NETWORK REVENUE MANAGEMENT GAME IN THE RAIL FREIGHT INDUSTRY

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Doctor of Philosophy

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ABSTRACT

The study aims to design the optimal track access tariff to coordinate the relationship between an Infrastructure Manager (IM) and a Freight Operating Company (FOC) in a vertical separated railway system. In practice, the IM takes advantage of leader position in determining the prices to unilaterally maximise its profits without the collaboration with the FOC, which leads to a sub-optimal situation.

The interaction between the IM and the FOC is modelled as a network-based Stackelberg game. First, a rigorous bilevel optimisation model is presented that determines the best prices for an IM to maximise its profits without any collaboration with the FOC. The lower level of the bilevel model contains binary integer variables representing the FOC's choices on the itineraries, which is a challenging optimisation problem not resolved in the literature. The study proposes a uniquely designed solution method involving both gradient search and local search to successfully solve the problem. Secondly, an inverse programming model is developed to determine the IM's prices to maximise the system profit and achieve global optimality. A Fenchel cutting plane based algorithm is developed to solve the inverse optimisation model. Thirdly, a government subsidy based pricing mechanism is designed. To identify the optimal amount of subsidy, a double-layer gradient search and local search method is developed. The proposed mechanism can lead to the global optimality and ensure that the IM and the FOC are better off than the above two scenarios.

Numerical cases based on the data from the UK rail freight industry are conducted to validate the models and algorithms. The results reveal that both the optimal prices obtained via inverse optimisation and the subsidy contract outperform the non-cooperation case in the current industrial practice; and that the cooperation between the IM and the FOC in determining track access tariff is better than non-cooperation.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

GLOSSARY OF TERMS

Abbreviation	Definition
The IM	Infrastructure Manager of railway system
The FOC	Freight Operation Company
ORR	Office of Rail and Road
DfT	Department for Transport
NR	Network Rail
LP	Linear Programming
MILP	Mixed Integer Linear Programming Problem
HGV	Heavy Goods Vehicles
CP	Control Period
RM	Revenue Management;
SCM	Supply Chain Management;
RS	Revenue Sharing
GHG	Greenhouse Gas

Chapter 1 Introduction

Rail plays an important role in the intermodal freight transportation network due to its economical and technological advantages (Islam, Ricci and Nelldal, 2016). In the UK and some other western countries, vertical separation governance structure has been adopted as an innovative method to prevent infrastructure owners from being monopolists (Nash *et al.*, 2014; Islam and O, 2016). After the separation of railway ownership from operation, a topical issue arising is how an Infrastructure Manager (IM) should charge Freight Operating Companies (FOC) for the use of rail infrastructure. This study will make contribution to designing a better pricing mechanism for a vertical separated railway system.

In this chapter, the context of the research will be explained. Four aspects of the research context will be considered: 1) the role of rail in the multimodal freight transportation system; 2) the technological and economical advantages of rail freight; 3) the governance structures in the rail freight industry; 4) the development of UK rail freight industry. Following on from this research context, the research questions, objectives and novelty will be presented at the end of the chapter.

1.1 The Role of Rail in the Multimodal Freight Transportation System

Freight transportation is essential for the development of a modern society. It enables the movement of raw materials, semi-finished, as well as finished products from one point to another. It acts as the lifeblood system of human society in association with the passenger transportation system (Engström, 2016). The transportation modes by which freight can achieve mobility come in four forms: land (that includes road, rail and multimodal), water (that includes maritime, coastal and inland waterways), pipeline and air (Figure 1). As shown in Figure 2 the cargoes can be delivered from an origin point to a destination point carried by a single transport mode only or multimodal transport mode. Multimodal transport provides a door-to-door service where road transport is normally used for the pickup and "last mile" delivery service and the other transportation modes, e.g., rail, water, and air, are used for long-haul transportation.

Freight transport has brought undoubted benefits such as improved mobility of goods and services to society, however it also generates some negative impacts such as greenhouse gas (GHG) emission, congestion resulting in unreliable transport or journeys and accidents resulting in damage to goods/materials, injuries and fatalities. Compared to road transport, rail transport is more friendly to the environment as it produces less GHG and causes less congestion and fatalities (Nelldal, Ricci and Islam, 2017; Mortimer and Islam, 2017).

The European Commission's Transport White Paper 2011 has set a target of modal shift, by 2030, 30% of road freight over 300km should shift to other modes such as rail or waterborne transport, and more than 50% by 2050. This indicates there are efficiency and capacity requirements for rail freight transportation over long distances (European Commission, 2011). The policy target for rail freight transport also suggests that rail will play a more important role than it is playing currently. This signals a valuable opportunity for the rail freight industry. In the meantime, all the stakeholders involved in the rail freight industry, including but not limited to infrastructure managers and freight service operators have been facing the increasing pressure of competition from other transportation modes, in particular roads, which forces the stakeholders to review and optimise their business management process, as well as their pricing strategy, to keep their competitive market position (Network Rail, 2006).

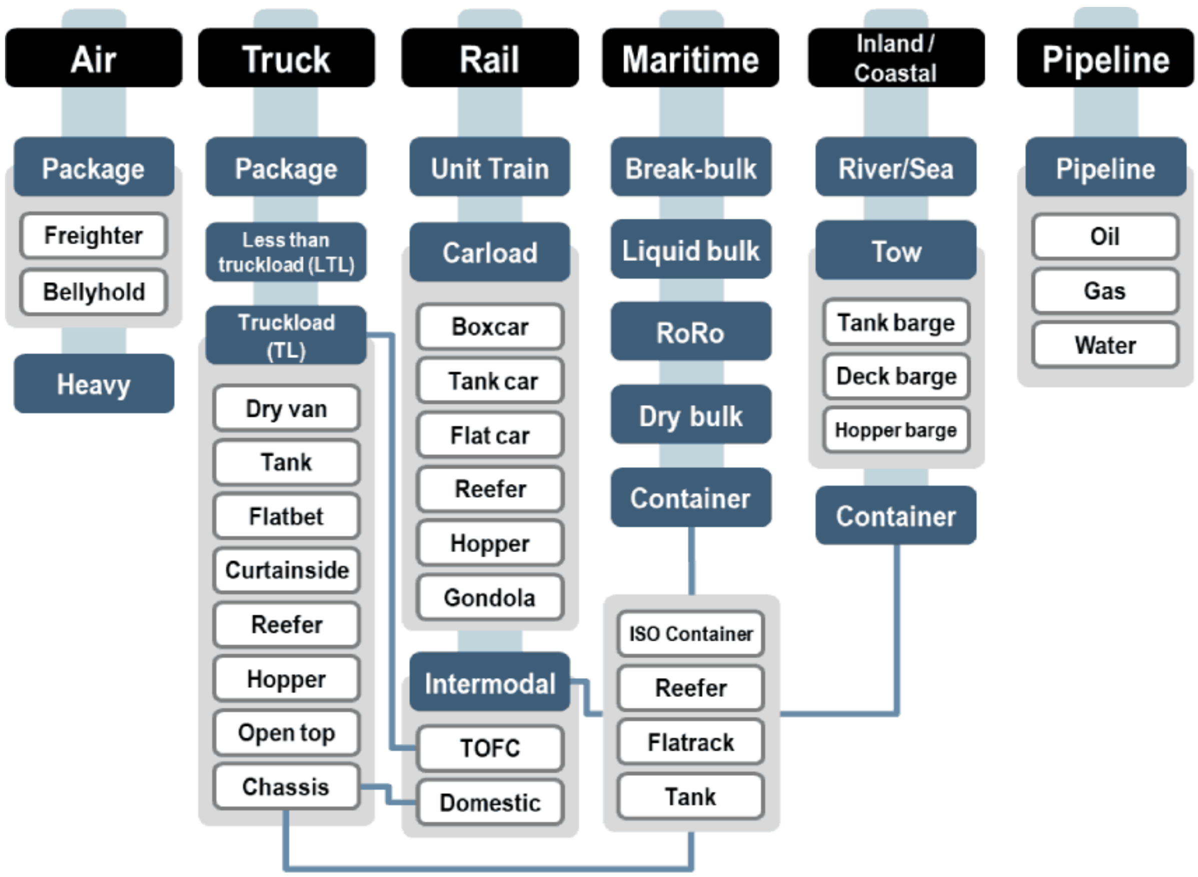

Figure 1 The Main Freight Modal Options

Source: (Slack, 2016)

