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Choquet OK?

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# Choquet OK?

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## Abstract

There is a large theoretical literature in both economics and psychology on decision making under ambiguity (as distinct from risk) and many preference functionals proposed in this literature for describing behaviour in such contexts. However, the empirical literature is scarce and largely confined to *testing* between various proposed functionals. Using a new design, in which we create genuine ambiguity in the laboratory and can control the amount of ambiguity, we generate data which enables us to *estimate* several of the proposed preference functionals. In particular, we fit Subjective Expected Utility, Prospect Theory, Choquet Expected Utility, Maximin, Maximax, and Minimum Regret preference functionals, and examine how the fit changes when we vary the ambiguity. We find that the Choquet formulation performs best overall, though it is clear that different decision makers have different functionals. We also identify new decision rules which are not explicitly modelled in the literature.

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## 1. Introduction

The theory of decision-making under ambiguity (or uncertainty), as distinct from that under risk, is well-developed in the literature. An ambiguous (uncertain) situation is when objective probabilities are not specified. Under certain assumptions (see, in particular, Savage 1954), agents act as if they attach subjective additive probabilities to the various events. However, the seminal work of Ellsberg (1961), and many subsequent experiments, have shown that people behave in a way which suggests that they are not able to attach subjective (Savage-type) probabilities to ambiguous or uncertain events. In particular, it seems to be the case that whatever measures of ‘probability’ are used to describe behaviour, they violate the laws of probability and specifically that of additivity. As a consequence, many economists regard the assumptions of the Savage theory as being excessively strong, and the conclusion that subjective probabilities are additive as unrealistic. From the large experimental literature on the Ellsberg paradox a substantial theoretical literature has developed, which relaxes the Savage assumptions and has led to a large number of alternatives to the Subjective Expected Utility (SEU) preference functional proposed by Savage. An early survey can be found in Camerer and Weber (1992), while some more recent articles are cited in Halevy (2007).

The experimental literature is extensive. It can be roughly grouped in three streams: first, experiments reproducing the Ellsberg paradox under various conditions (for example, Sarin and Weber 1993); second, experiments testing particular theories<sup>4</sup> (for example, Tversky and Fox 1995, Di Mauro and Maffioletti 1996); third, experiments reproducing the Ellsberg paradox and testing theories by using real events<sup>5</sup> rather than a random device in order to represent uncertainty. The rationale for this latter line of research is the presupposition that any random device used to represent uncertainty can be reduced to a second order probability (Keppe and Weber 1995, Chow

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<sup>4</sup> A good recent example is Halevy (2007), which tests between Choquet Expected Utility, Maximin Expected Utility, Recursive Non-Expected Utility and Recursive Expected Utility.

<sup>5</sup> For instance, the temperature at noon in Tokyo or San Francisco.

and Sarin 2001, 2002, Abdellaoui, Vossman and Weber, 2005) – the conclusion being that these experiments do not and cannot replicate the Ellsberg paradox in its original and simple form.

The research reported in this paper is located in the second stream outlined above – consisting of work that empirically investigates alternative models to the Savage theory of Subjective Expected Utility theory. Most of existing literature in this stream is devoted to *testing* between various competing models. However, to the best of our knowledge, there is no study which attempts to *estimate* the various proposed preference functionals and determine which fits the best.

This is what this present study does. Moreover, it does so using a unique experimental design in which we create genuine ambiguity in the laboratory, and in which we have different treatments with differing amounts of ambiguity.

The paper is structured as follows. In sections 2 and 3 we describe our method for producing genuine ambiguity in the laboratory and the experimental procedure that we implemented. In section 4 we discuss the various preference functionals that we estimate. The estimates themselves are presented in section 5. Section 6 is devoted to the issue of the additivity of our estimates. Section 7 concludes.

## **2. Creating Ambiguity in the Laboratory**

The key issue when designing an experiment on ambiguity is that of creating ambiguity in the laboratory. The classic Ellsberg method, of simply saying that “there are 100 balls in an urn but the number of black and white is not known”, would be considered too simple given modern experimental standards. Subjects are now trained to be naturally suspicious and expect to be given full information. Moreover, given that experiments in economics nowadays invariably have monetary incentives, and lotteries are played out, there will come a time when the Ellsberg urn has to be shown. *Ex ante* subjects will ask “how was the composition of the balls determined?”. What

does the experimenter say? What, in fact, does he or she do to compose the Ellsberg Urn? If there is some drawing of a random number from the set  $[0,1,\dots,100]$  to determine the number of black balls in the urn, then the situation is no longer ambiguous: it is transformed from true ambiguity to (either objective or subjects' interpretation of) second-order probabilities. Is that what Ellsberg had in mind? Is that an ambiguous situation? Moreover, if the experimenter refuses to answer questions from the subjects, then quite understandably they get suspicious and try and imagine ways of constructing the Ellsberg urn which would minimise the payment from the experimenter.

Ways of generating ambiguity in the laboratory which do not use a random device have other problems. The classic way is to have statements about which the subject may know nothing. For example, "state 1 occurs if the stock exchange index in Accra at 12 noon tomorrow is above 36777". But what happens if you have a Ghanaian in your subjects? Or a world expert on stock exchanges in strange places? There is no longer ambiguity for all the subjects. The experimenter has lost control.

The key issues are: (1) that the experimenter cannot (and cannot be seen to) manipulate the implementation of the ambiguity device; (2) that it is transparent; (3) that probabilities cannot be calculated on an objective basis; and (4) that the existence and amount of ambiguity is not subject-specific.

Our method of creating ambiguity in the laboratory is simple – we use an old-fashioned British Bingo Blower. If you did not grow up in Britain during the 1900's you may not be aware of what this is – but the amusement arcades in all the seaside resorts always had such a Bingo Blower. By the time we realised that this was what we needed for our ambiguity experiments, they were almost defunct and we found one (on eBay) only after a four-year search. A picture does not do justice to it – as it is only when it is in action does it reveal its true merits. In our case, the Bingo Blower is a rectangular-shaped, glass-sided, object some 3 feet high and 2 feet by 2 feet in horizontal section. Inside the glass walls are a set of balls – in *continuous motion* – being moved

about by a jet of wind from a fan in the base. When the time comes to eject one of the balls from the Blower, a transparent tube is rotated and a ball expelled at random up the tube through the pressure of the wind created by the fan. It is all physical, nothing electronic and un-manipulatable. There is no way that the identity of the ball being ejected can be selected – by either the experimenter or the subject. Moreover - and this is crucial to our design - all the balls inside the Blower can at all times be seen by people outside, but - unless the number of balls in the Blower is low - the number of balls of differing colours *can not be counted*: they are continually moving around. Hence the balls in the bingo can be seen but not counted, and the information available is not sufficient to calculate objective probabilities<sup>6</sup>. Moreover, recalling what Ellsberg (1961 p 657) wrote, in addition to the likelihood of the event and the desirability of the payoff, a third dimension is important:

*“...the nature of one’s information concerning the likelihood of events. What is at issue might be called the ambiguity of this information.....giving rise to one’s degree of “confidence” in an estimate of the relative likelihood” Ellsberg (1961) p.657*

We can manipulate this third factor by varying the total number of balls in the Blower while keeping the relative numbers constant.

Finally we note that a further advantage of this way of creating ambiguity in the laboratory is the fact that the information available is the same for all subjects. Hence there is no role for ‘comparative ignorance’ (Fox and Tversky 1995), and hence we can exclude such a factor as a possible explanation of behaviour.

