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# The dual process account of reasoning: historical roots, problems and perspectives [1]

## Abstract

Despite the great effort that has been dedicated to the attempt to redefine expected utility theory on the grounds of new assumptions, modifying or moderating some axioms, none of the alternative theories propounded so far had a statistical confirmation over the full domain of applicability.
Moreover, the discrepancy between prescriptions and behaviors is not limited to expected utility theory. In two other fundamental fields, probability and logic, substantial evidence shows that human activities deviate from the prescriptions of the theoretical models.
The paper suggests that the discrepancy cannot be ascribed to an imperfect axiomatic description of *human choice*, but to some more general features of *human reasoning* and assumes the "dual-process account of reasoning" as a promising explanatory key. This line of thought is based on the distinction between the process of *deliberate reasoning* and that of *intuition*; where in a first approximation, "intuition" denotes a mental activity largely automatized and inaccessible from conscious mental activity.
The analysis of the interactions between these two processes provides the basis for explaining the persistence of the gap between normative and behavioral patterns. This view will be explored in the following pages: central consideration will be given to the problem of the interactions between rationality and intuition, and the correlated "modularity" of the thought.

## *1 Introduction: rationality and intuition*

Beginning with the work of Allais in the early 1950s, psychologists and economists have discovered a growing body of evidence on the discrepancy between the prescriptions of expected utility theory and real human behavior. The accumulated experimental evidence has reached a critical point in which a large number of economists share the sharp opinion expressed by Reinhardt Selten (1999):

"Modern mainstream economic theory is largely based on an unrealistic picture of human decision making. Economic agents are portrayed as fully rational Bayesian maximizers of subjective utility. This view of economics is not based on empirical evidence, but rather on the simultaneous axiomization of utility and subjective probability. In the fundamental book of Savage the axioms are consistency requirements on actions with actions defined as mappings from states of the world to consequences (Savage 1954). One can only admire the imposing structure built by Savage. It has a strong intellectual appeal as a concept of ideal rationality. However, it is wrong to assume that human beings conform to this ideal."

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[1] I am very grateful to Laura Arrighi for her careful assistance in the preparation of this work. Correspondence concerning the present chapter should be addressed to Massimo Egidi, Luiss Guido Carli University, via Pola 12 - 00198 Rome. e-mail: megidi@luiss.it.

Despite the great effort that has been dedicated to the attempt to redefine expected utility theory on the grounds of new assumptions, modifying or moderating certain axioms, none of the alternative theories propounded so far had a statistical confirmation over the full domain of applicability. Moreover, the discrepancy between prescriptions and behaviors is not limited to expected utility theory. In two other fundamental fields, probability and logic, substantial evidence shows that human activities deviate from the prescriptions of the theoretical models.
Therefore one may suspect that the discrepancy cannot be ascribed to an imperfect theoretical description of *human choice*, but to some more general features of *human reasoning*. Along this line, implicitly held by D. Kahnemann in his Nobel Lecture, one of the more innovative approach is the recent development of the dual-process account of reasoning. This line of thought is based on the distinction between the process of *deliberate reasoning* and that of *intuition*; where in a first approximation, "intuition" denotes a mental activity largely automatized and inaccessible from conscious mental activity.
The analysis of the interactions between these two processes provides the basis for explaining the persistence of the gap between normative and behavioral patterns: interestingly, this approach echoes the original view of Laplace and other mathematicians who paved the way for the theory of probability, insofar as they considered intuition as an internal psychical force deviating individual's choices from pure rationality.
This view will be explored in the following pages: central consideration will be given to the problem of the interactions between rationality and intuition, by describing the experimental and neurological basis of the interactions and reading this approach from the perspective of a cognitive approach to rationality.

## *2 Historical roots*
### *2.1 Probability*

Historians usually consider a letter that Blaise Pascal wrote to Pierre Fermat in 1654 about a gambling problem as being the founding document of mathematical probability. The correspondence between Pascal and Fermat was exclusively devoted to gambling problems, and for a long period the most relevant probabilistic problems solved by mathematicians were of this sort. It is perhaps due to this fact that, since its early beginnings, the theory of probability was intended as a powerful tool to help players and gamblers see beyond the illusions, prejudices and emotions influencing their evaluations and to provide them with a "rational" strategy for gambling. Therefore, probability theory developed as a rational approach to risk and uncertainty, and the great mathematicians that gave fundamental contributions to this theory in the XVIII and XIX century implicitly assumed that their models should have allowed persons exposed to risky decisions to behave rationally. Interestingly, the efforts of mathematicians were only partially successful:

"Some crusading spirits, Daniel Defoe among them, hoped that mathematicians might cure the reckless of their passions for cards and dice with a strong dose of calculation (Defoe 1719). The mathematicians preached the folly of such pursuit along with the moralists, but apparently most gamblers had little appetite for either sort of edification."(Gingerenzer et al. 1989, p. 19)

Despite the efforts of mathematicians, the progressive edification of the theory of probability did not remove some "irrationalities" in gamblers' behavior: even today there are some typical lotteries' conditions in which gamblers exhibit systematic discrepancies from the normative prescriptions of the theory. The best known phenomenon in this respect is now called "gambler's fallacy": when the sequence of numbers extracted in repetitive runs of a lottery appears to gamblers not to be random. In a random sequence of tosses of a fair coin, for example, gamblers expect sequences where the proportion of heads and tails in any short segment stays far closer to .50 than probability theory would predict. In other words, gamblers also expect that a short sequence of heads on the toss of a coin will be balanced by a tendency for the opposite outcome (e.g., tails) and will bet accordingly.

That some gamblers' beliefs about probability are systematically biased becomes more and more evident in parallel with the progressive construction of the theory. Since his "Essai Philosophique sur les Probabilités" was published in 1796, Pierre Simon de Laplace was already concerned with errors of judgment to such a point that he included a chapter regarding "Des illusions dans l'estimation des probabilités". It is here that we find the first published account of the gambler's fallacy.

« Lorsqu'à la loterie de France un numéro n'est pas sorti depuis longtemps, la foule s'empresse de le couvrir de mises. Elle juge que le numéro resté longtemps sans sortir doit, au premier tirage, sortir de préférence aux autres. Une erreur aussi commune me parait tenir à une illusion par laquelle on se reporte involontairement à l'origine des événements. Il est, par exemple, très peu vraisemblable qu'au jeu de croix ou pile on amènera croix dix fois de suite. Cette invraisemblance qui nous frappe encore, lorsqu'il est arrivé neuf fois, nous porte à croire qu'au dixième coup pile arrivera. Cependant le passé, en indiquant dans la pièce une plus grande pente que pour pile, rend le premier dé ces événements plus probable que l'autre; il augmente, comme on 1'a vu, la probabilité d'amener croix au coup suivant. » (Laplace 1814, introduction, CXIII) [2]

The idea that humans had to learn to "calculate" their probabilities to make the right decision and that pure rationality might be "deviated" by emotion and intuition permeated the whole XIX century. Laplace maintains that the introduction of probability theory may help individuals to rationally correct the *illusions* generated by the "sensorium"(in today's terms the cognitive system); according to Laplace, the brain's processing of perceptions leads to an internal representation of reality that can be misleading. While admitting that his analysis of the role of perception, memorization, attention and other psychological elements, is largely "imperfect" - the state of psychology at that time - Laplace gives many vivid illustrations of the fact that psychological processes may produce biased evaluations. (Laplace 1814, introduction CXII, CXV) This point has been re-evocated in recent literature on cognitive psychology and neuro-economics, which we will consider later (Camerer, Loewenstein, Prelec, 2004), limiting the discussion to a particular kind of potential sources of biases, the "illusions" of Laplace's analysis.

