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The Importance of Industrial Policy in Quality-Ladder Growth Models

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Abstract

We extend the class of quality-ladder growth models (Grossman and Helpman, 1991, Segerstrom, 1998 and others), to encompass an economy with asymmetric fundamentals. In contrast to the standard framework, in our model industries may differ in terms of their innovative potential (quality jumps and arrival rates) and consumers' preferences. This extension allows us to bring industrial policy back into the realm of the growth policy debate. We first show that it is always possible to raise the long-run growth rate and the social welfare of the economy through a cost-less tax/subsidy scheme reallocating resources towards the relatively more promising industries. We then prove that, in certain economies, even a mere profit taxation policy increases economic growth and social welfare above the laissez-faire.

KEYWORDS: innovation-driven growth, asymmetric fundamentals, industrial policy

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1 Introduction

Since their very appearance, vertical R&D-driven growth models have focused on structurally symmetric economies, and on symmetric equilibria in R&D and in the final (or intermediate) goods sector. Elaborating on the basic quality-ladder growth framework - see, among others, Grossman and Helpman (1991), Segerstrom (1998) -, we develop a model for an economy with asymmetric fundamentals, that is, for an economy whose fundamentals may differ across industries. In the research sector, heterogeneity is introduced both in the quality jumps, i.e. the improvement in utility following each innovation, and in the arrival rates, i.e. the probabilities of innovating per unit of labor. Finally, the unit contribution to the consumer's utility of any final good is also assumed to be industry-specific. We eliminate the 'scale effect' (Jones, 1995), which penalized the first generation of these models, by incorporating the idea of increasing complexity in the innovation process. In particular, we follow the formalization proposed by Dinopoulos and Segerstrom (1999). As we will see, the existence of different quality jumps and utility weights across industries breaks down the symmetric structure of market demands and profits. Furthermore, asymmetric arrival rates cause the profitability of engaging in R&D to vary accordingly. This asymmetric structure of profitability is then reflected in an asymmetric configuration of R&D efforts in stationary equilibrium. In particular, for each industry the 'better' the fundamentals (for example, the higher the quality jump), the larger the equilibrium amount of R&D efforts. We characterize the new balanced growth path and carry out the comparative statics analysis.

The characterization of steady state equilibrium for 'asymmetric economies' opens up the possibility of reconsidering the role of industrial policy in enhancing economic performance. The symmetric structure of the economy in standard quality-ladder models prevents the policy maker from implementing industry-specific policies potentially capable of affecting the economy's performance. Conversely, when technological and preference differences across industries are taken into account, a policy targeting specific industries has significant effects on the rate of economic growth and on social welfare.

Market equilibrium in standard quality-ladder models is not Pareto-optimal. The amount of resources devoted to R&D may be higher or lower than the welfare-maximizing amount, depending on the specific values of the parameters involved and on the model specifications. Our extension adds a new form of sub-optimality for market equilibrium. Agents tend to invest too little in 'good' industries and too much in 'bad' industries, where 'good' and 'bad' refer to the technological and preference characteristics of the industries. The reason

for this market failure is related to the forward-looking nature of the agents' R&D investment decisions across industries. The amount of *future* investment that agents expect in each industry adversely affects their expected duration of the monopolistic position acquired through innovation, and hence their current investment decisions (a *creative destruction* effect). In equilibrium, agents tend to expect relatively high (low) levels of future R&D investments in relatively 'good' ('bad') industries. Since from a social perspective the expected duration of the monopoly does not play any role, agents allocate investments sub-optimally across industries by investing too little in 'good' industries and too much in 'bad' industries. The sub-optimal allocation of resources calls for a sensible industrial policy capable of inducing a redistribution of these resources from 'bad' industries to 'good' industries.

In particular, in a slightly simplified version of our model - in which only differences in quality jumps across industries are assumed - we carry out two policy exercises. In the first we compare the effects on the steady state growth rate of the economy and on social welfare of two different tax/subsidy schemes on profits. A 'symmetric' rule, under which all industries are equally taxed/subsidized, and an 'asymmetric' rule, under which industries are taxed or subsidized according to their specific quality jump (the higher the quality jump, the lower the tax or the higher the subsidy). Keeping equal the amount of public revenue/expenditure under the two rules, our result is that the steady state growth rate associated with the 'asymmetric' rule is unambiguously higher than that associated with the 'symmetric' rule. Moreover, since the total amount of consumption in equilibrium turns out to be the same under the two rules, the 'asymmetric' rule is also welfare-enhancing with respect to the 'symmetric' rule. An important corollary of this result is that a *zero-cost* industrial policy, which simply redistributes resources from 'bad' industries to 'good' industries while leaving the public budget balanced, unambiguously increases both economic growth and welfare above the *laissez-faire* equilibrium. Hence, whatever the optimal amount of R&D expenditure, an industrial policy relatively favoring the more promising industries is worth implementing.

In the second policy exercise we assume that the policy maker is only allowed to tax but not to subsidize across industries. We prove that, if the difference between 'good' and 'bad' industries is 'big enough', even a mere taxation policy - which only taxes 'bad' industries without subsidizing the 'good' ones - is capable of enhancing both economic growth and social welfare. This 'paradox of growth' - that is, the fact that taxation on R&D returns fosters growth - arises because taxation frees up resources from the 'bad' industries, which the market allocates partly to manufacture (so that final consumption

unambiguously increases) and partly to the 'good' industries. If the set of 'good' industries is sufficiently more productive than the set of 'bad' industries which are being taxed, the gain in productivity for the economy more than offsets the loss due to the decrease in the overall amount of resources devoted to R&D (which parallels the increase in consumption). This result is reminiscent of Segerstrom's (2000) discussion of the long-run growth effects of R&D subsidies. In Section 3.2 we draw a comparison between his and our model.

The policies mentioned above are all welfare-enhancing. The welfare-maximizing solution is found in Section 4 where we show that, given the asymmetric structure of the economy, it is socially optimal to concentrate the total amount of R&D investment in the one most profitable industry and leave the other industries to stagnate forever. The corresponding optimal policy would then be to turn off (through taxation with tax rates equal to one) all industries but the best one and then, as usual in quality-ladder models, to tax or subsidize the latter depending on the parameters of the model. It is worth remarking however that, while in the model the preference and technological characteristics are time-invariant and perfectly known by the policy maker, in reality that is most probably not the case. We thus find this policy option too risky to be implemented.

The paper is organized as follows. In Section 2 we develop the model, analyze its steady state properties and draw the main comparative statics results. In Section 3 we carry out the policy analysis. In Section 4 we develop the welfare analysis, while in Section 5 we conclude with some remarks.

2   The Model

Let us assume a continuum of industries producing final goods indexed by $\omega \in [0, 1]$. In each industry firms are distinguished by the quality index j of the goods they supply, with the quality of their goods being increasing in the integer j. At time $t = 0$ in each industry some firm knows how to produce a $j = 0$ quality product and no other firm can offer a better one. In order to develop higher quality versions of any product firms engage in R&D races. The winner of an R&D race becomes the sole producer of a good whose quality is one step ahead of the previous quality leader.

2.1 Households

We assume a fixed number of dynastic households (normalized to one) whose members grow at constant rate $n > 0$. Each member shares the same intertemporally additively separable utility $\log u(t)$ and is endowed with a unit of labor she supplies inelastically. Therefore each household chooses its optimal consumption path by maximizing the discounted utility

$$U \equiv \int_0^\infty L(0) e^{-(\rho-n)t} \log u(t) dt \tag{1}$$

where $L(0) \equiv 1$ is the initial population and $\rho > n$ is the common rate of time preferences.