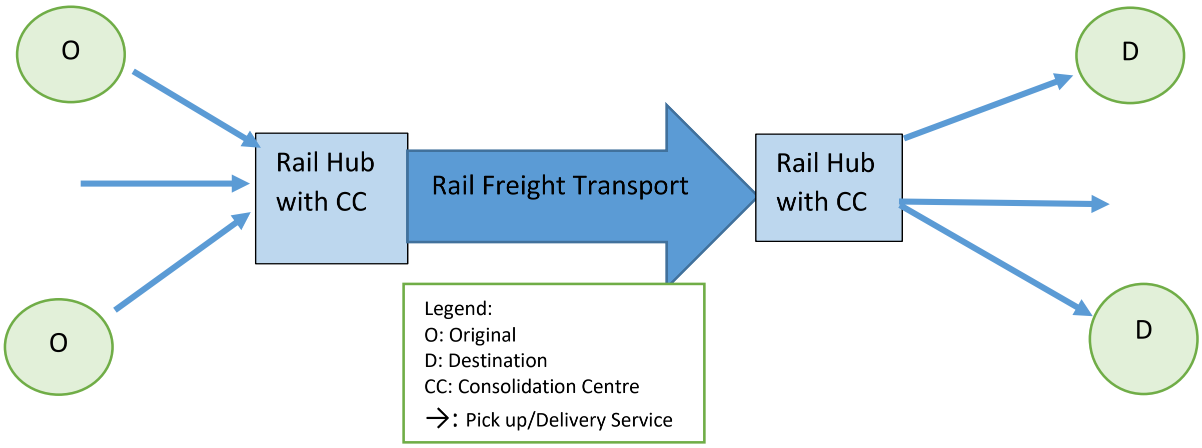

Figure 2 An example of a Multimodal Rail Freight Service Operation

Source: (Islam, 2014)

1.2 Technological and Economical Features of Rail Freight

In spite of the competition from the other transportation modes, rail freight has unique competitive advantages. With its pros and cons, rail plays a vital role in the economy. It is the engine of real economic activity and moves goods along supply chains. It brings benefits to the economy through productivity and environmental benefits such as reduced road congestion and emissions. The most obvious properties of rail freight transportation include:

Cost-Efficiency

Consistent with the common purchase principle, freight customers select an appropriate transportation mode which has the highest cost-efficiency. Comparing road transport with rail freight transport, shippers can determine that, in many cases, rail freight transport is a better choice resulting from economies of scale, particularly reflected in low value bulk cargo transportation such as coal and container transportation. In recent years there are signs of increasing volume of containerised cargo transport by rail (Department for Transport (DfT), 2016b).

Longer distance is an important factor for rail (and waterways) transportation option in terms of a break-even point. Researchers (Bontekoning, Macharis and Trip, 2004;(Mortimer and Islam, 2017) found that rail freight transport is cheaper than road transport for transportation tasks with distances more than 150km. Moreover, on average a gallon of fuel will move a tonne of goods 246 miles on rail but only 88 miles by road (Network Rail, 2017a), therefore, multimodal transport including rail, i.e. motor-rail multimodal transport can provide a cheaper inland freight service compared with truck only transport.

Reliability

Nowadays competition exists in every corner of the economic field. In order to improve competitiveness, companies need to make use of all key elements. When considering competitiveness, people may naturally think about cost. It is true that cost-efficiency is a key indicator influencing customers' choices on a products or service; however, reliability is another important element of competitiveness that needs special attention. These two elements of competitiveness are not independent. In many cases, the reliability can reduce the cost and therefore improve the competitiveness. The reason for this is that, in many industries such as manufacturing and retailing, high availability needs to be guaranteed. To satisfy this requirement, there are two different strategies: one is to keep a high level inventory and the other is to gain access to reliable transportation management. For the first plan, it is obvious that the company will incur inventory costs including additional warehouse cost and financial cost etc. on top of the normal transportation cost. For the second plan, the ideal situation is to achieve zero or near to zero inventory which requires a reliable transportation plan. It is doubtless that the second plan will lead to lower total costs and results in a competitive position for the product in the market. From this point of view, it is vital for the company to choose a reliable transportation method which can fulfil their customer demand in a timely manner, minimise total costs and achieve maximum competitiveness.

For most companies, especially those which supply businesses related to some seasonal products or short shelf life products, a very important criterion to select a carrier is the reliability of its transportation service system. In order to compete with other transportation modes and satisfy customers, rail freight stakeholders aim to increase unctuality rates through more efficient operations, including timetabling arrangements, blocking (wagons fulfilment management) and wagon routing plans, locomotive assignment and empty car management to reduce the possible delays in cargoes transport process. Related research can be found from the literature (Morvant, 2015; Khaled *et al.*, 2015; Murali, Ordóñez and Dessouky, 2016).

The report 'Value of Rail Freight' – UK Network Rail, 2010 shows that the UK rail freight performance is improving year on year. Between 2005/06 and 2008/09 the percentage of freight trains arriving on time rose by six points. More than 80 percent of freight trains completed their journey on time. As shown in Figure 3, train delay measured by minutes per 100 km show a declining trend . In recent years, the indicator fluctuates in a range of 9 -15 minutes per 100 km. For road transportation, there are more uncertainties, such as congestion, vehicle problems, traffic accidents etc., resulting in truck transport having an inherent lower reliability.