### *2.2 Logic*

Deviations from the logically correct reasoning, or "fallacies", have been widely analyzed since the XII Century; with the translation into Latin of *De Sophisticis Elenchis* (the last part of Aristotle's *Organon*) many scholars attempted to detect, describe, classify and analyze fallacious arguments. According to Hamblin,[3] "A fallacious argument, as almost every account from Aristotle onwards tells you, is one that seems to be valid but is not so"; how can it be that a wrong argument *seems* to be valid? As the studies in psychology of reasoning have shown, this discrepancy comes from the imperfect control by the human mind over the reasoning process: on the one hand errors may be inadvertently generated by individuals while developing the thinking process, on the other hand, hidden errors in a reasoning are detected with great difficulty. Playing on this difficulty, errors may also be consciously generated by one of the two opponents in a litigation to take advantage over the other party. As a matter of fact, a substantial motivation for identifying and classifying fallacies in

[2] Gamblers' fallacies have been carefully explored in recent times; see, among others, Tversky and Kahneman (1971) Roger (1998), Clotfelter and Cook (1993), Ayton and Fisher (2004)
[3] Hamblin (1970), p. 12

the Middle Ages was to avoid this eventuality and achieve fair conduct of the parties involved in a *dialectic* discussion, i.e. a discussion at the beginning of which it is unclear where the truth is. With a semantic shift from faulty to unfair, *"dialectic"* is nowadays meant as a potentially malicious reasoning, artificially created to persuade those listening; in medieval thought, the word *dialectic* was not necessarily labelled as negative: the existence of conflicting opinions was implicitly recognized as an intrinsic, unavoidable element of human reason. In the case of juridical questions the dialectic disputations were strictly regulated (Stump, 1989, Errera, 2006) and the classification of fallacies was therefore important to distinguish between valid reasoning and hidden (malicious or not) errors (Benzi, 2002). (See also the Appendix).

The distinction between logic and psychology of human reasoning grew increasingly clear, and in XVII century,

"Bacon said psychological and cultural factors – which he used to call "idols" - to be sources of errors in reasoning, as they generate distortions in human understanding. That view permeated the XVII century philosophical elaboration (see Locke, Cartesius, Arnauld and Nicolle, who co-authored the Port-Royal Logic), which considered psychology – not logic – to be the discipline dealing with errors […].

A turning point, that can be traced to Gottlob Frege and dates back to the end of the XIX century, was anti-psychologism in logic; this thesis draws a sharp distinction – which is typical in the prevalent contemporary view – between the roles of logic and psychology: logic should be conceived as the science of judgment, studying principles which allow one to assess and understand good, correct, valid reasoning and to distinguish it from its opposite; other disciplines - and especially psychology, but also ethnology and perhaps also other kinds of cultural analysis – are conceived as sciences studying the human reasoning processes, or people's mental processes. (Benzi, 2005, p.1).

### *2.3 Decision making*

A similar "anti-psychologistic" movement marked the evolution of the theory of rational decision making. Also in this field, the discrepancies between normative prescriptions and behavior were well known since the beginning but were considered a minor problem. In 1952, Friedman and Savage published a famous study in which they constructed an expected utility curve which, they claimed, provided a reasonably accurate representation of human behavior at the aggregate level. In this paper they consider the individual's expression of preferences as irrelevant and consequently not to be submitted to empirical control; deviations from rational decision making were supposed to be detectable only at the aggregate level, and many attempts were made to justify the persuasion that, on average, individuals behave rationally; in particular, Friedman suggested an evolutionary defense of full rationality by claiming that those who failed to conform to rational behavior would be gradually excluded by market selection . Therefore, according to this view, the psychological aspects of decision making were not considered worthy of investigation, because non-rational behaviors were thought to be a minor aspect of market economies.

The most serious challenge to this belief emerged with Allais' experiments. In 1953 Maurice Allais carried out experiments on individual preferences that showed systematic deviations from theoretical predictions. Drawing on his critical paper a growing body of evidence has revealed that individuals do not necessarily conform to the predictions of the theory of decision making but seem to depart from them systematically. The literature devoted to this issue, nowadays called the "heuristic and bias program" after the pioneering studies by Kahneman and Tversky, is incredibly vast; biases and deviations from the theoretical predictions have been tested by an enormous number of independent researchers and well confirmed "effects", like "preference reversal", "conjunction fallacy", "framing effect", and many others have been detected.

The XIX century view that considered logic and probability as theories representing the "laws" of human thought has therefore been severely challenged by experiments, and to a certain extent has been superseded by the clear-cut distinction between the normative and the behavioral.
This view seems to fit heuristic and bias program, because it holds a clear-cut differentiation between norms (rationality) and behaviors (reasoning). However, it opens a new problem: if there are important and systematic discrepancies between norms and behaviors, is it still realistic to aspire to predict human behavior?
If we do not accept that the gap between behavior and normative prescriptions indicates systematic human irrationality, a new, more accurate and sophisticated explanation of rational behavior can be provided. Scholars in the field of expected utility theory have followed two different paths: on the one hand, they have tried to modify some of the axioms of the theory in order to find predictions closer to real behaviors; on the other hand, they made a systematic attempt to respond to this critical problem by redefining the foundations of decision making via a deeper exploration into the nature of human intelligence; the latter method implies rethinking the relations between economics and psychology, along the lines initiated by H. Simon with his "bounded rationality" approach. (Simon, 1971). The road for a deeper exploration of psychological foundations of human thought has also been followed also in the fields of logic and probability, whereas very few attempts have been made to modify the axioms of these two theories in order to predict the real behavior of individuals.[4]
Since most evidence of the discrepancies occur within the framework of decision making and, more precisely, of expected utility theory, we will start from the phenomena which emerged in this field.

## *3 The separation between psychology and economics*

Since its first appearance, the notion of utility has been grounded in the psychological features of human choice. As reminded by Schumpeter in his *History of Economic Analysis*,
"In principle utility was considered a psychic reality, a sensation that became evident from introspection, independent of any external observation […] with directly measurable proportions. I believe this was Menger and Böhm-Bawerk's opinion".
The effort of economists consisted in building up a formal model – the utility theory – of human decisions, *descriptively valid*, to be used as the building block of the new emerging "neoclassical" theory of economic decision.
The separation of economics from psychology emerged only after heated debate in the first decades of XX century, with the full development of the theory of rational decision, that became progressively considered the core of the entire economic theory. A classic exposition of the theory is contained in the "Essay on the Nature and Significance of Economics" (1932) by Lionel Robbins, in which the author defines economics as the *science of choice*. According to this approach, 'calculation' is totally independent from individual psychological processes, and it takes place irrespective of the mental processes of individuals. The role of rational decision making theory is viewed as being fundamentally normative; such a view was shared by the vast majority of economists for about a century, basing on the assumptions and the definitions provided by Robbins. He codified the idea that economics and psychology were indeed autonomous disciplines, with independent scientific statutes.

The completion of the theory was achieved through the publication of "Theory of Games and Economic Behavior" by von Neumann and Morgenstern in 1944, in which the notion of expected utility was incorporated into the theory of choice. The original approach to decision making under risky conditions can be traced to a famous paper of 1738 by which Daniel Bernoulli introduced the idea of expected utility (see also Laplace, introduction, XX, XXV).

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[4] In his article "*Wason's Cards: What Is Wrong?*", Pei Wang has given an explanation of the phenomenon in terms of his non Aristotelian NARS inference system.

Prior to this paper, the rule suggested by mathematicians in order to provide a rational prescription for gamblers was to compare expected gains with expected losses, and claimed (later) by Laplace:

«Dans une série d'événements probables, dont les uns produisent un bien et les autres une perte, on aura l'avantage qui en résulte, en faisant une somme des produits de la probabilité de chaque événement favorable par le bien qu'il procure,- et en retranchant de-cette somme celle des produits de la probabilité de chaque événement défavorable par la perte qui y est attachée. Si la seconde somme l'emporte sur la première, le bénéfice devient perte, et l'espérance se change en crainte. » (Laplace, introduction XIX).

