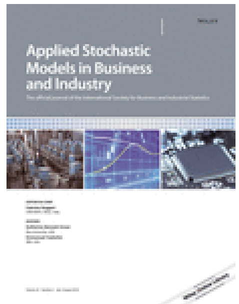

Options on Constant Proportion Portfolio Insurance with guaranteed minimum equity exposure

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Options on Constant Proportion Portfolio Insurance with guaranteed minimum equity exposure

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Abstract

In the present paper we study a new exotic option offering participation in a dynamic asset allocation strategy, which is an extension of the well-known *Constant Proportion Portfolio Insurance* (CPPI) strategy. Our novel approach consists in assuming that the percentage of wealth invested in stocks cannot go under a fixed level, called *guaranteed minimum equity exposure* (GMEE). In particular, our proposal ensures to overcome the so called *cash-in* risk, typically related to a standard CPPI technique, simultaneously guaranteeing the equity market participation. We look deeper into the valuation of call and put options linked to this new CPPI–GMEE strategy. A particular attention is devoted to the analysis of key parameters' value as to gain a better understanding of the sensitivities of the option prices, when changing, e.g., the embedded guarantee level. To show the effectiveness of our proposal we provide a detailed computational analysis within the Heston-Vasicek framework, numerically comparing the evaluation of the price of European plain vanilla options when the underlying is either a purely risky asset, a standard CPPI portfolio and a CPPI with guaranteed minimum equity exposure.

Keywords: Option on CPPI, CPPI, OBPI, portfolio insurance, stochastic volatility, guaranteed minimum equity exposure.
JEL classification: C63, G11, G13
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1 Introduction

During recent years, financial markets have been mainly characterized by the consequences of the great financial crisis happened in 2008 and, afterwards, by the linked increase of equity markets and related decrease of interest rate levels around the world, until a significant market drop in 2018 and the huge market volatility caused by the *Covid Crisis* in the first quarter of 2020.

Such kind of scenario implies an intrinsic difficulty to forecast assets prices' behavior and take related effective counter-moves to then establish opportune portfolio strategy according to specified risk profiles. It is worth to mention that this big challenge interests institutional entities as well as retail investors when looking for an *equity market participation* plus a *downside protection.*

Several attempts can be found in literature to solve this puzzle in terms of *portfolio insurance strategies,* at least starting from the early 70s of the last century, see, e.g., [6, 10, 11, 17], and references therein. Roughly speaking, a portfolio insurance strategy is a *protection blueprint*, based on the definition of a fixed threshold such that the terminal portfolio value always lies above it. This approach bypasses the risk of the actual return being below the expected return, or the uncertainty about the magnitude of that difference, see, e.g., [18, 22].

The portfolio insurance strategies were first introduced in [21], after the collapse of stock markets (the *New York Stock Exchange's Dow Jones Industrial Average* and the *London Stock Exchange's FT 30*, see, e.g., [14]) which implied the pension funds withdrawal. In particular, the authors noted ex-post that the presence of an insurance of the above mentioned type of risk could have convinced investors not to leave the market, guarantying them later the opportunity to take advantage of the rise of the same, an event that really happened just a couple of years later. In this context, the portfolio insurance can be interpreted as a put option on the whole portfolio.

Portfolio insurance strategies can be pigeonholed into three different classes, see, e.g., [25, 26]: an *option-based strategy,* an *option-duplicating strategy* and a *derivative-independent strategy.* A technical analysis, also in terms of performance evaluation, of such strategies can be found in [12], where bull, bear and no-trend markets are considered, while an overview of portfolio insurance strategies, along with their possible connections with financial instability, is provided in [23]. Moreover, an interesting numerical comparison of different approaches is provided in [20].

An early approach, related to the first class of strategies, is the so called *Option-Based Portfolio Insurance* (OBPI) method, see, e.g., [8], which consists of buying a zero-coupon bond with maturity equal to the investment time horizon plus an option written on the portfolio risky asset. As an alternative, in [3] a *minimum-cost portfolio insurance* strategy is presented. Here, the idea is to solve a portfolio optimization problem in incomplete markets by minimizing costs of a portfolio under the constraint that the payoff is greater than the insured one, avoiding losses and capturing gains.

It is worth to mention that low interest rate levels of today's markets are reducing the available risk budgets of such an OBPI approach significantly. This forces practitioners to rethink how to design the portfolio insurance strategy to offer a sustainable equity market participation plus a protection of the initial investment. In this direction, one choice consists in considering dynamic risk management tools to protect portions of the initial investment by dynamically allocating it both in risky and riskless assets, based on available portfolio risk budgets. In this framework the *Constant Portfolio Protection Insurance* (CPPI) is one of the most used techniques, see, e.g., [4, 7, 9, 10, 19, 24, 27], and references therein. Latter methods is realized by rebalancing an initial portfolio at each observation time, evaluating a present value of the aspired capital protection and then investing the available risk budget times a market-depending multiplier into risky assets, while investing the remaining part of the portfolio in time-congruent risk-free assets. An interesting analysis of portfolio insurance strategies, including the CPPI methodology, is provided in [5], where the authors exploit Value-at-Risk, Expected Shortfall and stochastic dominance to measure the portfolio performance of the above-mentioned techniques. By assuming that the CPPI portfolio evolves according to a Markov process, in [24] the authors focus on a discrete-time CPPI-based portfolio allocation method. More recently, a machine learning approach to determine the value of the parameters used to evaluate the correct proportion of wealth to be invested in stock is given in [15].

Despite a significant simplicity and a remarkable ease of implementation, the CPPI strategy suffers a fundamental drawback represented by the risk that, after a severe market drawdown, the risk budget drops to zero, and therefore the strategy offers no market participation afterwards. Practitioners call this case the *cash-in event.*

To overcome the latter scenario, most of the practitioners resort to different routines, such as using an inter-temporal risk budgeting which allows the full use of the available risk budget over time. Alternatively, they adopt the multiplier related to market volatility. However, while both methods can reduce the probability of *cash-in event* to happen, they cannot guarantee to avoid it within the traditional CPPI approaches.

To close this gap, we introduce new exotic option, which can be used within an OBPI portfolio structure and which offers participation in a CPPI allocation strategy with *guaranteed minimum equity exposure.* Our solution is based on the following idea: starting from a standard CPPI strategy, we define a minimum threshold, which is always invested in equity markets. We call it *guaranteed minimum equity exposure* (GMEE) , This then extending the CPPI strategy, since it can be used as underlying of a call or put option. We would like to highlight that, although such an approach has been selectively used in practice, no rigorous mathematical treatment of it has been provided up to now. Therefore, our work represents the first rigorous analytical treatment toward this direction.

