

# Discussion of “Equilibrium Yields” by Monika Piazzesi and Martin Schneider\*

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Section 1 of this discussion reviews the analysis of Piazzesi and Schneider (2006) (hereinafter PS). Section 2 analyzes alternative preference specifications. Section 3 derives term-structure implications using standard preferences but with a fractional integrated process for the inflation rate. Section 4 concludes pointing out some statistical evidence on term-structure data that needs to be further analyzed.

## 1 Introduction

No-arbitrage theory is based on the existence of some discount factor  $M_{t+1}$ , between generic periods  $t$  and  $t + 1$ , such that the return  $R_{t+1}^j$  of a generic asset  $j$ , between the same periods, satisfies the following moment condition

$$E_t[R_{t+1}^j M_{t+1}] = 1. \quad (1)$$

For a zero-coupon bond the return is given by the change between periods in the price of the bond. Let  $P_{n,t}$  denote the price at time  $t$  of a nominal bond with  $n$ -periods to maturity, (1) can be written as

$$E_t \left[ \frac{P_{n-1,t+1}}{P_{n,t}} M_{t+1} \right] = 1.$$

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Since the price of a zero-coupon bond at maturity is equal to 1, i.e  $P_{0,t} = 1$ , it is possible to write the price of a bond with  $n$ -periods to maturity as

$$P_{n,t} = E_t[M_{t+1}M_{t+2}M_{t+3}\dots M_{t+n}].$$

The yield to maturity on a bond with  $n$ -periods to maturity is defined as

$$y_{n,t} \equiv -\frac{1}{n} \ln P_{n,t}.$$

The theory of the term structure is nothing more than a theory of the stochastic discount factor. To have a model of the term structure that represents the data, it is necessary to specify a process  $\{M_t\}$ . This is the approach used in most of the term-structure literature in finance. (see among others Dai and Singleton, 2003.)

PS disentangle the problem using two steps. First, they specify the consumption preferences of some agent in the economy and derive the nominal stochastic discount factor based on these preferences. Preferences depend on macro variables, and consequently, so will the stochastic discount factor. Second, they estimate processes for the macro variables that make up the stochastic discount factor. In doing so, they are able to specify a process for the stochastic discount factor to have a model of the term structure that can be compared to the actual data.

Special to this procedure is that it is able to provide explanations regarding whether macro variables are important in driving term structure and whether preferences assumed in macro models are consistent with financial data.

Their first step consists of specifying preferences using a general family of isoelastic utility derived from the work of Kreps and Porteus (1979) and Epstein and Zin (1988). These preferences do not confuse behavior toward risk with that of intertemporal substitution as in the standard expected utility model. This makes it possible to distinguish the intertemporal elasticity of substitution from the risk-aversion coefficient.<sup>1</sup> PS fix the intertemporal elasticity of substitution to a unitary value which, together with other assumptions, has the advantage of implying a linear-affine model of the term structure. Utility at time  $t$  given by  $V_t$  is defined recursively as

$$V_t = C_t^{1-\beta} \{ [E_t V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}} \}^\beta$$

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<sup>1</sup>This is not the first paper to use this kind of preferences to study term-structure implications, but the first to take it seriously to the data. See among others Campbell (1999), Campbell and Viceira (2002), Restoy and Weil (2004).

where  $\gamma$  is the risk-aversion coefficient and  $\beta$  is the intertemporal discount factor.<sup>2</sup>

An important implication of the work of Tallarini (2000) is that risk aversion can be set as high as needed without significantly affects the relative variabilities and simultaneous movements of aggregate quantity variables in a business-cycle model.

Under this preference specification the nominal stochastic discount factor is given by

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{V_t}{[E_t V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\gamma} \frac{P_t}{P_{t+1}} \quad (2)$$

while its log implies

$$\begin{aligned} m_{t+1} &= \ln \beta - \Delta c_{t+1} - \pi_{t+1} - (\gamma - 1)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \beta^{j-1} \Delta c_{t+1+j} \\ &\quad - \frac{1}{2}(\gamma - 1)^2 \text{Var}_t(E_{t+1} \sum_{j=1}^{\infty} \beta^{j-1} \Delta c_{t+1+j}) \end{aligned} \quad (3)$$

where lower-case variables denote the log of the respective upper-case variable; and  $\pi_t$  is the inflation rate defined as  $\pi_t = \ln P_t / \ln P_{t-1}$ .

