definition 2 *An allocation, $\mathcal{A}$, is said to be constrained efficient (CE) if it is feasible, individually rational and it is not Pareto dominated by any other feasible and individually rational allocation.*

Now let $\{\theta_w; w \in W\}$ be a set of welfare weights, with $\theta_w > 0$ for all $w \in W$ and $\sum_w \theta_w \mu_w = 1$. I represent a CE allocation as an assignment $\mathcal{A}$ which maximizes the consumers' weighted *ex ante* expected utilities

$$U = \sum_w \left(\sum_j p_j u(x_{jw}) \right) \theta_w \mu_w,$$

subject to constraints (1), (4) and

$$y_w \geq y_w^R, \tag{5}$$

where y_w^R represents the reservation utility of the entrepreneur in a country with initial income w.

First order conditions

By the first order conditions for the maximization of U with respect to $\mathcal{A}$ subject to the given constraints, we get the following proposition.

Proposition 1 *If $\mathcal{A}$ is a CE allocation, there is a multiplier $\lambda > 0$ such that, for all $w \in W$, $\theta_w u'(x_{bw}) = \lambda$, $u'(x_{bw}) \geq u'(x_{gw})$, where the strict inequality holds if the individual rationality constraint, (4), is binding for w, and*

$$u'(x_{bw}) - u'(x_{gw}) = u'(x_{bw})(p_g f'(k_w) - \rho)/\beta p_g f'(k_w), \tag{6}$$

$$u'(x_{bw}) - u'(x_{gw}) \leq u'(x_{bw})\alpha/\beta. \tag{7}$$

Moreover, if $y_w > y_w^R$, equation (7) is satisfied with equality and, if (7) is satisfied with inequality, $y_w = y_w^R$.

Remark 1 (External debt) *Notice that, at a CE allocation $\mathcal{A}$, for all countries whose initial income w implies a binding individual rationality constraint, the pair $(\mathcal{H}, \mathcal{R})$ generating $\mathcal{A}$ is such that*

$$b_{hw} + d_w \geq (1-\gamma)w, \qquad R_{hw} = f(k_w) - y_w/\alpha.$$

Since $R_{hw} = f(k_w) - R_{fw} - y_w$, the latter implies $R_{fw} = y_w(1-\alpha)/\alpha$. Hence, in countries where the individual rationality constraint is binding, consumers are net debtors and entrepreneurs' ex-post repayments to the International Agency is proportional to their ex-post income.

First best allocations

A *first best allocation*, $\mathcal{A}^*$, maximizes U under the only constraints (1), (5) for all w. Evidently, this allocation is a CE allocation when the individual rationality constraint (4) is non binding for all w. Then, the first best allocation provides consumers with full insurance and it equalizes the marginal products of capital and the marginal rates of substitution between bad and good state consumption across countries. In particular, by the first order conditions (6) and (7), we get

$$k_w^* = k^*, \qquad x_{gw}^* = x_{bw}^* \equiv x_w^*, \ \forall w \in W,$$

where k^* is defined by $p_g f'(k^*) = \rho$.

By assumption 1, k^* is positive and well defined. Hence, using the individual rationality constraint (4), a CE allocation is a first best if and only if

$$x_w^* \geq \beta(f(k^*) - y_w^R/\alpha) \qquad \forall w.$$

To simplify the notation, let the world average of a variable be identified by an upper bar, so that

$$\bar{x}^* = \sum_w x_w^* \mu_w, \qquad \bar{y}^R = \sum_w y_w^R \mu_w.$$

Then, the feasibility constraint (1) at a first best allocation can be written as

$$\bar{x}^* = p_g f(k^*) - p_g \bar{y}^R + \rho(\bar{w} - k^*).$$

Evidently, a first best satisfies the individual rationality constraint only if $x_w^* \geq \beta(f(k^*) - y_w^R/\alpha)$ for all $w \in W$. Taking averages across countries and rearranging terms, the following proposition follow:

Proposition 2 *if a CE allocation attains the first best, must be*

$$\bar{y}^R \geq \alpha f(k^*) - \frac{\alpha}{\beta - \alpha p_g}[p_g(1-\alpha)f(k^*) + \rho(\bar{w} - k^*)]. \tag{8}$$

The above condition follows from the fact that, when the entrepreneurs' reservation utility is relatively high, consumers' have a relatively low level of consumption and, then, a lower incentive to breach the contract with the International Agency.

Remark 2 (The role of home lending) *A natural question is what role has home lending in this economy. Since home lending is a risky activity, one may wonder why this should ever be carried out at an efficient allocation. The answer is that home lending serves the purpose of relaxing the individual rationality constraints. In particular, recall that the individual rationality constraint for entrepreneurs reads* $y_w \geq \alpha(f(k_w) - R_{hw})$. *Now suppose that we want to implement the first best,* $\mathcal{A}^*$, *with* $b_{hw}^* = R_{hw}^* = 0$. *Since,* *at the first best, the entrepreneurs' utility is equal to the reservation value, we get the condition* $y_w^R \geq \alpha f(k^*)$ *for all* w*. By taking averages across countries, we get* $\bar{y}^R \geq \alpha f(k^*)$*. Evidently, the latter condition is more restrictive than condition (8) as long as*

$$p_g(1-\alpha)f(k^*) + \rho(\bar{w} - k^*) > 0.$$

Hence, even when the first best can be attained, home lending may be optimal. In other words, since consumers have the power to fully enforce home transfers, they can diminish the entrepreneurs' incentive to default on a foreign transfer by raising the direct repayments, R_{hw}*.*

Second best allocations

Now assume that condition (8) is not satisfied. Then, at the CE allocation the individual rationality constraint is binding for some w. In this case I say that the CE allocation is a "second best". Notice that, by the first order conditions and assumption 4, we have

$$p_g f'(k_w) > \rho \geq (1-\alpha)p_g f'(k_w), \qquad x_{gw} = \beta(f(k_w) - y_w/\alpha)$$

for all w such that the individual rationality constraint is binding. Moreover, if there is some w for which $y_w > y_w^R$, it must be $\beta > \alpha$ and, in addition to (6), the following must be satisfied

$$u'(x_{gw})/u'(x_{bw}) = (\beta - \alpha)/\beta, \qquad p_g f'(k_w) = \rho/(1-\alpha). \tag{9}$$

From now on I let $\hat{k}$ be defined by $p_g f'(\hat{k}) = \rho/(1-\alpha)$. Evidently, $\hat{k}$ defines a lower bound on investment, k_w, in a second best allocation. This implies an upper bound on the "default premium", $\pi(k_w) = (p_g f'(k_w) - \rho)/\rho$, and a lower bound on the "degree of insurance", $\delta_w = u'(x_{gw})/u'(x_{bw})$. Namely,

$$0 \leq \pi(k_w) \leq \frac{\alpha}{1-\alpha}, \qquad \frac{\beta-\alpha}{\beta} \leq \delta_w \leq 1,$$

where, by equation (6), $\delta_w = 1 - \pi(k_w)/\beta(1+\pi_w)$.