Let us recall that the analysis of factors that can potentially lead to gap risk in portfolio insurance framework, mainly taking into account the asset price behavior and the trading frequency, is described in [13]. Conversely, we refer to [16], resp. to [1], for studies on options based on a standard CPPI logic, resp. on options linked to the so called *VolTarget* strategies. Therefore, it turns out that the present paper realizes the first attempt to combine the above mentioned topics in a unified framework.

To better explain the concreteness as well as the goodness of our approach, we shall show how our proposal would have worked in the past, providing some historical simulations of structured products with CPPI and CPPI-GMEE. We scrutinize both versions of the CPPI logic in different market scenarios, to better appreciate the sensitivities of the strategies. Such an investigation enables to figure out the behavior of the risk–return profile, as well as of the asset allocation in different market cycles. Thereafter, we also determine the prices of the corresponding CPPI–GMEE options for different set of product levels for a market model where both the volatility and the interest rate parameters are assumed to be stochastic processes. In this work we aim to focus on a proper initial description of this new exotic option and how it behaves in market models with stochastic volatility and interest rates, without adding any suitable jump component, as for example considered in [28]. The latter being the subject of the authors' next research step.

The present paper is structured as follows: in Section 2 we recall the key concepts related to CPPI and OBPI strategies and we introduce the new CPPI–GMEE approach in more details; in Section 3 we compare the CPPI versus the CPPI with guaranteed minimum equity exposure approaches, mainly exploiting historical simulations; in Section 4 we provide numerical experiments to show the differences of plain vanilla options, options on CPPI and options on CPPI–GMEE; in Section 5 we recap results obtained in the paper also giving an outlook to related future developments.

2 The CPPI–GMEE portfolio allocation strategy

Throughout the paper, we let $0 < T < \infty$ be the investment's time horizon, while $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ is a filtered probability space, with $\mathbb{F} = \{\mathcal{F}_t\}_{0 \leq t \leq T}$, and we assume that all the processes introduced in what follows are $\mathbb{F}$-adapted.

Moreover, we state that there exists a measure $\mathbb{Q} \sim \mathbb{P}$, with respect to the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{Q})$ related to the portfolio allocation strategy used by a financial agent investing his wealth in one risk-less asset, e.g. a bond, and in one risky asset, e.g., an equity index, over a time interval $[0, T]$, $T > 0$

In particular, we consider a risk-free asset B_t whose dynamics reads as follows

$$\mathrm{d}B_t = r_t B_t \mathrm{d}t \, , \tag{2.1}$$

having a return r_t at time t, and a risky asset S_t, such that

$$\mathrm{d}S_t = r_t S_t \mathrm{d}t + \sqrt{v_t} S_t \mathrm{d}Z_t^S \, , \tag{2.2}$$

$$\mathrm{d}v_t = k(\theta - v_t)\mathrm{d}t + \sigma_v \sqrt{v_t} \mathrm{d}Z_t^v \, , \tag{2.3}$$

$$\mathrm{d}r_t = \nu(\beta - r_t)\mathrm{d}t + \sigma_r v_t^\gamma \mathrm{d}Z_t^r \, . \tag{2.4}$$

The stochastic processes Z_t^S, Z_t^v, Z_t^r are three correlated $(\mathbb{F}, \mathbb{Q})$-adapted Wiener processes, with

$$corr\left(\mathrm{d}Z_t^S, \mathrm{d}Z_t^v\right) = \rho_{S,v} \, ,$$
$$corr\left(\mathrm{d}Z_t^S, \mathrm{d}Z_t^r\right) = \rho_{S,r} \, ,$$
$$corr\left(\mathrm{d}Z_t^v, \mathrm{d}Z_t^r\right) = \rho_{v,r} = \rho_{S,v}\rho_{S,r} \, ,$$

where v_t, resp. r_t, represents the volatility, resp. the interest rate, stochastic process, with positive speed of reversion k, resp. ν, long-term mean levels $\theta > 0$, resp. $\beta > 0$, and variance

σ_v, resp. σ_r. Within such a financial scenario, we consider a European contingent claim X with maturity T whose payoff is a real-valued function $f = f(S_t, v_t, r_t)$. The payoff function f may either depend just on the final value of the underlyings, namely S_T and v_T, or on the whole underlying paths over $[0, T]$. In the former case we refer to plain vanilla call/put options, otherwise we consider path-dependent options on CPPI.

At any rate, we can define the price of the contingent claim as

$$\mathcal{O}_t(X) = \frac{1}{B_t}\mathbb{E}_t^{\mathbb{Q}}\left[f(S_T, v_T, r_T)\right], \forall t \in [0, T], \tag{2.5}$$

where $\mathbb{E}_t^{\mathbb{Q}}$ is the conditional expectation taken with respect to the initial filtration $\mathbb{F}$ to which the Wiener processes Z_t^S, Z_t^r, Z_t^v have been adapted and under the risk-neutral measure $\mathbb{Q} \sim \mathbb{P}$. Equivalently, we may consider a contingent claim $\hat{X}$ with maturity $T > 0$, whose payoff is a real-valued function $f = f(V_t^{CPPI}, v_t, r_t)$, relying upon the CPPI portfolio strategy. In particular, in this latter case we assume that the underlying asset for $\hat{X}$ is measured in units of the CPPI strategy, instead of units of stock, see, e.g., [1, 16] for further details. In this case, the price of the contingent claim $\hat{X}$, at time t reads as follows

$$\hat{\mathcal{O}}_t(X) = \frac{1}{B_t}\mathbb{E}_t^{\mathbb{Q}}\left[f(V_T^{CPPI}, v_T, r_T)\right], \forall t \in [0, T]. \tag{2.6}$$

Our novel approach consists in introducing a new version of the CPPI that can be used as an underlying of standard call and put options. Before going into details about the concrete realization of our proposal, let us first briefly recall the *standard* OBPI and CPPI portfolio allocation mechanisms.

The OBPI portfolio allocation strategy. The OBPI strategy is a portfolio insurance procedure characterized by ensuring a minimum terminal portfolio value, see, e.g., [30].

According with standard literature, see, e.g., [8], we define the OBPI portfolio process $V^{OBPI} = \{V_t^{OBPI}\}_{t\in[0,T]}$, with initial value V_0^{OBPI}, as follows

$$V_t^{OBPI} = qB_t + p\,Call(t, S_t, K), \text{ for all } t \in [0, T],$$

where q represents the number of riskless assets acquired by the investor to protect the capital, $Call(t, S_t, K)$ is the call option at time t, written on S_t,, having strike price K and maturity T,, while $p \geq 0$ is the number of calls which can be purchased at time $t = 0$, given the risk budget, see, e.g., [2].