It is possible to make predictions about the term structure simply by specifying the processes for consumption growth and inflation since the stochastic discount factor depends only on these two variables. Let  $z'_t = [\Delta c_t \ \pi_t]$ , PS estimate a process for  $z_t$  of the form

$$z_{t+1} = \mu_z + x_t + e_{t+1}, \quad (4)$$

$$x_{t+1} = \phi_x x_t + \phi_x K e_{t+1}. \quad (5)$$

Matrices and vectors are presented in their paper and the variance-covariance matrix of the innovation  $e_t$  is given by  $\Omega$ . One of the main findings of PS is that this two-step procedure is successful in reflecting statistical properties of the yield curve, especially for the average yield curve.

This discussion will first analyze the implications of alternative preference specifications given the estimated process and then moves to analyze an alternative process given standard preference specifications.

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<sup>2</sup>I am assuming an infinite horizon economy differently from PS finite-horizon model.

## 2 Preferences

### 2.1 What is preventing the standard expected utility model from working?

Under the standard isoelastic expected utility model with preferences given by

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{C_t^{1-\rho}}{1-\rho},$$

the nominal stochastic discount factor is

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_t}{P_{t+1}}, \quad (6)$$

where  $\rho$  is the risk-aversion coefficient which now coincides with the inverse of the intertemporal elasticity of substitution. In this case, the price of a bond with  $n$ -periods to maturity can be written as

$$P_{n,t} = E_t \left[ \beta^n \left( \frac{C_{t+n}}{C_t} \right)^{-\rho} \frac{P_t}{P_{t+n}} \right].$$

This price can be relatively low either when future prices or consumptions are expected to be relatively high. Under these conditions future marginal utility of nominal income is low. Agents dislike assets that pay when they do not need extra nominal income. The prices of these assets will be relatively low and agents require a premium to hold them. Following this line of reasoning, nothing should prevent standard preferences from reproducing, at least, the upward-sloping average yield curve dictated by the data. However, this is not the case under the estimated processes (4) and (5).

The first problem one can expect to face when working with standard preferences is in matching moments of the short-term interest rate  $i_{1,t}$ . This is given by

$$i_{1,t} = -\ln \beta + \rho E_t \Delta c_{t+1} + E_t \pi_{t+1} - \frac{\rho^2}{2} \text{var}_t \Delta c_{t+1} - \frac{1}{2} \text{var}_t \pi_{t+1} - \rho \text{cov}_t(\Delta c_{t+1}, \pi_{t+1})$$

which implies an unconditional mean

$$\mu_1 = -\ln \beta + \rho \mu_c + \mu_\pi - \frac{\rho^2}{2} \sigma_c^2 - \frac{1}{2} \sigma_\pi^2 - \rho \sigma_{c\pi}. \quad (7)$$

When the values of the parameters  $\beta$  and  $\rho$ , along with the vector of means  $\mu_z$  and the variance-covariance matrix  $\Omega$  from the estimated system (4) and (5) are known, it is possible to calculate a value for the unconditional mean. The estimated variance-covariance matrix does not play a large role in (7) since its magnitude is negligible compared to the means. According to the data  $\mu_c = 3.29\%$  and  $\mu_\pi = 3.70\%$ . In order for the unconditional mean of the short-term rate  $\mu_1 = 5.15\%$  to reflect the data, either  $\beta$  should be greater than one or  $\rho$ , the risk-aversion coefficient, should be less than one. If  $\beta$  is not allowed to be greater than one and is set arbitrarily at 0.999, then  $\rho$  should be 0.32.<sup>3</sup>