### **3. The Precise Implementation**

We now had to decide the form of the experiment. Rather than getting subjects to *value* lotteries defined on the Bingo Blower, we decided to give them a set of  $n$  pairwise choice questions

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<sup>6</sup> Though, of course, there *are* objective probabilities which the subjects could know if they could count the numbers of balls of different colours – but they cannot (unless the total number of balls in the Blower is small).

between (pairs of) lotteries, and incentivate them by choosing at random, after they had expressed their preference on each of the  $n$  questions, one of the  $n$  questions and playing out their preferred lottery on that question. The reasons for this are that: pairwise choice questions are easier to explain to subjects; easier for them to understand; and less prone to problems of understanding associated with the various mechanisms (such as the Becker-DeGroot-Marschak method) for eliciting valuations.

It was decided that there would be balls of  $m$  different colours in the Blower, and lotteries would be defined in terms of amounts of money and an associated colour. For example, the lottery (£100, pink; £10, blue; -£10, yellow) defines a lottery in which the subject would be paid £100 if the ball ejected was pink, £10 if the ball ejected was blue, and would lose £10 if the ball ejected was yellow. We then had to decide on  $n$  and  $m$ . The value of  $m$  clearly determines the number of possible states: with  $m = 2$  then there are just four states:  $\emptyset$ ,  $a$ ,  $b$ , and  $a \cup b$  (where  $a$  and  $b$  are the two colours); if  $m = 3$  then there are 8 states:  $\emptyset$ ,  $a$ ,  $b$ ,  $c$ ,  $a \cup b$ ,  $a \cup c$ ,  $b \cup c$  and  $a \cup b \cup c$  (where  $a$ ,  $b$  and  $c$  are the three colours). In general, with  $m$  colours, there are  $2^m$  states. Clearly  $m = 2$  is (relatively) uninteresting, while with  $m$  greater than 3 there are more than 15 different states. As with certain of the models that we are going to fit, and in particular the Choquet Expected Utility model, we need to estimate the weights on each of the possible states, we need to keep  $m$  low in order to conserve on degrees of freedom. We chose  $m = 3$  and hence had 3 colours: pink, blue and yellow. We also decided that there would be three amounts of money – three possible prizes –  $x_1$ ,  $x_2$  and  $x_3$ . This implies that we need, in addition, to estimate one utility value – that of  $x_2$  – normalising the other two to 0 and 1 respectively.

We now had to decide on the number  $n$  of pairwise choice questions. With 3 colours and with 3 amounts of money there are  $3^3 = 27$  possible lotteries that can be composed and hence  $27 \times 26 / 2 = 351$  possible different pairwise choice questions. Eliminating those questions in which there is stochastic (first-order) dominance, leaves us with 162 questions. This appears a lot.

However, extensive simulation work before the implementation of the experiment convinced us that we needed this many to get accurate estimates. Moreover, even if we forced subjects to spend 30 seconds answering each question (which we did), the substantive part of the experiment lasted just 81 minutes.

Next we had to decide on the three outcomes:  $x_1$ ,  $x_2$  and  $x_3$ . We chose the numbers in the example above; -£10, £10 and £100. In addition we gave the subjects a participation fee of £10. Thus their take-away earnings at the end of the experiment were either £0, £20 or £110. Such high amounts of money were necessary to get an incentive to take the experiment seriously. For a risk-neutral subject, who knew the correct probabilities, his or her expected earnings from the experiment were £46.63 (plus the participation fee), while if this subject just answered the questions at random his or her expected earnings would be £33.33 (plus the participation fee).

Finally we had to decide the actual numbers of balls in the Bingo Blower. For obvious reasons we did not want the same number of each colour. Moreover, we wanted different treatments in which the amount of ambiguity varied. We chose:

Treatment 1: 2 pink, 5 blue, 3 yellow

Treatment 2: 4 pink, 10 blue, 6 yellow

Treatment 3: 8 pink, 20 blue, 12 yellow.

In Treatment 1, it is actually possible to count the balls of each colour – so this is a situation of risk. In Treatment 2, it is just about possible to count the number of pink balls and begin to guess the number of yellow balls but it is impossible to count the number of blue balls. In Treatment 3 it is impossible to count the balls of any colour. We would say that the amount of ambiguity increases as we go through the treatments: it is effectively zero in Treatment 1, positive in Treatments 2 and 3 but higher in Treatment 3.

This completes the description of the design. We recruited 48 subjects – 15 on Treatment 1, 17 on Treatment 2 and 16 on Treatment 3. As we have already noted, this was an expensive



experiment and we paid out a total of £2130 – equal to £44.37 per subject<sup>7</sup>. Subjects were recruited using the ORSEE (Greiner 2004) software and the experiment was conducted in the EXEC laboratory at the University of York. In the laboratory the Bingo Blower was on display, in action, in the middle of the room, throughout the whole of the experimental session. In addition, images of the Blower were projected via a video camera onto two big screens in the lab. Subjects were free at any stage to go closer to the Blower to examine it as much as they wanted. At the beginning of the experiment, subjects were taken into the laboratory and given written Instructions (available in the Appendix). They were then allowed to turn to the computer, which repeated the Instructions. The experimenter then responded to any questions, and the subjects were allowed to begin answering the 162 pairwise choice questions<sup>8</sup>. The software was designed so that they had to spend a *minimum* of 30 seconds before they could move on to the next question (though they could take more time if they wanted). When they had answered all 162 questions, they called over an experimenter and drew a numbered ticket from a box containing tickets numbered from 1 to 162. The computer then recalled their answer to that question. At that point the subject and the experimenter went over to the Bingo Blower and expelled one ball. The colour of the ball, the question picked at random and their answer to that question determined their payment. They filled in a brief questionnaire, were paid, signed a receipt and were free to go.

#### **4. The Preference Functionals Estimated**

This section describes the various preference functionals that we fitted to the data. We note that none of these functionals involve the use of second order probabilities<sup>9</sup>. There are two reasons

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<sup>7</sup> It is interesting to note that this is considerably larger than the expected payment if subjects just answered randomly – suggesting that they did not. We shall test this formally later.

<sup>8</sup> The order of the questions and the left-right juxtaposition of the two lotteries on the screen were randomised.

<sup>9</sup> There is a good discussion of such models in Halevy (2007). Key references are: Gilboa and Schmeidler (1989) who introduced Maxmin Expected Utility theory; Segal (1987), who introduced Recursive Non-Expected Utility theory; and Klibanoff *et al* (2005) and others who explored Recursive Expected Utility theory.

for this. First, that the experimental setting does not invoke the use of such second order probabilities. Second, and perhaps more importantly, even if the subjects were using some second-order probability rule, we do not have enough data to estimate the various second-order distributions<sup>10</sup>. Moreover, none of these functionals include any reference to ‘comparative ignorance’ (Fox and Tversky 1995), because all subjects were equally informed.

As we have already noted there were three possible outcomes in the experiment ( $x_1$ ,  $x_2$  and  $x_3$ ). We normalise the highest to have a utility of 1 and the lowest to have a utility of 0; we denote the utility of the middle outcome by  $u$ . We denote the three colours by  $a$ ,  $b$  and  $c$ . A lottery can be denoted by

$$L = (x_1, S_1; x_2, S_2; x_3, S_3) \quad (1)$$

Here  $S_i$  is the state (one of  $\emptyset$ ,  $a$ ,  $b$ ,  $c$ ,  $a \cup b$ ,  $a \cup c$ ,  $b \cup c$  and  $a \cup b \cup c$ ) in which the lottery pays out  $x_i$ . We now describe the various preference functionals estimated.