The instantaneous utility function is a logarithmic Cobb-Douglas. We let the utility weights $(\alpha(\omega))$ vary across industries, so as to represent a possible heterogeneity of consumers preferences among the set of commodities. As the $\alpha(\omega)$'s represent the relative weights of the goods in the utility function, we can normalize them in such a way that $\int_0^1 \alpha(\omega)d\omega = 1$. If we define $\lambda(\omega)$ as the size of quality improvements (the so-called 'quality jump'), assumed to be industry-specific to allow for asymmetry in the technical evolution of each line, $j^{\max}(\omega, t)$ as the highest quality reached by product ω at time t, and $d(j, \omega, t)$ as the consumption of product ω of quality j at time t, then the instantaneous utility function can be written as

$$\log u(t) \equiv \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j, \omega, t) d\omega, \tag{2}$$

and the static maximization problem can be represented as

$$\max_d \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j, \omega, t) d\omega \tag{3}$$

$$s.t. \quad C(t) = \int_0^1 \left[\sum_{j=0}^{j^{\max}(\omega,t)} p(j, \omega, t) d(j, \omega, t) \right] d\omega$$

where $p(j, \omega, t)$ denotes the price of product ω of quality j at time t and $C(t)$ is the total expenditure at time t.

At each point in time consumers maximize static utility by spreading their expenditure across industries proportionally to the utility contribution of each

product line $(\alpha(\omega))$. Since they perceive vertically differentiated products in a given industry ω as perfect substitutes once adjusted for quality differences, they will purchase in each product line those products with the lowest price per unity of quality. As we will see in the next subsection, in each product line the $j^{\max}(\omega, t)$ quality product is the unique good with the minimum price-quality ratio. Hence, the individual static demand functions are

$$
d(j, \omega, t) = \begin{cases} \dfrac{\alpha(\omega)C(t)}{p(j, \omega, t)} & \text{for } j = j^{\max}(\omega, t) \\[2em] 0 & \text{otherwise} \end{cases} . \tag{4}
$$

Substituting (4) into (2) and (2) into (1), we state the intertemporal maximum problem as

$$
\max_C U = \int_0^\infty e^{-(\rho - n)t} [\log C(t) + \int_0^1 \alpha(\omega)[\log \alpha(\omega) + \log [\lambda(\omega)]^{j^{\max}(\omega, t)}
$$
$$
- \log p(j, \omega, t)] d\omega] dt
$$

$$
s.t. \int_0^\infty e^{-\int_0^t [r(s) - n] ds} C(t) dt \leq W(0)
$$

where $r(s)$ is the instantaneous interest rate at time s and $W(0)$ is the present value of the stream of incomes plus the value of initial wealth at time $t = 0$. The solution to this problem obeys the differential equation

$$
\frac{\dot{C}(t)}{C(t)} = r(t) - \rho. \tag{5}
$$

2.2   Manufacture

Each good is produced by employing labor through a constant returns to scale technology. In order to produce one unit of any good firms hire l_m units of labor regardless of quality.

In each industry the $j^{\max}(\omega, t)$ quality product can only be manufactured by the firm owning the blueprint. As usual, since firms engage in Bertrand competition, the quality leader monopolizes its relative market until a successive innovation is introduced in its product line. Indeed, having a quality

advantage over its competitors, it can charge a price higher than its unit cost, with the quality-adjusted price being still ε-lower than those of its followers. Moreover, because of the 'Arrow effect', the quality leader does not engage in R&D races (see Grossman and Helpman 1991, p. 47). Hence, it is always one step ahead of its immediate follower, and the limit price that still monopolizes the market is $\lambda(\omega)w(t)l_m$ where $w(t)$ is the wage rate set equal to 1^1. Given the individual demand functions $d(j,\omega,t)$, the market demand of good ω at time t is $D(\omega,t) = d(j,\omega,t)L(t) = \alpha(\omega)C(t)L(t)/p\left[j^{\mathrm{max}}(\omega,t),\omega,t\right]$. Its unit elastic structure makes the quality leader exactly set the limit price (see Grossman and Helpman 1991, p. 46). Then

$$p\left[j^{\mathrm{max}}(\omega,t),\omega,t\right] = \lambda(\omega)l_m.$$

We can now calculate the profit flow in each industry as

$$\pi(\omega,t) = p\left[j^{\mathrm{max}}(\omega,t),\omega,t\right]D(\omega,t) - l_m D(\omega,t) = \frac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega)C(t)L(t).$$

2.3 R&D Races

The research sector is characterized by the efforts of R&D firms to develop better versions of the existing products in order to displace the current monopolists. We assume free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. In this sector we depart from the standard framework by allowing for different arrival rates across industries. With this adjustment we intend to formally introduce into the analysis the possibility that some industries are more promising than others in terms of their chances of discovering a new commodity. Any firm hiring l_u units of labor in industry ω at time t acquires the instantaneous probability of innovating of $A(\omega)l_u/X(\omega,t)$, where $X(\omega,t)$ is the R&D difficulty index.

Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation (or research intensity) is

$$\frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} \equiv i(\omega,t) \tag{6}$$

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[1] Here labor is the numeraire.

where $L_I(\omega, t) = \sum_u l_u(\omega, t)$. The R&D difficulty index $X(\omega, t)$ describes the evolution of technology. It is assumed to increase over time in order to rule out the 'scale effect' (Jones, 1995), that is, to allow for constant growth rates even with a growing population. In what follows we will assume it to evolve in accordance with the specification suggested by Dinopoulos and Segerstrom (1999)[2]. This formulation has been developed to formalize the increasing difficulty of introducing new products in more crowded markets

$$X(\omega, t) = kL(t) \tag{PEG}$$

where k is a positive constant.

Whenever a firm succeeds in innovating, it acquires the uncertain profit flow that accrues to a monopolist, that is, the stock market valuation of the firm. Let us denote it by $v(\omega, t)$. Thus, the problem faced by an R&D firm is that of choosing the amount of labor input in order to maximize its expected profits

$$\max_{l_u} \left[\frac{v(\omega, t)A(\omega)}{X(\omega, t)} l_u - l_u \right]$$

which provides a finite, positive solution for l_u only when the arbitrage equation

$$\frac{v(\omega, t)A(\omega)}{X(\omega, t)} = 1$$

is satisfied. Notice that in this case, though finite, the size of the R&D firm is indeterminate because of the constant returns research technology.

Households own the monopolistic firms through an efficient stock market. In equilibrium the stock market valuation of these firms yields an expected rate of return equal to the riskless interest rate $r(t)$. This equality must hold because the presence of a perfectly efficient financial market allows risk-averse households to completely diversify their portfolio across industries and, hence, to care only about deterministic mean returns. The representative shareholder receives a dividend of $\pi(\omega, t)dt$ over a time interval of length dt, and the value of the monopoly appreciates by $\dot{v}(\omega, t)dt$ if no firm innovates in the unit of time dt. However, if an innovation occurs, which happens with probability

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[2]This specification is known under the acronym PEG, which stands for 'permanent effects on growth' of policy measures such as subsidies and taxes. It has been independently developed by Smulders and van de Klundert (1995), Young (1998), Dinopoulos and Thompson (1998), Peretto (1998) and Howitt (1999). We adopt the formalization suggested by Dinopoulos and Segerstrom (1999).