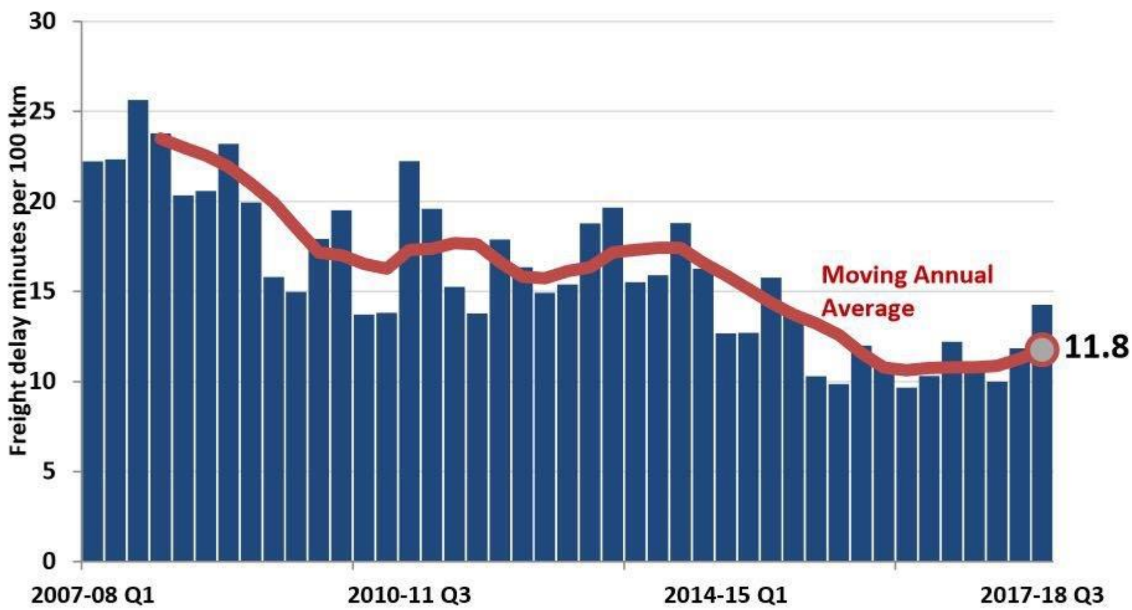

Figure 3 Normalised Freight delay per 100 train kilometres
Source: http://orr.gov.uk

Environment Effect

Worldwide the utmost priority is to urgently reduce greenhouse gas emissions. In order to contribute to this target, many countries have set up their carbon budget which places a restriction on the total amount of greenhouse gases the whole country can emit over a 5-year period. Many researchers have applied quantitative and qualitative methods to prove that rail freight is an environment friendly service, as it can help to mitigate noise, congestion and emissions compared to other modes in particular road transport (Kirschstein & Meisel, 2015; Pritchard et al., 2014; Alessandrini et al., 2012 ; Anderson et al., 2005 ; Dinwoodie, 2006).

The UK is one of the first countries to set legally binding carbon budgets (Priestley, 2019). Data from the EU statistical pocketbook suggests that transportation is the only sector where emissions are on the rise. Statistics show that in 2014 in the UK, domestic transportation emissions contributed 25% of total greenhouse gases emissions, where road freight makes up 17% of total transportation emissions. In contrast, rail, including passenger and freight are responsible for only 2% of total transportation emissions. The latest data released on 24th October 2017 by the UK Office of Rail and Road shows that in 2016-17 the CO2 emissions in the rail freight industry per freight tonne kilometre were 33.2g which is just 33% of the greenhouse gas of an equivalent journey by road (Goddard, 2018). It is universally accepted that rail transportation is greener and more environmentally friendly than road. There is no doubt that the policy “shift to rail” is an appropriate way to reduce the greenhouse gases emissions.

High Capacity

Data from Network Rail as shown in Table 1 indicates that a freight train can deliver many more commodities than a lorry, i.e. a full loaded train transporting construction materials can take 77 HGV (Heavy Goods Vehicles) from the road. By reducing the number of lorries, congestion on the national road will be improved.

Rail freight is also proven to be a cost-effective method that can apply economies of scale, in particular to low value cargo such as ore.

Commodity	Fully Loaded Train Potential	Equivalent Number of Heavy Goods Vehicles
Coal	1,500 tonnes	52
Metals and ore	1,000 to 2,500 tonnes	60
Construction materials	1,500 to 3,000 tonnes	77
Oil and petroleum	2,000 tonnes	69
Consumer goods	600 to 1,100 tonnes	43
Other traffic	1,000 to 1,500 tonnes	43

Table 1 Potential for A Fully Loaded Freight Train to Remove Lorries
Source: (Network Rail, 2017b)

1.3 Governance Structure in the Railway Industry

Governance structure has significant impact on the cost-effectiveness of the railway industry. There are three typical governance structures in the railway industry: Vertically Separated System, Vertically Integrated System, and Holding Company. The main difference between these governance structures is whether and to what extent infrastructure ownership is separated from operation. Based on the existing research in this field, the countries adopting each governance structures are summarised in Table 2 (Nash, 2008; Nash, 2014; Nash et al., 2013; Islam, 2014; Preston & Robins, 2013)

Vertically Separated System	**Vertically Integrated System**	**Holding Company**
Sweden	China	German
Britain	Ireland	Austria
Finland	Northern Ireland	Belgium
Denmark	U.S.	Italy
Netherlands	Japan	Latvia
Norway		Poland
Spain		Greece
Portugal		France
Slovakia		
Lithuania		

Table 2 Railway Operation Modes with Examples

Source: Nash (2008)

The first governance structure is termed 'Vertically Separated System'. The essence of this mode is the complete separation of infrastructure ownership from operation institutionally, functionally and financially (Nikitinas and Dailydka, 2016). Two separate legal persons without interrelation were established to govern the railway freight system: one is the Infrastructure Manager (IM); and the other is Freight Operating Company (FOC). In the vertically separated mode, the IM is responsible for developing and maintaining railway network and providing capacity for freight service; and the FOC generates revenue from selling rail freight service to end customers (shippers/consignees) by utilising the capacity acquired from the IM. One of the key decisions for vertical separation governance is how the IM should charge the FOC. From the perspective of governments, the pricing process needs to be regulated as the social benefit of the railway system needs to be considered. It is believed that the rail freight industry in the UK is a good example of this vertical separation governance structure.

'Vertically Integrated System' is another type of governance structure, which is totally different from vertical separation governance structure discussed above. Under this arrangement, a single company owns and operates the whole railway system. The pricing problem in the aforementioned vertically separated system does not exist as there is only a single company that has all the functions that the IM and the FOC provide. The IM and the FOC are two different departments in the same company in a Vertically Integrated System. China and the USA are two typical examples for this type of governance structure. In China, railway is fully state owned and operated, and there is no formal fee or separate accounting system between railway owner and operator (http://www.nra.gov.cn/). For the USA rail sector, it is primarily developed and maintained by private companies on a commercial basis. The relationship between the department representing the ownership and the department operating the railway are not typical service providers and customers since the two departments belong to a single company. Under the vertical integration governance structure, the rail company that owns and operates the railway system is very likely to be in a monopoly position in an intra-rail market which can lead to the absence of market-pricing.

The third type of governance structure is a Holding Company where the IM and the FOCs are separated companies but under the same ownership. The German rail sector is known as a typical example for this governance structure (Nikitinas and Dailydka, 2016) . The features of this governance structure include: (1) The IM and the FOCs have separate accounts; (2) independent institutions are established to determine infrastructure charges and distribute infrastructure capacities; (3) The IM and the FOCs remain business segments of a single company (Nikitinas & Dailydka, 2016; Zunder et al., 2013; Islam, 2014). The French railway sector adopts a holding company governance structure as well. The IM of French railway, Réseau ferré de France (RFF, French: French Rail Network) became SNCF Réseau from 1st January 2015 according to European Union Directive 91/440, it defines the guiding principles and procedures and sells train paths. SNCF Mobilités is the train operator in France. They all belong to SNCF, the historic state-owned railway company (Morvant, 2015).