Remark 3 (Why $k_w < \hat{k}$ cannot be optimal) *When* $\beta > \alpha$*, the International Agency is less protected in her relation with home consumers than she is in her relation with home entrepreneurs in each country or, in other words, it is more difficult to liquidate the assets of consumers than it is to liquidate the assets of entrepreneurs. In this case, the entrepreneur's expected consumption may be above his reservation level,* $\bar{y}_w$*, at a second best allocation. The intuition is the following.*

Consider an allocation that maximizes the Planner's problem with respect to bad and good state consumption and investment for all countries, but keeps every entrepreneurs' income at the reservation level. Then, this allocation satisfies the first order condition (6) for all w*. Now consider a rise in entrepreneurs' income in all countries with an initial income* $w = s$ *only. The effect on social welfare is measured by*

$$\frac{\partial U}{\partial y_s} = \sum_w \theta_w \left(\sum_j p_j u'(x_{jw}) \frac{\partial x_{jw}}{\partial y_s} \right) \mu_w.$$

Plugging in the first order conditions for a CE allocation, we get

$$\frac{1}{\lambda}\frac{\partial U}{\partial y_s} = \left[\sum_w \left(\sum_j p_j \frac{\partial x_{jw}}{\partial y_s} \right) \mu_w \right] - \left[\sum_w \left(\frac{p_g f'(k_w) - \rho}{\beta f'(k_w)} \right) \frac{\partial x_{gw}}{\partial y_s} \mu_w \right],$$

where λ is the Lagrange multiplier associated to the feasibility constraint (1). There are two parts in the above expression. The first part (between the first two square brackets) measures the effect of a rise in y_s on average consumption. By the feasibility constraint, (1), we get

$$\sum_w \left(\sum_j p_j \frac{\partial x_{jw}}{\partial y_s} \right) \mu_w = \sum_w (p_g f'(k_w) - \rho) \frac{\partial k_w}{\partial y_s} \mu_w - p_g \mu_s.$$

The second part (between the second pair of square brackets) measures the effect of a rise in y_s on the maximum insurance compatible with the individual rationality constraint. Namely, as long as a rise in y_s generates a fall (rise) in good state consumption, consumers are allowed to get more (less) insurance with no violation of the individual rationality constraint, (4). Since the latter is assumed to be binding,

$$\frac{\partial x_{gw}}{\partial y_s} = \beta \left(f'(k_w) \frac{\partial k_w}{\partial y_s} - \frac{1}{\alpha} \right).$$

The envelope theorem implies that the indirect effects, measured by $\partial k_w / \partial y_s$ for all w, cancel out and, hence, we can write

$$\frac{1}{\lambda}\frac{\partial U}{\partial y_s} = -p_g \mu_s + \left(\frac{p_g f'(k_w) - \rho}{\alpha f'(k_w)} \right) \mu_s. \tag{10}$$

Then, when considering a rise in y_s on social welfare at the optimal allocation, the Planner is facing a trade-off between two effects. On the one hand, average consumption goes down by $p_g \mu_s$, because more resources must be allocated to entrepreneurs in countries with initial income $w = s$ for the same level of capital. On the other hand, consumers in countries with that income level can get more insurance because the individual rationality constraint will be relaxed by a rise in y_s. The last effect is measured by the second expression on the right hand side of (10). By rewriting this equation, we conclude that a rise in y_s increases social welfare (i.e., the fall in average consumption is more than compensated by the benefit of greater insurance) if and only if $p_g f'(k_w) > \rho/(1-\alpha)$. Hence, $k_w < \hat{k}$ cannot be optimal.

4 Competitive allocations

I assume now that redistributions are performed through competitive asset markets. There is a large set of two type of financial intermediaries operating world-wide: mutual funds (or insurance companies) and banks. All of them make zero profits on average by pooling risks across countries. Mutual funds offer a security d with state contingent payoffs to consumers, banks offer a loan b_f to entrepreneurs. More formally, a redistribution, $\mathcal{R}$, is said to be performed through a competitive anonymous market if it is such that

$$z_{gw} = r_g d_w, \qquad z_{bw} = r_b d_w, \qquad R_{fw} = r_f b_{fw},$$

where $r = (r_g, r_b, r_f) \geq 0$ satisfy

$$\sum_j p_j r_j = \rho, \qquad \sum_w (p_g R_{fw} - \rho b_{fw})\mu_w = (p_g r_f - \rho) \sum_w b_{fw}\mu_w = 0.$$

Hence, all consumers earn the same per unit contingent rate of return on their assets and all entrepreneurs pay the same per unit loan rate on their debt.

In a competitive equilibrium, home transfers, (b_{hw}, R_{hw}), should be interpreted as being part of a *domestic contract*. For any given price vector, r, domestic contracts are selected by consumers so as to maximize their expected utility subject to the entrepreneurs' "participation constraint" (i.e., they take the form of a take-it-or-leave-it offer) and that they are contingent on the level of "foreign debt", b_{fw}. Hence, a domestic contract is, effectively, a triple (b_{hw}, R_{hw}, b_{fw}) defining the size of domestic and foreign loans and the amount of "repayment" from entrepreneurs to consumers. Since I am considering a decentralized allocation, it is natural to assume that entrepreneurs' will accept a domestic contract as long as this provides them with a non negative expected income (or profit) net of the opportunity cost of investment in the risky technology, i.e., as long as

$$p_g y_w - \rho\gamma w \geq 0.$$

An home transfer generated in this way will be called a domestic contract with non negative profits. Notice that this construction implies that domestic contracts are socially optimal for any given redistribution.

Decentralization through credit limits

In this section I will try to decentralize constrained efficient allocations as competitive equilibria with credit limits. Essentially, these allocations are characterized by redistributions performed through competitive markets (i.e., linear pricing and zero profits for international intermediaries), domestic contracts with non negative profits, upper limits on consumers' desired assets and upper limits on entrepreneurs' desired liabilities. I will show, though, that, despite the fact that we can impose appropriate constraints on agents' net asset positions, a constrained efficient allocation may not always be decentralizable as a competitive equlibrium. This failure occurs when a country's aggregate income, w, is particularly low. In these cases the decentralizability of a CE allocation can be resumed if we impose an appropriate *ex post* local transfer from consumers to entrepreneurs within successful countries.

A set of upper limits on the holdings of consumers' assets (credit limits) is an array $\mathcal{L} = (\bar{d}_w, \bar{b}_{hw}, \bar{b}_{fw}; w \in W)$. A set of *ex post* domestic transfers from consumers to entrepreneurs in a good state is an array $\mathcal{T} = (T_w; w \in W)$. A competitive equilibrium is an array $\mathcal{E} = (\mathcal{H}, \mathcal{R}, r)$ (a set of home transfers, a redistribution and a set of prices).