$i(\omega, t)dt$, the shareholder suffers a loss of $v(\omega, t)$. Therefore, the expected rate of return from holding a share of a monopolistic firm per unit of time is

$$\frac{\pi(\omega, t) + \dot{v}(\omega, t)}{v(\omega, t)} - i(\omega, t),$$

which needs be equal to the interest rate $r(t)$. From that equality we can derive the firm's market valuation as

$$v(\omega, t) = \frac{\pi(\omega, t)}{r(t) + i(\omega, t) - \dfrac{\dot{v}(\omega, t)}{v(\omega, t)}}$$

so that the R&D equilibrium condition is

$$\frac{\pi(\omega, t)A(\omega)}{X(\omega, t)\left[r(t) + i(\omega, t) - \dfrac{\dot{v}(\omega, t)}{v(\omega, t)}\right]} = 1. \tag{7}$$

2.4   The Labor Market

Since in each industry the market demand $D(\omega, t) = \alpha(\omega)C(t)L(t)/\lambda(\omega)l_m$ requires $D(\omega, t)l_m$ units of labor in order to be produced, the total employment in the manufacturing sector is

$$\int_0^1 \frac{\alpha(\omega)C(t)L(t)}{\lambda(\omega)} d\omega.$$

Then, the labor market-clearing condition implies

$$L(t) = \int_0^1 \frac{\alpha(\omega)C(t)L(t)}{\lambda(\omega)} d\omega + \int_0^1 L_I(\omega, t) d\omega \tag{8}$$

where $\int_0^1 L_I(\omega, t) d\omega$ is the total employment in the research sector.

2.5   Balanced Growth Path

We now focus on the balanced state growth path where the endogenous variables all grow at constant rates. The steady state requires that the distribution of resources between manufacturing and research be constant. In turn,

$\dot{C}(t)/C(t) = 0$, and from the Euler equation we have $r(t) = \rho$. Moreover, from the definition of $v(\omega, t)$ it follows that its steady state growth rate is $\dot{v}(\omega, t)/v(\omega, t) = n$. The arbitrage equation (7) then becomes

$$\frac{\pi(\omega, t)A(\omega)}{X(\omega, t)\left[\rho + i(\omega, t) - n\right]} = 1.$$

The rational expectations equilibrium requires that expectations on research intensities be equal to their actual values. Moreover, as our model assumes increasing complexity, the steady state analysis makes these intensities constant over time. From (7) we determine the following expression for them

$$i(\omega, t) = \frac{\pi(\omega, t)A(\omega)}{X(\omega, t)} - \rho + n = \frac{\lambda(\omega) - 1}{\lambda(\omega)k}\alpha(\omega)A(\omega)C - \rho + n = i(\omega).$$

Notice that the probabilities of innovation $i(\omega)$ in steady state are affine transformations of industry-specific profits and research technologies. If we define the sectoral population-adjusted research employment as $l_I(\omega, t) \equiv L_I(\omega, t)/L(t)$, from the definition of the research intensity it follows that $l_I(\omega) = i(\omega)X(\omega, t)/L(t)A(\omega)$. Then, by plugging the expression for $i(\omega)$ given above, we obtain

$$l_I(\omega) = \frac{\lambda(\omega) - 1}{\lambda(\omega)}\alpha(\omega)C - \frac{k(\rho - n)}{A(\omega)}. \tag{9}$$

The population-adjusted steady-state resource condition is given by

$$1 = \int_0^1 \frac{\alpha(\omega)C}{\lambda(\omega)}d\omega + \int_0^1 l_I(\omega)d\omega. \tag{10}$$

The two equations (9) and (10) define the steady state values of per-capita expenditure C and of the population-adjusted research employment in each industry $l_I(\omega)$. Using (9) to substitute for $l_I(\omega)$ into (10), we obtain the steady state value for C

$$C^* = 1 + k(\rho - n)\int_0^1 \frac{1}{A(\omega)}d\omega. \tag{11}$$

Plugging the expression above into (9) we then determine the steady state value for $l_I(\omega)$

$$l_I^*(\omega) = \frac{\lambda(\omega) - 1}{\lambda(\omega)} \alpha(\omega) \left(1 + k(\rho - n) \int_0^1 \frac{1}{A(\omega)} d\omega\right) - \frac{k(\rho - n)}{A(\omega)}.$$

As expected, the research efforts in the steady state equilibrium are now industry-specific. Comparative statics analysis confirms the standard results obtained in symmetric quality-ladder models. In each industry the population-adjusted research effort is an increasing function of the quality jump, the utility weight and the arrival rate. The total amount of research is negatively correlated with the rate of time preferences and positively correlated with the population growth rate[3]. Finally, given that $i(\omega) \equiv A(\omega)L(t)l_I(\omega)/X(\omega, t)$, we can immediately find the steady-state values for the research intensites as

$$i^*(\omega) = \frac{A(\omega)}{k} \frac{\lambda(\omega) - 1}{\lambda(\omega)} \alpha(\omega) \left(1 + k(\rho - n) \int_0^1 \frac{1}{A(\omega)} d\omega\right) - (\rho - n).$$

Since the growth rate of individual utility can be thought of as the measure of the economy's growth rate, we now solve for its steady state value. Substituting for $p(j, \omega, t) = \lambda(\omega)l_m$ and $C(t) = C$, the steady state value of the utility is

$$\log u(t) = \log C + \int_0^1 \alpha(\omega) \left[\log \alpha(\omega) + \log [\lambda(\omega)]^{j^{\max}(\omega, t)} - \log [\lambda(\omega)l_m]\right] d\omega =$$

$$\log C + \int_0^1 \alpha(\omega) \log \alpha(\omega) d\omega + \int_0^1 \alpha(\omega) \log [\lambda(\omega)]^{j^{\max}(\omega, t)} d\omega$$

$$- \int_0^1 \alpha(\omega) \log [\lambda(\omega)l_m] d\omega.$$

Given that

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[3]See Appendix A for a brief derivation of these results.

$$\int_0^1 \alpha(\omega) \log [\lambda(\omega)]^{j^{\max}(\omega,t)} \, d\omega \;=\; \int_0^t \int_0^1 [i(\omega,\tau)\alpha(\omega) \log \lambda(\omega)d\omega] \, d\tau =$$

$$t \int_0^1 i(\omega)\alpha(\omega) \log \lambda(\omega)d\omega$$

(where $\int_0^t i(\omega,\tau)d\tau$ represents the expected number of successes in industry ω up to time t), differentiating $\log u(t)$ with respect to time yields

$$\frac{\dot{u}}{u} = \int_0^1 i(\omega)\alpha(\omega) \log \lambda(\omega)d\omega.$$

To sum up, the steady state equilibrium is characterized by a set of constant prices $p(j,\omega,t) = \lambda(\omega)l_m$, $w(t) = 1$, $r(t) = \rho$; and constant per capita quantities. In particular, total expenditure is

$$C^* = 1 + k(\rho - n) \int_0^1 \frac{1}{A(\omega)}d\omega,$$

sectoral production is

$$d(j,\omega,t) = \frac{\alpha(\omega)C^*}{\lambda(\omega)l_m},$$

while sectoral investment in R&D is

$$l_I^*(\omega) = \frac{\lambda(\omega)-1}{\lambda(\omega)}\alpha(\omega) \left(1 + k(\rho - n) \int_0^1 \frac{1}{A(\omega)}d\omega \right) - \frac{k(\rho-n)}{A(\omega)}.$$

Utility is the only per capita variable growing in steady state. Since growth in utility is a linear function of research intensities, in the next section we investigate how industrial policy can affect the economy's growth and welfare by influencing the distribution of investments in R&D.