This view implicitly considers the balance between gains and losses (weighted by probabilities), i.e. the "expected value" of the prospect as the leading element to take a rational decision.
In the same period, Bernoulli propounded a more sophisticated means of analyzing how a gambler may have taken a rational decision. To solve a chance game devised by his cousin Nicholas in 1713, known as the St. Petersburg paradox, Daniel Bernoulli suggested that the more a person is wealthy the less "useful" he considers increments in income, so that a gain would increase utility less than a loss of the same magnitude would reduce it. As regards a person with such an attitude to risk, it can be easily proved that the utility of the expected value of a fair gamble is lower than the utility of not playing; a gambler that behaves in this way is classified as "risk adverse", because he prefers a certain given outcome to an uncertain outcome of the same expected value; the opposite behavior is called "risk proclivity".
Bernoulli's idea implicitly assumes that in order to predict a gambler's behavior we should compare the expected utility from engaging in the gamble to the expected utility from refusing to play. That suggests that we can compute the utility of a risky prospect as the sum of the utilities of the gains and losses weighed by the probabilities. This is in fact the notion of expected utility, explicitly suggested by von Neumann and Morgenstern in" Theory of Games and Economic Behavior" about two centuries after Bernoulli's work. With the publication of the "Theory of Games" in 1944, the formal incorporation of risk into economic theory opened the door to important applications to game theory and other relevant fields.
After von Neumann and Morgenstern's book, the notion of expected utility entered a vast number of applied fields in economics, notwithstanding the scarce attention devoted by scholars to the empirical aspects: one well known example is a paper by Friedman and Savage published in 1948. Here the authors try to construct an expected utility curve that is supposed to represent the individual average behavior in the face of risky choices reasonably well. They combine two basic characteristics of the everyday (average) behavior of individuals: on the one hand, the fact that a large number of individuals risk small sums taking part in lotteries, i.e. are risk takers; on the other hand, the fact that those same individuals usually insure themselves, which means that are risk adverse. The first property requires a convex utility function, while the second requires a concave function. To take both these features of people's behavior into account, Friedman and Savage (1952) suggest that the expected utility curve must have an "S" shape. Nevertheless, Friedman constructs a general shape of the curve without testing the two characteristics on a real population. In fact, precise data on insurance were not taken into consideration and reliable data relating to the income of gamblers do not even exist. Therefore, despite the great potentialities and the vast number of important applications, a sound testing of the expected utility assumptions was missing until a paper by Maurice Allais was published in 1952.
At a symposium held in Paris, he presented two studies in which he criticized the descriptive and predictive power of the choice theory upheld by the "American school" and by Friedman in particular (Allais, 1953). He exposed experiments in which subjects faced with alternative choices under conditions of risk systematically violated the assumptions of the expected utility theory. Violations like the one discovered by Allais imply inconsistency in the choices of individuals: the question is such violations are systematic or not: in fact, rationality does not mean that the behavior

of individuals fits perfectly normative prescriptions of the expected utility theory; it means that individuals do not systematically deviate from normative prescriptions; that is perhaps why the initial reaction to Allais' experimental result was lukewarm: at the beginning the experiments were thought to reflect extreme cases rather than systematic deviations from rational behavior, caused by the particularly large sums at stake; the experimental result did not have much impact for a while and its recognition was categorized under the label of "Allais paradox" for many years; it was only years later, after repeated experiments with players to whom modest sums were conceded, that the phenomenon emerged once again and its systematic nature was finally recognized.
Since experiments showed a violation of the axioms, it was natural to suspect that this violation resulted from stringent characteristics imposed by the definition of the expected utility function. The reactions to Allais's experiments led in fact to the creation of more sophisticated versions of the utility theory in conditions of uncertainty, modifying or moderating certain axioms or generalizing their characteristics. Many such proposals arose, especially from the mid 70s onwards, all of which were based on the attempt to relax or slightly modify the original axioms of expected utility theory. Among the best known are perhaps the Weighted Utility Theory (Chew and MacCrimmon1979), that assumes a weaker form of the axiom of independence; the Rank Dependent Model (Quiggin, 1993) and the Disappointment Theory, suggested by Gul (1991). Again, the approach propounded by Mark Machina (1982) redefines the theory without resorting to the independence axiom. Finally, the Regret Theory proposed by Loomes and Sugden (1982) has received careful attention. None of these proposals had a statistical confirmation over the full domain of applicability. A comprehensive classification of the different alternatives is provided by Camerer (1995).
Therefore this response to Allais' criticism did not prove successful (or at least not thus far). Only gradually economists came to recognize the complex consequences of the systematic discrepancies between the predictions of expected utility theory and economic behavior; this opened an "embarassing" and still unsolved question: how to model in a more realistic way human behavior in economics.
An alternative approach to the question was opened – in the same period as Allais' experiments - by the idea of "bounded rationality" advanced by Simon, who explored the limits of rationality and began to shed light on the decision mechanisms by investigating the cognitive processes involved. Modifications of the axioms of the expected utility theory seemed to him and to many other scholars unfit to hold out the prospect of a future overall theory of decision-making: the alternative root was the complexity of the underlying psychological phenomena.

## *4 Bounded rationality: a new road map*

The research conducted by Kahneman and Tversky was directed to make crucial reference to the mental processes involved. Their approach coherently fits within the analytical frame of Simon's Bounded Rationality, as the two authors explicitly acknowledge. Moving beyond Allais' experiments, they made clear that under risky decisions, individuals show a systematic inconsistency related to the framing of the decision. Among an incredibly vast number of important families of biases they have discovered, the most celebrated is the so-called framing effect, in which individuals exhibit propensity to risk when a choice is presented in terms of loss, while they show aversion to risk if the same problem is presented in terms of gain.
The proposal that Kahneman and Tversky advance to explain the results of the experiments is twofold: on the one hand, they advance the "prospect theory", which assumes a strong inconsistency in individuals' behavior, on the other, they pave the way for a new explanation of biases based on a *dualistic approach to reasoning.* Prospect theory is analyzed by Giraud in the present volume; therefore we will consider this first strand first and than the second in more detail.

The framing effect can be interpreted as saying that we perceive outcomes as gains and losses relative to a reference point; this target point usually corresponds to the current asset position, but might also be an aspired wealth position. The representation in terms of losses and gains relative to a reference point is crucial, because the risk-taking attitude of individuals proves to be different above versus below the reference point. More precisely, in the prospect theory, the subjective value of an edited prospect is assumed to be convex for losses and concave for gains (Kahneman and Tversky, 1979).
The introduction of the idea of a reference point is in accordance with a widespread phenomenon characterizing perception: the perception is *reference-dependent*, i.e. the attributes of a stimulus reflects the contrast between that stimulus and a context of prior and concurrent stimuli. By transferring this basic property of perception to the field of choice, Kahneman claims that *Bernoulli was wrong*, because he assumed the attitude to risk to be reference-independent.

"The facts of perceptual adaptation were in our minds when Tversky and I began our joint research on decision making under risk. Guided by the analogy of perception, we expected the evaluation of decision outcomes to be reference-dependent. We noted, however, that reference-dependence is incompatible with the standard interpretation of Expected Utility Theory, the prevailing theoretical model in this area. [….] The idea that decision makers evaluate outcomes by the utility of final asset positions has been retained in economic analyses for almost 300 years. This is rather remarkable, because the idea is easily shown to be wrong; I call it Bernoulli's error.
Bernoulli's model is flawed because it is reference-independent: it assumes that the value that is assigned to a given state of wealth does not vary with the decision maker's initial state of wealth. This assumption flies against a basic principle of perception, where the effective stimulus is not the new level of stimulation, but the difference between it and the existing adaptation level. The analogy to perception suggests that the carriers of utility are likely to be gains and losses rather than states of wealth, and this suggestion is amply supported by the evidence of both experimental and observational studies of choice" (Kahneman, 2002).