The OBPI approach is said to be *static* in the sense that no trading occurs in $(0, T)$, so that the unique portfolio values we are interested in are

$$V_0^{OBPI} = qB_0 + p\,Call(0, S_0, K) \text{ and } V_T^{OBPI} = qK + p\max\{S_T - K, 0\},$$

therefore, at maturity, the client gets the capital qK plus p times any positive performance of S_T greater than K. In case $q = 1$, $p = 1$ and $S_T > K$, the client gets exactly the performance of the underlying asset.

The CPPI portfolio allocation strategy. In order to define the CPPI portfolio process we begin by specifying the so called *floor* F representing the lowest acceptable value of the portfolio. In particular, we consider the process $F = \{F_t\}_{t \in [0,T]}$ with dynamic

$$\mathrm{d}F_t = r_t F_t \mathrm{d}t \ ,$$

and initial value $F_0 = \mathcal{C} \exp \left\{ \int_0^T r_u \mathrm{d}u \right\}$, where $\mathcal{C}$ is the fixed amount of capital guaranteed at maturity.
We define the process $V^{CPPI} = \{V_t^{CPPI}\}_{t \in [0,T]}$ with initial value V_0^{CPPI}, representing the portfolio value associated to the CPPI strategy, namely

$$V_t^{CPPI} = \alpha_t S_t + \beta_t B_t \ , \tag{2.7}$$

where the model dynamics is defined in eq. (2.2) and α_t, resp. β_t, represents the portfolio proportion invested in the risky, resp. in the risk-less asset.
By assuming that the portfolio strategy is self-financing, the dynamics of the CPPI portfolio can be easily obtained from eq. (2.7) as follows

$$\mathrm{d}V_t^{CPPI} = \alpha_t \mathrm{d}S_t + \beta_t \mathrm{d}B_t \ . \tag{2.8}$$

Moreover, we assume $\mathcal{C} < V_0^{CPPI} \exp \left\{ \int_0^T r_u \mathrm{d}u \right\}$, that is, the guaranteed return must be less than the market interest rate.
Since we are interested in determining the optimal allocation, then, for all $t \in [0, T]$, we evaluate the excess of the portfolio value V^{CPPI} over the floor F, dubbed *cushion*, as

$$C_t := \begin{cases} V_t^{CPPI} - F_t, & V_t^{CPPI} \geq F_t \\ 0, & \text{otherwise} \end{cases} , \tag{2.9}$$

so that $C_t = \max\{0; V_t^{CPPI} - F_t\}$, for all $t \in [0, T]$.
The investment in stock represents the *exposure*, which is given by

$$E_t = M \cdot C_t = M \cdot \max \left\{ V_t^{CPPI} - F_t; 0 \right\} , \quad \text{for all } t \in [0, T] , \tag{2.10}$$

the constant M being a *multiplier* representing the factor by which the risk budget is amplified, giving rise to the risky asset.

Remark 2.1. *Let us note that since we are dealing with a dynamic leverage adjustment mechanism, if we consider a general setting, the multiplier can be represented in terms of a suitable continuous function M_t, depending on different model parameter, see, e.g., [27]. While, for the sake of simplicity, in our case we will consider a constant multiplier $M := \frac{1}{ONR}$, where ONR factor represents the* Over-Night risk *of the risky asset. The market practice usually assumes $ONR = 25\%$ for a given equity index that serves as underlying, implying that $M = 4$.*

2.1 The proposal: CPPI with Guaranteed Minimum Equity Exposure

Our proposal is articulated according to the following steps.

Step 1. We first consider that the investor can not suffer losses related to the amount of her initial wealth. Therefore, in what follows, we will extensively make use of the so called *protection level* (PL), defined as the percentage of the initial capital guaranteed at maturity. Hence, following a standard portfolio insurance strategy procedure, we assume to invest a fixed initial capital, say C, e.g., equal to the $100\% - PL$, so that the strategy will pay at least C at maturity.

In order to ensure such a payment, the present value of C is invested in a Zero-Coupon-Bond (ZCB), while the remaining part is invested in a European option. The former is referred to as the *discounted value of the guarantee* (DG), while the latter is the *risk budget* (RB). The unit of option purchased for RB represents the so called *participation rate.*

Step 2. We select an appropriate investment strategy for RB, so that the latter does not show negative changes, after some adjustments in the model parameters for the underlying. More precisely, we decided to choose plain vanilla options linked to the CPPI mechanism.

As regards the evaluation of the percentage of wealth to be invested in the risky asset, eq. (2.10) gives

$$\alpha_t = \frac{M \cdot C_t}{V_t}, \qquad \beta_t = V_t^{CPPI} - \alpha_t \,, \tag{2.11}$$

by assuming $V_t^{CPPI} \neq 0$ $\mathbb{P}$-a.e.

In particular, eq. (2.11) implies that the investment in the risky asset might be potentially unbounded. To limit such a potential leverage effect in the optimal allocation, the market practice suggests to introduce the so called *maximum leverage factor* L_{max} in the equity weights α_t, such that

$$\alpha_t := \max\left\{\min\left\{L_{max}; \frac{M \cdot C_t}{V_t^{CPPI}}\right\}; 0\right\} \,. \tag{2.12}$$

Motivated by some regulatory constraints, see, e.g., [29] , L_{max} is typically set to $L_{max} = 150\%$, or $L_{max} = 200\%$.

Step 3. It is worth to mention that, due to abrupt changes of market's levels, the risky asset loses significantly in value. As a result, the risk budget drops to zero. This implies that, from that moment on, the manager has to invest the total portfolio in risk-less securities until the contract expires. This situation is referred to as *cash-in risk.*

To avoid such a scenario, we introduce the so called *guaranteed minimum equity exposure* (GMME) α_{min} with $0 \leq \alpha_{min} \leq 1$ in eq. (2.12), obtaining

$$\alpha_t^{CPPI} = \max\left\{\min\left\{L_{max}; \frac{M \cdot C_t}{V_t^{CPPI}}\right\}; \alpha_{min}\right\} \,. \tag{2.13}$$

Consequently, the CPPI portfolio whose equity exposure is given by eq. (2.13), is labeled as *CPPI–GMEE* portfolio.

Let us note that exploiting equations (2.12) and (2.13), we easily get that the case $\alpha_{min} = 0$ coincides with the traditional CPPI logic.