3 The Role of Industrial Policy in Enhancing Economic Performance and Social Welfare

Market equilibrium of quality-ladder growth models is not optimal. The existence of the well-known distortionary effects, namely the 'consumer surplus

effect', the 'intertemporal spillover' and the 'businness stealing effect', (see Grossman and Helpman 1991, p. 52) makes it necessary to introduce either a tax or subsidy (depending on the values of the parameters and on the model specifications) to induce the welfare-maximizing R&D expenditure[4]. The asymmetric structure of our economy brings an additional failure of the market forces in delivering the social welfare optimum. The total amount of resources devoted to R&D (whether optimal or not) is inefficiently distributed across industries. The source of this industry-specific sub-optimality is still associated with the 'intertemporal spillover'. As is well known, the forward-looking nature of the market equilibrium makes the arrival of the next innovation in every industry exercise a negative effect on the market value of the incumbent - because of its 'creative destruction' effect. As a result, in our asymmetric economy higher R&D efforts in the relatively more profitable industries generate higher creative destruction in those industries, thus lowering their expected returns. However, from a social perspective the expected duration of the monopolistic position acquired by an innovation is irrelevant, and the arrival of the successive innovation enhances consumer welfare unambiguously. Hence, pursuing industries with higher utility more intensively improves overall welfare. Our asymmetric policies go in this direction by switching the incentive to invest from the least to the most profitable industries.

The argument is developed along two lines. In the next subsection we show that, under the same public budget constraint, a *selective* policy intervention - which favors the relatively better industries - is always unambiguously better than a *uniform* one for both economic growth and the agents' welfare in the steady state. As a particular case of this reasoning we also compare the laissez-faire solution with a costless asymmetric policy rule which, by taxing some industries and subsidizing others, is such that the whole tax revenue (or subsidy expenditure) is zero. This asymmetric rule increases the growth rate and the overall welfare of the economy above laissez-faire. In Subsection 3.2 we find that, under certain conditions, the same result may be obtained even through a pure taxation policy such that a subset of relatively bad industries are taxed and no subsidies are awarded.

For reasons of tractability let us consider a slightly simplified version of the model developed in the last section. In this version the heterogeneity in both the utility weights ($\alpha(\omega)$) and the technology parameters ($A(\omega)$) is removed,

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[4]Particular attention has been devoted to the relation between the optimal subsidy and the size of innovation. Grossman and Helpman (1991) found it to be n-shaped; in Segerstom (1998) the optimal subsidy is a monotonic negative function of the innovation size, while Li's (2003) model implies the optimal subsidy to be positive for 'low' and 'high' levels of innovation size but not for intermediate ones.

while the quality jumps $\lambda(\omega)$ are still left as industry-specific[5]. In other words, we are considering an economy where some industries might be more attractive for R&D investments in terms of the monopoly profits they can generate once the innovation is brought into the market.

3.1 The Tax/Subsidy Scheme

Let us introduce profit taxation/subsidy. Taxes (subsidies) are assumed to be transferred (withdrawn) to (from) the representative household, so that its intertemporal budget constraint remains unaffected. We will derive our result by comparing two distinct policy rules. The first rule imposes that each industry be equally taxed (subsidized), while the second requires that the tax (subsidy) rate be inversely (directly) related to its quality jump, that is, the higher the quality jump the lower the tax (the higher the subsidy). Let $\sigma(\omega)$ be the industry-specific tax (subsidy) rate, then the two rules can formally be expressed as (a) $\sigma(\omega) = \sigma = (1-s)$, (b) $\sigma(\omega) = (1-m\lambda(\omega))$. When $s, m\lambda(\omega) \in (0,1]$, then $\sigma(\omega) \in R_+$ and represents a *tax* rate. When $s, m\lambda(\omega) \in [1, +\infty)$, $\sigma(\omega) \in R_-$ and represents a *subsidy* rate. Our goal is to compare the different effects on the economy's growth rate (and welfare) of these two rules under the constraint that the total amount of government revenue/expenditure be the same. This public budget constraint can be expressed as

$$\int_0^1 \pi^s(\omega, t)(1-s)d\omega \equiv \int_0^1 \frac{\lambda(\omega) - 1}{\lambda(\omega)} C^s L(t)(1-s)d\omega =$$

$$\int_0^1 \pi^{as}(\omega, t)(1 - m\lambda(\omega))d\omega \equiv \int_0^1 \frac{\lambda(\omega) - 1}{\lambda(\omega)} C^{as} L(t)(1 - m\lambda(\omega))d\omega \quad (12)$$

where π^s, C^s denote profits and expenditure under the symmetric policy rule, while π^{as}, C^{as} denote profits and expenditure under the asymmetric policy rule.

We can now determine the steady state values for the expenditure and the research effort in both cases[6]. In the presence of profit tax/subsidy, the agents evaluate the opportunity of investment in R&D on the basis of the *after-tax*

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[5] Our result is actually robust against the choice of which variable to leave as industry-specific.

[6] In what follows we will not run through all the steps as we did in the last Section: this model is a special case of the one already illustrated, except for the introduction of the two tax/subsidy rules on profits.

(or *after-subsidy)* profit flow they can gain in the case of successful innovation. Hence, the new steady state arbitrage equation under the asymmetric rule becomes

$$\frac{A\dfrac{\lambda(\omega) - 1}{\lambda(\omega)}C^{as}m\lambda(\omega)}{k\left[\rho + \dfrac{A}{k}l_I^{as}(\omega) - n\right]} = 1.$$

Once again notice that $m\lambda(\omega) < 1$ represents a tax, while $m\lambda(\omega) > 1$ represents a subsidy. Solving for $l_I^{as}(\omega)$ we obtain

$$l_I^{as}(\omega) = (\lambda(\omega) - 1)C^{as}m - \frac{k}{A}(\rho - n)$$

and plugging it into the new resource condition

$$1 = C^{as}\int\limits_0^1 \frac{1}{\lambda(\omega)}d\omega + \int\limits_0^1 l_I^{as}(\omega)d\omega$$

we can derive the steady state expenditure under the asymmetric policy rule

$$C^{as} = \frac{1 + \dfrac{k}{A}(\rho - n)}{\int\limits_0^1 \dfrac{1}{\lambda(\omega)}d\omega + m\int\limits_0^1 (\lambda(\omega) - 1)d\omega}.$$

Analogously, given that the after-tax (subsidy) steady state arbitrage equation under the symmetric rule is

$$\frac{A\dfrac{\lambda(\omega) - 1}{\lambda(\omega)}C^s s}{k\left[\rho + \dfrac{A}{k}l_I^s(\omega) - n\right]} = 1,$$

and using the resource condition as before, we can solve for C^s and obtain

$$C^s = \frac{1 + \dfrac{k}{A}(\rho - n)}{\int\limits_0^1 \dfrac{1}{\lambda(\omega)}d\omega + s\int\limits_0^1 \dfrac{\lambda(\omega) - 1}{\lambda(\omega)}d\omega}.$$

C^{as} and C^s are functions of, among others, the two parameters m and s respectively, which measure the direction and the intensity of the government intervention. So far we have not established any relation between them, so that C^{as} and C^s are in fact incomparable. However, we know that the constraint on total tax revenue (12) must be satisfied. We now show that, when this constraint is imposed, the relation between s and m is such that the overall expenditure is exactly the same under the two different taxation schemes (that is, $C^{as} = C^s$). Plugging the expression for C^{as} and C^s just derived into (12) we can solve for s as a function of m and obtain

$$s = m \frac{\int\limits_0^1 (\lambda(\omega) - 1) d\omega}{\int\limits_0^1 \frac{\lambda(\omega) - 1}{\lambda(\omega)} d\omega}. \tag{13}$$

Substituting for s into the equilibrium value of C^s, it can then be immediately verified that $C^s = C^{as} \equiv C$. This result is not surprising. Since the government revenue/expenditure is constrained to be the same under both tax regimes, the decision on whether to invest in research or to employ resources in the manufacturing sector is not affected.