#### ***A. Subjective Expected Utility theory (SEU)***

In this subjects choose between lotteries on the basis of their expected utility, calculated on the basis of the subject’s subjective probabilities attached to the various states. The expected utility of the lottery  $L$  is given by

$$EU(L) = p_2u + p_3 \quad (2)$$

where  $p_i$  is the (subjective) probability of state  $i$ . If we use the notation that  $p_i = P(S_i)$  where  $S_i$  denotes the state in which the lottery pays out  $x_i$ , then we have

$$\begin{aligned} P(\emptyset) &= 0 \\ P(a) &= p_a \quad P(b) = p_b \quad P(c) = p_c \\ P(a \cup b) &= p_a + p_b \quad P(a \cup c) = p_a + p_c \quad P(b \cup c) = p_b + p_c \\ P(a \cup b \cup c) &= p_a + p_b + p_c = 1 \end{aligned} \quad (3)$$

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<sup>10</sup> Indeed one could argue that many of these second-order models simply are not identifiable.

where  $p_a$ ,  $p_b$  and  $p_c$  are the subject's subjective probabilities for the three colours. In this model we estimate  $u$ ,  $p_a$ ,  $p_b$  and  $p_c$  (subject to the constraint that  $p_a + p_b + p_c = 1$ ).

### **B. Prospect Theory (PT)**

This is a preference functional 'between' that of SEU and the Choquet Expected Utility functional. We should say at the outset that we are hesitant about the acceptability of this term being used in this context, but it seems appropriate. Prospect Theory (see Kahneman and Tversky 1979) envisages utilities being weighted by some function of the 'true' probabilities. If there are true probabilities of the various colours  $\pi_a$ ,  $\pi_b$  and  $\pi_c$  then Prospect Theory envisages them being replaced by  $f(\pi_a)$ ,  $f(\pi_b)$  and  $f(\pi_c)$ . If we denote these respectively by  $p_a$ ,  $p_b$  and  $p_c$  then we get this specification. It is precisely the same as the Expected Utility preference functional except for the fact that the 'probabilities' are not additive. In this model we estimate  $u$ ,  $p_a$ ,  $p_b$  and  $p_c$  (but no longer subject to the constraint that  $p_a + p_b + p_c = 1$ ). We note that this preference functional may not satisfy dominance (though it does so in this context), unlike the Choquet preference functional, which does. It could also be interpreted as a model in which the decision-maker has several possible probabilities for each of the three colours, and works with the *minimum probability* for each colour.

### **C. Choquet Expected Utility theory (CEU)**

Here the Choquet Expected Utility of the lottery  $L$  is given by

$$CEU(L) = w_{23}u + w_3(1-u) = (w_{23} - w_3)u + w_3 \quad (4)$$

where  $w_i$  is the Choquet capacity (or weight<sup>11</sup>) of state  $i$ . If we use the notation that  $w_i = W(S_i)$  where  $S_i$  denotes the state in which the lottery pays out  $x_i$ , then we have, in order to satisfy the Choquet conditions, that

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<sup>11</sup> We borrow this term from Rank Dependent Expected Utility theory, which has strong affinities with Choquet Expected Utility theory.

$$\begin{aligned}
W(\emptyset) &= 0 \\
W(a) &= w_a \quad W(b) = w_b \quad W(c) = w_c \\
W(a \cup b) &= w_{ab} \quad W(a \cup c) = w_{ac} \quad W(b \cup c) = w_{bc} \\
W(a \cup b \cup c) &= 1
\end{aligned} \tag{5}$$

Here  $w_a, w_b, w_c, w_{ab}, w_{ac}$  and  $w_{bc}$  are the subject's Choquet capacities (or weights) for the various possible states. In this model we estimate  $w, w_a, w_b, w_c, w_{ab}, w_{ac}$  and  $w_{bc}$ . Note that there is no necessity that  $w_{de} = w_d + w_e$  for any  $d$  or  $e$ . That is, there is no necessity that the weights are additive (probabilities). Indeed this is the main difference between Expected Utility theory and Choquet Expected Utility theory. In our estimates we will be particularly interested in the degree of additivity of the estimated weights.

#### **D. Maximin**

In this, the decision maker is presumed to follow the rule of choosing the lottery for which the worst outcome is the best. We assume that the rule is followed lexicographically, so that we get the following rule, where  $l_1, l_2$  and  $l_3$  denote the three outcomes on one of the two lotteries,  $L$ , ordered from the worst to the best, and  $m_1, m_2$  and  $m_3$  denote the outcomes on the other lottery,  $M$ , also ordered from the worst to the best:

$$\begin{aligned}
&\text{if } l_1 > m_1 \text{ then } L \succ M \\
&\text{if } l_1 < m_1 \text{ then } L \prec M \\
&\text{if } l_1 = m_1 \text{ and } l_2 > m_2 \text{ then } L \succ M \\
&\text{if } l_1 = m_1 \text{ and } l_2 < m_2 \text{ then } L \prec M \\
&\text{if } l_1 = m_1, l_2 = m_2 \text{ and } l_3 > m_3 \text{ then } L \succ M \\
&\text{if } l_1 = m_1, l_2 = m_2 \text{ and } l_3 < m_3 \text{ then } L \prec M \\
&\text{if } l_1 = m_1, l_2 = m_2 \text{ and } l_3 = m_3 \text{ then } L \sim M
\end{aligned} \tag{6}$$

We note that there are no parameters to be estimated in this model, though we do assume that the decision maker ranks £100 as the best outcome, £10 as the second best and -£10 as the worst.

### ***E. Maximax***

In this the decision maker is presumed to follow the rule of choosing the lottery for which the best outcome is the best. We assume that the rule is followed lexicographically, so that we get the following rule, using the same notation as above:

$$\begin{aligned} &\text{if } l_3 > m_3 \text{ then } L \succ M \\ &\text{if } l_3 < m_3 \text{ then } L \prec M \\ &\text{if } l_3 = m_3 \text{ and } l_2 > m_2 \text{ then } L \succ M \\ &\text{if } l_3 = m_3 \text{ and } l_2 < m_2 \text{ then } L \prec M \\ &\text{if } l_3 = m_3, l_2 = m_2 \text{ and } l_1 > m_1 \text{ then } L \succ M \\ &\text{if } l_3 = m_3, l_2 = m_2 \text{ and } l_1 < m_1 \text{ then } L \prec M \\ &\text{if } l_3 = m_3, l_2 = m_2 \text{ and } l_1 = m_1 \text{ then } L \sim M \end{aligned} \tag{7}$$

Again there are no parameters to estimate.

### ***F. Minimax Regret***<sup>12</sup>

With this preference functional, the decision maker is envisaged as imagining each possible ball drawn, calculating the regret associated with choosing each of the two lotteries, and choosing the lottery for which the maximum regret is minimized. Again there are no parameters to estimate, though it is assumed that there is a larger regret associated with a larger difference between the outcome on the chosen lottery and the outcome on the non-chosen lottery.

## **5. The Estimated Preference Functionals**

Estimation was carried out using maximum likelihood and implemented with GAUSS. We made the following assumptions about the stochastic specification. For the SEU, CEU and PT preference functionals we assumed a Fechnerian error story. That is, we assumed that the preference functionals were measured with error, such that the difference in the evaluations of the two lotteries

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<sup>12</sup> See Luce and Raiffa (1957).

was normally distributed with mean 0 and standard deviation  $s$ . We estimate the parameter  $s$  along with the other parameters. For the remaining preference functionals, where the implementation is lexicographic, we adopted a *tremble* model (see Loomes *et al* 2002): that is, we assumed that the subject implemented the desired choice with probability  $(1-w)$  and chose at random with probability  $w$ . We estimate the parameter  $w$  along with the other parameters.