Remark 2.2. *Looking at the proposed strategy, we first observe that Step 1 has the standard operating scheme characterizing typical portfolio insurance strategies, hence according with what is already known in the literature.*
Secondly, it is interesting to note that Step 2 of the proposed algorithm represents a recent, albeit already widely exploited, variant of the portfolio insurance strategies. Indeed, this represents an alternative to the risk budget investments in either plain vanilla or exotic options, linked to standard indexes.
Moreover, we observe that, thanks to eq. (2.13)*, the equity participation will never go below* α_{min} *even in case of a severe market drop, but, at the same time, it would mean that this adjusted CPPI allocation implemented in a real portfolio might not be able to protect the invested capital. Hence, we suggest to use the adjusted CPPI mechanism within an OBPI-based portfolio approach, where the call option is linked to the CPPI allocation logic with GMEE.*
Finally, we would like to emphasize that the novelty of our proposal rests on the role played by the GMEE within the strategy, namely Step 3. In this view, to corroborate our intuition about the relevance of GMEE, we are going to compare the prices of European options linked to the CPPI mechanism, both in the presence and absence of the aforementioned further condition. We leave the investigation on the allocation strategies' performances for future research.

3 Historical simulation of structured products with CPPI vs. CPPI with guaranteed minimum equity exposure

In order to verify weather the presence of the GMEE threshold permits to dodge cash-in events, in this section we compare CPPI portfolio allocation strategies, with and without guaranteed minimum equity exposure, linked to European equity markets. To better capture the sensitivities of the proposed strategies, we propose two different scenarios: namely we first consider $PL = 100\%$ from 2007 and until 2017, then we analyze the $PL = 90\%$ case, within the time period from 2000 to 2010.

The 100%-protection case. The CPPI simulation has been conducted assuming the following

- the underlying is given by a *Euroland large and mid cap equity index* as risky asset between 31st of December 2007 and 29th of December 2017;
- the CPPI mechanism should protect 100% of the initial investment after 10 years. For the sake of simplicity, we assume a constant risk-free rate of 3%,, since the risk-free rates in 2008 were close to such a level. Moreover, taking into consideration interest rates' levels near to zero, the related risk budget would be also close to zero. In this framework, one can either extend the investment time horizon, to benefit from higher interest rates as a result of longer maturities, or set up the investment w.r.t. a lower protection level, e.g. at 90%, instead of 100%;
- We choose a multiplier $M = 3$ and a maximum leverage factor of $L_{max} = 150\%$, while the guaranteed minimum equity exposure α_t is set to 30%.

Figure 1: Historic Simulation for a standard CPPI and a CPPI with guaranteed minimum equity exposure linked to European equity markets between 2007 and 2017.

In Fig. 1 we report the obtained results to better motivate our key idea.

The left hand axis gives the performance of the risky asset, w.r.t. a standard CPPI approach and a CPPI approach with guaranteed minimum equity exposure, also providing the present value of the guarantee in percentage of the initial investment. The right hand axis shows the risky asset exposure over time for a CPPI and a CPPI–GMEE portfolio in percentage of the overall portfolio allocation.

The key findings can be summarized as follows: looking at the risky asset itself, we see that the considered market index significantly lost in value between 2007 and March 2009. In fact, during this period the index lost close to 50% of its initial value. After that, the equity index recovered nicely and over the full 10 year horizon the index generated a positive performance of more than 20%. Nevertheless, from the point of view of a conservative investor, such an equity investment might be too volatile. Focusing on a traditional CPPI allocation logic, we can see that the red line gives the performance of a CPPI strategy linked to this equity index as risky asset, exploiting the parameters mentioned above. Without loss of generality, we do not consider transaction costs.

The standard CPPI has an initial exposure to the risky asset of more than 70%. Due to the extreme losses in the risky asset, the risk budget quickly decreases, and the CPPI needs to reduce the risky asset exposure to less than 30% after 10 months. The CPPI approach itself can limit the losses successfully compared to the pure risky asset investment in this time, but it cannot participate in any upwards markets afterward. Again looking at the risky asset linked volatility dynamic over the next 4 years, we can see that its allocation drops to zero at the end of 2012, hence no market participation exists afterward. Overall, the CPPI can achieve $PL = 100\%$, but it cannot benefit from the overall positive return in the equity market. The CPPI with guaranteed minimum equity exposure starts with the same risky asset allocation as the standard CPPI. Also this CPPI approach is forced to reduce its risky asset exposure

Figure 2: Historic Simulation for a standard CPPI and a CPPI with guaranteed minimum equity exposure linked to European equity markets between 2000 and 2010.

significantly due to negative risky asset performance. Nevertheless, by definition, the risky asset exposure never drops below the predefined threshold of 30%. Therefore, the proposed CPPI alternative approach can benefit from rising equity markets again and over the full remaining life time, the CPPI with guaranteed minimum equity exposure achieves a return which is quite comparable to the pure risky asset.

At the same time, such a CPPI idea would not be able to achieve the PL of 100% by itself. If markets would continue to fall, the GMEE would lead to further losses in the portfolio. Therefore the CPPI-GMEE approach can only be used as portfolio insurance strategy, if one uses an OBPI portfolio structure and uses an call option linked to the CPPI-GMEE strategy.

The 90%-protection case. In this second example we slightly adjust the CPPI parameters as follows.

- We use a *Euroland* large and mid cap equity index as risky asset, looked into the time interval from 31st of December 1999 to 31st of December 2009;
- the considered CPPI mechanism should protect 90% of the initial investment after 10 years, and we assume a constant risk-free rate of 4%;
- within the CPPI we choose a more conservative multiplier $M = 2$, capping the maximum leverage factor at $L_{max} = 100\%$. The guaranteed minimum equity exposure α_t is set at 30%, and we do not consider any transaction costs.

As can be seen in Fig. 2, the risky asset during this period of time starts positively, then achieving a return of close to 20%, in the first year. But, between 2000 and 2002, the equity index loses about 50% of its initial value. Then, it well recovers until mid of 2007, when the worldwide financial crisis causes new severe losses. Therefore, after 10 years, the index loses about 15% of its initial value.

The standard CPPI linked to this index has an initial exposure of about 80%, as a result of higher risk budget given the lower protection level of just 90% and a higher risk-free rate of 4% compared to the previous case. When the equity index increases initially in value then the exposure of the standard CPPI increases to close to 90%. But, during following years, the exposure is significantly reduced to less than 20% in 2003. When markets are recovering, also the risky asset exposure increases to roughly 40%, but during the financial crisis it falls below 10% in early 2009. Therefore, after 10 years, the exposure is at 10% and the performance of the CPPI ends with −5%, namely it could achieve a higher return than the index itself and than the aspired capital protection level of 90%, but still the investor made a loss.