This means that when the first point of the model average yield curve corresponds to the data, all the parameters are already tied down, making it harder for the model to match other facts as the upward-sloping average yield curve. Indeed for these parameters and processes, the risk-premia on holding long-term maturity bonds is negative and not positive. The no-arbitrage condition (1) implies that the expected log excess return on a bond with  $n$ -periods to maturity ( $er_{n,t}$ ) corrected for Jensen inequality term is given by

$$er_{n,t} = E_t r_{n,t+1} + \frac{1}{2} Var_t r_{n,t+1} - i_{1,t} = -cov_t(r_{n,t+1}, m_{t+1}).$$

Assets that command a positive risk-premium are those, of which, their return covaries negatively with the discount factor. In particular, for a zero-coupon bond with  $n$ -periods to maturity the between-period return is given by  $r_{n,t+1} = p_{n,t+1} - p_{n,t}$ . Under the assumptions (4), (5) and (6) bond prices are linear affine in the state vector  $x$

$$p_{n,t} = -A(n) - B(n)'x_t$$

where

$$A(n) = A(n-1) - \ln \beta + v' \mu_z - \frac{1}{2} [B'(n-1) \phi_x K + v'] \Omega [B'(n-1) \phi_x K + v']'$$

$$B(n)' = B(n-1)' \phi_x + v'$$

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<sup>3</sup>Standard procedures require first to set  $\rho$  and then derive  $\beta$  but this would violate the upper bound on  $\beta$ . PS finite-horizon model allows for  $\beta$  to be greater than the unitary value. The fact that by rising the risk-aversion coefficient the mean of the short-rate increases is the mirror image of the equity premium puzzle. This is the risk-free rate puzzle, see Weil (1989).

$$v' = [\rho \ 1].$$

It follows that the expected excess return on a bond with  $n$ -periods to maturity is given by

$$er_{n,t} = -B(n-1)' \phi_x K \Omega v$$

which given their estimated matrixes is slightly negative for all maturities. This explains the downward-sloping trend of the average yield curve shown in the third line of Table 1.

The preference specification (3) used by PS adds an extra factor to standard preferences that allows for greater flexibility. Under these preferences, the prices of the bonds with different maturities are still linear-affine, but

$$\begin{aligned} A(n) &= A(n-1) - \ln \beta + i' \mu_z + \frac{1}{2} (\gamma - 1)^2 e_1' Z \Omega Z' e_1 + \\ &\quad - \frac{1}{2} [B'(n-1) + i' + (\gamma - 1) e_1' Z] \Omega [B'(n-1) + i' + (\gamma - 1) e_1' Z]' \end{aligned}$$

and

$$B(n)' = B(n-1)' \phi_x + i'$$

with

$$\begin{aligned} i' &\equiv [1 \ 1] & e_1' &\equiv [1 \ 0] \\ Z &\equiv I + \beta(I - \phi_x \beta)^{-1} \phi_x K. \end{aligned}$$

The expected excess return is given by

$$er_{n,t} = -B(n-1)' \phi_x K \Omega i - (\gamma - 1) B(n-1)' \phi_x K \Omega Z' e_1$$

which shows an additional term that helps to generate a positive risk premium. This new term is multiplied by the risk-aversion coefficient, which can be freely moved to produce an upward-sloping average yield curve, as shown in the second line of Table 1.<sup>4</sup>

As discussed in Cochrane (2006), drawing empirical facts from financial data using a stochastic discount factor based on consumer preferences

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<sup>4</sup>The second line of Table 1 reports the results of the finite-horizon model of PS. Note that in the infinite-horizon case to generate a positive risk premium it is sufficient to assume a value of  $\gamma$  above 2, but to match the risk-premium of the data  $\gamma$  should be 59. A high risk-aversion coefficient also reduces the unconditional mean of the short rate helping to reduce the value of  $\beta$  needed to match the first point of the average yield curve.

requires that additional factors be added to the standard expected utility model. I will now investigate the implications for the yield curve of traditional extensions to standard preferences which have been used to explain the equity-premium puzzle.