Review of the worldwide railway governance structure can be found in the literature (Laurino et al., 2015; Nash, 2008; Nash et al., 2013). Debates on the advantages and disadvantages of the separation between the IM and the FOCs also exist in literature. Emmanuel & Crozet (2014), Cui & Besanko (2016) thought that vertical separation is better than vertical integration in terms of cost reduction while others expressed the opposite opinion. Similarly, Nash et al. (2013) compared the railway transport models in Britain, Sweden, and Germany~~.~~ They concluded that the governance structure in the UK, which adopts a vertical separation system is the best, with Sweden next and Germany least successful. Manuel & Andrade (2013) compared the difference between U.S. and European freight railways. Jensen & Stelling (2007) discussed the effect of vertical seperation based on the Swedish example where a longitudinal econometric approach for railway deregulation process was applied. They concluded that vertical separation leads to increased costs, whilst the introduction of competition may lower costs. The combined impact on cost efficiency is an improvement. Conversely, Mizutani & Shoji (2004) found that vertically separated systems do not have significant advantages over vertically integrated ones in terms of infrastructure maintenance costs.

This research will focus on the pricing process for vertical separation governance structures adopted in the UK and some other countries as highlighted above. The following will provide an overview of the rail freight industry and its development and governance structure in the UK.

1.4 Rail Freight Industry in the UK

In the UK, rail freight started its renaissance after rail liberalisation (Woodburn, 2001). After several decades' development, Britain's rail freight network connects almost all major freight origins and destinations of the island-country. Figure 4 *Map showing the Strategic Freight Sites of the UK* shows the strategic freight sites of the UK.

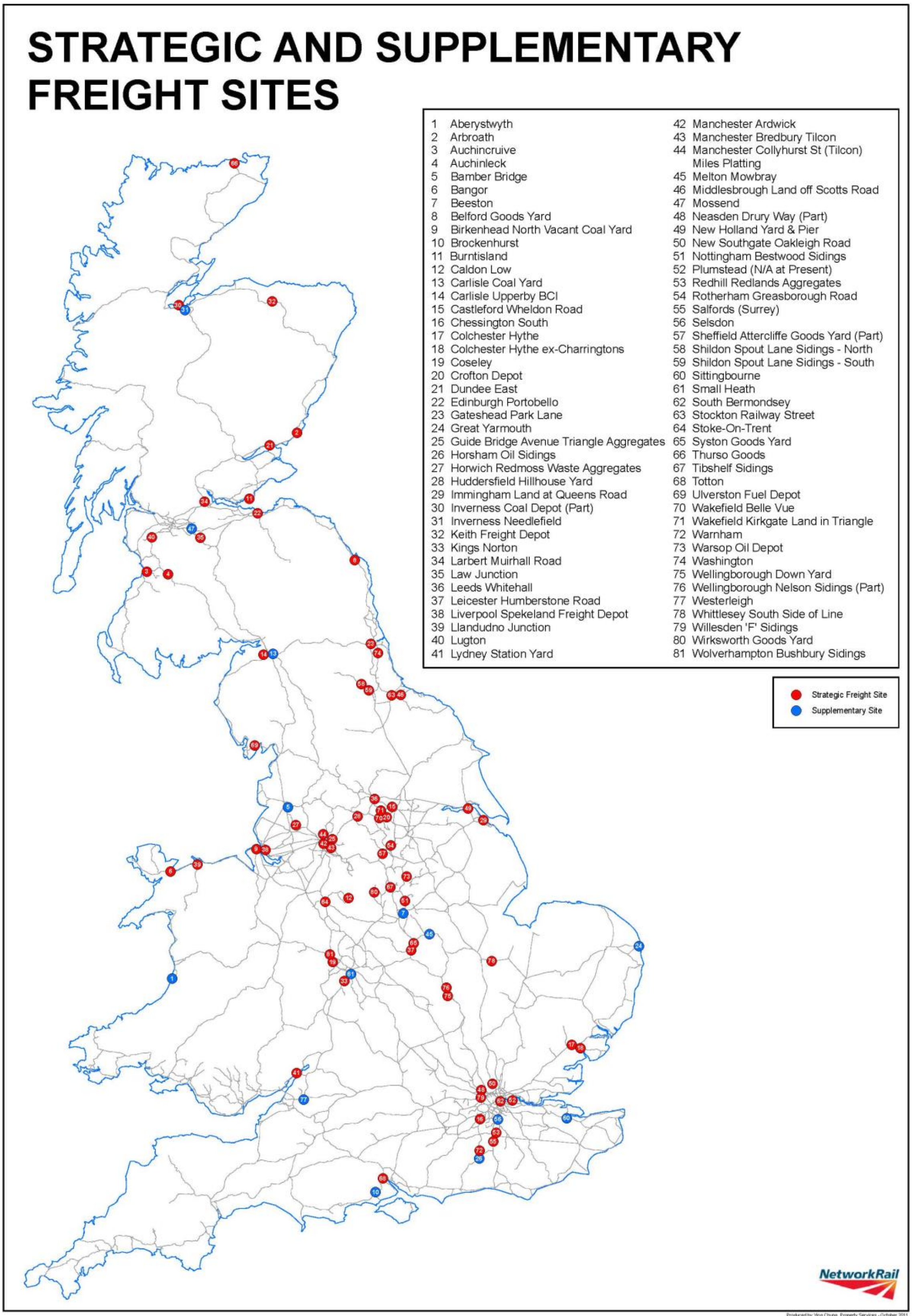

Figure 4 Map showing the Strategic Freight Sites of the UK

Source: (Network Rail, 2017b)

More and more companies such as Jaguar Cars, Tesco and Sainsbury's make rail freight their first transportation choice. Information from Network Rail shows that in the past eight years, the amount of consumer goods delivered by rail has increased by 75%. The consumer goods market has become the greatest growth freight market (Network Rail, 2017b). Moreover, data collected for Felixstowe shown in Figure 5, from Network Rail shown in Table 3 National Railway Freight Moved By Commodityand from ORR shown in Figure 6, suggests that rail freight maintained a fast growth trend for a very long period in the UK.

As shown in Figure 7, the rail freight traffic forecast for the near future published by Network Rail in their Strategic Business Plan on 9th Feb 2018 suggests that rail freight traffic will have a very significant increase in the period from 2019 to 2024. This information indicates that in the future, the rail freight industry in the UK will continue to play an important role in the development of economy and will be crucial to the prosperity of the country.

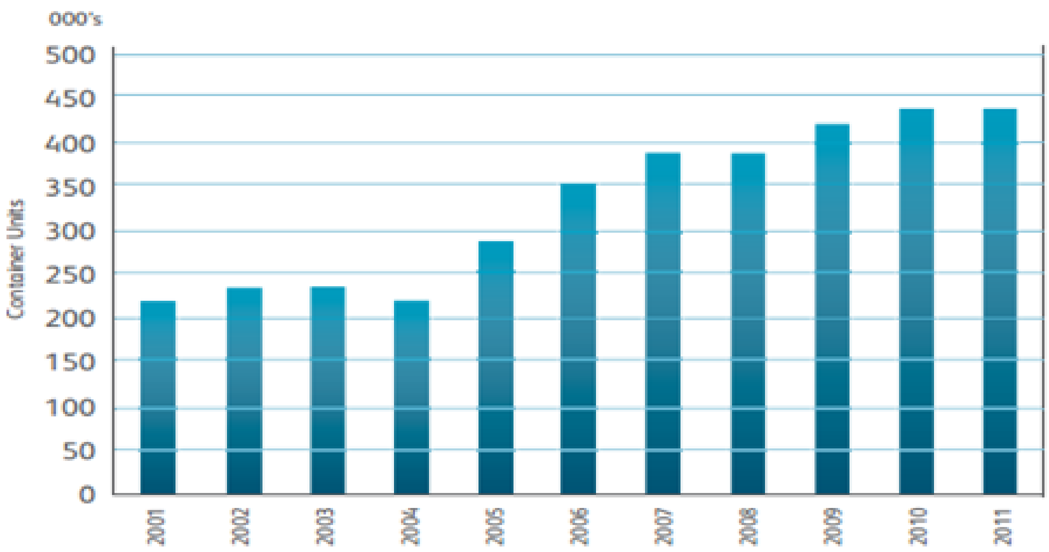

Figure 5 Growth in Rail Freight at Felixstowe

Source: (Network Rail, 2017b)