First, we compare the fitted preference functionals in terms of their log-likelihoods. Table 1 reports these log-likelihoods<sup>13</sup>. The asterisk indicates the model with the highest log-likelihood. It should, however, be noted that this does not take into account the different degrees of freedom of the various functional forms. We have the following numbers of estimated parameters: SEU 4; PT 5; CEU 8; Maximin 1; Maximax 2; Minimum Regret 1. The SEU model is nested within the PT model and that, in turn, is nested within the CEU model. We can therefore carry out standard likelihood ratio tests comparing these three models. The results are in Table 2. We also carry out Vuong tests (see Vuong 1989) comparing the non-nested models. This considers two models and tests the null hypothesis that the two models are equally close to the ‘true model’. The test statistic has approximately a unit normal distribution. Positive significance indicates that the first model is closer to the true model while negative significance indicates that the second is closer to the true model. Combining the results from Tables 2 and 3 we get Table 4 which indicates the ‘best-fitting model’ in each case. We adopted a lexicographic process to arrive at this table. If, on the log-likelihood ratio tests CEU fitted significantly better than SEU and PT, and on the Vuong tests CEU was closer to the true model than Maximin, Maximax and Minimax Regret, then we declared CEU the best model. If, however, CEU was not significantly better than SEU on the log-likelihood ratio tests but SEU was better than PT while SEU was closer to the true model than Maximin, Maximax and Minimax Regret, then we declared SEU the best model. And so on.

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<sup>13</sup> We note that all the log-likelihoods reject the null hypothesis that the subjects are responding at random.

Several subjects are interesting. Subject 25 is clearly risk-neutral (the estimated value of  $u$  is 0.1) and is working with what he or she thinks are the correct probabilities (but which are not). Subjects 35 and 36 have values  $1/3$  for the weights on the colours individually and  $2/3$  on the various pairwise combinations and an estimated value for  $u$  equal to  $0.5$ <sup>14</sup>. These subjects, while having behaviour consistent with CEU, are obviously using a simple rule of thumb: for each lottery they give 0 points for any outcome of  $-\pounds 10$ , 1 point for an outcome of  $\pounds 10$ , and 2 points to an outcome of  $\pounds 100$ . They then add these points and chose the lottery for which the sum of the points is the largest. Subject 41, while apparently using Maximax, can also be interpreted (since he or she has values  $1/3$  for the weights on the colours individually and  $2/3$  on the various pairwise combinations and an estimated value for  $u$  equal to  $1/3$ ) as following a similar rule but with 3 points for an outcome of  $\pounds 100$ . Such rules of thumb do not, to the best of our knowledge, appear in the literature. Classifying these subjects as using Rules of Thumb and summarising Table 4 we get the following bottom line:

<b>Model(s)</b>	<b>Treatment 1</b>	<b>Treatment 2</b>	<b>Treatment 3</b>	<b>TOTALS</b>
<b>SEU</b>	4	4	4	12
<b>PT</b>	2	6	3	11
<b>CEU</b>	9	5	5	19
<b>Maximin</b>	0	1	0	1
<b>Maximax</b>	0	0	1	1
<b>Minimax Regret</b>	0	0	0	0
<b>Rule of Thumb</b>	0	1	3	4
<b>TOTALS</b>	15	17	16	48

## 6. Additivity

In the previous literature, there were no direct estimates of the parameters of the CEU preference functional, and hence attitude to ambiguity was measured simply through capacities not being additive. However, since we are able to directly estimate the parameters of the CEU

<sup>14</sup> The estimated parameters are in an Appendix, available on request.

preference functional, we can explore our results in terms of measures of additivity. There are various measures that one can calculate. For the CEU estimates to be consistent with SEU we need the following conditions (obviously not independent of each other) to hold:

$$\begin{aligned}
 S_1 &= w_a + w_b + w_c = 1 \\
 S_2 &= (w_{bc} + w_{ac} + w_{ac}) / 2 = 1 \\
 S_3 &= (w_a + w_{bc}) = 1 \\
 S_4 &= (w_b + w_{ac}) = 1 \\
 S_5 &= (w_c + w_{ab}) = 1
 \end{aligned} \tag{8}$$

We define  $S_1$  through  $S_5$  as our additivity measures. All should equal 1 for SEU to hold. Figure 1 shows a scatter of  $S_2$  against  $S_1$  by treatment. Table 5 reports the means and standard deviations of all five measures by treatment. Interestingly the measures for Treatment 1 (effectively a case of risk rather than ambiguity) are all a little (but not significantly) over 1.0, and average 1.0322. In Treatment 2, a case of ambiguity, they are on average (1.0646) a little higher, while in Treatment 3, with higher ambiguity, they are all well below 1, averaging 0.9348. In this latter case there is clear sub-additivity – as the theorists suspect.

We can also look at the individual attitudes towards ambiguity as measured through these additivity measures. In table 6 we show for each treatment and each definition of additivity ( $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$  or  $S_5$ ) the percentages of subjects who are ambiguity-loving or ambiguity-averse<sup>15</sup>. It emerges that there are 29 ambiguity-averse subjects ( $S_1 < 1$ ) and 19 ambiguity-loving subjects ( $S_1 > 1$ ) using the usual definition of additivity. As we can also see from the same tables there is a slight increase in the number of subjects who change from ambiguity-aversion to ambiguity-loving as the number of balls contained in the bingo increases – that is, as the ambiguity increases. It can also be noted that the different measures of additivity, which depend on different partitions of the same event space are not the same. This latter result is consistent with Support Theory (Tversky and

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<sup>15</sup> We define as ambiguity-neutral a subject whose sum of probability of disjoint events is equal to 1, as ambiguity-averse a subject whose sum of probability of disjoint events is less than 1 and as ambiguity-loving a subject whose sum of probability of disjoint events is more than 1. We note that none of our subjects, using the parameter estimates for the CEU functional, were exactly ambiguity-neutral.



Koehler 1994) and other psychological theories (Fox and Rottenstreich 2003), that allow for judgemental probability or individual belief to depend on the description of the events.

Figure 2 shows further interesting evidence: in all graphs  $S_I$  is on the horizontal axis; in 2.1  $S_3$  is on the vertical axis; in 2.2  $S_4$  is on the vertical axis; in 2.3  $S_5$  is on the vertical axis; the sloping line is the 45° line. There are separate scatters in each figure for the different treatments. One thing seems clear: as a general rule we see that  $S_3 > S_I$ ,  $S_4 > S_I$  and  $S_5 > S_I$  when the measures indicate *sub-additivity* and  $S_3 < S_I$ ,  $S_4 < S_I$  and  $S_5 < S_I$  when the measures indicate *super-additivity*.

It is instructive to examine these latter results in more detail. If  $S_i > S_I$  for  $i > 2$ , it follows that  $w_{ab} > w_a + w_b$  for some pair of colours  $a$  and  $b$ . So the weight on the union of  $a$  and  $b$  is greater than the sum of the weights on  $a$  and  $b$  separately. We see that this generally occurs when  $S_I < 1$ , that is, when the subject displays ambiguity-aversion. These subjects, according to our estimates, generally have a weight on the union of two colours greater than the sum of the weights on the same two colours individually. This seems to accord with intuition concerning the meaning of ambiguity-aversion. In contrast, for those subjects who display ambiguity-loving, their weight on the union is less than the sum of the weights individually. These findings seem to make good sense.