The key aspect emerging from the framing effect is that individuals give different answers to isomorphic problems, i.e. they evaluate differently two representations of the same problem. Kahneman suggests that framing must be considered a special case of the more general phenomenon of *dependency from the representation*: the question is how to explain the fact that different representations of the same problem yield different human decisions. While the notion of "representation" has received a deep treatment in Artificial Intelligence literature and in cognitive psychology, where it has been put in relation with the notions of abstraction and categorization (Giunchiglia, Villafiorita, Walsh, 1997), there is still a lack of experimental works on these issues. We owe the first studies in this field to Simon and Hayes (1976): they have explored the "representation" problem by constructing a collection of transformation puzzles, all formally identical to the tower of Hanoi problem, and found that 'problem isomorphs' varied greatly in difficulty. The authors note that "It would be possible for a subject to seek that representation which is simplest, according to some criterion, or to translate all such problems into the same, canonical, representation..." (Simon & Hayes, 1976, p 183) but subjects will adopt the representation that constitutes the most straightforward translation of the original description of the puzzle.
"There is also a piece of evidence […] that a subject may abandon a difficult representation for one that makes solving the problem easier" (Simon & Hayes, 1976, p 190).

There is evidence in many other problem solving tasks that, to reduce the mental load, individuals will try to simplify the problem's representation. This implies that they try to use prior knowledge to re-codify in a simpler way a given problem. Therefore the process of recalling the memorized knowledge is a crucial aspect of problem representation and may be an important source of bias; in fact *accessibility*, i.e. the ease with which a memorized element comes to the mind (Higgins,1996) is not guided by a controlled process; there is not a mechanical "data mining" process, a mechanical introspection with which individuals could decide which memorized element to recall to conscious attention and which not. Therefore

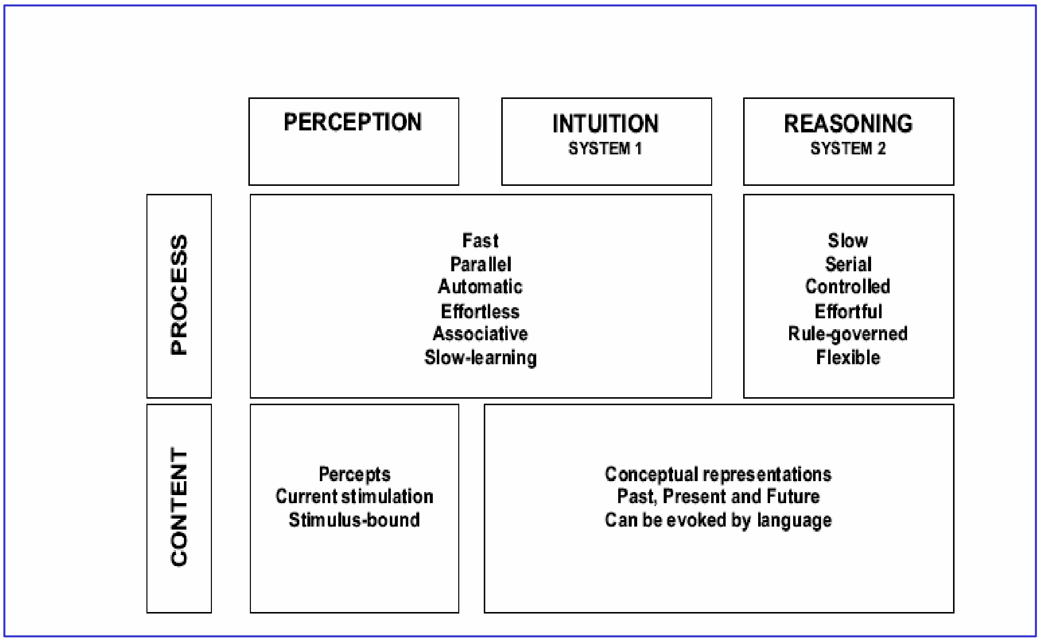
"Invariance cannot be achieved by a finite mind. The impossibility of invariance raises significant doubts about the descriptive realism of rational-choice models (Tversky & Kahneman, 1986). In the absence of a system that reliably generates appropriate canonical representations, intuitive decisions will be shaped by the factors that determine the accessibility of different features of the situation. Highly accessible features will influence decisions, while features of low accessibility will be largely ignored. Unfortunately, there is no reason to believe that the most accessible features are also the most relevant to a good decision." (Kahneman, 2002).

## *5 The dual approach to reasoning*

As hinted above, the framing effect may be considered as a particular case of the more general phenomenon of *dependency from representation*: different representations of the same problem may lead individuals to different answers to the same problem. How can we explain this phenomenon? There are two main answers in the literature: on the one hand, the claim that *representation governs accessibility*: the relevance of an element is induced by the features of representation. This means that, to some extent, a representation (a frame) leads us to (unconsciously) recall from long-term memory items that are relevant to the solution of a given task. As a consequence, precodified knowledge may be filtering and distorting the path to achieve a solution and may induce individuals to inadvertently commit errors.
A second line which will be illustrated in section 6, claims that a large number of mental processes are governed by special-purpose systems—often called 'modules' dedicated to solving domain-specific problems; following this view, different representations of the same problem may trigger different reactions because are processed by different *domain specific* modules.
To fully understand these lines of research, we must resort to the *dual model of thinking*, an approach common to most psychologists in the present day. The dual view is based on the wide evidence that a large part of neural activity is related to 'automatic' processes, which are faster than conscious deliberations and which occur with little or no awareness of effort.
Thinking is supposed to be composed of two different cognitive processes: on the one hand a controlled, deliberate, sequential and effortful process of calculation; on the other a non-deliberate process, which is automatic, effortless, parallel and fast. The two processes have been described in many different ways, by different authors, but there is considerable agreement nowadays among psychologists on the characteristics that distinguish them.
The distinction was raised by Posner and Synder (1975) by wondering what level of conscious control individuals have over their judgements and decisions; among other authors who considered this question, Schneider and Shiffrin (1977) defined the two processes respectively as "automatic" and "controlled"; since then, many analogous two-system models have been developed under different names, as discussed by Camerer (2004). Stanovich and West (2000), call them respectively System 1 and System 2.[5]
The question is also addressed by Kahneman in his Nobel Lecture, where he calls the two modes of thinking respectively intuition (automatic) and reasoning (controlled) (see Fig.1)

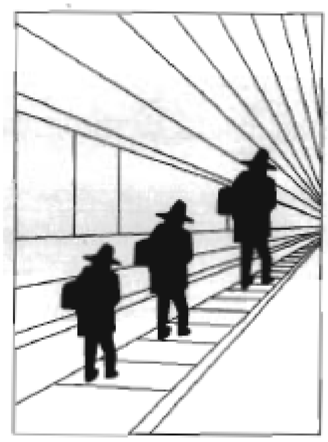
Fig.1

[5] The dualistic approach to cognition boasts a very rich literature; among the contributions relevant to the present discussion, see Bargh, J. A., Gollwitzer, P. M.(1994) ,Bargh, J. A., Chen, M., & Burrows, L. (1996), Chartrand, T. L., Bargh, J. A. (1996),(1999), Chen, M., Bargh, J. A. (1997), (1999), Bargh, J. A., Chartrand, T. L. (1999), Bargh J. A., Ferguson M.J. (2000), Evans (2003).




Kahneman emphasizes the strong similarities between perception and intuition, both being automatic and difficult to control, and the consequent similarities between biases in perceptions and biases in intuitive judgments. Biases in perceptions are *reference-dependent*: as Fig. 2 shows, people consider the three individuals walking along a pathway to be of different size, whereas in fact they are identical: the reference is "disturbing" the evaluation of the size of the individuals, and this bias is automatic. Very few people see the three individuals as identical, but everybody will recognize this by turning the figure upside down.