## 2.2 Habit Model as in Abel (1991).

Consider the model proposed by Abel (1991) in which the utility function is given by

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left( \frac{C_T^j}{C_{T-1}^\theta} \right)^{1-\rho}$$

where the utility flow does not only depend on individual consumption, but on consumption relative to past aggregate consumption. This model can be interpreted as a relative habit model, or better as a “keeping up with the Joneses” model. The parameter  $\theta$  measures the importance of others’ aggregate consumption and is such that when  $\theta = 0$ , standard isoelastic expected-utility preferences are nested. The nominal stochastic discount factor implied by these preferences is

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{C_{t-1}}{C_t} \right)^{\theta(1-\rho)} \frac{P_t}{P_{t+1}},$$

from which it follows that the short-term interest rate is given by

$$\begin{aligned} i_{1,t} = & -\ln \beta + \rho E_t \Delta c_{t+1} + \theta(1-\rho) \Delta c_t + E_t \pi_{t+1} - \frac{\rho^2}{2} \text{var}_t \Delta c_{t+1} - \frac{1}{2} \text{var}_t \pi_{t+1} \\ & - \rho \text{cov}_t(\Delta c_{t+1}, \pi_{t+1}) \end{aligned}$$

and its unconditional mean by

$$\mu_1 = -\ln \beta + [\rho + \theta(1-\rho)]\mu_c + \mu_\pi - \frac{\rho^2}{2}\sigma_c^2 - \frac{1}{2}\sigma_\pi^2 - \rho\sigma_{c\pi}.$$

Assuming that  $\theta = 1$ , it is now possible to increase the value of the risk-aversion coefficient without necessarily increasing the unconditional mean of the short-term rate. This will increase the risk-premium and generate an upward sloping yield curve. In particular, set  $\beta = 0.999$  and  $\rho = 24.7$  to reflect the unconditional mean of the short-term rate. As shown in the fourth line of Table 1, together with the estimated processes (4) and (5),

this preference specification can now generate an upward sloping yield curve. However, the shape of the curve does not correspond to that of the data. The curve is too steep at short-term maturities and lies above data levels afterward. Most importantly, as shown in Table 2, this model fails to generate the proper volatility of the yields since it exhibits substantially high volatility for the short-term rate.

TABLE 1 HERE

TABLE 2 HERE

### 2.3 External Shock as in Gallmeyer et al. (2005).

To explore the implications of a more sophisticated model of habit as presented by Gallmeyer, Hollifield and Zin (2005), which falls under the class of habit models discussed in Campbell and Cochrane (1999), consider a utility flow of the form

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{C_T^{1-\rho}}{1-\rho} Q_T$$

where  $Q_t$  is a preference shock that follows a martingale, i.e.  $E_t(Q_{t+1}/Q_t) = 1$ . In this case, the nominal stochastic discount factor is given by

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{Q_{t+1}}{Q_t} \right) \frac{P_t}{P_{t+1}}.$$

The shock  $Q_t$  is modelled in a way that

$$-\Delta q_{t+1} = (\phi_c \Delta c_t)(\Delta c_{t+1} - E_t \Delta c_{t+1}) + \frac{1}{2}(\phi_c \Delta c_t)^2 \text{var}_t \Delta c_{t+1},$$

where, as previously, lower-case letters denote logarithms and  $\phi_c$  is a parameter. It follows that the nominal stochastic discount factor can be written as

$$M_{t+1} = k_t \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \left( \frac{C_t}{C_{t-1}} \right)^{-\phi_c \xi_{t+1}} \frac{P_t}{P_{t+1}}, \quad (8)$$

where

$$\xi_{t+1} = (\Delta c_{t+1} - E_t \Delta c_{t+1})$$



captures unexpected surprises in consumption at time  $t + 1$  and

$$k_t \equiv \exp\{1/2(\phi_c \Delta c_t)^2 \text{var}_t \Delta c_{t+1}\}.$$

Past consumption matters as it did in the standard habit model, but now its weight depends on the magnitude of the unexpected consumption surprises.

This preference specification, together with the processes (4) and (5), generates an affine linear yield curve, in which risk-premia are now time varying.