We had hoped to see some differences in the degree of additivity across the three treatments, but it is difficult to see much difference between the various scatters. It should, however, be recalled that we only have a total of 48 observations.

## 7. Conclusions

In this paper we have provided what we believe are the first estimates of preference functionals under ambiguity using data from an experiment in which there is genuine ambiguity in the experimental task. Moreover, we have different treatments with different degrees of ambiguity. Our results show that some 25% of the subjects act in accordance with Subjective Expected Utility

theory, and hence attach (additive) subjective probabilities to the various states. Some 23% of the subjects use (our interpretation of) Prospect Theory (in its original form) and hence attach non-additive subjective probabilities to the various simple states. A further 38% of the subjects seem to use Choquet Expected Utility and hence attach non-additive capacities (or weights) to the various states (both simple and compound). A few use Maximin, Maximax and simple Rules of Thumb. Indeed we have identified a new Rule of Thumb which is easy to implement in this context.

We discover a range of attitudes towards ambiguity, with both ambiguity-averse and ambiguity-loving subjects in our sample. Moreover, those subjects who are ambiguity-averse (loving) seem to have a weight on the union of two events greater (less) than the sum of the weights attached to the individual events.

What we failed to find clear evidence on was the effect of increasing ambiguity on the preference functionals used. Treatment 1 was essentially a case of risk while Treatments 2 and 3 showed ambiguity, with more ambiguity in the latter. However, there is no obvious difference between behaviour in the three treatments, except for a slight tendency for more Rules of Thumb to be used when there is an increase in ambiguity. This could be a result of the small numbers of subjects in our sample. Further work is required to explore further the implications.

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**Table 1: The Log-Likelihoods of the various functional forms (\* indicates the highest LL)**

subject	SEU	PT	CEU	Maximin	Maximax	Minimax Regret
1	-16.3936	-13.5371	-3.8593*	-37.5349	-88.2474	-88.2474
2	-31.9309	-26.3077	-25.0640*	-77.6248	-52.2226	-58.5595
3	-26.1971	-22.3794	-16.1582*	-80.5076	-80.5076	-87.0474
4	-24.0772	-24.0573	-14.2056*	-79.0862	-45.2583	-56.5108
5	-6.2671	-6.2671	-3.9524*	-56.5108	-98.4463	-98.4463
6	-17.1767	-12.8252	-8.9046*	-10.7765	-74.5783	-72.9909
7	-37.9959	-31.0934	-29.8623*	-64.3577	-47.6564	-58.5595
8	-27.4427	-22.0363	-21.0722*	-79.0862	-84.5417	-90.5449
9	-26.5198	-16.1746	-12.1479*	-71.3590	-90.5449	-92.7095
10	-33.3195	-32.7294	-30.5403*	-60.5485	-77.6248	-79.0862
11	-33.2107	-25.7538	-23.5967*	-58.5595	-76.1226	-81.8901
12	-41.7651	-36.8833	-32.5486*	-64.3577	-85.8124	-91.6435
13	-16.7974	-16.7974	-15.9764*	-58.5595	-99.2966	-98.4463
14	-30.4775	-25.9931	-20.2295*	-87.0474	-69.6813	-76.1226
15	-30.2564	-29.8099	-25.8302*	-40.2045	-89.4130	-89.4130
16	-69.0147	-68.5726	-65.6452*	-104.4464	-96.6566	-97.5665
17	-17.9054	-12.7410	-10.8748*	-52.2226	-80.5076	-83.2345
18	-29.5882	-17.7773	-16.0873*	-76.1226	-72.9909	-79.0862
19	-21.6397	-21.6224	-17.7990*	-52.2226	-85.8124	-87.0474
20	-24.6320	-24.3416	-23.0134*	-40.2045	-84.5417	-85.8124
21	-47.0512	-41.5665	-35.7003*	-67.9563	-80.5076	-87.0474
22	-53.7313	-50.0000	-49.2475*	-74.5783	-74.5783	-81.8901
23	-28.3288	-28.0443	-27.4709	-22.3128*	-74.5783	-72.9909
24	-54.6523	-49.7872	-44.4599*	-74.5783	-76.1226	-83.2345
25	0.0000*	0.0000*	0.0000*	-96.6566	-52.2226	-64.3577
26	-22.4209	-22.3374	-21.5572*	-49.9763	-79.0862	-80.5076
27	-35.5261	-32.8166	-29.2171*	-66.1824	-93.7433	-95.7164
28	-32.1557	-30.1588	-28.9741*	-64.3577	-98.4463	-97.5665
29	-35.8260	-33.0205	-27.1318*	-91.6435	-79.0862	-84.5417
30	-53.1288	-52.8768	-48.1509*	-79.0862	-56.5108	-66.1824
31	-31.2604	-27.8191	-26.6412*	-67.9563	-85.8124	-88.2474
32	-37.3533	-20.4654	-19.4992*	-69.6813	-58.5595	-67.9563
33	-22.3461	-22.0537	-17.0730*	-64.3577	-80.5076	-83.2345
34	-29.7881	-27.3112	-23.9688*	-34.7586	-84.5417	-85.8124
35	-28.7992	-28.7763	-23.0808*	-60.5485	-37.5349	-52.2226
36	-36.0849	-32.2506	-22.4238*	-37.5349	-62.4805	-62.4805
37	-27.4019	-21.4020	-18.6183*	-54.3996	-93.7433	-92.7095
38	-25.3586	-25.3055	-24.2999*	-88.2474	-96.6566	-98.4463
39	-89.0304	-21.3118	-20.5653	-77.6248	0.0000*	-18.7554
40	-13.8012	-13.8012	-12.2077*	-18.7554	-83.2345	-83.2345
41	-6.6484	-6.4681	-3.9830	-77.6248	0.0000*	-22.3128
42	-26.9787	-16.8200	-15.9348*	-67.9563	-90.5449	-92.7095
43	-33.4483	-13.2807	-12.3331*	-56.5108	-42.7764	-52.2226
44	-25.0722	-25.0475	-24.7430*	-91.6435	-54.3996	-64.3577
45	-4.3845	-4.3819	-3.9005*	-54.3996	-97.5665	-97.5665
46	-39.8947	-37.8061	-37.3866*	-85.8124	-67.9563	-74.5783
47	-26.7598	-20.5777	-18.1830*	-45.2583	-72.9909	-79.0862
48	-22.6738	-22.2003	-16.8821*	-87.0474	-89.4130	-93.7433

Note: Treatment 1, subjects 1 to 15; Treatment 2, subjects 16 to 32; Treatment 3, subjects 33 to 48.