Fig. 2



"From its earliest days, the research that Tversky and I conducted was guided by the idea that intuitive judgments occupy a position – perhaps corresponding to evolutionary history – between the automatic operations of perception and the deliberate operations of reasoning. Our first joint article examined systematic errors in the casual statistical judgments of statistically sophisticated researchers (Tversky & Kahneman, 1971). Remarkably, the intuitive judgments of these experts did not conform to statistical principles with which they were thoroughly familiar."

It has been argued that brain activities corresponding to automatic and controlled processes develop in distinguishable areas. Patients with lesions in the prefrontal cortex have difficulties with decision

making and problem solving in ill-structured situations, particularly problem types involving planning and look-ahead components. Controlled processes seem to correspond to neural activations mainly in the orbital and prefrontal parts of the brain. (Goel, V., Grafman, J.,1995, 2000), (Lieberman, Gaunt, Gilbert and Trope, 2002).
“The prefrontal cortex is sometimes called the "executive" region, because it draws inputs from almost all other regions, integrates them to form near and long-term goals, and plans actions that take these goals into account (Shallice and Burgess, 1998). Regions that support cognitive automatic activity are concentrated in the occipital, parietal and temporal parts of the brain. The amygdala, buried below the cortex, is responsible for many important automatic affective responses, especially fear.” (Camerer, Loewenstein and Prelec, 2005)

The intuitive process, which automatically elicits prior knowledge, is therefore considered as a basic source of errors in reasoning; many experiments show that the cognitive self-monitoring, i.e. the control of the deliberate system over the automatic one, is quite light and allows automatic thought to emerge almost without control. According to Kahneman, errors in intuitive judgments involve failures in both systems: the automatic system, that generates the error, and the deliberate one, which fails to detect and correct it (1982). Along this line, Camerer et al. (2005) hold that because the person has little or no introspective control over the automatic system, intuitive judgments are not the outcome of a fully deliberate process, and do not conform to normative axioms of inference and choice.

## *6 Automatic reasoning and modularity*

According to Schneider and Shiffrin, automatic processing is activation of a *learned sequence* of elements in long-term memory that is initiated by appropriate inputs and then proceeds automatically without subject control, without stressing the capacity limitations of the system, and without necessarily demanding attention. It may happen that the sequences are the outcome of a perfectly rational reasoning, like for example rational strategies to solve simple problems. Their memorization in long-term memory leads to an automatic retrieval and use, in contexts that are not necessarily identical to the situations in which they originated.
The experimental data available on puzzle solving, for example, show that most individuals, once they have been able to identify one strategy and use it repetitively until it becomes familiar, do not abandon it even in new contexts where better strategies are available.
This tendency has been proved by Luchins (1942) and Luchins and Luchins (1950) who conducted experiments with subjects exposed to mathematical problems that had solutions at different levels of efficiency. The authors show that subjects, having identified a simple solution of a task in a given context, may “automatically” and systematically use such a solution applying it also to contexts where it proves to be sub-optimal. This process is called “mechanization of thought”.
They used water jar problems where participants had three jars of varying sizes and an unlimited water supply and were asked to obtain a required amount of water (Fig. 3). Everyone received a practice problem. People in the experimental group then received five problems (problems 2-6) prior to critical test problems (7, 8, 10, and 11). People in the control group went straight from the practice problems to problems 7-11. Problems 2-6 were designed to establish a "set" (Einstellung) for solving the problems in a particular manner (using containers b-a-2c as a solution). People in the experimental group were highly likely to use the Einstellung Solution on the critical problems even

though more efficient procedures were available. In contrast, people in the control group used solutions that were much more direct.

Fig. 3

| Problem | Given Jars of the Following Sizes | | | Obtain the Amount |
|---|---|---|---|---|
| | A | B | C | |
| 1 | 29 | 3 | | 20 |
| 2 Einstellung 1 | 21 | 127 | 3 | 100 |
| 3 Einstellung 2 | 14 | 163 | 25 | 99 |
| 4 Einstellung 3 | 18 | 43 | 10 | 5 |
| 5 Einstellung 4 | 9 | 42 | 6 | 21 |
| 6 Einstellung 5 | 20 | 59 | 4 | 31 |
| 7 Critical 1 | 23 | 49 | 3 | 20 |
| 8 Critical 2 | 15 | 39 | 3 | 18 |
| 9 | 28 | 76 | 3 | 25 |
| 10 Critical 3 | 18 | 48 | 4 | 22 |
| 11 Critical 4 | 14 | 36 | 8 | 6 |

Possible Answers for Critical Problems (7, 8, 10, 11)

| Problem | Einstellung Solution | Direct Solution |
|---|---|---|
| 7 | 49 — 23 — 3 — 3 = 20 | 23 — 3 = 20 |
| 8 | 39 — 15 — 3 — 3 = 18 | 15 + 3 = 18 |
| 10 | 48 — 18 — 4 — 4 = 22 | 18 + 4 = 22 |
| 11 | 36 — 14 — 8 — 8 = 6 | 14 — 8 = 6 |

As seen above, Luchins and Luchins show that when subjects have identified the best solution of a task in a given context, they automatically transfer it to contexts where it is sub-optimal. The experiments demonstrate that, once a mental computation deliberately performed to solve a given problem has been repeatedly applied to solve analogous problems, it may become "mechanized". Mechanization enables individuals to pass from deliberate effortful mental activity to partially automatic, unconscious and effortless mental operations.

Therefore, the experiments by Luchins and Luchins fully match the distinction between controlled and automatic processes: they show how a process of controlled reasoning – typically composed of slow, serial and effortful mental operations – comes to be substituted by an effortless process of automatic thinking. The sequences, once memorized, can be considered as the building blocks of the intuitive process.

Luchins' experiments allow us to provide evidence that elementary cognitive skills may be stored in memory as *automatic routines*, or *programs* that are triggered by a pattern matching mechanism and are executed without requiring mental effort or explicit thought. In some contexts these specialized skills, automatically activated, have been called *specialized modules*.

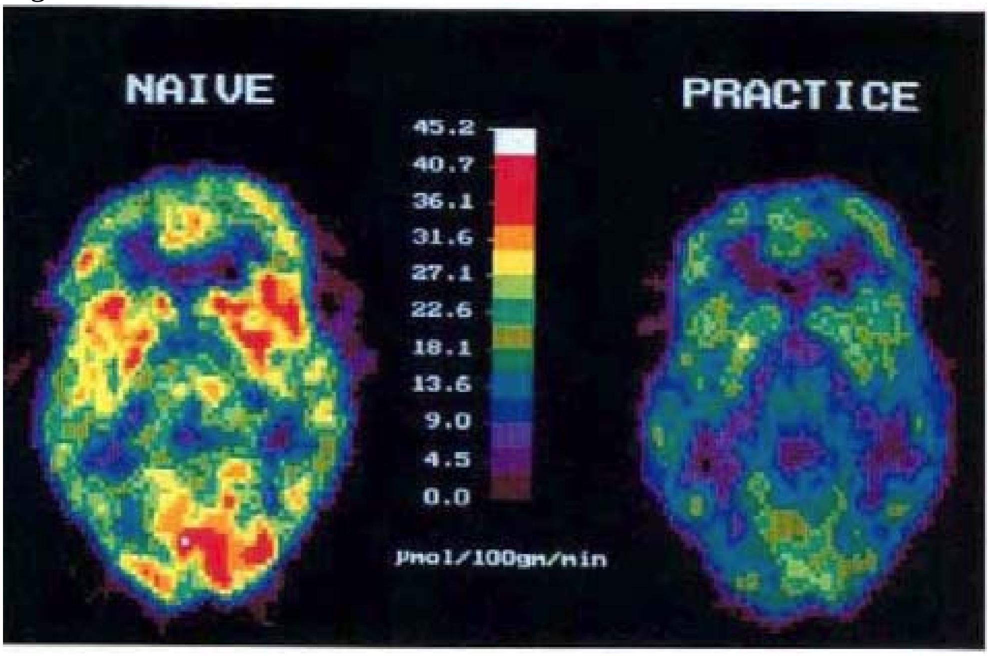
It is important to note that this notion of specialized modules neither coincides with the notion of modules as innate mental devices proposed by Fodor (1983), nor does it fit the definition proposed by evolutionary psychologists who argue that mental modules are pervasive and the products of natural selection (Cosmides and Tooby,1992).