Definition 3 *A competitive equilibrium with credit limits $\mathcal{L}$ and domestic transfers, $\mathcal{T}$, is an array $\mathcal{E}(\mathcal{L}, \mathcal{T})$ such that*

(CC) consumers select an insurance, d_w, and a contract, (b_{hw}, R_{hw}, b_{fw}), by maximizing $\sum_j p_j u(x_{jw})$ subject to $d_w \leq \bar{d}_w$, $b_{hw} \leq \bar{b}_{hw}$, $b_{fw} \leq \bar{b}_{fw}$, where

$$\begin{aligned} x_{gw} &= \rho((1-\gamma)w - b_{hw} - d_w) + r_g d_w + R_{hw}, && (11) \\ x_{bw} &= \rho((1-\gamma)w - b_{hw} - d_w) + r_b d_w, && (12) \\ R_{hw} &\leq f(\gamma w + b_{hw} + b_{fw}) - r_f b_{fw} - \rho\gamma w/p_g - T_w. && (13) \end{aligned}$$

(ZP) international intermediaries make zero profits, i.e., $\sum_j p_j r_j = \rho$ and $p_g r_f = \rho$,

(MK) markets clear, i.e.,

$$\sum_w \left(\sum_j p_j x_{jw} + p y_w - p f(k_w) - \rho(w - k_w) \right) \mu_w \leq 0.$$

First of all, I provide a condition that prevents first best allocations to be attainable in a competitive equilibrium.

Proposition 3 *A competitive equilibrium with credit limits cannot be first best efficient when*

$$(\beta - p) f(k^*) + \rho k^* > \left(\frac{\gamma\beta}{p\alpha} + (1-\gamma) \right) \rho \bar{w},$$

where k^* is the first best level of investment, i.e., $pf'(k^*) = \rho$.

Proof. Remember that first best efficiency implies $y_w = y_w^R$ for all $w \in W$. By the definition of a competitive equilibrium with credit limits, $y_w^R = \rho\gamma w / p_g$. Using equation 2, we immediately derive the proposition. **Q.E.D.**

Proposition 4 *Under the restrictions $\beta \geq 1 - p_g(1-\alpha)$, $r_g > \rho > r_b$, a CE allocation can be decentralized as a competitive equilibrium with credit limits and domestic transfers and:*

1. *there are two income levels, $w^* > w^m \geq 0$ (with $w^m > 0$ if and only if $\beta > \alpha$), such that, for all $w < w^*$, full insurance cannot be attained and credit limits are binding;*
2. *there are two continuous functions, $\kappa : [w^m, w^*] \to \mathbb{R}_+$ and $\zeta : [0, w^m] \to \mathbb{R}_+$, such that the CE equilibrium allocation has the following characterization:*

$$(k_w, y_w) = \begin{cases} (\hat{k}, \zeta(w)) & \text{for} \quad w \in (0, w^m], \\ (\kappa(w), \rho\gamma w/p_g) & \text{for} \quad w \in [w^m, w^*], \end{cases}$$

   where $\kappa(w)$ is increasing, $\zeta(w)$ is decreasing and $\kappa(w^m) = \hat{k}$, $\kappa(w^*) = k^*$, $\zeta(w^m) = \rho\gamma w/p_g$,

3. *income transfers are $T_w = \zeta(w) - \rho\gamma w/p_g$ (i.e., they are positive for $w \in (0, w^m)$ and zero for $w \geq w^m$).*

Proposition 4 shows that, for a competitive equilibrium to be decentralizable as a second best CE allocation, intermediaries must impose upper limits on consumers' demand for insurance and on the supply for loans. Moreover, when countries are particularly poor ($w < w^m$), credit constraints may not be sufficient. Consumers' must be forced to transfer some amount of their *ex post* income to entrepreneurs when investment is successful.

The proposition also shows that, in a CE competitive allocation, a rise in a country's total wealth, w, has a positive effect on investment, k_w, when the entrepreneur's participation constraint, $p_g y_w \geq \rho\gamma w$, is binding (i.e., when $w \geq w^m$). Otherwise, the (local) effect on k_w of a rise in w is zero. To see why, let me simplify the notation by setting $\beta = 1$ (the same logic applies in the more general case $\beta \leq 1$, except that more notation is involved). With $\beta = 1$, we can write the first order condition at a CE allocation as

$$1/\delta_w \equiv u'(x_{bw})/u'(x_{gw}) = p_g f'(k_w)/\rho \equiv 1 + \pi(k_w),$$

where δ_w measures the degree of risk sharing and $\pi(k_w)$ is the default premium. When the CE allocation is decentralized as a competitive equilibrium with zero entrepreneurs' profits ($p_g y_w = \rho\gamma w$), good state consumption is increasing in k_w (the risky asset) and decreasing in w, whereas bad state consumption is decreasing in k_w and increasing in w. Hence, $1/\delta_w$ is increasing in k_w and decreasing in w. On the other hand, by the assumption of decreasing marginal productivity, $\pi(.)$ is decreasing in k_w. Hence, we may be able to determine the CE value of k_w from the intersection between $1/\delta$ and $1 + \pi(k_w)$ (see figure 2 at the end of the paper). In this case, a small decrease in w shifts up $1/\delta_w$, determining a fall in k_w and a rise in the optimal values of both $1/\delta_w$ and $\pi(k_w)$. However, remember (from section 2) that, when $\beta = 1$, constrained efficiency implies that both $1+\pi(k_w)$ and $1/\delta_w$ cannot be greater than $1/(1-\alpha)$. Then, when w is sufficiently small, we cannot hope to find an intersection between $1/\delta_w$ and $1+\pi(k_w)$ for values of δ_w and $\pi(k_w)$ satisfying the above restrictions. In these cases, an intersection satisfying the restrictions can be found if we set $k_w = \hat{k}$ and we allow for $p_g y_w > \rho\gamma w$. In other words, when $\beta > \alpha$ and a country is particularly poor, it is optimal to redistribute resources from consumers/lenders to entrepreneurs/borrowers (i.e., make within country debt cheaper) so as to keep the degree of insurance across states (and investment) from falling below a minimum.