**Table 2 : Log-Likelihood Ratio Tests of the Nested Models (significant at \*\*1%, \*5%)**

subject	CEU v SEU	CEU v PT	PT v SEU
1	25.0686**	19.3556**	5.7130*
2	13.7338**	2.4874	11.2464**
3	20.0778**	12.4424**	7.6354**
4	19.7432**	19.7034**	0.0398
5	4.6294	4.6294*	0.0000
6	16.5442**	7.8412*	8.7030**
7	16.2672**	2.4622	13.8050**
8	12.7410*	1.9282	10.8128**
9	28.7438**	8.0534*	20.6904**
10	5.5584	4.3782*	1.1802
11	19.2280**	4.3142*	14.9138**
12	18.4330**	8.6694*	9.7636**
13	1.6420	1.6420	0.0000
14	20.4960**	11.5272**	8.9688**
15	8.8524	7.9594*	0.8930
16	6.7390	5.8548*	0.8842
17	14.0612**	3.7324	10.3288**
18	27.0018**	3.3800	23.6218**
19	7.6814	7.6468*	0.0346
20	3.2372	2.6564	0.5808
21	22.7018**	11.7324**	10.9694**
22	8.9676	1.5050	7.4626**
23	1.7158	1.1468	0.5690
24	20.3848**	10.6546*	9.7302**
25	0.0000	0.0000	0.0000
26	1.7274	1.5604	0.1670
27	12.6180*	7.1990*	5.4190*
28	6.3632	2.3694	3.9938*
29	17.3884**	11.7774**	5.6110*
30	9.9558*	9.4518*	0.5040
31	9.2384	2.3558	6.8826**
32	35.7082**	1.9324	33.7758**
33	10.5462*	9.9614*	0.5848
34	11.6386*	6.6848*	4.9538*
35	11.4368*	11.3910**	0.0458
36	27.3222**	19.6536**	7.6686**
37	17.5672**	5.5674*	11.9998**
38	2.1174	2.0112	0.1062
39	136.9302**	1.4930	135.4372**
40	3.1870	3.1870	0.0000
41	5.3308	4.9702*	0.3606
42	22.0878**	1.7704	20.3174**
43	42.2304**	1.8952	40.3352**
44	0.6584	0.6090	0.0494
45	0.9680	0.9628	0.0052
46	5.0162	0.8390	4.1772*
47	17.1536**	4.7894*	12.3642**
48	11.5834*	10.6364*	0.9470

Note: Treatment 1, subjects 1 to 15; Treatment 2, subjects 16 to 32; Treatment 3, subjects 33 to 48.

**Table 3: Vuong Tests between the non-nested models (significant at: \*1%; !5%)**

subject	SEU versus			PT versus			CEU versus			MaxiMax versus		Maximin v
	MaxiMax	Maximin	MiniMax Regret	MaxiMax	Maximin	MiniMax Regret	MaxiMax	Maximin	MiniMax Regret	Maximin	MiniMax Regret	MiniMax Regret
1	8.38*	1.58	8.36*	8.64*	1.67	8.64*	10.32*	1.97!	10.32*	-6.18*	0.00	6.18*
2	1.69	4.45*	2.46!	2.04!	5.09*	2.76*	1.19	4.28*	1.93	2.66*	1.36	-2.09!
3	5.95*	6.12*	7.04*	5.81*	5.68*	6.84*	5.88*	5.82*	7.03*	0.00	1.89	0.90
4	1.54	5.63*	2.79*	1.25	5.34*	2.51!	1.58	5.88*	2.92*	3.54*	1.95	-2.41!
5	15.21*	5.13*	15.21*	14.75*	4.83*	14.75*	14.67*	4.24*	14.67*	-6.00*	0.00	6.00*
6	4.75*	-2.56*	4.56*	5.63*	-2.64*	5.42*	5.80*	-3.22*	5.56*	-6.40*	-0.45	6.19*
7	-0.73	1.58	1.28	-0.08	1.99!	1.58	-1.69	-0.01	-0.42	1.63	2.77*	-0.40
8	6.07*	5.96*	7.15*	6.62*	6.27*	7.77*	5.83*	5.50*	6.95*	-0.73	1.89	1.62
9	6.59*	4.58*	6.97*	8.19*	5.39*	8.64*	8.71*	5.12*	9.28*	-2.60*	0.81	2.98*
10	4.24*	2.51!	4.43*	3.93*	2.22!	4.11*	3.34*	1.60	3.52*	-1.86	0.45	2.09!
11	4.54*	2.52!	5.57*	5.19*	2.78*	6.16*	4.45*	2.06!	5.37*	-1.86	1.65	2.64*
12	4.31*	2.10!	5.49*	4.61*	2.19!	5.73*	4.33*	1.79	5.40*	-2.62*	1.88	3.55*
13	10.83*	3.79*	10.57*	10.46*	3.51*	10.21*	9.66*	2.80*	9.42*	-5.91*	-0.45	5.82*
14	3.97*	6.38*	4.88*	4.15*	6.44*	5.10*	3.77*	6.27*	4.66*	2.23!	1.65	-1.47
15	6.45*	0.30	6.71*	6.21*	0.03	6.45*	5.69*	-0.41	5.99*	-6.10*	-0.00	6.10*
16	2.06!	4.03*	2.20!	2.24!	3.30*	2.34!	1.55	2.63*	1.66	1.77	0.45	-1.62
17	7.30*	3.43*	7.86*	7.75*	3.53*	8.30*	6.93*	2.82*	7.48*	-3.07*	0.82	3.49*
18	4.51*	5.08*	5.40*	5.48*	5.83*	6.39*	4.75*	5.06*	5.66*	0.36	1.65	0.37
19	8.18*	3.15*	8.47*	7.79*	2.81*	8.08*	7.27*	2.24!	7.53*	-3.93*	0.45	4.23*
20	6.54*	1.04	6.91*	6.33*	0.74	6.68*	5.58*	-0.08	5.83*	-5.02*	0.38	5.17*
21	3.14*	1.93	4.23*	3.53*	2.20!	4.64*	3.39*	1.91	4.50*	-1.48	1.89	2.38!
22	1.61	1.67	2.58*	1.71	1.74	2.64*	0.89	0.91	1.79	0.00	1.91	0.90
23	4.20*	-2.24!	3.97*	4.08*	-2.69*	3.83*	3.33*	-3.79*	3.08*	-5.17*	-0.45	4.99*
24	2.05!	1.62	3.23*	2.33!	1.84	3.49*	1.81	1.49	2.85*	-0.18	1.91	1.10
25	5.31*	16.77*	7.03*	5.01*	16.29*	6.72*	4.10*	14.85*	5.77*	6.17*	2.56*	-4.58*
26	6.33*	2.64*	6.57*	6.01*	2.30!	6.25*	5.22*	1.42	5.45*	-3.08*	0.45	3.36*
27	6.80*	2.80*	7.23*	6.56*	2.65*	6.93*	6.06*	2.16!	6.42*	-3.79*	0.81	4.17*
28	6.77*	2.57*	6.62*	6.46*	2.38!	6.30*	5.92*	1.78	5.75*	-4.96*	-0.45	4.86*
29	4.91*	6.42*	5.74*	5.01*	6.46*	5.87*	4.75*	6.82*	5.62*	1.83	1.64	-1.10
30	-0.54	1.81	0.66	-0.80	1.54	0.37	-1.18	1.32	0.03	2.49!	2.31!	-1.53
31	5.86*	3.61*	6.30*	5.96*	3.57*	6.36*	5.07*	2.79*	5.46*	-2.23!	0.82	2.61*
32	1.84	3.06*	3.30*	3.46*	4.97*	4.83*	2.61*	4.10*	3.95*	1.10	2.30!	-0.19
33	5.85*	3.81*	6.39*	5.64*	3.56*	6.16*	5.52*	3.31*	6.01*	-1.85	0.82	2.24!
34	5.73*	-0.37	6.01*	6.09*	-0.41	6.54*	5.52*	-1.03	5.87*	-5.68*	0.45	6.12*
35	0.14	2.62*	2.01!	-0.18	2.35!	1.68	-0.43	2.18!	1.38	1.88	2.62*	-0.73
36	2.07!	-0.83	2.15!	2.24!	-0.60	2.28!	-5.57*	-5.70*	-5.57*	-2.12!	-0.00	2.12!
37	7.54*	2.28!	7.27*	8.73*	2.79*	8.50*	8.45*	2.19!	8.20*	-5.22*	-0.45	5.09*
38	9.31*	7.81*	9.89*	9.02*	7.52*	9.59*	8.13*	6.55*	8.65*	-1.46	0.81	1.81
39	-53.30*	-2.34!	-10.30*	-16.82*	5.56*	-1.67	-10.08*	4.40*	-2.40!	10.60*	2.58*	-6.51*
40	8.35*	-0.48	8.36*	8.01*	-0.93	8.01*	7.15*	-1.88	7.16*	-7.76*	0.00	7.76*
41	-4.90*	7.81*	0.98	-5.48*	7.48*	0.69	-12.77*	7.28*	0.07	10.60*	2.94*	-5.83*
42	7.44*	4.47*	7.91*	8.86*	5.06*	9.35*	7.91*	4.27*	8.38*	-3.00*	0.81	3.38*
43	0.23	1.78	1.49	2.03!	3.49*	3.02*	1.34	2.80*	2.33!	1.11	2.08!	-0.38
44	2.83*	8.04*	4.19*	2.50!	7.72*	3.86*	1.55	6.70*	2.90*	4.78*	2.32!	-3.67*
45	15.24*	5.02*	15.25*	14.79*	4.72*	14.80*	13.87*	3.91*	13.87*	-6.08*	0.00	6.08*
46	2.32!	4.43*	3.88*	2.21!	4.27*	3.73*	1.23	3.46*	2.70*	2.36!	2.22!	-1.10
47	5.07*	1.52	6.39*	5.66*	1.89	6.87*	4.97*	1.19	6.08*	-2.68*	1.65	3.54*
48	8.67*	7.72*	10.25*	8.28*	7.27*	9.82*	8.18*	7.28*	9.80*	-0.36	1.62	1.09