As shown in Table 1, this model is more successful in producing an upward-sloping yield curve and toward this aspect of the data performs as well as the benchmark model of PS. In particular the parameter  $\beta$  is set equal to 0.9999 while  $\phi_c = -11250$ . The latter number is not large since  $\xi_{t+1}$  is very small. The standard deviation over the sample of  $\phi_c \xi_{t+1}$ —which is the variable what matters in (8)—is 36. Note the similarities between these preferences and the ones used in PS. Both add an additional martingale to the stochastic discount factor. This additional term can be interpreted as a distortion in the initial probability measure as in the risk-sensitive control literature. (see Hansen and Sargent, 2006)

### 3 Processes for consumption and inflation

In the previous section, the estimated processes for consumption and inflation were maintained as those in the specification of PS. It was shown that in order to match an upward sloping average yield curve, the standard isoelastic expected utility model had to be modified to include additional terms. However, the models discussed thus far have all failed to properly represent one important aspect of the data regarding the volatility of the yields, as shown in Table 2. Every model has implied a progressively decreasing trend, even though the volatility of the yields over the full sample of data does not actually decrease with longer maturities. This result greatly depends on the estimation of the processes (4) and (5). The estimation is performed on demeaned data, which imposes stationarity on the variables influencing the stochastic discount factor. As discussed in Backus and Zin (1993), when the state vector is stationary, the volatility of the yields with longer maturities converge to zero. Since the data does not show this pattern, this indicates some non-stationarity in the factors influencing the yield curve for at least some part of the sample.

An obvious candidate of this non-stationary behavior is the inflation-rate process. Firstly, because if a raw unit-root test is performed on the data taken from 1952 to 19xy where xy is above 70, a unit root cannot be rejected for some years. Also because recent literatures on inflation forecasting discussed in Mayoral and Gadea (2005) have argued that inflation processes for many OECD countries can be described well by fractionally-integrated processes. This class of processes implies longer memory and as discussed in Backus and Zin (1993) can generate a non-decreasing volatility of yields.

A careful multivariate fractional integration approach to consumption and inflation is out of this discussion's scope. Yet, I will explore the implication of a fractionally integrated process for inflation and show that even the standard isoelastic expected utility model can reconcile at the same time an upward sloping yield curve with the non-decreasing volatility of the yields.

First consider a fractional integrated process for inflation of order  $d$  as

$$(1 - L)^d \pi_t = \xi_{\pi,t}$$

which is equivalent to

$$\sum_{j=0}^{\infty} a_j \pi_{t-j} = \xi_{\pi,t}$$

where the coefficients  $a_j$  solve the following recursion

$$a_j = \left[ 1 - \frac{1+d}{j} \right] a_{j-1}$$

with  $a_0 = 1$ . I set  $d = 0.72$  as it is found in Mayoral and Gadea (2005) and consider a maximum lag of 19. I estimate a bivariate VAR with one lag for the vector  $(\Delta c_t, \xi_{\pi,t})$ . Next I construct a process for a state vector  $z_t = (\Delta c_t, \pi_t, \pi_{t-1} \dots \pi_{t-18})$ .<sup>5</sup> I compute the implications for term structure of assuming this state process under the stochastic discount factor (6) implied by standard isoelastic expected utility. In particular I set  $\beta = 0.9999$ ,  $\rho = 0.28$  in order to match the unconditional mean of the short-term rate. The results are presented in the last lines of Tables 1 and 2. Now, the standard isoelastic expected utility model is able to match an upward-sloping yield curve in accordance with the data.<sup>6</sup> Most importantly, the volatilities of the yields are higher than in the previous case and still declining, but at a slower pace.

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<sup>5</sup>To further simplify the analysis, I keep only the significant coefficients from the VAR estimates.

<sup>6</sup>The result that the standard expected utility model can be also succesfull in generating

## 4 What have we learnt?

There are two important messages that PS's paper conveys that can be useful for macro modelling. First, the paper suggests that standard expected utility preferences are not satisfactory. This is a common leitmotiv in the current finance literature which relies on preference specifications. The second message concerns the mechanism through which the term structure is upward sloping. It is emphasized that bad news on inflation are also bad news on current and future consumption. However, nothing has been said about whether this mechanism is consistent with a macro model nor on the driving shocks and forces behind this relationship.