Remark 4 (Competitive external debt) *I have already noticed, in remark 1, that, when w is such as to imply a binding individual rationality constraint (i.e., when a country is unable to fully insure), it is*

$$\rho((1-\gamma)w - d_w - b_{hw}) \leq 0, \qquad R_{fw} = y_w(1-\alpha)/\alpha.$$

Within a competitive allocation, this means that consumers have positive net liabilities with respect to the international asset market and entrepreneurs' net liabilities with respect to foreign lenders are proportional to their ex post income. In particular, since in a competitive allocation we have $R_{fw} = (\rho/p_g)b_{fw}$, we can evaluate the ex post total liabilities with respect to external lenders of a country with initial income w as

$$D^f_{jw} = \rho(d_w + b_{hw} - (1-\gamma)w) + \epsilon_j y_w(1-\alpha)/\alpha,$$

where $j = b, g$ and $\epsilon_j \in \{0, 1\}$. Since $b_{hw} = k_w - \gamma w - b_{fw}$ and $b_{fw} = (p_g/\rho)y_w(1-\alpha)/\alpha$,

$$D^f_{jw} = \rho(d_w + k_w - w) + (\epsilon_j - p_g)y_w(1-\alpha)/\alpha,$$

where d_w can be derived from

$$\beta(f(k_w) - y_w/\alpha) = \rho((1-\gamma)w - b_{hw} - d_w) + r_g d_w + f(k_w) - r_f b_{fw} - y_w,$$

which equates the competitive equilibrium good state consumption to the constrained efficient consumption. Since $r_f b_{fw} = y_w(1-\alpha)/\alpha$, we get

$$\rho(d_w + k_w - w) = \frac{\rho}{\rho - r_g}\left(r_g(w - k_w) + (1-\beta)f(k_w) + \frac{\beta + p_g(1-\alpha) - 1}{\alpha}y_w\right).$$

As an example, consider the case $r_g = 0$. Then,

$$D^f_{gw} = (1-\beta)f(k_w) + \frac{\beta - \alpha}{\alpha}y_w, \qquad D^f_{bw} = (1-\beta)f(k_w) - \frac{1-\beta}{\alpha}y_w.$$

By taking the average over idiosyncratic states, we can compute the external average (constrained efficient) debt of countries with initial income w at a competitive equilibrium as

$$\sum_j p_j D^f_{jw} = (1-\beta)f(k_w) + [\beta + p_g(1-\alpha) - 1]y_w/\alpha.$$

Then, by the assumption in proposition 4 and by the statements in the proposition, $\sum_j p_j D^f_{jw}$ is decreasing with w up to w^m and it is increasing thereafter. However, in general, the shape of a country's debt depends on the value of r_g, for which we have the only restriction $r_g < \rho$. For instance, a monotonic increasing relation between external debt and income can be obtained for $r_g > 0$. This may reconcile the model with the empirical correlation verified in Lane (2004) and Gertler and Rogoff (1989).

Comparing output volatility across countries

For each w, output volatility can be evaluated by the standard deviation of output (consumption) across the two states of nature, $\sigma_{cw} = \sqrt{p_g p_b}(x_{gw} - x_{bw})$. Hence, in this section I will evaluate $V(w) = (x_{gw} - x_{bw})$ as a function of w for the same world average income (and consumption) under the assumption that the CE allocation does not attain the first best.

In anticipation of a more formal analysis, let me provide some intuition about the relation between output volatility and the degree of risk sharing as a function of initial wealth. Recall that $\pi_w = (p_g f'(k_w) - \rho)/\rho$ has been defined as a default premium on investment. The "extent of incomplete risk sharing" at a constrained efficient allocation can be measured by π_w. I will show below that, with partial risk sharing ($\pi_w > 0$) and decreasing marginal productivity of capital, a higher income implies a higher investment, a smaller risk premium and, then, a fall in π_w.

Using a first order Taylor expansion of marginal utilities, one can show that

$$\sigma_{cw} \sim \left(\frac{\sqrt{p_g p_b}}{A_{bw}}\right)\left(\frac{\pi_w}{\beta - (1-\beta)\pi_w}\right),$$

where A_{bw} is the absolute degree of risk aversion at the bad state level of consumption. Since higher income countries enjoy a lower risk premium, they tend to have a smaller σ_{cw}. In an equal treatment allocation, bad state consumption is equalized across countries (cf. next section) and, hence, A_{bw} is invariant with respect to individual income. Instead, the competitive allocation is such that bad state consumption is positively affected by individual income and, thus, A_{bw} goes up or down with w according to whether absolute risk aversion is increasing or decreasing in consumption. This explains why σ_{cw} may not be decreasing with w. However, the model says more than this. It also shows that, at a constrained efficient allocation, the degree of insurance (and the level of investment) in each country is bounded below. In other words, when $\beta > \alpha$ and a country is particularly poor, it is optimal to redistribute resources from consumers/lenders to entrepreneurs/borrowers (i.e., make the within country debt contracts cheaper) so as to keep the degree of insurance across states (and investment) from falling below a minimum. Hence, when a country is very poor, a rise in his income may leave π_w almost or completely unchanged. In this case, any change in the volatility of consumption would be caused by changes in the absolute degree of risk aversion.

A more formal analysis is provided by the following proposition. Before stating the proposition, let me define, for $j = g, b$,

$$A_{jw} = -u''(x_{jw})/u'(x_{jw}), \qquad A_{fw} = -f''(k_w)/f'(k_w).$$

Proposition 5 *At a competitive equilibrium, σ_{cw} is strictly decreasing for all $w < w^*$ if absolute risk aversion is non decreasing. If, on the other hand, absolute risk aversion is decreasing, σ_{cw} is increasing when: either $w < w^m$ or $w \in (w^m, w^*)$ and $\beta(A_{bw} - A_{gw}) > \Gamma_f(w)A_{fw}$, for some $\Gamma_f(w) > 0$ (whose value depends on w and f).*

Since σ_{cw} must be eventually zero for $w \geq w^*$, when the degree of risk aversion is decreasing in consumption, consumption variance will be typically hamp shaped, except when it has more than one critical point. For instance, consider the following simple parametrization:

$$u(x) = \log x, \qquad f(k) = 3\sqrt{k}, \qquad p = 2/3, \qquad \rho = 1, \qquad \gamma = 0.$$

In this case we have

$$k_w = \max\left\{(1-\alpha)^2, \frac{w}{2}\right\}, \qquad y_w = T(w) = \max\left\{2\alpha(1-\alpha) - \frac{\alpha}{1-\alpha}w, 0\right\}$$

for $w \leq w^* = 2$ and $k_w = 1$, $y_w = 0$ for $w \geq 2$. Moreover,

$$\sigma_{cw} = \begin{cases} (1-\alpha)\alpha + w\alpha/(1-\alpha) & \text{if} \quad w \leq 2(1-\alpha)^2, \\ (3/2)(\sqrt{2w} - w) & \text{if} \quad w \in [2(1-\alpha)^2, 2], \\ 0 & \text{if} \quad w \geq 2, \end{cases}$$

so that σ_{cw} is hamp shaped with a single critical point at $w^c = 1/2$ (see figure 3).

Equal treatment

An *equal treatment allocation* is a solution to the Planner's problem under the condition $\theta_w = 1$ for all $w \in [0, 1]$. The first order conditions imply that this allocation is characterized by the same amount of bad state consumption for all agents, i.e., $x_{bw} = \bar{c}_b$ for all $w \in W$. Assuming that full insurance cannot be attained at a CE allocation and using the feasibility condition, we get

$$\bar{c}_b = \left((1-\beta)\sum_w p_g f(k_w)\mu_w + \rho(\bar{w} - \bar{k}) + p_g\bar{y}(\beta - \alpha)/\alpha\right)/p_b,$$

where $\bar{k} = \sum_w k_w\mu_w$, $\bar{y} = \sum_w y_w\mu_w$.