**Table 4: The Best Fitting Functionals**

<b>Treatment 1</b>		<b>Treatment 2</b>		<b>Treatment 3</b>	
<i>Subject</i>	<i>Model</i>	<i>Subject</i>	<i>Model</i>	<i>Subject</i>	<i>Model</i>
1	CEU	16	SEU	33	CEU
2	PT	17	PT	34	CEU
3	CEU	18	PT	35	CEU*
4	CEU	19	SEU	36	CEU*
5	SEU	20	SEU	37	CEU
6	CEU	21	CEU	38	SEU
7	CEU	22	PT	39	Maximax
8	PT	23	Maximin	40	SEU
9	CEU	24	CEU	41	Maximax*
10	SEU	25	SEU	42	PT
11	CEU	26	SEU	43	PT
12	CEU	27	CEU	44	SEU
13	SEU	28	PT	45	SEU
14	CEU	29	CEU	46	PT
15	SEU	30	CEU	47	CEU
		31	PT	48	CEU
		32	PT		

\*The subjects indicated with an asterisk can be interpreted as following some other rule (see the text).



**Table 5: Means and Standard Deviations of Additivity Measures by Treatment**

Treatment	means					<i>standard deviations</i>				
	S1	S2	S3	S4	S5	<i>S1</i>	<i>S2</i>	<i>S3</i>	<i>S4</i>	<i>S5</i>
1	1.021	1.040	1.008	1.059	1.033	<i>0.718</i>	<i>0.275</i>	<i>0.402</i>	<i>0.416</i>	<i>0.392</i>
2	0.988	1.115	1.046	1.104	1.070	<i>0.286</i>	<i>0.124</i>	<i>0.137</i>	<i>0.173</i>	<i>0.173</i>
3	0.926	0.941	0.964	0.918	0.925	<i>0.385</i>	<i>0.233</i>	<i>0.196</i>	<i>0.281</i>	<i>0.271</i>

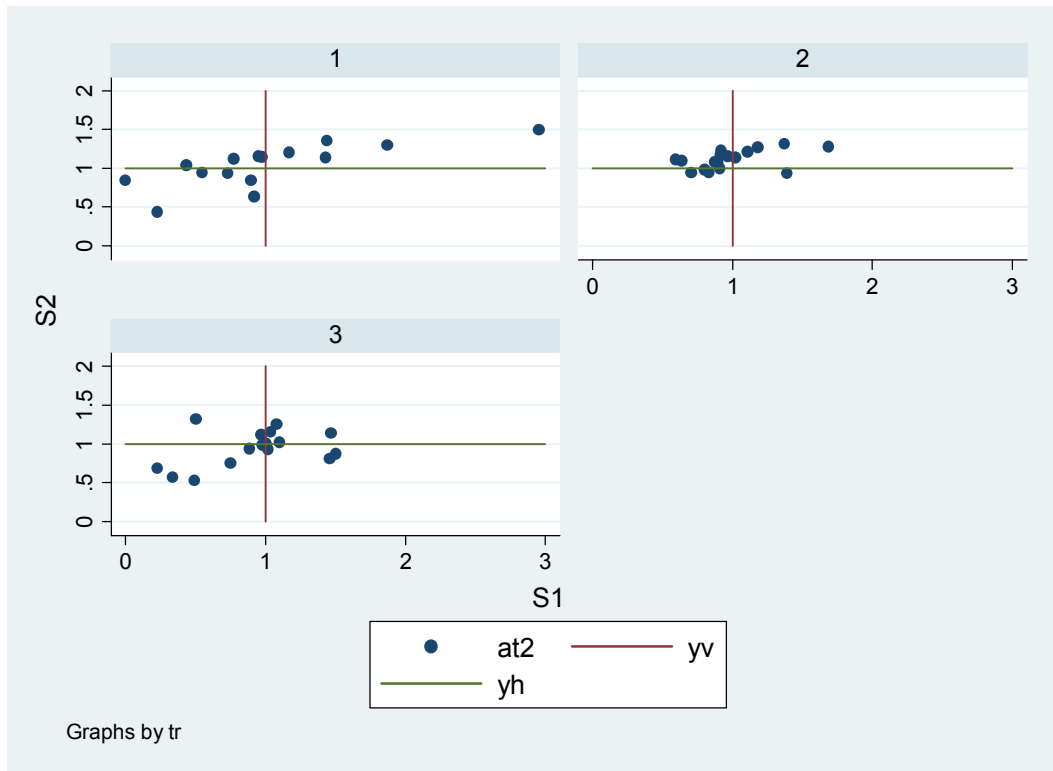
**Table 6: Percentage of Individuals Classified According to Ambiguity Attitudes and Treatment (number of subjects in brackets)<sup>16</sup>**

		Treatment 1	Treatment 2	Treatment 3	All treatments
S1	ambiguity-averse	67% (10)	64% (11)	50% (8)	60% (29)
	ambiguity-loving	33% (5)	36% (6)	50% (8)	40% (19)
S2	ambiguity-averse	40% (6)	24% (4)	56% (9)	40% (19)
	ambiguity-loving	60% (9)	76% (13)	44% (7)	60% (29)
S3	ambiguity-averse	47% (7)	41% (7)	44% (7)	44% (21)
	ambiguity-loving	53% (8)	59% (10)	56% (9)	66% (27)
S4	ambiguity-averse	47% (7)	29% (5)	44% (7)	40% (19)
	ambiguity-loving	53% (8)	71% (12)	56% (9)	60% (29)
S5	ambiguity-averse	53% (8)	41% (7)	44% (7)	46% (22)
	ambiguity-loving	47% (7)	59% (10)	56% (9)	54% (26)

<sup>16</sup> The data are derived from the additivity measures, as defined in equation (8)