Luchins' experiments show how the process of acquisition of specialized modules - intended as automatic decision-action sequences - takes place; they form gradually, as a result of repeated

experience, and constitute specialized skills that, if applied to the original problem, give rise to an effortless response; the setting up of specialized skills is the explanation for experts being able to elaborate strategies for solving problems more efficiently than novices.
The essential role of *specialized modules* is testified by the experiments with chess players. An approximate estimate of chess grandmasters' capacity to store different possible board setups in memory is around 10000 positions. Gobet and Simon (1996) tested memory for configurations of chess pieces positioned on a chess board, showing that superiority of experts over novices in recalling meaningful material from their domain of expertise vanishes when random material is used. They found that expert chess players were able to store the positions of players almost instantly, but only if they were in positions corresponding to a plausible game. For randomly arranged chess pieces, the experts were not much better than novices. Importantly, not only were experts able to recognize the boards, but to react instantly and automatically. Therefore, both the requirements - domain specificity and automatic activation – for specialized modules to be activated were satisfied in chess competitions; as we will see later, the automatic activation of specialized modules is a potential cause for biases.
As Kahneman (2002) notes, "The acquisition of skills gradually increases the accessibility of useful responses and of productive ways to organize information, until skilled performance becomes almost effortless. This effect of practice is not limited to motor skills. A master chess player does not see the same board as the novice, and visualizing the tower in an array of blocks would also become virtually effortless with prolonged practice".
As we have noted, the term "module" in cognitive psychology has two different meanings. According to Richard Samuels,
"Until recently, even staunch proponents of modularity typically restricted themselves to the claim that the mind is modular at its periphery. So, for example, although the discussion of modularity as it is currently framed in cognitive science derives largely from Jerry Fodor's arguments in *The Modularity of Mind*, Fodor insists that much of our cognition is subserved by non modular systems. According to Fodor, only input systems (those responsible for perception and language processing) and output systems (those responsible for action) are plausible candidates for modularity. By contrast, 'central systems' (those systems responsible for 'higher' cognitive processes such as reasoning, problem-solving and belief-fixation) are likely to be non modular".
In contrast with this view, evolutionary psychologists reject the claim that the mind is only peripherally modular, in favor of the Massive Modularity Hypothesis, which proposes that the human mind is largely or even entirely composed of Darwinian modules. On this line Tooby and Cosmides (1995) claim that "On this [the modular] view, our cognitive architecture resembles a confederation of hundreds or thousands of functionally dedicated computers (often called modules) designed to solve adaptive problems (Tooby and Cosmides, 1995, p. xiii). [….] Each of these devices has its own agenda and imposes its own exotic organization on different fragments of the world. There are specialized systems for grammar induction, for face recognition, for dead reckoning, for construing objects and for recognizing emotions from the face. There are mechanisms to detect animacy, eye direction, and cheating. (ibid., p. xiv)
The last characterization of the term "modules" is strongly controversial, and does not allow a clear experimental way to identify their existence and their role in the process of skill creation. On the contrary, in recent times experiments with neuroimaging procedures seems to confirm the emergence of modules – intended as *specialized cognitive capabilities emerged from experience*:

"In a process that is not well understood, the brain figures out how to do the tasks it is assigned efficiently, using the specialized systems it has at its disposal. When the brain is confronted with a new problem it initially draws heavily on diverse regions, including, often, the prefrontal cortex (where controlled processes are concentrated). But over time, activity becomes more streamlined, concentrating in regions that specialized in processing relevant to the task. In one study (Richard Haier et al. 1992), subjects' brains were imaged at different points in time as they gained experience with the computer game Tetris, which requires rapid hand-eye coordination and spatial reasoning. When subjects began playing, they were highly aroused and many parts of the brain were active (Figure 3, left panel). However, as they got better at the game, overall blood flow to the brain decreased markedly, and activity became localized in only a few brain regions (Figure 4, right panel). [.....] the brain seems to gradually shift processing toward brain regions and specialized systems that can solve problems automatically and efficiently with low effort. (Camerer, Loewenstein, Prelec, 2005)

Fig. 4

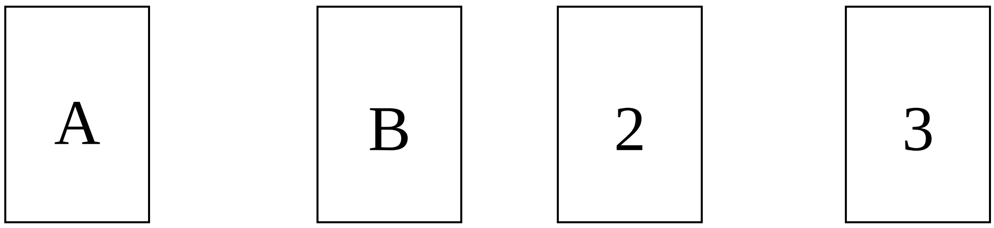



This experiment confirms the emergence of stable modularized skills that are domain specific and automatically triggered, while evidently their persistence cannot be attributed to some innate or genetic property, as claimed by evolutionary psychologists.

### *7 Domain specific versus domain general mental competences*

That a cognitive structure is domain-specific means that it is dedicated to solving a class of problems in a restricted domain. For instance, the simple algorithm learnt by individuals exposed to Luchins' experiment, once memorized and automatized, is a domain specific competence; the same happens for chess players, who react in a specific way to given chessboards. By contrast, a cognitive structure that is domain-general is one that can be brought into play in a wide range of different domains. The traditional approach to rationality implicitly assumes that people have general cognitive capabilities that can be applied to any type of problem, and hence that they will perform equivalently on problems that have similar structure. On the contrary, the dual model

approach predicts that performances will be strongly dependent upon the matching between the given problem and the capabilities acquired by previous experience.
As a side effect, if the dual model approach predictions are correct, acquired (routinized) capabilities may interfere with each other and more crucially, may interfere with System 2 (intuition) general capabilities. One of the most investigated reasoning problems in the literature in which the dual model's prediction have been tested, is the Wason selection task. While it is known to be very difficult in its conceptual version, if represented in a different, "deontic", version is it is quite easy and – interestingly - may lead either to the right or the wrong response depending on the form in which is presented.
Wason's "four-card selection task" (1966) may be described in this way: subjects are presented with four cards lying on a table; they know that these cards have been taken from a pack in which each card has a letter on one side and a number on the other.




They are given the following conditional rule:

"If a card has an A on one side, then it has a 2 on the other side"

Subjects are requested to say which card(s) have to be turned to check if the proposed rule is right. The majority of respondents propose either to turn only A or to turn A and 2 ; very few of them (less than 10%) suggest the right solution, that is to turn A and 3.[6]
Johnson-Laird, Legrenzi and Legrenzi (1972) provided a series of examples of the fact that when the rule expresses a duty or a right resulting from social arrangements, the number of subjects that correctly selecting the right cards increases.
In the version proposed by Griggs and Cox later (1982) the number of correct answers increased dramatically: around 75% of the subjects responded successfully to a version of the Wason selection task described in the following way:
"Imagine that you are a police officer on duty. It is your job to ensure that people conform with certain rules. The cards in front of you have information about four people sitting at a table. On one side of a card is a person's age and on the other side of the card is what a person is drinking. Here is a rule:

if a person is drinking beer, then the person must be over 19 years of age.