	Coal	Metals	Construction	Oil & petroleum	International	Domestic intermodal	Other	Total
1996/97	3.9	..	..	..	..	..	11.2	15.1
1997/98	4.4	..	..	..	..	..	12.5	16.9
1998/99	4.5	2.1	2.1	1.6	1.1	3.5	2.5	17.3
1999/00	4.8	2.2	2	1.5	1	3.9	2.7	18.2
2000/01	4.8	2.1	2.4	1.4	1	3.8	2.6	18.1
2001/02	6.2	2.4	2.8	1.2	0.6	3.5	2.6	19.4
2002/03	5.7	2.6	2.5	1.2	0.5	3.4	2.7	18.5
2003/04	5.8	2.4	2.7	1.2	0.5	3.5	2.8	18.9
2004/05	6.7	2.6	2.9	1.2	0.5	4	2.5	20.3
2005/06	8.3	2.2	2.9	1.2	0.5	4.3	2.3	21.7
2006/07	8.6	2	2.7	1.5	0.4	4.7	1.9	21.9
2007/08	7.7	1.8	2.8	1.6	0.4	5.1	1.7	21.2
2008/09	7.9	1.5	2.7	1.5	0.4	5.2	1.4	20.6
2009/10	6.2	1.6	2.8	1.4	0.4	5.5	1	19.1
2010/11	5.5	2.2	3.2	1.3	0.4	5.7	0.9	19.2
2011/12	6.4	2.2	3.5	1.2	0.4	6.3	1	21.1
2012/13	7.5	1.8	3.1	1.2	0.4	6.3	1.2	21.5
2013/14	8.1	1.8	3.6	1.3	0.5	6.2	1.4	22.7
2014/15	6.5	1.8	3.9	1.2	0.6	6.5	1.7	22.2

Table 3 National Railway Freight Moved By Commodity

Source: (Network Rail, 2017b)

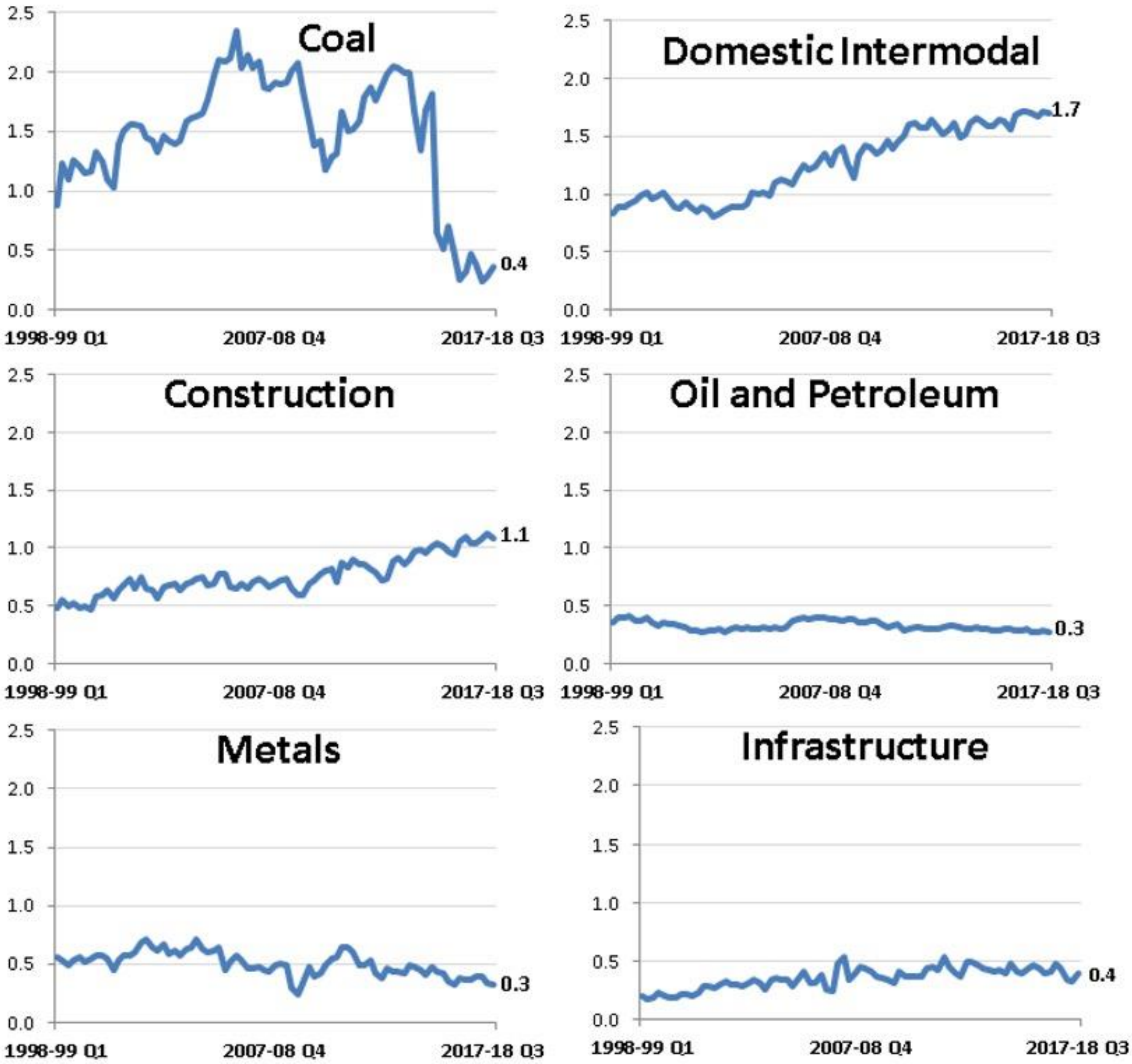

Figure 6 The volume of rail freight moved (billion net tonne km)

Source: (Office of Rail and Road, 2019b)