Since in the equal treatment allocation the bad state consumption of individual consumers' is invariant with respect to their initial income, consumption variance across states of nature in any single country varies with this country's good state consumption only, as long as aggregate variables stay constant. This implies that consumption variance, $V(w)$, as a function of a single country's income, w, cannot be increasing. This is shown in the next proposition, under the assumption that $y_w^R = \rho\gamma w/p_g$. This assumption is made to make the equal treatment allocation as close as possible to the competitive allocation.

Proposition 6 *In an equal treatment allocation $V(w)$ is non increasing. More specifically, there is a value $w_e^m \geq 0$, a value $w_e^* > w_e^m$ (with $w_e^m > 0$ if $\beta > \alpha$), such that $V(w)$ is positive and invariant with respect to w for all $w \leq w_e^m$, $V(w) = 0$ for all $w \geq w_e^*$ and $V(w)$ is strictly decreasing in w for all $w \in (w_e^m, w_e^*)$.*

5 Conclusions

In this paper I have studied the constrained efficient allocation of capital and consumption across states and income levels in a world economy where countries can insure incompletely against idiosyncratic shocks and they are subject to borrowing constraints as a consequence of limited commitment on contract obligations. I have shown that, whereas the degree of risk sharing and the default premium are, respectfully, increasing and decreasing with income, the standard deviation of consumption may be a non monotonic function of income. This happens when the constrained efficient allocation is decentralized as a competitive equilibrium with credit limits and it is a more likely outcome if the legal protection of lenders' rights is more problematic when the borrower is a consumer rather than an entrepreneur. This may explain why the correlation between output volatility and GDP in a cross section of countries is negative and significant for a dataset including developed and less developed economies and much less significant when the dataset includes relatively poor economies only.

A byproduct of this analysis is that, when lenders' protection is more problematic with respect to consumers rather than entrepreneurs, credit limits are not sufficient to decentralize a constrained efficient allocation as a competitive equilibrium. The reason is that, in this case, a country with low income may get more access to international lending by shifting resources from consumers to entrepreneurs. However, this redistribution cannot be implemented through credit limits only.

Although my model is completely static, one can embed the same basic framework in a dynamic environment by assuming (as in Galor and Zeira (1993) and Piketti (1997)) an overlapping generations structure where agents derive utility from bequests. Under this extension, it is not difficult to show that the time path of the development process is crucially affected by the stochastic realizations of the risky projects and, hence, slow growth may be a consequence of misfortune (as in Acemoglu and Zilibotti (1997)).

6 Appendix

Proof of proposition 4

A solution to consumers' choice problem exists and is unique for all w under the condition $r_b > \rho > r_g$. Moreover, the following conditions are verified

$$\begin{aligned}
\sum_j p_j u'(x_{jw})(r_j - \rho) &\geq 0, & (14)\\
p_g u'(x_{gw})(f'(k_w) - \rho) - p_b u'(x_{bw})\rho &\geq 0, & (15)\\
f'(k_w) - r_{fw} &\geq 0, & (16)\\
f(k_w) - r_f b_{fw} - R_{hw} - \rho\gamma w/p_g - T_w &= 0, & (17)
\end{aligned}$$

where $k_w = \gamma w + b_{hw} + b_{fw}$. It is immediate to verify that equation (14) holds under the assumption $r_b > \rho > r_b$ when the marginal utilities, $u'(x_{bw})$ and $u'(x_{gw})$, satisfy (6). In particular, the equation holds with equality if and only if w allows for full insurance. In fact, by the zero profit condition, $\sum_j p_j r_j = \rho$, the unrestricted individually optimal level of d_w provides full insurance. Hence, the constraint $d_w \leq \bar{d}_w$ must be binding for all w at which full insurance cannot be attained under second best efficiency. Using the zero profit condition $p_g r_f = \rho$ in equation (16) and plugging the first order condition at a constrained efficient allocation, (6), in equation (15), we can derive the following inequalities:

$$\rho/(1-\beta) \geq f'(k_w) \geq \rho/p_g. \tag{18}$$

Since the efficient investment, k_w, must be greater than or equal to $\hat{k}$, which implies $p_g f'(k_w) \leq \rho/(1-\alpha)$, inequalities (18) hold for all efficient values of k_w if $\rho/(1-\beta) \geq \rho/p_g(1-\alpha)$, i.e., $\beta \geq 1 - p_g(1-\alpha)$.

Now suppose that w is such that full insurance cannot be attained at the constrained efficient allocation. Then, the first order conditions (14)-(16) hold with inequality and the optimality of a competitive allocation is insured by the triple $(\bar{d}_w, \bar{b}_{hw}, \bar{b}_{fw})$ satisfying the following equations

$$\begin{aligned}
\beta(f(k_w) - y_w/\alpha) &= \rho((1-\gamma)w - \bar{b}_{hw} - \bar{d}_w) + r_g\bar{d}_w + f(k_w) - r_f\bar{b}_{fw} - y_w, & (19)\\
k_w &= \gamma w + \bar{b}_{hw} + \bar{b}_{fw}, & (20)\\
y_w &= \rho\gamma w/p_g + T_w. & (21)
\end{aligned}$$

Using the zero profit condition $\sum_j p_j r_j = 0$ and solving (19) and (20) for $\bar{d}_w$ and $\bar{b}_{hw}$, we get

$$x_{bw} = [\rho(w - k_w) + (1-\beta)p_g f(k_w) + p_g y_w(\beta - \alpha)/\alpha)]/p_b. \tag{22}$$

Now I try to decentralize a CE allocation as a competitive equilibrium with $T_w = 0$. In this case, $y_w = \rho\gamma w/p_g$ and

$$x_{bw} = [p_g(1-\beta)f(k_w) + \rho(w-k_w) + \rho\gamma w(\beta-\alpha)/\alpha)]/p_b.$$

Since I am assuming that w is such that full insurance cannot be achieved, must be $x_{gw} > x_{bw}$. In turn, this inequality holds if and only if

$$w < v(k_w) \equiv \frac{(\beta - p_g)f(k_w) + \rho k_w}{\rho(1-\gamma+\gamma\beta/p_g\alpha)}.$$

Notice that, by proposition 3, the assumption that the first best cannot be attained implies that

$$\bar{c}^* < \beta f(k^*) - \frac{\gamma\beta}{p_g\alpha}\rho\bar{w}. \tag{23}$$