The labor market-clearing condition, on the other hand, requires that the sum of per capita expenditure and of per capita aggregate research be constant. Hence, the total amount of research is also exactly the same under the two policy rules. We can finally find the steady state industry-specific research intensities under the two different policy rules (we plug (13) into the expression for $i^s(\omega)$ in order to render the two quantities directly comparable)

$$i^{as}(\omega) = \frac{A}{X(\omega)} L_I(\omega) = \frac{A}{k}[\lambda(\omega) - 1] \frac{1 + \frac{k}{A}(\rho - n)}{\int\limits_0^1 \frac{1}{\lambda(\omega)} d\omega + m \int\limits_0^1 (\lambda(\omega) - 1) d\omega} m - (\rho - n)$$

$$i^s(\omega) = \frac{A}{X(\omega)} L_I(\omega) = \frac{A}{k} \frac{\lambda(\omega) - 1}{\lambda(\omega)} \frac{1 + \frac{k}{A}(\rho - n)}{\int\limits_0^1 \frac{1}{\lambda(\omega)} d\omega + m \int\limits_0^1 (\lambda(\omega) - 1) d\omega} m \cdot$$

$$\cdot \frac{\int\limits_0^1 (\lambda(\omega) - 1) d\omega}{\int\limits_0^1 \frac{\lambda(\omega) - 1}{\lambda(\omega)} d\omega} - (\rho - n),$$

and show that the growth rate is higher under the asymmetric policy rule. In this simplified version of the model the growth rate is

$$\frac{\dot{u}}{u} = \int\limits_0^1 i(\omega) \log \lambda(\omega) d\omega.$$

This depends on the research intensities and, in particular, on the effect of the policy rule on the research intensities. Our reasoning now goes as follows.

First notice that, since the overall R&D effort is constant, the total research intensity $(\int_0^1 i(\omega)d\omega = (A/k) \int_0^1 l_I(\omega)d\omega)$ is constant as well. Let us now define $\Delta i(\lambda(\omega)) \equiv i^{as}(\omega) - i^s(\omega)$. It is easy to show that

$$\Delta i(\lambda(\omega)) \gtrless 0 \Leftrightarrow \lambda(\omega) \gtrless \frac{\int\limits_0^1 (\lambda(\omega) - 1)d\omega}{\int\limits_0^1 \frac{\lambda(\omega) - 1}{\lambda(\omega)}d\omega}.$$

The asymmetric policy affects the distribution of the intensities by reallocating resources towards industries with relatively higher quality jumps. Since the growth rate of the economy is the sum of the log of the quality jumps each weighted with the research intensity of its industry, the steady state growth rate associated with the asymmetric policy is unambiguously higher than the one associated with a symmetric policy. The underlying intuition is that heterogeneity in the quality jumps determines an asymmetric structure of profits and, then, of research intensities across industries. The asymmetric rule amplifies this effect by polarizing the distribution of the R&D resources towards the industries characterized by higher quality jumps. Notice also that an increase in the steady state growth rate, coupled with an unchanged level of total expenditure C, leads to an unambiguous increase in social welfare[7].

This result contains an important corollary. If we are to suggest a certain policy (namely, an asymmetric one), we may find it reasonable to compare it with its 'natural' reference point, the laissez-faire policy. This comparison is exactly what we obtain when we set $s = 1$ in the policy analysis we have developed above. That condition, together with (12), implies

---

[7]Cozzi and Impullitti (2005) develop an asymmetric quality-ladder model similar to ours. Their focus is however on the effects of fiscal policy on the skill premium. They find that a change in the composition of public expenditure in favor of the most profitable sectors would imply a rise in the skill premium.

$$\hat{m} = \frac{\int\limits_0^1 \dfrac{\lambda(\omega) - 1}{\lambda(\omega)} d\omega}{\int\limits_0^1 (\lambda(\omega) - 1) d\omega} \ .$$

While $s = 1$ identifies the 'laissez-faire policy', $\hat{m}\lambda(\omega)$ describes an asymmetric 'zero-sum policy', that is, a policy which can only redistribute resources from one industry to another, leaving the public budget exactly balanced. Interestingly, a comparison of the research intensities across the two policy rules shows[8] that it is always possible to improve upon the laissez-faire equilibrium through a costless system of tax/subsidy which reallocates resources towards the relatively more promising indutries. By continuity, it is then possible to implement a tax/subsidy scheme which enhances economic performance as well as social welfare and, at the same time, guarantees a strictly positive tax revenue for the government. In the next subsection we go even further by showing that, in some economies, the same result may be achieved even when the policy maker is only allowed to tax, but not to subsidize, across industries.

3.2   The Pure Taxation Scheme

In the simplified framework developed above - in which only the quality jumps are industry-specific - we now introduce a particular ordering of industries such that $\lambda(\cdot)$ is an increasing function of the industry index ω, that is, an ordering from the least to the most innovative industry as ω goes from 0 to 1. Let us partition the set of industries into two subsets, $[0, \bar{\omega}]$ and $(\bar{\omega}, 1]$. The suggested policy rule consists of taxing uniformly[9] the first subset of (relatively less innovative) industries, while leaving the second subset of (relatively more innovative) industries unaffected. Under the adopted notation the industry-specific tax rate is $\sigma(\omega) = (1-s) \in (0, 1) \ \forall \omega \in [0, \bar{\omega}]$ and $\sigma(\omega) = 0 \ \forall \omega \in (\bar{\omega}, 1]$. The new set of arbitrage equations is now given by

$$\frac{A\dfrac{\lambda(\omega) - 1}{\lambda(\omega)} C^T s}{k\left[\rho + \dfrac{A}{k} l_I^T(\omega) - n\right]} = 1 \ \ \forall \omega \in [0, \bar{\omega}],$$

---

[8] As a particular case of the one above, the procedure to obtain the result is straightforward after substituting for $\hat{m}$ into the expressions for the equilibrium research intensities $i^s(\omega)$, $i^{as}(\omega)$.

[9] For simplicity we consider a uniform policy instead of an asymmetric one. The latter would indeed enlarge the set of economies for which our proposition holds.

$$\frac{A\dfrac{\lambda(\omega)-1}{\lambda(\omega)}C^T}{k\left[\rho+\dfrac{A}{k}l_I^T(\omega)-n\right]}=1 \quad \forall \omega \in (\bar{\omega},1].$$

Solving for $l_I^T(\omega)$ and plugging it into the usual resource condition we find the equilibrium value for expenditure to be

$$C^T=\frac{1+\dfrac{k}{A}(\rho-n)}{\left(\int\limits_0^{\bar{\omega}}\dfrac{1}{\lambda(\omega)}d\omega-\bar{\omega}\right)(1-s)+1}.$$