Select the card, or cards that you definitely need to turn over to determine whether or not people are violating the rule.

[6] The evidence from the Wason experiment shows that individuals do not search for the cards that might falsify the propounded rule, they suggest to turning the cards that can verify the rule. The majority of individuals, by suggesting turning A and 2, follow a verification principle, and are not stimulated to elicit the falsification principle.

| | | | |
|---|---|---|---|
| **Drinking Beer** | **Drinking Coke** | **16 Years Old** | **22 Years Old** |

This version of the task is a transformation of the original one, but is represented in “deontic form” i.e. as prescriptions related to a norm (in this case a social norm).
Two elements have been invoked to explain the success of Cox and Griggs’ version of selection task: the “familiarity” and the “deontic” character. As Sperber, Cara and Girotto (1995) note, initially it was hypothesized that versions with concrete, familiar content, close to people's experience, would "facilitate" reasoning and elicit correct performance. This hypothesis proved wrong, however, when versions that were familiar and concrete but non-deontic failed to elicit the expected good performance (Manktelow & Evans, 1979) and when, on the contrary, abstract deontic rules (Cheng & Holyoak, 1985, 1989) or unfamiliar ones (Cosmides, 1989; Girotto, Gilly, Blaye & Light, 1989) were successful.
The deontic format seemed therefore to be the key to explaining successful performance, as capable of eliciting the right reaction: Leda Cosmides (1989) argued that this deontic representation elicits a domain-specific human capacity, the ability to detect cheaters; if applied to the Cox and Griggs version of the selection task, this capacity leads to an “automatic” selection of the right cards; in this particular version, in fact, subjects searching for cheater turn the cards that should cover up a *violation* of the rule, i.e. “drinking beer” and “16 years old”.
Cosmides suggested an evolutionary explanation of the cheating detection mechanism: she argued that, for cooperation to have stabilized during human evolution, humans must have developed *reciprocal altruism* and – at the same time - domain-specific cognitive capacities that allowed them to detect cheaters. She argued that the cognitive capacities in question consisted of a *social contract module* allowing people to detect those not respecting the terms of the contract. Moreover she argued that not all deontic rules elicit correct selections, but only those which are processed by means of underlying evolved *modules* such as the social contract algorithm.[7]
Along this line, considering both content and the context of the problem as essential, the “perspective effect” introduced by Gigerenzer and Hug presents a deontic version that elicits the cheating detection “module” within an employer-employee contractual relation and explores the effects of changing the role of the subjects involved in the contract.
Gigerenzer’s thesis is that a "cheating detection mechanism" guides reasoning in the following type of selection task: *If the conditional statement is coded as a social contract, and the subject is cued into the perspective of one party in the contract, then attention is directed to information that can reveal that party being cheated.* (Gigerenzer and Hug 1992). This thesis can be proved or falsified by comparing two different versions of the selection task by changing the subject that can be cheated in the contractual relation.
The “perspective effect” version of the Wason selection task presents the following basic instructions:

[7] Of course in this context modules are intended as *genetically innate* elements, contrary to the definition we assumed in the context of Luchins experiment .

“The cards below have information about four employees. Each card represents one person. One side of the card tells whether the person worked on the weekend, and the other side tells whether the person got a day off during the week.
Given the following rule: “If an employee works on the week-end, than that person gets a day off during the week” indicate only the card(s) you definitely need to turn over to see if the rule has been violated.”

| | | | |
|---|---|---|---|
| **Worked on the weekend** | **Did get a day off** | **Did not work on the weekend** | **Did not get a day off** |

In addition to the basic instructions, two context stories were used. One of them cued the subjects into the employee’s perspective, the other into the employer’s.

*First perspective: the subject identifies himself as an employee*

The employee version stated that working on the weekend is a benefit for the employer, because the firm can make use of its machines and be more flexible. Working on the weekend, on the other hand, is a cost for the employee. The context story was about an employee who had never worked on the weekend before, but who is considering working on Saturdays from time to time, since having a day off during the week is a benefit that outweighs the costs of working on Saturday.

“There are rumors that the rule has been violated before. The subjects’ task was to check information about four colleagues to see whether the rule has been violated before.”

According to the cheating detection mechanism, the cards that have to be turned to detect if the *employer* did not respected the rule are “worked on the weekend” and “did not get a day off”. These are also the cards that have to be turned in a correct logical reasoning. Gigerenzer and Hug report that typically 75% of subjects chose the right cards.
They then switched the perspective from *employee* to *employer* but held everything else constant.

*Second perspective: the subject identifies himself as the employer*

For the employer, being cheated meant that the employee "did not work on the weekend and did get a day off" that is, from this perspective, subjects should select cards that are *not* the right ones in a logical reasoning.
As the two authors report, “The results showed that when the perspective was changed, the cards selected also changed in the predicted direction. The effects were strong and robust across problems.” In the employee perspective of the day-off problem, 75% of the subjects selected "worked on the weekend" and "did not get a day off," but only 2 % selected the other pair of cards. In the employer perspective, this 2% (who selected "did not work on the weekend" and "did get a day off") rose to 61%.
Therefore, the experiment shows a clear and strong impact of the representation of the problem and particularly the effect of the semantic content on the process of reasoning. Cosmides ascribes the domain-specific ability of detecting cheaters to an innate, genetically developed “cognitive module”; in her view, cooperation would not work without a *module* for directing an individual’s

attention to information that could reveal that it (or its group) is being cheated. Whether this happens automatically through some module specifically designed for social contracts, as claimed by Gigerenzer and Hug (1992) or as interaction between domain-specific skills and the domain-general reasoning process, as dual account process suggests, is debatable. According to Sperber, Cara and Girotto (1995) we do not consider convincing the assumption of the Massive Modularity Hypothesis (the idea that the mind is a bunch of domain-specific Darwinian modules) held by evolutionary psychologists (Tooby and Cosmides). Sperber et al. have offered a different interpretation of the context sensitivity of the Wason selection task. By introducing the Relevance Theory, they suggest that individuals infer from the selection task rule testable consequences, consider them in *order of accessibility*, and stop when the resulting interpretation of the rule meets their expectations of relevance. Order of accessibility of consequences and expectations of relevance vary with rule and context, and so, therefore, does subjects' performance.[8]

This view seems coherent with Kahneman's assumptions that automatic and controlled cognitive operations compete for the control of overt responses:
"The central characteristic of agents is not that they reason poorly but that they often act intuitively. And the behavior of these agents is not guided by what they are able to compute, but by what they happen to see at a given moment.[….] Judgment heuristics, which explain many systematic errors in beliefs and preferences are explained [..] by a process of attribute substitution: people sometimes evaluate a difficult attribute by substituting a more accessible one. Variations in the ability of System 2 to correct or override intuitive judgments are explained by variations in the accessibility of the relevant rules" (Kahneman 2002).
Therefore the dichotomy between automaticity and computability suggests that performance will depend critically on the interaction between domain specific and domain general reasoning: domain specific cognitive skills that are evoked in reasoning in relation to a particular frame or solving a particular problem will be considered, as in Luchins' experiment, as basic building blocks created by routinizaton of a mental process. Experimental evidence shows that individuals, once they have memorized the basic building blocks that proved useful to solve problems in a given context, tend to apply them automatically to analogous conditions (Luchins, 1942, Egidi and Narduzzo,1997); the evidence leads us to assume that when a specialized module (intended as *specialized cognitive capability developed from experience*) exists and is applied to a particular task, processing is rapid and the task goal seems to be almost effortlessly achievable.
At the same time, however, the use of automatic modules in contexts that are slightly different from the original ones – as in Luchins' experiments - may explain why automaticity can easily lead to errors in judgment, reasoning and problem solving. In fact, errors may be generated at two different levels: on the one hand, errors may be related to the use of "wrong" modules, that is modules which do not perfectly fit the context; more importantly, when new modules are going to be created, errors may be generated by the representation and decomposition of the problem.