By equation (23)), $v(k^*) > \bar{w}$. Now, for $k \in [\hat{k}, k^*]$, $w < v(k)$, define

$$G(k,w) = u'(x_b)/u'(x_g) - \beta p_g f'(k)/(\rho - (1-\beta)p_g f'(k)).$$

Since $\beta \geq \alpha$, $G(k,w)$ is strictly increasing in k and strictly decreasing in w and

$$G(k^*, v(k^*)) = 0, \qquad G(k^*, w) > 0,$$

for all $w \in [0, v(k^*)]$. By the definition of $\hat{k}$, $\beta p_g f'(\hat{k})/(\rho-(1-\beta)p_g f'(\hat{k}))$ is well defined and equal to $\beta/(\beta-\alpha)$ if and only if $\beta > \alpha$. I let $G(\hat{k}, w)$ be the limit of $G(k,w)$ as $k \to \hat{k}$ when $\beta = \alpha$. Since $G(\hat{k}, v(\hat{k})) < 0$ and $G(k,w)$ is continuous and strictly decreasing in w for all $k > \hat{k}$,

- if $G(\hat{k}, 0) < 0$, we have $G(\hat{k}, w) < 0$ for all $w \in [0, v(\hat{k})]$ and,
- if $G(\hat{k}, 0) > 0$, there exists a unique value $\hat{w} \in (0, v(\hat{k}))$ such that $G(\hat{k}, \hat{w}) = 0$ and $G(\hat{k}, w) < 0$ for all $k > \hat{w}$.

Now let $w^* = v(k^*)$ and $w^m = 0$ if $G(\hat{k}, 0) < 0$, $w^m = \hat{w} > 0$ if $G(\hat{k}, 0) > 0$. We can state that, for all $w \in (w^m, w^*)$, $G(k^*, w) > 0$ and $G(\hat{k}, w) < 0$. Hence, by continuity and monotonicity, there exists a unique value, $k = \kappa(w)$, such that $G(\kappa(w), w) = 0$ for all $w \in (w^m, w^*)$. By the implicit function theorem, $\kappa(w)$ is continuously differentiable, strictly increasing and such that $\kappa(w^*) = k^*$ and $\kappa(w^m) \geq \hat{k}$ (with $\kappa(w^m) = \hat{k}$ when $G(\hat{k}, 0) < 0$). Hence, for all $w \in [w^m, w^*]$, $k_w = \kappa(w)$ is a CE level of investment at a competitive equilibrium, since, for this value of k, the first order condition (6) is satisfied.

On the other hand, if $\beta > \alpha$ and $w < w^m$, we get $\kappa(w) < \hat{k}$, which is not compatible with constrained efficiency. Then, in this case, I try to decentralize the CE allocation as a competitive equilibrium with a positive *ex post* transfer T_w. Namely, assume $\beta > \alpha$, $w^m > 0$ (i.e., $G(\hat{k}, 0) > 0$), set $k_w = \hat{k}$ and define the function

$$H(y) = \frac{u'\left(\left(\rho(w-\hat{k}) + p_g(1-\beta)f(\hat{k}) + p_g y(\beta-\alpha)/\alpha\right)/p_b\right)}{u'(\beta(f(\hat{k}) - y/\alpha))} - \frac{\beta}{\beta-\alpha}.$$

Evidently, the competitive state contingent consumption, (x_{bw}, x_{gw}), satisfy the first order conditions (9) for a CE allocation if and only if $H(\zeta(w)) = 0$ for some $\zeta(w) \geq \rho\gamma w/p_g$. In this case, I set $T_w = \zeta(w) - \rho\gamma w/p_g$. Notice that

$$H(\rho\gamma\hat{w}/p_g, \hat{w}) = G(\hat{k}, \hat{w}) = 0,$$

and $H(y^o(w), w) < 0$ for some $y^o(w) > \gamma\rho w/p_g$, $w \leq \hat{w}$, where $y^o(w)$ is the value of y_w at which $x_{gw} = x_{bw}$, for $k_w = \hat{k}$. Since $H(y, w)$ is decreasing in y and w, for all $y \geq \rho\gamma w/p_g$ and $w \leq \hat{w}$, we get

$$H(\rho\gamma\hat{w}/p_g, w) > 0, \qquad H(y^o(w), w) < 0.$$

By continuity, there exists a unique value, $\zeta(w) > \rho\gamma w/p_g$, such that $H(\zeta(w), w) = 0$ for all $w < \hat{w} = w^m$. Moreover, $\zeta(w)$ is strictly decreasing for $w \in (0, w^m)$. This completes the proof of the proposition. In particular, by the first order conditions for a CE allocation, we get that $p_g y_w = \rho\gamma w$, $T_w = 0$ for all $w \geq w^m$ and $\zeta_w = y(w)$, $T_w = \zeta(w) - \rho\gamma w/p_g \geq 0$ for $w \in [0, w^m]$. **Q.E.D.**

Proof of proposition 5

By proposition 4, at a competitive allocation,

$$\begin{aligned} x_{gw} &= \beta(f(k_w) - y_w/\alpha), \\ x_{bw} &= \left((1-\beta)p_g f(k_w) + \rho(w-k_w) + (\beta-\alpha)p_g y_w/\alpha\right)/p_b, \end{aligned}$$

Then,

$$\sigma_{cw} = \left[(\beta - p_g)f(k_w) - \rho(w-k_w) - y_w(\beta - p_g\alpha)/\alpha\right]/p_b.$$

Now, let $A_{jw} = -u''(x_{jw})/u'(x_{jw})$, $(j = g, b)$, $A_{fw} = -f''(k_w)/f'(k_w)$ and recall that, for $w \leq w^m$, $k_w = \hat{k}$ and $y_w = \zeta(w)$, where $\zeta(w)$ is defined in the proof of proposition 4. Taking derivatives, we get

$$\zeta'(w) = -\rho\alpha A_{bw}/[p_g(\beta-\alpha)A_{bw} + p_b\beta A_{gw}].$$

Since, for $w \leq w^m$, $k_w = \hat{k}$, we get

$$\frac{\partial\sigma_{cw}}{\partial w} = \frac{\rho\beta(A_{bw} - A_{gw})}{p_g(\beta-\alpha)A_{bw} + p_b\beta A_{gw}}.$$

This expression proves the first part of the proposition.

Now let $w \in (w^m, w^*)$. Then, $k_w = \kappa(w)$, as defined in proposition 4, and $y_w = \gamma\rho w/p_g$. Taking derivatives, we get

$$\kappa'(w) = \rho \frac{\left(1-\gamma+\frac{\beta\gamma}{\alpha}\right) A_{bw} + \frac{\beta\gamma}{p_g\alpha} p_b A_{gw}}{\frac{\rho p_b}{\rho-(1-\beta)p_g f'(k_w)} A_{fw} + (\rho-(1-\beta)p_g f'(k_w))A_{bw} + \beta p_b f'(k_w) A_{gw}}.$$

Then, σ_{cw} is increasing in w for $w \in (w^m, w^*)$ iff $\beta(A_{bw} - A_{gw}) > \Gamma_f(w) A_{fw}$, where

$$\Gamma_f(w) = \frac{\rho\,(1-\gamma+\beta\gamma/p_g\alpha)}{((1-\gamma+\gamma/\alpha)f'(k_w) - \rho\gamma/p_g\alpha)\,(\rho - p_b f'(k_w))}.$$

Q.E.D.