Since $s<1$ implies $\left(\int_0^{\omega}1/\lambda(\omega)d\omega-\bar{\omega}\right)(1-s)+1<1$, the taxation policy raises expenditure above the laissez-faire. Hence, given the resource constraint, it lowers the total amount of resources employed in R&D below the laissez-faire. Furthermore, this policy has an additional effect on the industry-specific research intensities which in equilibrium are given by

$$i^T(\omega)=\frac{\lambda(\omega)-1}{\lambda(\omega)}\frac{\dfrac{A}{k}+\rho-n}{\left(\int\limits_0^{\bar{\omega}}\dfrac{1}{\lambda(\omega)}d\omega-\bar{\omega}\right)(1-s)+1}s-(\rho-n) \quad \forall \omega \in [0,\bar{\omega}], \quad (14)$$

$$i^T(\omega)=\frac{\lambda(\omega)-1}{\lambda(\omega)}\frac{\dfrac{A}{k}+\rho-n}{\left(\int\limits_0^{\bar{\omega}}\dfrac{1}{\lambda(\omega)}d\omega-\bar{\omega}\right)(1-s)+1}-(\rho-n) \quad \forall \omega \in (\bar{\omega},1]. \quad (15)$$

The research intensities in the subset $(\bar{\omega},1]$ are positively affected by the tax policy. The reason is that higher expenditure raises profits in manufacturing, and hence raises the incentive to invest in the industries which are not directly affected by the taxation. Given the resource constraint, the increase in the resources devoted to consumption and to the more innovative industries is exactly balanced by the reduction of investment in the taxed industries. Then our policy rule has two opposite effects on the growth rate of utility. On the one hand, it reduces the total resources devoted to the research sector (a negative effect). On the other hand, by reducing the profitability of investing in the

less innovative industries, it favors a partial reallocation of research investment towards those industries with a higher contribution to household utility (a positive effect). There exist economies where the second effect more than offsets the first effect and, hence, for which a pure taxation policy increases the long-run growth rate.

Let $(\dot{u}/u)^T = \int_0^1 i^T(\omega) \log \lambda(\omega) d\omega$ and $(\dot{u}/u)^{LF} = \int_0^1 i^{LF}(\omega) \log \lambda(\omega) d\omega$ respectively be the growth rates under the tax policy and under laissez-faire (where $i^{LF}(\omega)$ can be obtained from $i^T(\omega)$ by posing $s = 1$). Given (14) and (15), and since $s < 1$

$$(\frac{\dot{u}}{u})^T > (\frac{\dot{u}}{u})^{LF} \iff$$

$$\int_0^{\bar{\omega}} \tfrac{\lambda(\omega)-1}{\lambda(\omega)} \log \lambda(\omega) d\omega < \int_0^1 \tfrac{\lambda(\omega)-1}{\lambda(\omega)} \log \lambda(\omega) d\omega \cdot \int_0^{\bar{\omega}} \tfrac{\lambda(\omega)-1}{\lambda(\omega)} d\omega. \qquad (16)$$

Given that

$$\int_0^1 \tfrac{\lambda(\omega)-1}{\lambda(\omega)} \log \lambda(\omega) d\omega = \int_0^{\bar{\omega}} \tfrac{\lambda(\omega)-1}{\lambda(\omega)} \log \lambda(\omega) d\omega + \int_{\bar{\omega}}^1 \tfrac{\lambda(\omega)-1}{\lambda(\omega)} \log \lambda(\omega) d\omega >$$

$$\int_0^{\bar{\omega}} \tfrac{\lambda(\omega)-1}{\lambda(\omega)} \log \lambda(\omega) d\omega,$$

and that $\int_0^{\bar{\omega}} (\lambda(\omega) - 1)/\lambda(\omega) d\omega < 1$, the inequality may or may not hold. As expected, whether it holds or not depends 1. on the fundamentals of the economy (that is, on 'how much better' are industries in $(\bar{\omega}, 1]$ with respect to those in $[0, \bar{\omega}]$ in terms of quality jumps $\lambda(\omega)$), and 2. on $\bar{\omega}$ (that is, on 'how many' industries are taxed). The inequality above is more likely to be satisfied the higher the difference in the quality jumps between the two subsets of industries, and/or the smaller the mass of taxed industries. Therefore, in a generic economy it is not always the case that a mere taxation policy is capable of yielding both an improvement in the growth rate and higher social welfare. However, with a sufficiently polarized structure of the quality jumps, there exists a threshold value for $\bar{\omega}$ of taxable industries such that this result can be achieved by taxing up to $[0, \bar{\omega}]$. Interestingly, notice that whether this result holds or not does not depend on the intensity of taxation (that is, on s).

As an illustrative example let us find a simple family of economies for which taxation fosters long-run growth. Let us consider a set of economies indexed

by i in which, given the industry interval $[0, 1]$, for reasons of tractability we assume that the fraction of industries $[0, \bar{\omega}_i]$ is characterized by identical quality jumps λ^L, while the fraction of industries $(\bar{\omega}_i, 1]$ is characterized by identical quality jumps $\lambda^H > \lambda^L$ (L stands for 'low', H for 'high'). Each economy i is then distinguishable from the others via the industry $\bar{\omega}_i$ separating the set of less innovative industries from the set of more innovative industries. The policy suggested in each economy is that of taxing the set of less innovative industries. We now find the set of economies for which taxing the fraction $[0, \bar{\omega}_i]$ of industries raises the growth rate of the economy. Given the new configuration, (16) becomes

$$
\begin{aligned}
(\tfrac{\dot{u}}{u})_i^T &> (\tfrac{\dot{u}}{u})^{LF} \iff \\
\bar{\omega}_i \tfrac{\lambda^L-1}{\lambda^L} \log \lambda^L &< \left[\bar{\omega}_i \tfrac{\lambda^L-1}{\lambda^L} \log \lambda^L + (1-\bar{\omega}_i) \tfrac{\lambda^H-1}{\lambda^H} \log \lambda^H \right] \bar{\omega}_i \tfrac{\lambda^L-1}{\lambda^L},
\end{aligned}
$$

from which we obtain

$$
0 < \bar{\omega}_i < \frac{\frac{\lambda^H-1}{\lambda^H} \log \lambda^H - \log \lambda^L}{\frac{\lambda^H-1}{\lambda^H} \log \lambda^H - \frac{\lambda^L-1}{\lambda^L} \log \lambda^L} \equiv \bar{\omega}, \tag{17}
$$

which essentially tells us that, in order for the long-run growth rate of the economy to be positively affected by the tax policy, the economy must be such that the mass of the less innovative industries to be taxed cannot be higher than $\bar{\omega}$. Notice that, in writing (17) as the solution to the inequality above, we have implicitly assumed that the following condition on the parameters of the model holds (which guarantees that $\bar{\omega}_i > 0$)[10]:

$$
\tfrac{\lambda^H-1}{\lambda^H} \log \lambda^H - \log \lambda^L > 0
$$

Once again, with a sufficiently polarized structure of quality jumps, that is, if λ^H is 'sufficiently higher' than λ^L, a pure taxation directed towards the industries characterized by λ^L has a positive effect both on the steady-state

---

[10]If $\left(\lambda^H - 1 \right)/\lambda^H \log \lambda^H - \log \lambda^L < 0$, the solution to the inequality would be

$$
\frac{\frac{\lambda^H-1}{\lambda^H} \log \lambda^H - \log \lambda^L}{\frac{\lambda^H-1}{\lambda^H} \log \lambda^H - \frac{\lambda^L-1}{\lambda^L} \log \lambda^L} < \bar{\omega}_i < 0.
$$

Since $\bar{\omega}_i < 0$ is not economically meaningful, fostering growth through a pure taxation policy is impossible. Incidentally notice that the other condition, $\bar{\omega}_i < 1$, is instead always satisfied.

growth rate of the economy and, given the increase in consumption, on social welfare.