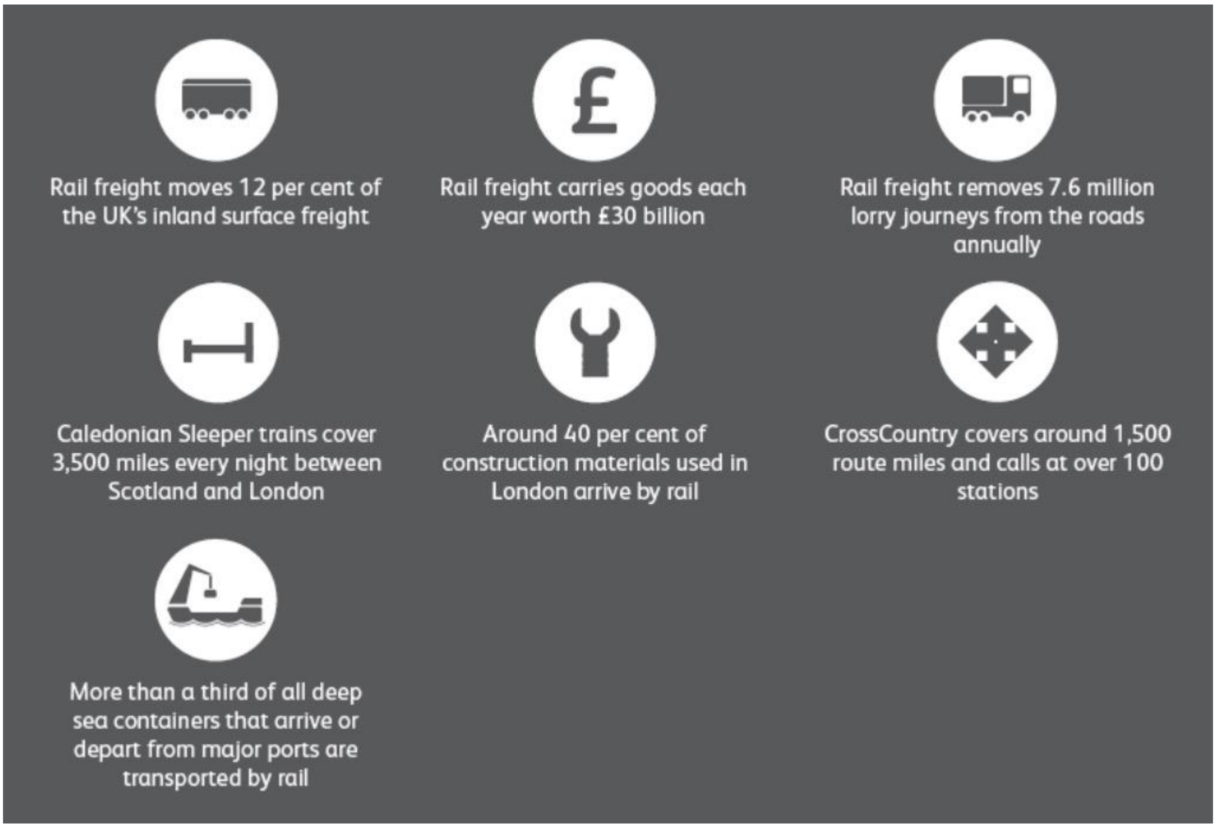

Figure 7 Strategic Business Plan 2019 – 2024: Freight

Source: (Network Rail, 2017b)

In recent years, to reduce the emission of greenhouse gases, the UK has been encouraging the shift of traffic volumes from road to rail. One of the important measures is to move cargoes through intermodal transport, involving rail in two or more modes of transport in the conveying process. As shown in Figure 8 below, there are a total of 63 intermodal rail terminals in the UK, which provide freight services to the customers. The terminals are run by different operators (including freight operating companies such as DB Cargo) shown in Table 4 *Intermodal Terminal Operators in GB.*

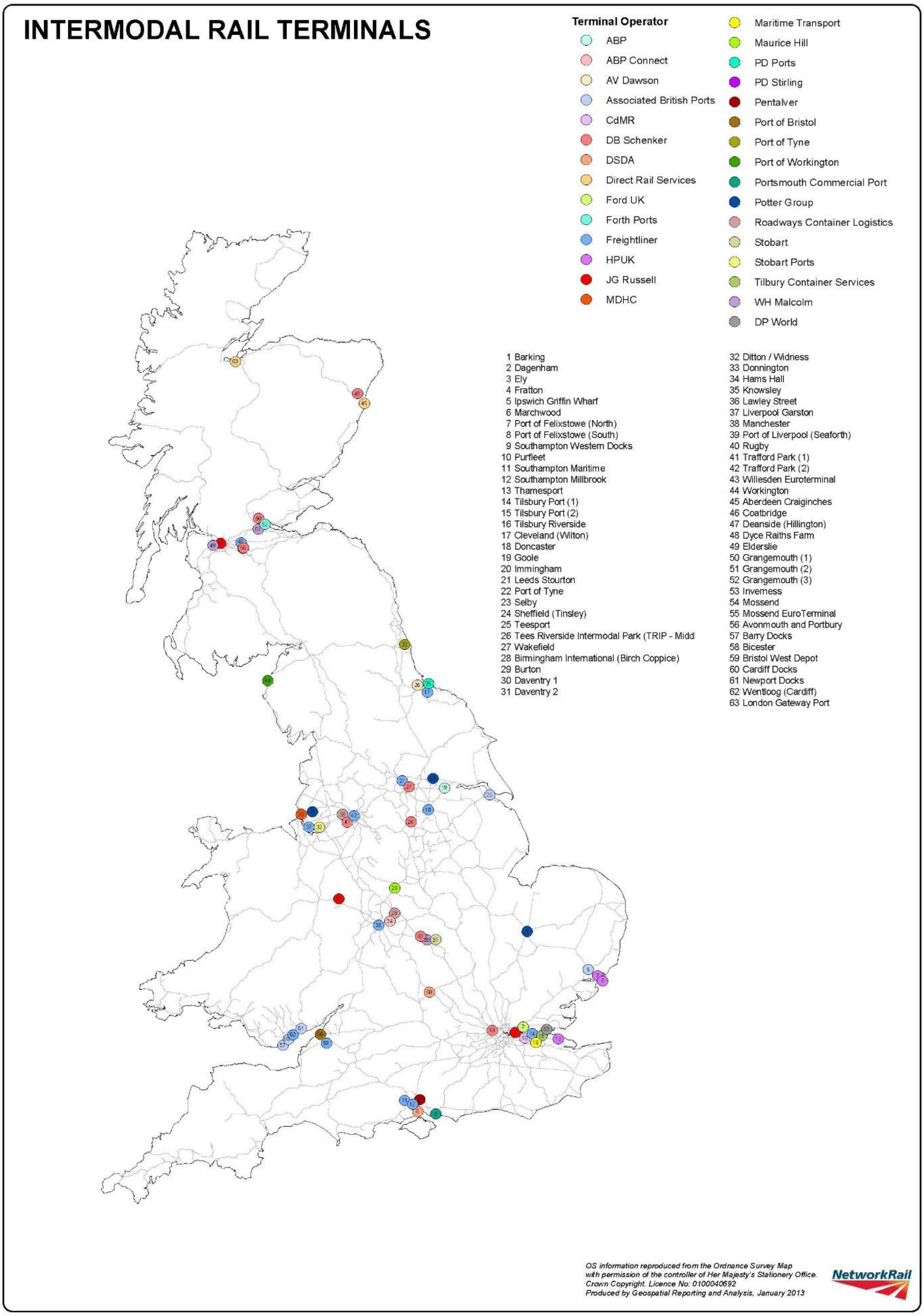

Figure 8 Intermodal Rail Terminals in Great Britain

Source: (Network Rail, 2017b)

Operator	Link
ABP- Associated British Ports	http://www.abports.co.uk/
ABP Connect	http://www.abports.co.uk/custinfo/other_op/abp_connect.htm
AV Dawson	http://www.av-dawson.com/
Bristol Port Company	http://www.bristolport.co.uk/
CdMR	http://www.cobelfret.com/terminals_purfleet.html
DB Schenker	http://www.dbschenker.com/site/logistics/dbschenker/com/en/start.html