Proof of proposition 6

For the most part of this proof I will consider w as the initial income of a single country for which full insurance cannot be attained. Hence, by the previous discussion, constrained efficiency implies

$$x_{gw} = \beta(k_w - y_w/\alpha), \qquad y_w \geq \rho\gamma w/p_g.$$

First, I prove the following lemma.

Lemma 1 *Suppose that the first best cannot be attained. In an equal treatment allocation there is a value $w_e^m \geq 0$, a value $w_e^* > w_e^m$ (with $w_e^m > 0$ if $\beta > \alpha$) and an increasing continuous function $\kappa_e(w)$, such that $k_w = \hat{k}$ and $y_w = \hat{y} \geq \rho\gamma w_e^m/p_g$ for all $w \leq w_e^m$, $k_w = \kappa_e(w)$ and $y_w = \rho\gamma w/p_g$ for all $w \in [w_e^m, w_e^*]$ and $k_w = k^*$, $y_w = \rho\gamma w/p_g$ for all $w \geq w_e^*$.*

Proof. Notice that, for an equal treatment allocation, $x_{gw} \geq \bar{c}_b$ if and only if $w \leq s(k)$, where

$$s(k) = (f(k) - \bar{c}_b/\beta)p_g\alpha/\rho\gamma.$$

$s(k^*) > 0$ is guaranteed by the assumption that the first best cannot be attained (i.e., by equation (23)). In fact, $s(k^*) > 0$ iff $\beta f(k^*) > \bar{c}_b$ and, by (23),

$$\bar{c}_b < \sum_w \left(\sum_j p_j x_{jw} \right) \mu_w < \bar{c}^* < \beta f(k^*).$$

Finally, for $k \in [\hat{k}, k^*]$, $w \in [0, s(k))$, define

$$F(k,w) = u'(\bar{c}_b)/u'(\beta(f(k) - \gamma\rho w/p_g\alpha)) - \beta p_g f'(k)/(\rho - (1-\beta)p_g f'(k)).$$

Notice that $F(k,w)$ is strictly increasing in k and strictly decreasing in w and

$$F(k^*, s(k^*)) = 0, \qquad F(k^*, w) > 0, \ \forall w \in [0, s(k^*)].$$

By the definition of $\hat{k}$, $\beta p_g f'(\hat{k})/(\rho - (1-\beta)p_g f'(\hat{k}))$ is well defined and equal to $\beta/(\beta-\alpha)$ if and only if $\beta > \alpha$. From now on, we take $F(\hat{k}, w)$ as the limit of $F(k,w)$ as $k \to \hat{k}$ when $\beta = \alpha$. Notice that, since $F(k,w)$ is continuous and strictly decreasing in w for all $k > \hat{k}$ and $F(\hat{k}, s(\hat{k})) < 0$,

- if $F(\hat{k}, 0) < 0$, we have $F(\hat{k}, w) < 0$ for all $w \geq 0$ and,
- if $F(\hat{k}, 0) > 0$, there exists a unique value $\hat{w}_e \in (0, s(\hat{k}))$ such that $F(\hat{k}, \hat{w}_e) = 0$ and $F(\hat{k}, w) < 0$ for all $k > \hat{w}_e$.

Now denote $w_e^* = s(k^*)$, $w_e^m = 0$ if $F(\hat{k}, 0) < 0$, $w_e^m = \hat{w}_e$ if $F(\hat{k}, 0) > 0$. We can state that, for all $w \in (w_e^m, w_e^*)$, $F(k^*, w) > 0$ and $F(\hat{k}, w) < 0$. By continuity and monotonicity, there exists a unique value, $k = \kappa_e(w)$, such that $F(\kappa_e(w), w) = 0$ for all $w \in (w_e^m, w_e^*)$. By the implicit function theorem, $\kappa_e(w)$ is continuously differentiable, strictly increasing and such that $\kappa_e(w_e^*) = k^*$ and $\kappa_e(w_e^m) \geq \hat{k}$ (with $\kappa_e(w_e^m) = \hat{k}$ when $F(\hat{k}, 0) < 0$).

Using the first order condition (6), we can claim that $k_w = \kappa_e(w)$ for all $w \in [w_e^m, w_e^*]$, $k_w = k^*$ for all $w \geq w_e^*$ and $k_w = \hat{k}$ for all $w \leq w_e^m$. Moreover, for $w \leq w_e^m$, $y_w = \hat{y}$, where $\hat{y}$ is the unique value of y_w such that

$$u'(\bar{c}_b)/u'(\beta(f(\hat{k}) - y_w/\alpha)) - \beta/(\beta - \alpha) = 0.$$

Then, by the first order conditions for a CE allocation, we get that $y_w = \rho\gamma w/p_g$ for all $w \geq w_e^m$ and $y_w = \hat{y}$ for $w \in [0, w_e^m]$. **Q.E.D.**

Now notice that $x_{gw} = \bar{c}_g$ for all $w \geq w_e^*$, $x_{gw} = \beta(f(\hat{k}) - \hat{y}/\alpha)$ for $w \leq w_e^m$ and

$$u'(\bar{c}_b)/u'(x_{gw}) - \beta p_g f'(\kappa_e(w))/(\rho - (1-\beta)p_g f'(\kappa_e(w))) = 0$$

for $w \in [w_e^m, w_e^*]$. Then, taking the derivative of the above expression with respect to $w \in (w_e^m, w_e^*)$,

$$f'(k)u''(x_{gw})\frac{\partial x_{gw}}{\partial w} + \frac{\rho f''(k)u'(x_{gw})}{\rho - (1-\beta)p_g f'(k)}\kappa_e'(w) = 0.$$

Since $\kappa_e'(w) > 0$, the above shows that x_{gw} is decreasing in w for $w \in (w_e^m, w_e^*)$. **Q.E.D.**