The CPPI with guaranteed minimum equity exposure shows initially a similar behavior, hence like the traditional CPPI. In particular, it also starts with an equity exposure of around 80%, which increases to even 90% and then falls to the guaranteed minimum equity exposure of 30% in 2002. The equity exposure remains there, between 2005 and 2007 it increases again to ca. 40%. During the financial crisis it drops back again, but, by definition, not below the minimum exposure of 30%. As a result, the CPPI with guaranteed minimum equity exposure could limit losses from equity markets in 2001 and 2002, and also during the financial crisis. Nevertheless, it could especially benefit from recovering markets more than a standard CPPI. After 10 years the new CPPI approach generated a positive return of more than 3%. Moreover, it could achieve $PL = 90\%$ and it could generate a much better risk-return profile than a pure equity investment.

4 Numerical Pricing of Options on CPPI with guaranteed minimum equity exposure

We now consider options on CPPI and CPPI–GMEE strategies. More precisely, in what follows we provide an in-depth numerical analysis related to the its implementation in the option pricing context.

4.1 Options on CPPI and CPPI-GMEE

In this section we are going to compare At-The-Money (ATM) European call/put option prices, evaluated both in the standard case, i.e. when the underlying is the stock dynamics, and when the underlying equals the CPPI portfolio allocation strategy, the second type of computations has been conducted both with and without Guaranteed Minimum Equity Exposure. Let us assume the following:

- for normal CPPI-based strategies, the $PL = 100\%$ at the end of the option life time, the multiplier is $M = 4$, while the Maximum Leverage is equal to $L_{max} = 150\%$;
- for options linked to CPPI strategies with guaranteed minimum equity exposure, the protection level, the multiplier and the Maximum Exposure are as in the previous case, while the guaranteed minimum equity exposure is $\alpha_{min} = 30\%$.

Table 1: Model parameters used in the numerical experiments. The second column refers to the stochastic interest rate model (Vasicek model). The last column refers to the stochastic volatility model (Heston model).

	Vasicek model	Heston model
Long-run mean	$\beta = 0.05$	$\theta = 0.04$
Rate of mean reversion	$k = 1.25$	$\nu = 1.25$
Volatility	$\sigma_r = 0.025$	$\sigma_v = 0.2$
Correlation	$\rho_{S,r} = -0.2$	$\rho_{S,v} = -0.5$

We assume no transaction costs and dividends are directly reinvested into the strategy. We also assume that all the CPPI strategies are re-allocated each business day. From a practical perspective, this implies that the manager assumes that trading actions are discrete in time. Then, subdividing $[0, T]$ in n intervals, not necessarily of the same length, according with the following series of ordered time points $\{t_i\}_{i=0,..,n}$: $0 = t_0 < t_1 < \ldots < t_n = T$, each of which represents a fixed trading date, we have

$$V_{t_i}^{CPPI} = \alpha_{t_i} V_{t_{i-1}}^{CPPI} \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} + (1 - \alpha_{t_i}) V_{t_{i-1}}^{CPPI} (1 + r_{t_i}) \ , \tag{4.1}$$

with initial condition $V_0^{CPPI} = V_0$.

Remark 4.1. *Let us note that there is not a standard rule to determine the time grid that has to be used for CPPI portfolio re-balancing purposes. In the market practice often a so called* trading filter *is applied. Namely, as long the real allocation deviates not significantly to the theoretical CPPI allocation, no trading happens. This is to avoid trading on noise and also to reduce transaction costs.*
Overall a weekly allocation is probably a reasonable assumption. In case of financial distress the re-balancing frequency can increase, and managers might have to trade daily or, in extreme cases, even two or three times a day.

Concerning the numerical values for the parameters, we consider four time horizons (measured in years) $T = \{1, 2, 5, 10\}$, an initial interest rate $r_0 = 3\%$, and an initial volatility level $v_0 = 20\%$. The remaining parameters are described in Table 1.

4.1.1 Comparison of call options

In what follows, numerical results for European call options linked to the CPPI strategy as underlying are provided. We compare the values of European call options within the Vasicek-Heston model when the underlying is a pure risky asset, the CPPI strategy, and the CPPI with guaranteed minimum equity exposure, respectively. In Fig. 3 we report the call option price for different maturities in each of the aforementioned scenarios.
Starting with an initial volatility level of 20% and an initial interest rate level of 3%, we observe that the price obtained by taking the pure risky asset as the derivative's underlying results is the most expensive strategy. Instead, the option pricing with respect to the CPPI strategy leads to a reduction of the option price. As expected, the CPPI with guaranteed minimum

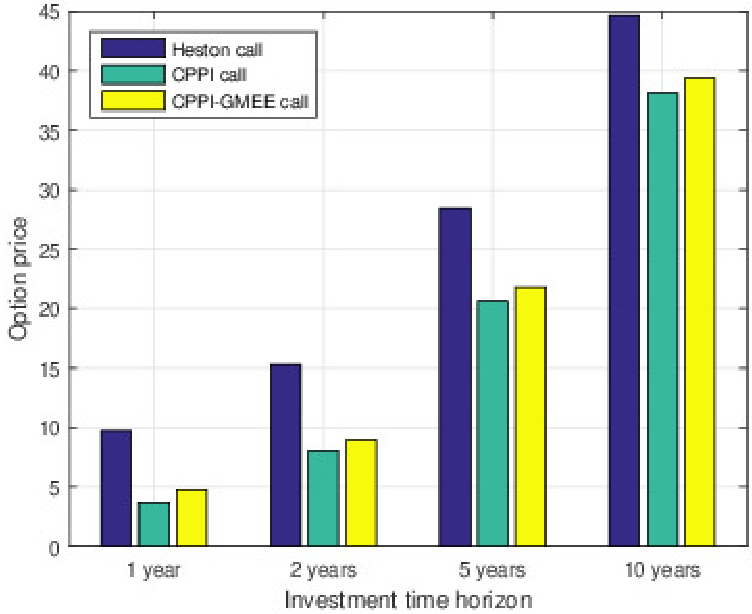

Figure 3: ATM call option pricing under different underlyings and different maturities. The Heston model parameters being set as follows: $\sigma_0 = 0.2$, $k = 1.25$, $\theta = \sigma_0^2$, $\sigma_v = 0.2$, $\rho_{S,v} = -0.5$, while the interest rate model parameters are: $r_0 = 0.03$, $\nu = 1.25$, $\beta = r_0$, $\sigma_r = 0.025$, $\gamma = 0.5$, $\rho_{S,r} = -0.2$, and evebntually the CPPI strategy parameters are: $L_{max} = 100\%$, $\alpha_{min} = 30\%$, $M = 4$, $PL = 100\%$.