The relevance of erroneous representations emerges clearly in problem solving contexts: problem-solving generally implies the decomposition of problems into parts to be solved separately. The decomposition process is susceptible to being repeatedly applied until elementary and easy sub-problems are identified, and elementary modules are created and memorized. Some progress in understanding the properties of decomposition has been achieved in the recent years, after the original works of Simon et al. (1961), mainly with the support of applications to games and puzzles: the Hanoi tower and Rubik cube are typical contexts in which decomposition has been analyzed in depth to discover solution strategies.

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[8] Intuition of relevance is activated by a pragmatic mechanism involved in comprehending the task; people trust these intuitions and select cards accordingly.

However, it has been shown (Egidi, 2006) that when a problem is decomposed into elementary parts and the solutions are discovered and memorized into elementary modules, errors that players – whether consciously or not – make have two sources: the architecture of the decomposition and the transfer of modules beyond their domain of applicability. The first happens as a consequence of a general property of decomposition, related to Bellman principle: when we decompose a problem, although we discover and apply the optimal solution to each sub-problem, we generally get a sub-optimal solution to the global problem. The conditions for optimal decomposition are quite restrictive and generally do not emerge in the course of the natural human process of problem solving; roughly speaking, this happens because individuals, searching for a decomposition pattern, make use of prior modules, that have been discovered and memorized in different contexts. This means that categorizations and classifications which have been used to solve one problem are extended beyond the original domain and applied to areas where can turn out to be inappropriate. Consequently, the errors in the mental representation of a problem can be the natural effect of the categorization and identification of building blocks (modules) beyond their "right" domain. This inappropriate extrapolation can be viewed in the "mechanization of thought" process shown by the Luchins and Luchins (1950) experiments. Hence, an important source of biases in decision making originates from the representation of the problem through decomposition.

## *8 Concluding remarks*

While many writers have emphasized the impact of emotions in "disturbing" reasoning, we have paid more attention to the "illusions", that is, according to Laplace, to the erroneous representations of problems that automatically come to the mind of individuals and elicit erroneous decisions. According to Kahneman, the term "intuition" has been used here to describe the process of automatic mental editing that characterizes a large part of human reasoning; the dualism between reasoning and intuition has therefore been considered as the source of misrepresentations and biases in decision and reasoning.

The dual model we have been discussing so far is a new and promising attempt to explain the origin of biases. While some aspects of the dual approach have been clarified by neurophysiology - particularly by using brain imaging techniques - traditional psychological experiments have provided the greater quantity of significant evidence; although early results seem to be encouraging, neuropsychological studies on reasoning are still in their infancy and substantial research effort is needed to develop a better understanding of the neurological basis of reasoning. Of course, many aspects of the thinking process are still unexplored or escape our present understanding; in particular, the relationship between automatic mental activities and conscious calculation is more complex than it was emphasized in recent literature: many experiments show that the relation is not limited to the control of the deliberate over the automatic system and, therefore, to a "correction" of errors: as Simon's experiments in chess show, chess masters' performance depends crucially upon the memorized boards and correlated automatic skills (modules): the master deliberately uses the modules combining them to build up his strategy; as a consequence, if any of the modules does not perfectly fit to the solution, the strategy will deviate from the (theoretical) optimum. Moreover, even if all modules fit, their combination may lead to errors as a consequence of a wrong representation of the problem; errors are therefore nested in the reasoning process because automatic modules are the elementary "words" of the process: unconscious automatic processes and deliberate calculation are inextricably connected in complex reasoning. As a consequence, errors and erroneous frames may persist with remarkable stability even when they have been falsified. This leads us to a consideration of rationality which differs from the one traditionally assumed (optimizing capacity) while converging with Popper and Hayek’s views: in our discussion, in fact, rationality can be essentially considered as the capacity to get rid of our errors.

APPENDIX

The rediscovery of Aristotle's thought in the XII century not only provided a fundamental technique for solving juridical disputations. A second, not less important consequence of the rebirth of Aristotle's logic and epistemology was the rise of a philosophical program intended to "logically calculate" the truth of a statement from shared premises, and therefore to provide stable and undisputable foundations to many domains of human knowledge. The most striking effect of this program was the attempt to define the foundations of Christian theology through an axiomatic approach. Guillaume d'Auxerre, with his *Summa de officiis ecclesiastici*s, initiated some ideas along this line but, as everybody knows, this view owes its full development to Thomas Aquinas who, mainly in his *Summa Theologiae* (1273), built the pillars of a theory of a "rational theology".
An unpleasant consequence of providing a logically coherent - and therefore undisputable - base to theology was to classify as heretical a large number of otherwise questionable philosophical and theological propositions. This may help understand the atmosphere of passions and antagonisms in which the theory of fallacies grew up, sometimes leading to regrettable facts: the opposition to the study of fallacies, passionately upheld by Pierre de la Ramée (Ramus) around 1540, ruined him, as it is shown in the following biographical note.

From CERPHI Centre d'études en rhétorique, philosophie et histoire des idées

Pierre La Ramée, dit Ramus
Notice biographique

Pierre De La Ramée dit Ramus (1515-1572): Né dans le Vermandois, d'une famille très modeste. Il monte à Paris à douze ans pour suivre les cours du collège de Navarre en devenant le domestique d'un riche écolier : il sert son maître le jour et étudie la nuit. À vingt et un ans, il devient maître ès arts en soutenant une thèse anti-aristotélicienne. Il débute au collège du Mans, puis, avec Omer Talon et Barthélemy Alexandre, il ouvre des cours publics où on lit des auteurs grecs et latins, où l'éloquence n'est pas séparée de la philosophie ; la foule y accourt. Ramus entreprend alors une révision critique de toutes ses études, en commençant par la logique. Son anti-aristotélisme prend une forme aiguë dans ses *Dialecticae partitiones* de 1543 et ses *Aristotelicae animadversiones*, qui lui valent les attaques de Joachim Périon et d'Antoine de Govea. Censurés par la faculté de théologie, ses ouvrages sont condamnés par un édit royal du 1er mars 1544.
L'enseignement de la philosophie lui est même interdit, mais, protégé par le Cardinal de Lorraine, il est nommé en 1545 principal du collège de Presle. Malgré les attaques du recteur Charpentier, Ramus est nommé en août 1551 lecteur au Collège royal. Il consacre les huit premières années de son enseignement à la grammaire, la rhétorique et la logique, puis passe aux mathématiques. Il se convertit alors au protestantisme. Les guerres de religion l'obligent à fuir. À la paix d'Amboise (1563), il reprend possession de sa chaire, continue son enseignement des mathématiques et fait un cours sur la physique et la métaphysique d'Aristote. Il refuse une chaire à Bologne, continue à batailler contre Charpentier. Après avoir à nouveau dû fuir pendant la seconde guerre civile, il rentre à Paris en 1568. Mais sa situation n'est plus sûre. Il demande un congé pour visiter les principales universités d'Europe, où il est accueilli avec honneur. La religion est devenue sa grande préoccupation. Rappelé une troisième fois à Paris après le traité de Saint-Germain (1570), l'Université l'empêche d'enseigner en raison de son appartenance calviniste ; il souhaite gagner Genève, mais Bèze l'éconduit. Deux jours après la Saint-Barthélemy, il est assassiné par des égorgeurs à gage ; son corps est traîné dans les rues de Paris et jeté à la Seine.
( http://www.cerphi.net/biblio/ramus.htm)

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