Here, for the purpose of providing further insights on yield-curve characteristics relevant to a macroeconomic perspective, some statistical analysis on PS's data is presented. I compare the full sample (1952-2004) to the Great Moderation period (1984-2004), the pre Great-Moderation (1962-2004), the Greenspan period (1987-2004) and the last decade (1995-2004). Table 3 presents the means of consumption growth and inflation for the various subsamples as well as the means of the one-quarter, 3-year and 5-year yields. The main difference between the first and the second half of the sample for the two macro variables considered is in the lower mean of inflation in the second part. The average yield curve is always upward sloping for all the subsamples considered and relatively flatter for the periods 1952-1984 and 1995-2004.

TABLE 3 HERE

Most interesting is the analysis of volatilities shown in Table 4. The Great Moderation period and the Greenspan period are characterized by a fall in the volatilities of consumption growth and inflation. The most important trend of these periods is the fact that the volatilities of the yields have also decreased. This means that there could be common factors affecting the macro variables and the yield curve which is promising evidence for the research agenda attempting to link macroeconomics and finance more tightly together. An

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a positive risk-premium is in some way consistent with PS learning experiment in which the estimation procedure can account for possible breaks in the consumption and inflation processes. Indeed, in their final example of section 5, they need a parameter of risk aversion  $\gamma = 4$  which is close to imply the standard expected utility model.

additional interesting fact found in Table 4 is the non-decreasing volatility of the yield curve, due mostly to the first part of the sample. Particularly in the Greenspan period and the last decade, the volatility of the yield curve is downward sloping. This is clearly a consequence of some important changes in the inflation process.

TABLE 4 HERE

This evidence points toward asking whether it is possible that changes in the conduction of monetary policy in the last decades are responsible of the changes observed in the term structure. Futhermore, is there a model that can rationalize this evidence? Perhaps one in which monetary policy actions become more credible, or in which the instrument and targeting rules change or in which monetary policymakers acquire a better understanding of the model economy.

PS's intuition for an upward sloping yield curve relies on the correlation between consumption growth and inflation. This relationship is negative if the full sample is considered.

TABLE 5 HERE

However, table 5 shows that this negative relationship is a feature of only the first-part of the sample and that it becomes statistically insignificant toward the last parts of the sample. As well, other correlations are strong for the first part of the sample and insignificant during the Greenspan period. This is the case for the correlations between the short-term rate and inflation, and the short-term rate and consumption. Moreover Figure 1 replicates their Figure 1 but just for the sample 1987-2005 showing that the cross covariances are small in magnitude and perhaps not significant.

FIGURE 1 HERE

Perhaps, this is no longer supporting their intuition that negative inflation shocks lead to negative future consumption growth which is puzzling since even in this subsample the average yield curve is upward sloping.

Several questions and issues are left open for further research.

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**Table 1**

Average Nominal Yield Curve

	1 quarter	1 year	2 year	3 year	4 year	5 year
Data	5.15	5.56	5.76	5.93	6.06	6.14
Benchmark Model	5.15	5.33	5.56	5.78	5.97	6.14
Expected Utility	5.15	5.15	5.14	5.13	5.11	5.10
External Habit	5.15	6.75	7.07	7.17	7.22	7.24
External Shock	5.15	5.29	5.51	5.74	5.95	6.14
Fractional Process	5.15	5.42	5.64	5.84	6.19	6.40

**Table 2**

## Volatility of Yields

	1 quarter	1 year	2 year	3 year	4 year	5 year
Data	2.91	2.92	2.88	2.81	2.78	2.74
Benchmark Model	1.80	1.64	1.47	1.34	1.22	1.12
Expected Utility	2.04	1.92	1.75	1.60	1.47	1.35
External Habit	30.3	10.19	5.48	3.77	2.92	2.41
External Shock	2.00	1.86	1.67	1.50	1.34	1.20
Fractional Process	2.18	2.06	1.98	1.92	1.80	1.74