equity exposure leads to higher prices rather than the simple CPPI, this due to the *cost* we pay for having fixed a minimum guaranteed equity exposure at 30%.
In order to stress the role of this guaranteed minimum exposure, we study the call option value as a function of the parameter α_{min} for different maturity levels.
In Fig. 4 we observe that the option price increases as the minimum guarantee threshold raises. The lowest price is reached when the minimum guaranteed exposure is zero, which corresponds to the case of a standard CPPI strategy as underlying of the option. The case $\alpha_{min} = 100\%$ coincides with the plain vanilla call option, as in this case the allocation to risky asset is always 100%, since we assumed $L_{max} = 100\%$ within the CPPI logic. The effect of a rising option price with a rising minimum guaranteed equity exposure is also true for longer maturities.
In Table 2 we report the results obtained by exploiting the different allocation strategies for the evaluation of the call option prices, w.r.t. different initial interest rates and volatilities, and considering a short investment time horizon, i.e. taking $T = 1$ year. In Panel A we reported the plain vanilla call option prices, while Panel B contains the normal CPPI-based options, and Panel C refers to the options linked to the CPPI with guaranteed minimum equity exposure. Let us underline that, comparing the results reported in Panel A, B and C of Table 2, the call options linked to the traditional CPPI-based approach remain almost constant for different volatility levels, no matter about the initial interest rate value. Furthermore, higher volatilities might increase the option price for a pure risky asset underlying, and higher volatilities for a CPPI strategy increase the risk of a cash-in event such that a higher number of simulated paths ends up with the minimum protection level of 100%.
Let us also note that options linked to CPPI-based strategies are significantly cheaper than plain

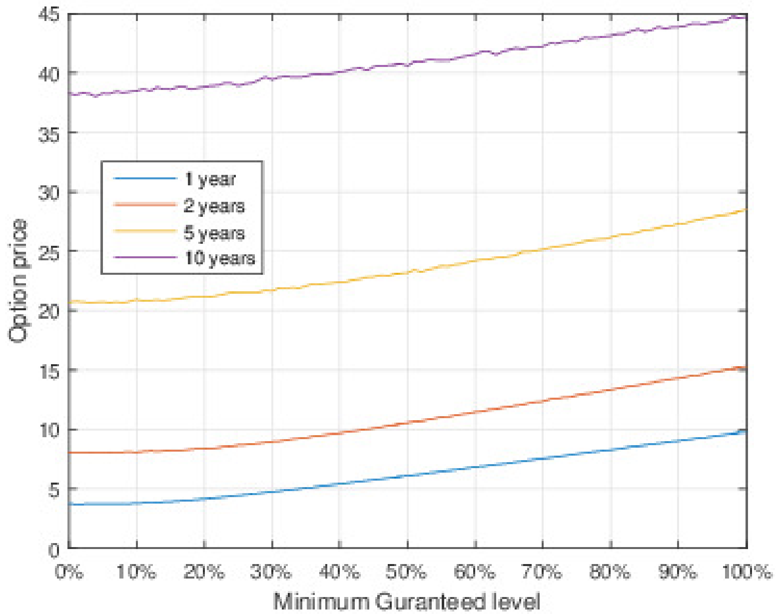

Figure 4: ATM call option price when the underlying is a CPPI strategy with guaranteed minimum equity exposure as a function of the minimum exposure parameter. The CPPI strategy parameters are: $L_{max} = 100\%$, $M = 4$, $PL = 100\%$.

Table 2: ATM call option prices for different values of initial interest rate and initial volatility. The Heston model parameters are: $k = 1.25$, $\theta = \sigma_0^2$, $\sigma_v = 0.2$, $\rho_{S,v} = -0.5$, while the interest rate model parameters are: $\nu = 1.25$, $\beta = r_0$, $\sigma_r = 0.025$, $\gamma = 0.5$, $\rho_{S,r} = -0.2$, and the CPPI strategy parameters are: $L_{max} = 100\%$, $\alpha_{min} = 30\%$, $M = 4$, $PL = 100\%$.

Panel A: option on pure risky asset					
Initial interest rate (r_0)	Initial annual volatility (v_0)				
	0.10	0.20	0.30	0.40	0.50
0.01	5.29	9.19	13.06	16.89	20.68
0.03	6.07	9.77	13.57	17.38	21.18
0.05	6.84	10.35	14.14	17.88	21.57
Panel B: option on CPPI					
Initial interest rate (r_0)	Initial annual volatility (v_0)				
	0.10	0.20	0.30	0.40	0.50
0.01	2.64	2.64	2.65	2.65	2.62
0.03	3.72	3.72	3.71	3.72	3.66
0.05	4.78	4.78	4.78	4.78	4.72
Panel C: option on CPPI with Guaranteed Minimum Equity Exposure					
Initial interest rate (r_0)	Initial annual volatility (v_0)				
	0.10	0.20	0.30	0.40	0.50
0.01	3.02	3.97	5.07	6.28	7.55
0.03	3.97	4.74	5.76	6.94	8.27
0.05	4.94	5.55	6.50	7.64	8.96

vanilla ones, thanks to the embedded risk management features. Moreover, below an annual market volatility of 20% the CPPI with and without minimum exposure give a comparable price range. When the volatility exceeds 20% the former becomes more expensive.

4.1.2 Comparison of put options

In this subsection we provide numerical results for the put option case. Fig. 5 shows the put

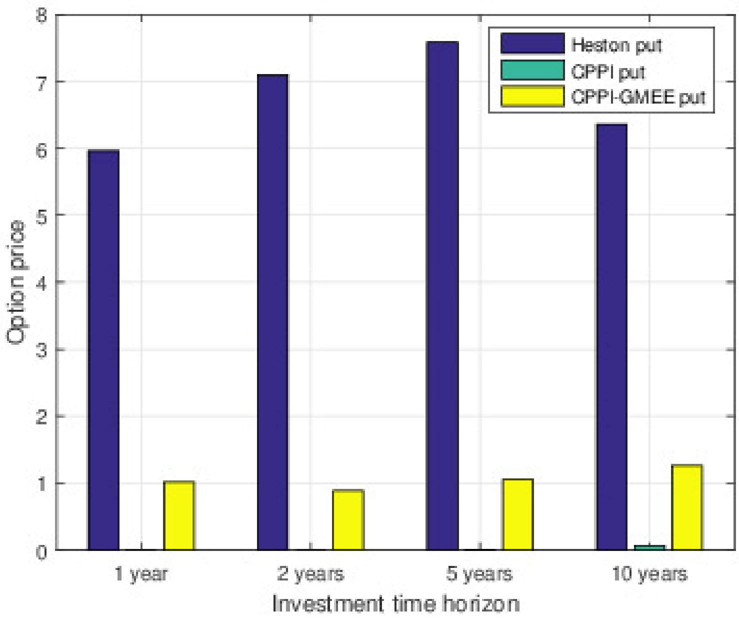

Figure 5: ATM put option pricing under different underlyings and different maturities. The Heston model parameters are: $\sigma_0 = 0.2$, $k = 1.25$, $\theta = \sigma_0^2$, $\sigma_v = 0.2$, $\rho_{S,v} = -0.5$, while the interest rate model parameters are: $r_0 = 0.03$, $\nu = 1.25$, $\beta = r_0$, $\sigma_r = 0.025$, $\gamma = 0.5$, $\rho_{S,r} = -0.2$, the CPPI strategy parameters being: $L_{max} = 100\%$, $\alpha_{min} = 30\%$, $M = 4$, $PL = 100\%$.

option price for different maturities when the underlying is represented by a pure risky asset, resp. by a standard CPPI portfolio, resp. by a CPPI strategy with guaranteed minimum equity exposure.