Table 4 Intermodal Terminal Operators in GB

Source: (Network Rail, 2017b)

The British rail freight industry is a distinctive example of vertical separation governance. There are many companies and organisations involved in the operation of the rail freight industry in the UK including infrastructure manager (IM, refer to Network Rail); freight operating companies (FOCs), e.g. Colas Rail, DB Cargo UK, Devon and Cornwall Railways, Direct Rail Services, GB Rail Freight, Freightliner Group; regulating organisations, the Office of Rail and Road (ORR); Rail Safety and Standards Board - RSSB; terminal operators and industry suppliers. Network Rail is the only IM (although after the establishment of High Speed 2, there will be another IM - High Speed Two (HS2) Limited) in the UK. It is responsible for establishing, maintaining and providing safe and reliable railway paths for the FOCs. The FOCs are licenced to operate the freight service as per, among others, the path allocation and will have to pay for using the path. All the potential operators need to satisfy a series of requirement before being licensed by the Office of Rail and Road (ORR). Office of Rail and Road is an independent regulator on behalf of government (Nash *et al.*, 2014).

The separation of infrastructure ownership from operation in the UK mainly aims to establish the competition mechanism between the FOCs. So far it is operationally working well and to some extent, it offers efficiency (from an asset utilisation point of view) to the British railway system (Nash, 2014). But it is only part of the story. The separation also leads to a suboptimal operating system due to the fact that the separation results in the infrastructure managers and freight operators making their operational decisions independently and in some cases selfishly, which is not rare in the commercial world. This feature is very distinct in their pricing process.

1.5 Research Motivation

One of the major decision making issues for a vertically separated railway system is how to set up an appropriate pricing mechanism between the IM and the FOC for the use of rail infrastructure. In what follows, the UK rail freight industry will be used as an example to explain the issue.

At present, the UK Department for Transport (DfT) is responsible for the planning and investment in the transportation infrastructure. It is a Ministerial Department supported by 19 agencies and public bodies in which the Office of Rail and Road (ORR) takes charge of the railway industry. To set up the price, Network Rail firstly proposes an initial industry plan that contains the information on construction, maintenance, improvement and price. This document is then submitted to ORR for approval. The updated price information will be released by ORR after a proper consultation process with direct and indirect stakeholders. The consultation process involves safety bodies, expert advisors, related government departments, rail freight customers, industrial companies and freight operating companies. Britain's current transportation system regulation structure is summarised in Figure 9 and the pricing process for rail freight itinerary is shown in Figure 10. The principle followed by Network Rail in determining the price is to recover their costs generated by using the railway infrastructure. This principle complies with the rule of DIRECTIVE 2012/34/EU of the European Parliament and of the Council and Commission Implementing Regulation (EU) 2015/909 (The European Parliament, 2012) (The European Commission, 2015).