**Table 3**

Means and sub-samples

	$\mu(\Delta C)$	$\mu(\pi)$	$\mu(y_{1q})$	$\mu(y_{3yr})$	$\mu(y_{5yr})$
1952-2004	3.29	3.70	5.14	5.93	6.14
1952-1984	3.44	4.18	5.30	5.88	5.99
1984-2004	3.05	3.03	4.97	6.06	6.41
1987-2004	2.96	2.99	4.55	5.55	5.88
1995-2004	3.13	2.54	3.78	4.57	4.86

**Table 4**

Volatility and Sub-samples

	$\sigma(\Delta C)$	$\sigma(\pi)$	$\sigma(y_{1q})$	$\sigma(y_{3yr})$	$\sigma(y_{5yr})$
1952-2004	1.88	2.51	2.91	2.81	2.73
1952-1984	2.18	3.01	3.30	3.13	3.06
1984-2004	1.29	1.24	2.26	2.32	2.24
1987-2004	1.30	1.25	2.03	1.89	1.75
1995-2004	1.09	0.98	1.76	1.51	1.27

**Table 5**

Correlations and sub-samples

	$c(\Delta C, \pi)$	$c(y_{1q}, \pi)$	$c(y_{1q}, \Delta C)$
1952-2004	-0.35**	0.67**	-0.15**
1952-1984	-0.44**	0.74**	-0.27
1984-2004	-0.13	0.43**	0.10
1987-2004	-0.19	0.44**	0.00
1995-2004	-0.06	-0.12	0.26

\*\*=1% significance level

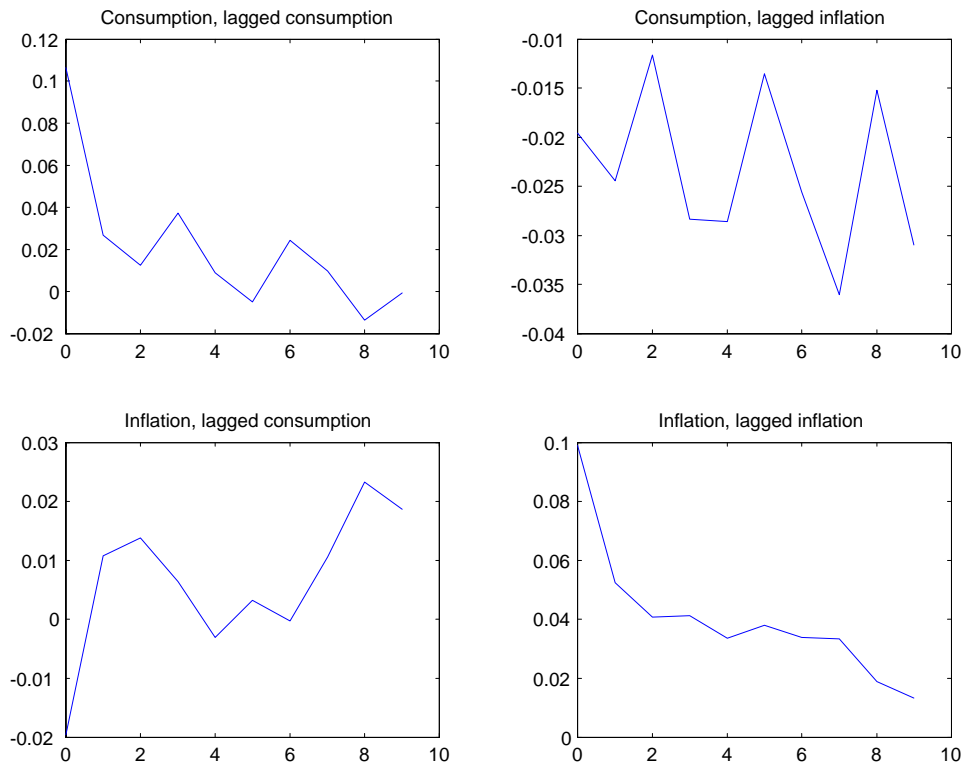


Figure 1: Covariance function computed from the raw data for the sample 1987-2004. (See PS figure 1 for the full sample)