As expected, the put option linked to the standard CPPI strategy provides a value close to zero. The rare cases in which also the value of the put option linked to a standard CPPI is positive implies that we have some paths for which the CPPI logic does not achieve the protection level of 100%. The latter is due to the fact that, especially for longer time horizon, the risk increases that in certain cases the overnight loss is higher than the assumptions embedded in the multiplier M.

As seen in Sect. 4.1.1, we obtain that the put options linked to CPPI-based strategies, with and without guaranteed minimum equity exposure, are cheaper than the standard options linked to a pure equity index underlying.

Moreover, in Fig. 6 we study the put option value as a function of the minimum equity exposure α_{min} for different maturity levels. As before, the case of $\alpha_{min} = 0\%$ coincides with the put option linked to a standard CPPI. For $\alpha_{min} = 100\%$ the results coincide with put option prices using a pure risky asset as underlying.

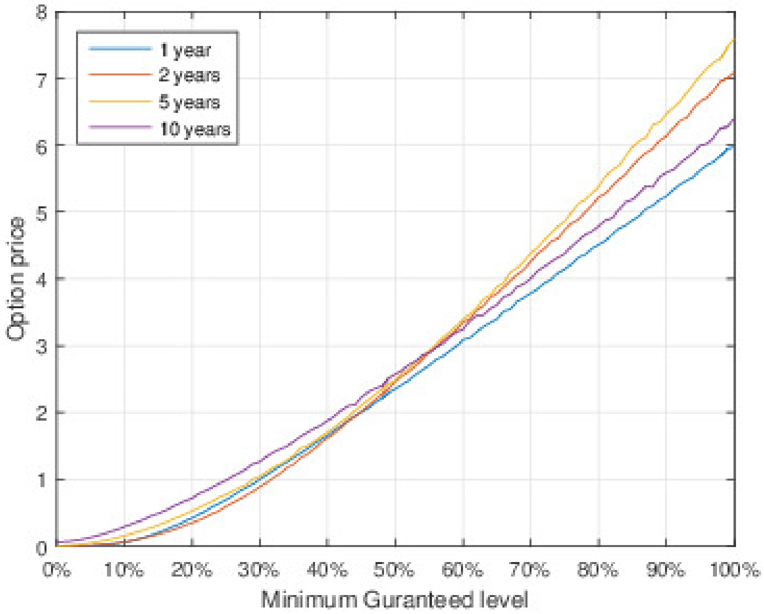

Figure 6: ATM put option price when the underlying is a CPPI strategy with guaranteed minimum equity exposure as a function of the minimum exposure parameter. The CPPI strategy parameters are: $L_{max} = 100\%$, $\alpha_{min} = 30\%$, $M = 4$, $PL = 100\%$.

Table 3 reports our evaluations for the ATM put option prices for different initial interest rates and volatilities, assuming an option maturity $T = 1$ year. Panel A refers to put options on a pure risky asset, while Panel B refers to put options linked to the standard CPPI case, and Panel C reports data for options related to a CPPI with guaranteed minimum equity exposure. It can be seen that put options linked to CPPI-based strategies with minimum exposure to the risky asset are positive and significantly more expansive than put options on a normal CPPI. These results show that, as expected, there exists a number of paths in which the CPPI with guaranteed minimum exposure can lead to real losses and cannot achieve capital preservation like the standard CPPI approach in itself.

4.1.3 CPPI-based option pricing strategies for different protection levels

An alternative way to adjust the portfolio allocation is to modify the protection level. In particular, we consider a CPPI-GMEE allocation with different protection levels, ranging between 0% and 100%. A reduction of the protection level increases the risk budget and, consequently, the equity exposure of the portfolio. For the standard CPPI strategy the protection level remains at 100%. The numerical results for ATM call/put options are provided in Fig. 7 and 8.
We observe that:

- by reducing the protection level of the CPPI-GMEE approach from 100% to 90% the CPPI-GMEE strategy gets riskier. This implies that the corresponding option price increases significantly. The same behavior can be spotted when the protection level is even more reduced, e.g. when we consider $PL = 50\%$.

  Such a circumstance is more evident in the case of put options, where the option price

Table 3: ATM put option prices for different values of initial interest rate and initial volatility. The Heston model parameters are: $k = 1.25$, $\theta = \sigma_0^2$, $\sigma_v = 0.2$, $\rho_{S,v} = -0.5$, while the interest rate model parameters are: $\nu = 1.25$, $\beta = r_0$, $\sigma_r = 0.025$, $\gamma = 0.5$, $\rho_{S,r} = -0.2$, the CPPI strategy parameters being: $L_{max} = 100\%$, $\alpha_{min} = 30\%$, $M = 4$, $PL = 100\%$.

Panel A: option on pure risky asset					
Initial interest rate (r_0)	Initial annual volatility (v_0)				
	0.10	0.20	0.30	0.40	0.50
0.01	2.62	6.49	10.35	14.18	18.02
0.03	2.29	5.99	9.80	13.57	17.38
0.05	1.97	5.49	9.25	13.00	16.74
Panel B: option on CPPI					
Initial interest rate (r_0)	Initial annual volatility (v_0)				
	0.10	0.20	0.30	0.40	0.50
0.01	0.00	0.00	0.00	0.00	0.00
0.03	0.00	0.00	0.00	0.00	0.00
0.05	0.00	0.00	0.00	0.00	0.00
Panel C: option on CPPI with Guaranteed Minimum Equity Exposure					
Initial interest rate (r_0)	Initial annual volatility (v_0)				
0.01	0.37	1.31	2.41	3.62	4.93
0.03	0.24	1.01	2.03	3.21	4.51
0.05	0.15	0.76	1.71	2.85	4.13

doubles when the protection level halves;

- the case in which $PL = 0\%$ equals the case with a pure risky asset as the derivative underlying, hence we see the same option price;
- there exists the risk that the CPPI strategy ends below 100, implying that also the put option price on standard CPPI is greater than zero for long maturities.