As we discussed above, in R&D-driven growth models the market equilibrium is not efficient as the amount of resources devoted to R&D may be too high or too low with respect to the optimal level. As a result, the optimal policy may require either a subsidy or a tax on R&D investment. However, in these models the effect of tax/subsidy activity is ambiguous on social welfare but is usually clear on the long-run growth rate of the economy. A subsidy indeed raises the growth rate while a tax lowers it. Segerstrom (2000) shows that, under certain conditions, this result can be reversed. Our result goes in the same direction but is driven by a different mechanism. Segerstrom (2000) elaborates on Howitt (1999), who constructs a model with both vertical and horizontal innovation, in which the scale effect is eliminated while the effect of R&D subsidies on long-run growth is preserved (as opposed to Jones, 1995). While Howitt (1999) restricts the analysis to an R&D technology where returns from vertical innovation are higher than those from horizontal innovation, Segerstrom (2000) generalizes this model by relaxing this assumption. Moreover, he allows for complexity in research activities to grow at different speeds. In steady state, given the population growth constraint, a uniform subsidy to both vertical and horizontal research raises (lowers) the quality growth rate by reducing (increasing) the variety growth rate when returns to R&D are higher (lower) in the vertical sector than in the horizontal one. Depending on how strongly the leading-edge quality increases the complexity of research, either of the results will increase the overall growth rate of the economy. For example, if complexity grows relatively fast and the returns from R&D are higher in the vertical sector, a uniform subsidy to R&D lowers the growth rate. The paradox of growth in turn depends on the pace of increasing complexity in research, and on the technology assumed in the vertical and in the horizontal R&D sectors. Our model only considers vertical innovation. It is the asymmetric configuration of technological parameters (the quality jumps) across industries which is responsible for the paradox of growth. Taxation of the less innovative sectors frees resources which will be partly reallocated to the more innovative ones, thus possibly increasing the growth rate despite the reduction in total research investment.

4   Welfare Analysis

We can now go back to the original model developed in Section 2 and derive the optimal steady state growth rate and, in turn, the subsidy/tax scheme

which maximizes steady state welfare. From Subsection 2.5 remember that the value of the instantaneous utility in steady state is given by

$$
\log u(t) = \log C + \int_0^1 \alpha(\omega) \log \alpha(\omega) d\omega + \int_0^1 \alpha(\omega) \log \left[\lambda(\omega)\right]^{j^{\max}(\omega,t)} d\omega
$$
$$
- \int_0^1 \alpha(\omega) \log \left[\lambda(\omega) l_m\right] d\omega
$$

and that

$$
\int_0^1 \alpha(\omega) \log \left[\lambda(\omega)\right]^{j^{\max}(\omega,t)} d\omega = t \int_0^1 i(\omega)\alpha(\omega) \log \lambda(\omega) d\omega.
$$

Our goal is to find the allocation of resources which maximizes steady state welfare, i.e.

$$
\max_{i(\omega)} U \equiv \int_0^\infty e^{-(\rho-n)t} \log u(t) dt
$$

$$
\text{s.t.} \quad 1 = C \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + k \int_0^1 \frac{i(\omega)}{A(\omega)} d\omega
$$

where the resource constraint is expressed in per capita terms and we use the definition $i(\omega) \equiv A(\omega)l_I(\omega)/k$. Solving the integrals we find

$$
U \equiv \frac{1}{\rho-n} \left[\log C - \int_0^1 \alpha(\omega) \log \left[\lambda(\omega)l_m\right] d\omega + \int_0^1 \alpha(\omega) \log \alpha(\omega) d\omega \right]
$$
$$
+ \frac{1}{(\rho-n)^2} \int_0^1 i(\omega)\alpha(\omega) \log \lambda(\omega) d\omega.
$$

From the resource constraint we now substitute for C into U to get

$$
U \equiv \frac{1}{\rho - n}\left[\log\left(\frac{1}{\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega} - \frac{k}{\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}\int_0^1 \frac{i(\omega)}{A(\omega)}d\omega\right) - \int_0^1 \log\left[\lambda(\omega)l_m\right]d\omega\right.
$$

$$
\left. + \int_0^1 \alpha(\omega)\log\alpha(\omega)d\omega\right] + \frac{1}{(\rho-n)^2}\int_0^1 i(\omega)\alpha(\omega)\log\lambda(\omega)d\omega.
$$

Let us now consider the derivative of U with respect to the research intensity in a given industry ω

$$
\frac{\partial U}{\partial i(\omega)} = \frac{-\frac{k}{(\rho-n)A(\omega)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}}{\frac{1}{\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega} - \frac{k}{\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}\int_0^1 \frac{i(\omega)}{A(\omega)}d\omega} + \frac{1}{(\rho-n)^2}\alpha(\omega)\log\lambda(\omega).
$$

Notice that the partial derivative of the total utility does not depend on the specific intensity $i(\omega)$, and is an increasing function in $A(\omega), \alpha(\omega), \lambda(\omega)$. Hence, there will exist one industry (denote it by $\tilde{\omega}$) characterized by $\alpha(\tilde{\omega}), \lambda(\tilde{\omega})$ and $A(\tilde{\omega})$ such that the partial derivative of total utility with respect to it is *always* the highest in the economy. This means that, whatever the distribution of resources between consumption and research, it is always the case that welfare can be improved by reallocating research investment towards industry $\tilde{\omega}$. In turn, optimality requires $i(\tilde{\omega}) = \int_0^1 i(\omega)d\omega$[11]. At this point it is simple to solve for the optimal level of research intensity $i^*(\tilde{\omega})$. In our economy there is now a single research sector, which implies the following total utility

---

[11]A positive mass of R&D expenditure cannot be concentrated in a zero measure industry. However, the result holds as the limit of a discrete case where $\tilde{\omega}$ has non-zero measure. Let I be the total amount of resources employed in R&D industries. Industries are indexed over the interval $[0,1]$ by ω_i, with $\omega_i = i/N$, $i = 0,1,...N$. Then, $I = \sum_{i=0}^{N} i(\omega_i)(\omega_{i+1} - \omega_i) = \sum_{i=0}^{N} i(\omega_i)(1/N)$, where $i(\omega_i)$ is the frequency of the distribution of research across industries. Since the partial derivative of the total welfare is always bigger for $\tilde{\omega}$ independently of the frequency of research in that industry, then it has to be that $i(\omega_i) = 0 \,\forall \omega_i \neq \tilde{\omega}$, so that $I = \sum_{i=0}^{N} i(\omega_i)(1/N) = i(\tilde{\omega})(1/N)$. If we couple this with the fact that from the definition of the integral: $\lim_{N\to\infty} \sum_{i=0}^{N} i(\omega_i)(1/N) = \int_0^1 i(\omega_i)d\omega$, then $\lim_{N\to\infty} i(\tilde{\omega})(1/N) = \int_0^1 i(\omega_i)d\omega$. In the limit, the frequency distribution of research across industries converges to a Dirac delta density centered in $\tilde{\omega}$.

$$U \equiv \frac{1}{\rho - n}\left(\log C - \int_0^1 \log\left[\lambda(\omega)l_m\right]d\omega + \int_0^1 \alpha(\omega)\log\alpha(\omega)d\omega\right)$$
$$+\frac{1}{(\rho-n)^2}i(\tilde{\omega})\alpha(\tilde{\omega})\log\lambda(\tilde{\omega})$$

while the new resource constraint is

$$1 = C\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + k\frac{i(\tilde{\omega})}{A(\tilde{\omega})}. \tag{18}$$