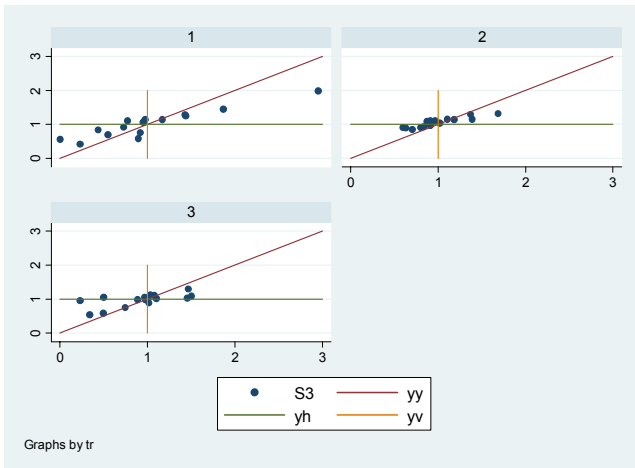
### Figure 1: Scatters of the First Two Additivity Measures

These are scatters of S2 against S1, by treatment

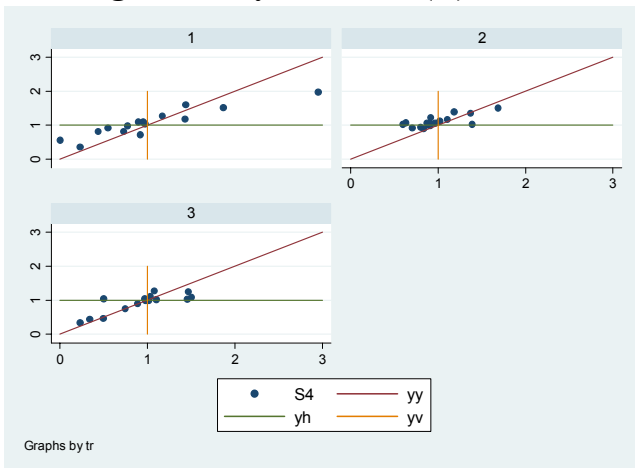


**Figure 2: Scatters of  $S_3$ ,  $S_4$  and  $S_5$  against  $S_I$**

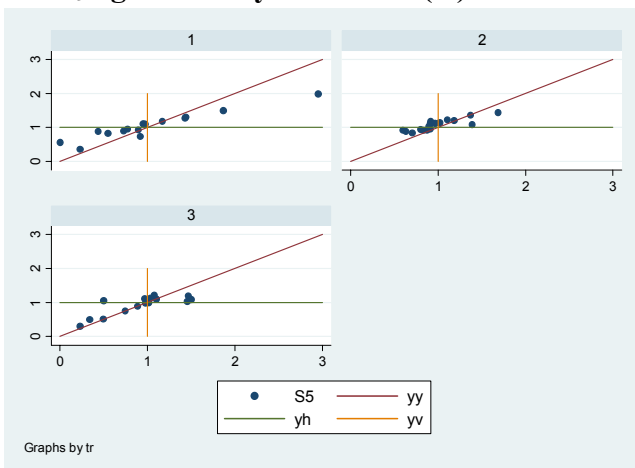
**2.1:  $S_3$  against  $S_I$  by treatment (tr)**



**2.2:  $S_4$  against  $S_I$  by treatment (tr)**



**2.3:  $S_5$  against  $S_I$  by treatment (tr)**



## Appendix 1: The Experimental Instructions

# EXEC

Centre for Experimental Economics at the University of York

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Welcome to this experiment. MIUR (the Italian ministry for the universities) has provided the funds to finance this research. Depending on your decisions you may earn a considerable amount of money which will be paid to you in cash immediately after the end of the experiment. This sum will be composed of the £10 participation fee plus your ‘earnings’ from a lottery. This latter could be a loss of £10, a gain of £10 or gain of £100. You cannot walk away from this experiment with less money than that with which you arrived, though you might walk away with £20 more or with £110 more.

There are no right or wrong ways to complete the experiment, but the decisions that you take will have implications for what you are paid at the end of the experiment. This depends partly on the decisions that you take during the experiment and partly on chance. So you will need to read these instructions carefully.

At the end of the experiment you will be asked to complete a brief questionnaire and to sign a receipt for the payment that you received, and to acknowledge that you participated voluntarily in the experiment. The results of the experiment will be used for the purpose of academic research and will be published and used in such a way that your anonymity will be preserved.

### Outline of the experiment

You will be asked 162 questions. Each will be of the same type. You will be presented with two lotteries and you will be asked which you prefer. After you have answered all 162 questions, one of them will be selected at random, the lottery that you said that you preferred on that question will be played out, and you will be paid the outcome: if the outcome is a loss of £10 you will leave the experiment with the same as when you came; if the outcome is a gain of £10 you will leave the experiment with £20 more than when you came; if the outcome is a gain of £100 you will leave the experiment with £110 more than when you came. If you did not express a preference on the selected question then one of the two lotteries will be selected at random and played out. It is clearly in your interests to answer each question as if that were the question to be played out.

### The Bingo Blower

You will have noticed a Bingo Blower in the laboratory. In this Blower there are balls of three different colours: pink, blue and yellow. The balls are constantly being blown about in the Blower. At the end of the experiment, when we come to play out your preferred choice on one of the questions, we will use this Bingo Blower to determine a colour: we will allow you to open the exit chute – this will lead to one ball being expelled. Obviously this expulsion will be done at random as there is no way that you can control the colour of the ball that emerges. The colour of the ball and the lottery that you chose on the question that was selected will determine your payment.

## **The Questions**

A sample question is illustrated in the Figure attached to these Instructions. In this figure, there are two lotteries – that on the left and that on the right. The lottery on the left would lead to a loss of £10 if the ball expelled was yellow, to a gain of £10 if the ball expelled was blue and to a gain of £100 if the ball expelled was pink. The lottery on the right would lead to a loss of £10 if the ball expelled was pink or blue and to a gain of £10 if the ball expelled was yellow. You have to decide for each question whether you prefer the lottery on the left or that on the right. You should indicate your choice by clicking on the box below the appropriate lottery. You will be given at least 30 seconds to make up your mind and you cannot proceed to the next question until these 30 seconds have elapsed. The number of seconds left to make your decision will be indicated at the bottom of the screen. If you want more time, simply click on ‘STOP THE CLOCK’; then click on ‘RESTART THE CLOCK’ when you are happy to proceed. If the 30 seconds have elapsed and you have not taken a decision then ‘no decision’ will be recorded for that question.

## **The end of the experiment**

After you have answered all 162 questions you will be asked to call over an experimenter. In front of him or her you will choose at random one of the questions - by picking at random a ticket from a set of cloakroom tickets numbered from 1 to 162. The computer will recall that question and your answer to it, and then you will play out your preferred choice on that question – in the manner described above. If you did not take a decision on that question then you will toss a coin to determine which of the two lotteries will be played out. You will then be asked to fill in a short questionnaire. We will then pay you, you will sign a receipt and then you will be free to go. Note that the experiment will take at least 81 minutes of your time. You can take longer and it is clearly in your interests to be as careful as you can when you are answering the questions.

**If you have any questions at any stage, please ask one of the experimenters.**

John Hey    Gianna Lotito    Anna Maffioletti

Question number 1

Instructions  
If you prefer the left lottery click on the left button. If you prefer the right, click on the right button. If you want more time, click on 'STOP THE CLOCK'.

**Left Lottery**

LOSS of £10 

GAIN of £10 

GAIN of £100 

**Right Lottery**

LOSS of £10  

GAIN of £10 

GAIN of £100

Click here if you prefer the Left lottery

STOP THE CLOCK

Click here if you prefer the Right lottery

You have 16 seconds left to make your decision unless you stop the clock