5 Conclusion

In this paper we have introduced a new exotic option, which offers a participation in a CPPI strategy with guaranteed minimum equity exposure. This option applied within an OBPI structure can build a portfolio insurance strategy with portfolio characteristics close to a standard CPPI, but avoiding the well-known cash-in event of a standard CPPI strategy. This represents a concrete innovation, both from the practitioner and from the literature point of view, when compared to the existing portfolio protection strategies based on OBPI with plain vanilla call options or a standard CPPI.

We have provided historical simulations showing how the new CPPI strategy differs from a standard CPPI portfolio logic, according to the market environment. In addition, we looked at option prices' behaviors under different frameworks, namely, when the underlying is a pure

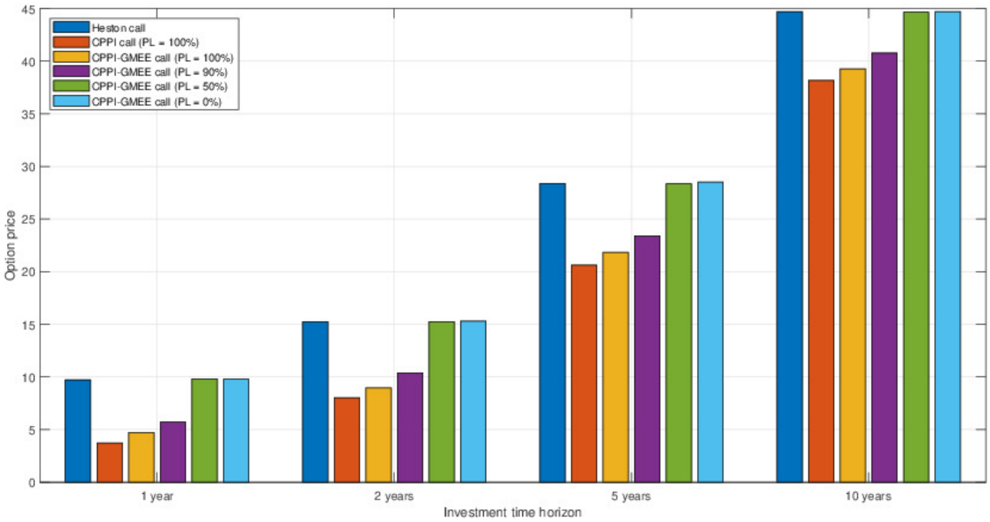

Figure 7: ATM call option pricing under different underlyings, different maturities and different protection levels. The Heston model parameters are: $\sigma_0 = 0.2$, $k = 1.25$, $\theta = \sigma_0^2$, $\sigma_v = 0.2$, $\rho_{S,v} = -0.5$, while the interest rate model parameters are: $r_0 = 0.03$, $\nu = 1.25$, $\beta = r_0$, $\sigma_r = 0.025$, $\gamma = 0.5$, $\rho_{S,r} = -0.2$, and the CPPI strategy parameters are: $L_{max} = 100\%$, $\alpha_{min} = 30\%$, $M = 4$, $PL = [0\%, 50\%, 90\%, 100\%]$.

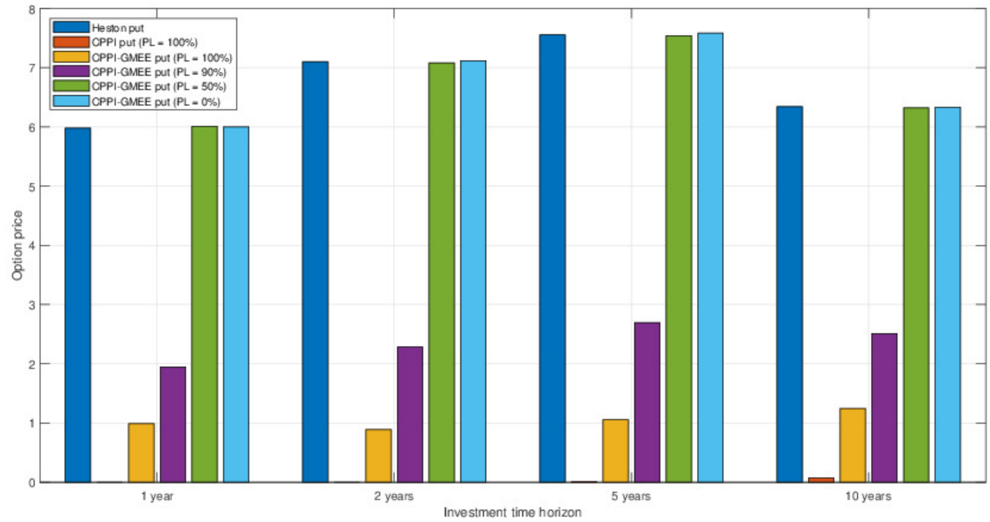

Figure 8: ATM put option pricing under different underlyings, different maturities and different protection levels. The Heston model parameters are: $\sigma_0 = 0.2$, $k = 1.25$, $\theta = \sigma_0^2$, $\sigma_v = 0.2$, $\rho_{S,v} = -0.5$, while the interest rate model parameters are: $r_0 = 0.03$, $\nu = 1.25$, $\beta = r_0$, $\sigma_r = 0.025$, $\gamma = 0.5$, $\rho_{S,r} = -0.2$, and the CPPI strategy parameters are: $L_{max} = 100\%$, $\alpha_{min} = 30\%$, $M = 4$, $PL = [0\%, 50\%, 90\%, 100\%]$.

risky asset, a CPPI strategy, or a CPPI–GMEE based one. Obtained results clearly illustrate that, depending on the parameters choice, our method provides a valuable compromise between an option linked to a pure risky asset investment strategy and an option linked to a traditional CPPI. In fact, it ensures to avoid the aforementioned cash-in risk of a standard CPPI, although it is rather more expensive than the options on standard CPPI.

We would like to underline that the present work represents a first step in our research agenda.

Further research include option pricing within a more complex volatility market models, which also assumes the presence of jumps. Furthermore, we plan to compare options on CPPI with guaranteed minimum equity exposure to options linked to other dynamic asset allocation strategies, such, e.g., Volatility Target Strategies, as well as to consider options on CPPI–GMEE strategies with lock-in elements, which can be also seen as the analysis of the so-called TIPP strategies in a standard CPPI framework. Eventually, we are going to examine the role played by transaction costs in the option valuation on CPPI-GMEE framework.
Given the remaining low interest rate environment in combination with high volatilities in the markets, also as a result of the Corona Crisis, we expect that practitioners will continue to look for new ways how to offer draw-down protection with extremely low risk budgets. New option payoffs with dynamic asset allocations as underlying are clearly one path, in which further innovations will be seen. Our idea of a CPPI with guaranteed minimum equity exposure is one example of this research direction.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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