By substituting for C given in the resource constraint into U, and taking the first order condition we can solve for $i(\tilde{\omega})$ and obtain

$$i^*(\tilde{\omega}) = \frac{A(\tilde{\omega})}{k} - \frac{(\rho - n)}{\alpha(\tilde{\omega})\log\lambda(\tilde{\omega})}.$$

Given the optimal amount and distribution of research, we can now move on to discuss the subsidy/tax scheme capable of making the market provide the optimal allocation. First we need to turn off research in all industries different from $\tilde{\omega}$. This can be easily accomplished by imposing the tax rate $\sigma(\omega) = 1\ \forall\omega \neq \tilde{\omega}$. Then we have to look for the value $\sigma(\tilde{\omega}) = 1 - s(\tilde{\omega})$ such that the market equilibrium intensity is exactly equal to $i^*(\tilde{\omega})$. Solving for the market equilibrium $i(\tilde{\omega})$, we obtain[12]

$$i(\tilde{\omega}) = \frac{\frac{A(\tilde{\omega})}{k}\frac{\lambda(\tilde{\omega})-1}{\lambda(\tilde{\omega})}\alpha(\tilde{\omega})s(\tilde{\omega}) - (\rho - n)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}{\frac{\lambda(\tilde{\omega})-1}{\lambda(\tilde{\omega})}\alpha(\tilde{\omega})s(\tilde{\omega}) + \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}.$$

Therefore the optimal subsidy/tax $\sigma^*(\tilde{\omega})$ has to be such that $s^*(\tilde{\omega})$ solves

$$\frac{\frac{A(\tilde{\omega})}{k}\frac{\lambda(\tilde{\omega})-1}{\lambda(\tilde{\omega})}\alpha(\tilde{\omega})s^*(\tilde{\omega}) - (\rho - n)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}{\frac{\lambda(\tilde{\omega})-1}{\lambda(\tilde{\omega})}\alpha(\tilde{\omega})s^*(\tilde{\omega}) + \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega} = \frac{A(\tilde{\omega})}{k} - \frac{(\rho - n)}{\alpha(\tilde{\omega})\log\lambda(\tilde{\omega})}.$$

---

[12]The value $i(\tilde{\omega})$ is found by imposing the arbitrage condition for industry $\tilde{\omega}$ and the new resource constraint (18).

Needless to say, the welfare analysis for the simplified version used for the policy analysis in Section 3 is a particular case of the one above, obtained by posing $A(\omega) = A$ and $\alpha(\omega) = \alpha$. In particular, the social optimum in this case requires that the unique industry where R&D is carried out be exactly the one with the highest quality jump.

5 Concluding Remarks

In the previous pages we have generalized a standard quality-ladder model with increasing complexity in order to encompass a typical trait of real economies, which is the presence of industries characterized by different - preference and technological - fundamentals. We have assumed an R&D sector characterized by industry-specific quality jumps ($\lambda(\omega)$) and arrival rates ($A(\omega)$). While the first assumption makes the mark-up charged by each monopolist (and then profits) vary across industries, the second alters the per-industry profitability of engaging in R&D. Furthermore, as in the standard case, the consumer's utility is represented by a logarithmic Cobb-Douglas function. However, asymmetric utility contributions ($\alpha(\omega)$) of each good are now assumed. The asymmetry in the fundamentals causes prices, market demands and profits to vary across industries. Accordingly, in the stationary equilibrium, an asymmetric composition of actual and expected R&D efforts is necessary to make engaging in R&D in each industry equally profitable. This extension does not alter the comparative statics results obtained in the standard symmetric models.

Our model is also aimed at bringing industrial policy back into the realm of the growth policy debate. We have shown that a policy favoring - either directly or indirectly - industries with higher innovative capacity fosters economic growth and welfare. However, our policy conclusion needs a couple of qualifications.

First, the actual implementation of the policy recommended requires that the policy-maker be both *able* and *willing* to 'pick winners'. Our assumption on the ability to recognize winners, at least in an economy where the structure of the quality jumps is time invariant, is indeed not unrealistic. Given the relation between quality jumps and mark-ups, the sectoral distribution of the $\lambda(\omega)$'s can be easily ascertained. For instance empirical estimates of sectoral markups for U.S. manufacturing can be found in Hall (1988) and more recently in Roeger (1995) and Martin, Scarpetta and Pilat (1996). Cozzi and Impullitti (2005) have calibrated sectoral quality jumps by using these estimates. The sectoral distribution of the $\alpha(\omega)$'s is even easier to know, as $\alpha(\omega)$ represents the market share of industry ω. Finally $A(\omega)$ stands for the expected number

of innovations per unit of time and per unit of labor. By dividing in each period the number of innovations occurred by the number of researchers in industry ω, we can easily obtain an estimate of $A(\omega)$. Willingness to pick winners can instead be threatened by the presence of lobbies. A policy based on the selection of specific industries bears the inevitable risk that the selection criteria be not inspired by economic efficiency and social welfare but rather by the special interests of particular firms capable of redirecting the policy maker's intervention in their favor.

Second, our model is characterized by full employment in every instant of time. An asymmetric policy intervention strengthens the market selection process of the best industries, inducing an instantaneous and zero-cost reallocation of resources (and, hence, of workers) from one industry to another. In the real world such a process can indeed not only take time, but also be socially painful. There may then be calls for social protection in favor of the (R&D) workers in the declining industries, and more generally for policies aimed at smoothing the shift of workers from those industries to the developing ones. The costs of such interventions should then be taken into account when evaluating the opportunity of implementing the policies recommended.

With these *caveats* in mind we have shown that market forces do not provide sufficient incentives to make agents exploit completely the differences in technological and/or preference fundamentals across industries, and that the policy-maker can (and should) intervene, through a sensible industrial policy, to cure over-investment in poor industries and under-investment in promising ones. We have proven that a zero-cost tax/subsidy policy and, under certain circumstances, even a mere tax policy unambiguously raise growth and welfare above the laissez-faire equilibrium.

A   Comparative Statics

Since the zero measure of each industry makes the contribution of the variation of a ω-specific λ to $\bar{C}$ negligible, then $d\bar{C}/d\lambda = 0$. Analogously $d\bar{C}/d\alpha = 0$. Then, for any given ω

$$\frac{dl_I}{d\lambda} = \frac{1}{\lambda^2}\frac{\alpha}{1-s}\bar{C} > 0$$

$$\frac{dl_I}{d\alpha} = \frac{\lambda(\omega) - 1}{\lambda(\omega)(1-s)}\bar{C} > 0$$

If we define

$$\bar{l}_I \equiv \int\limits_0^1 l_I(\omega)d\omega = 1 - \bar{C}\int\limits_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$$

then

$$\frac{d\bar{l}_I}{d\rho} = -\frac{d\bar{C}}{d\rho}\int\limits_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega \text{ and } \frac{d\bar{l}_I}{dn} = -\frac{d\bar{C}}{dn}\int\limits_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega$$

Therefore, since

$$\frac{d\bar{C}}{d\rho} = \frac{k\int\limits_0^1 \frac{1}{A(\omega)}d\omega}{\int\limits_0^1 \frac{\alpha(\omega)(\lambda(\omega)-s))}{\lambda(\omega)(1-s)}d\omega} > 0 \text{ and } \frac{d\bar{C}}{dn} = -\frac{k\int\limits_0^1 \frac{1}{A(\omega)}d\omega}{\int\limits_0^1 \frac{\alpha(\omega)(\lambda(\omega)-s))}{\lambda(\omega)(1-s)}d\omega} < 0$$

we can finally state

$$\frac{dL_I}{d\rho} < 0 \text{ and } \frac{dL_I}{dn} > 0.$$

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