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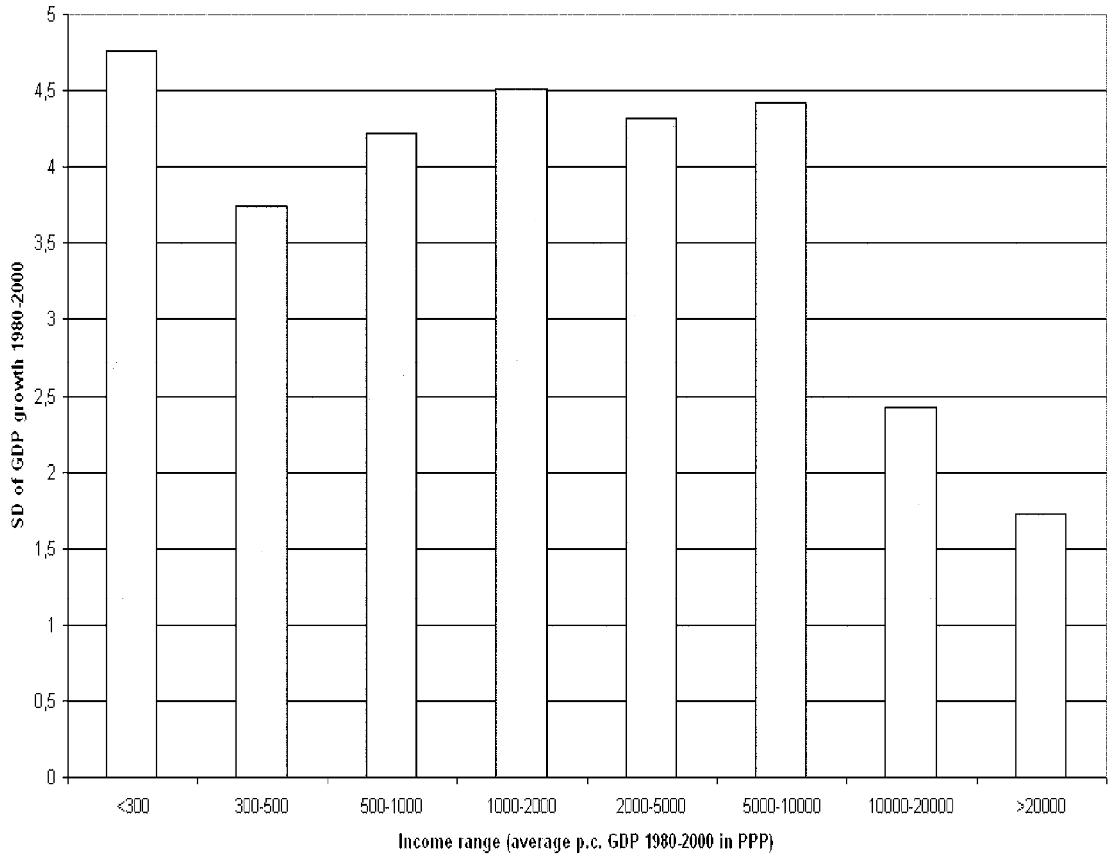

Figure 1: Average p.c. GDP and volatility in 1980-2000 for subsamples of countries (WDI data base).

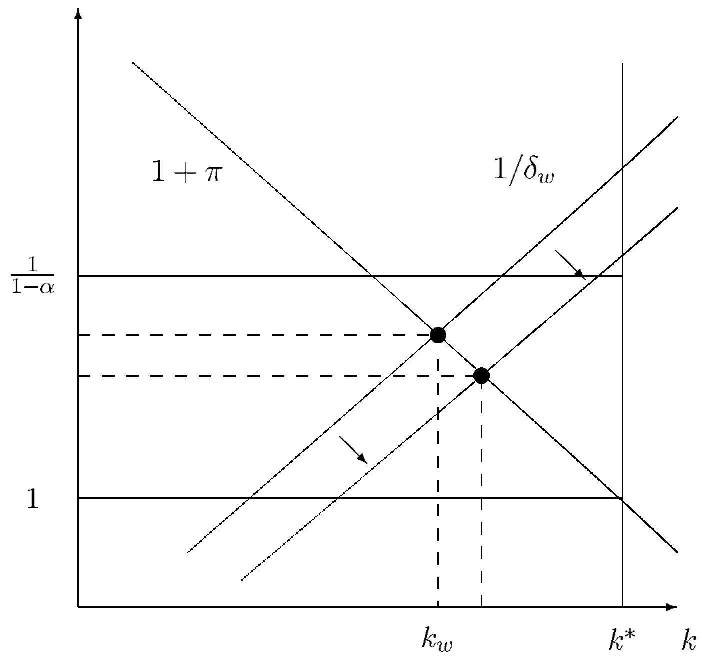

Figure 2: effect of increasing w.

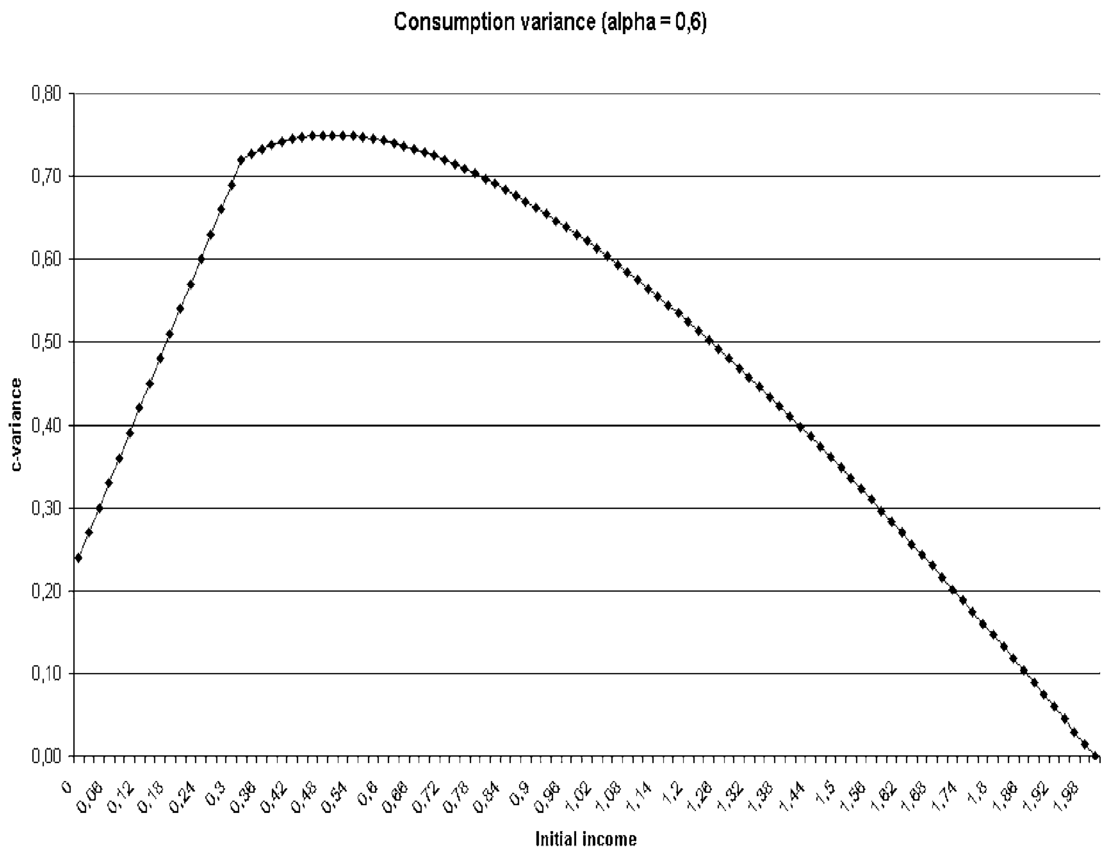

Figure 3: Consumption variance as a function of income.