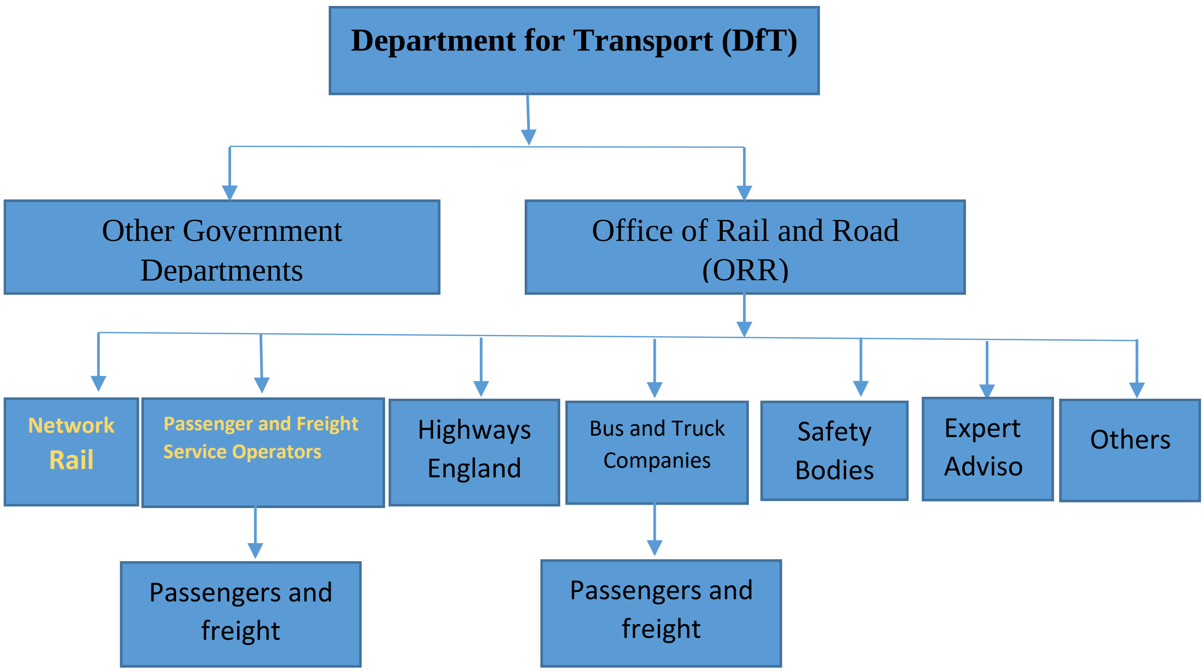

Figure 9 Regulation Structure of Transportation System in the UK

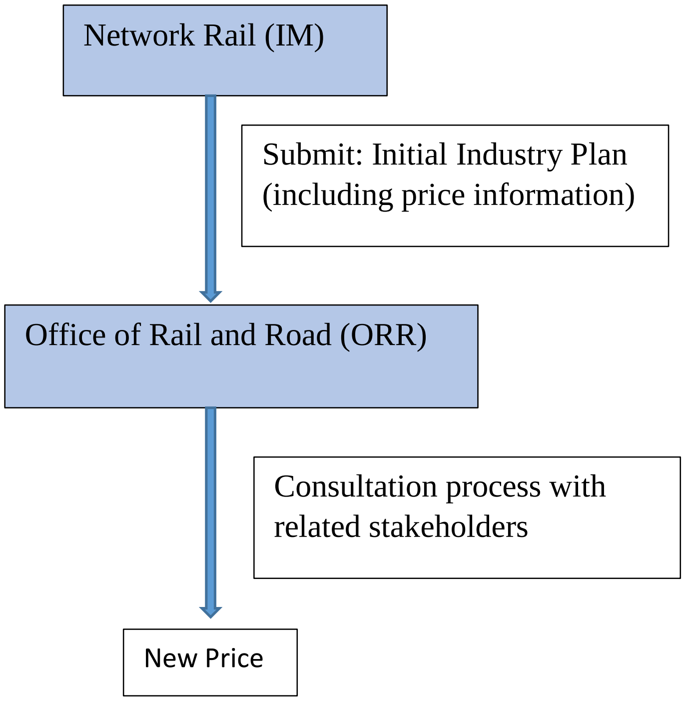

Figure 10 Current Pricing Procedure

There are a few drawbacks relating to the current pricing process. First, as an important stage prior to the ORR's approval, the consultation process on how the IM should charge the FOCs is very complex. The heated debates and conflicting interests between different stakeholders also make the review procedure very slow. For example, the consultation of Control Period 6 started on 18th May 2016, but the final determination was published in December, 2018. This decision-making process is very long, but the freight market changes very quickly. It is very likely that the outcomes of the consultation may be out of date by the time they are released. Second, in the pricing process, Network Rail acts as the leader to set up the price tariff for using the rail infrastructure; and then the freight operating companies make the decision on the purchase of the train itineraries as the followers. It is common practice that an IM may take advantage of its leader position in designing the tariff to unilaterally maximise its profits without caring about the FOCs' profits. According to the industrial visits to the UK rail freight industry, to prevent an IM from doing this, the UK government applies a certain cap on the IM's profit. As a countermeasure, the IM often chooses to exaggerate its cost, and attempts to gain additional profits. In such a battle between the government and the IM, the government is often not in a good position as it has great difficulty ascertaining the IM's genuine costs due to the complexity of railway system operations. Hence, the current pricing mechanism is not really as effective as expected. There is a need to design a better strategy to improve the pricing process of the rail freight transport system adopting the vertical separation governance structure. Third, in practice, an IM's tariff significantly affects not only the IM and FOCs' profitability but also the utilisation rate of the railway system, which should be maximised from the perspective of government and society. However, as an independent safety and economic regulator of Britain's railways, ORR's main responsibility is to ensure that the railway operators comply with health and safety law. Its function includes regulating Network Rail's activities to satisfy its funding requirements, regulating access to the railway network, licensing the operators of railway assets and publishing rail statistics (Office of Rail and Road, 2019b). ORR has no concern for the IM's profit, the FOCs' profit, and the profit of the entire freight system. There is no institution and mechanism to maximise the social benefits.

1.6 Research Questions

In recognition of this issue, this study aims to propose novel mathematical models to capture the complicated relationships between the IM and the FOC and design a better coordinated pricing mechanism for the vertically separated railway system. Firstly, the optimal prices that an IM can charge to maximise its own profits unilaterally without cooperation with FOCs are determined. This is also what an IM is trying to do in practice, but currently they do not have a rigorous mathematical tool to determine the prices of their train itineraries. They largely rely on their intuitive experience or manually change their prices repeatedly and choose the one that leads to the maximum profits, which is a "trial and error" approach. Secondly, alternative better pricing mechanisms are explored. More specifically, efforts are made for the identification of the pricing strategy that is able to lead to global optimality. It is then evaluated whether it is worthwhile for an IM to achieve global optimality initially, and share the corresponding profits with FOCs afterwards.

When developing the models to investigate the above issues, a stylised railway system that adopts the vertical separation governance structure is considered as a three-echelon service network based supply chain comprising an IM, a FOC, and end customers. The whole pricing process between the IM and the FOC is considered as a dynamic Stackelberg (leader-follower) game (Stackelberg, 2011). As the leader, the IM's decision is the price tariff for their network consisting of all the itineraries. The follower, the FOC needs to design a service network based on the IM's tariff and shippers' orders, which is a typical network design problem. Based on these, a mechanism (contract) will also be introduced for the IM and FOCs to collaborate. The target of the mechanism is to maximise the total profit of both the IM and FOCs. It will also guarantee that the IM's and the FOCs' profit under the mechanism is no less than their profit achieved without the contract.

In summary, the following research questions will be answered in this thesis:

1) What are the optimal prices of the IM-FOC game at the Stackelberg equilibrium? The solution at Stackelberg equilibrium specifies the prices that the IM can charge to maximise its profits unilaterally without any cooperation with the FOC.
2) What are the optimal prices that can maximise the overall profit of the whole rail freight system (supply chain) and lead to system optimality under the vertical separation operation structure?
3) What is the best contract that should be adopted to coordinate the whole rail freight system?

The aim of the research is to develop a pricing mechanism that can achieve global optimality and also make IM and FOCs better off than the pricing mechanism being used in the practice. Based on the aim, the following objectives are designed:

- Identify the equilibrium of the current pricing mechanism where there is no cooperation between IM and FOCs. This involves solving the Stackelberg game.
- Identify the solutions that lead to global optimality of the pricing process.
- Develop a mechanism to maximise the total profit of the freight system and allow each individual stakeholder to obtain profits no less than the existing mechanism. In designing the mechanism, the different results that the relevant stakeholders can obtain under cooperation or non-cooperation context will be compared.

The Novelty of the research is summarised as follows:

1) A vertically separated railway freight system is considered as a three-tier service supply chain consisting of an IM, a FOC and end customers. The conflicting interests of the IM, the FOC and the end customers are considered in designing the optimal pricing strategy for the IM and the optimal freight service network for the FOC;
2) The proposed game theoretic model contains integer or binary variables that model the FOC's purchasing choice on itineraries. This can better model the FOC's route/ itinerary purchasing choice, but leads to a big challenge to solve the model;
3) Both the equilibrium solution and the global optimal solution for the game models are obtained;

4) The uncertainty of customer demands is considered and a stochastic programming model is developed to accommodate the issue;
5) A proposed government subsidy based mechanism that can lead to global optimality is designed in the mechanism, the system optimality is achieved, and the government subsidy will be paid back to avoid government financial burden.

1.7 Structure of This Thesis

This thesis is divided into 8 chapters including the introduction as the first chapter.

In chapter 2, related research topics and publications are reviewed according to the identified research questions and objectives.

Chapter 3 presents the methodology to be applied in this research. Linear programming, stochastic linear programming, game theory, bi-level linear programming, inverse mixed integer linear programming and gradient search method are discussed in detail in this chapter.

Chapter 4 explains the industry practice on pricing procedure and provides the basis for model development. Two data sets in line with the industry practice will be prepared to validate the models and algorithms to be developed in this thesis.

Chapter 5 is devoted to the scenario where the IM makes decision on price selfishly without collaboration with the FOC. The scenario reflects the current pricing practice, and is also a Stackelberg game on networks. A bilevel optimisation model and a gradient search method is developed for obtaining the solution at Stackelberg equilibrium.

Chapter 6 presents the ideal scenario where the IM and the FOC have full cooperation, which will lead to global optimality of the game. Inverse mixed integer linear programming model and a Fenchel cut based solution method is developed to find out the optimal prices leading to the global optimality.

Chapter 7 aims to identify the best mechanism for the government to regulate the collaboration between the IM and the FOC. The aim of the mechanism is to ensure the IM sets up a price tariff that can lead to global optimality and also make the IM’s profits the same as that at

Stackelberg equilibrium. After comparing a number of contracts, a government subsidy based mechanism is choosen to achieve the target, and a double-layer gradient search based algorithm is developed to obtain the optimal subsidy rate.

In Chapter 8, conclusions are made based on the developed models, algorithms and numerical case studies. Recommendations for future work are given.

1.8 Summary of the Chapter

In this chapter, the role of rail in the broader context of intermodal transportation is discussed firstly; then the features of rail freight are introduced from the aspects of cost efficiency, reliability, environment and high capacity. Following on from this, the operation practice of the freight industry in the UK is reviewed. By analysing the current freight pricing process, the weaknesses that motivates this research is identified, and subsequently the research questions, objectives and novelty of this research are presented. The structure of the thesis is given at the end of this chapter.

Chapter 2 Literature Review

This chapter provides the knowledge base and a thorough literature review on all aspects of the proposed research questions in Chapter 1. It will inform the research gaps and justify the potential contributions of the study to the literature.

2.1 The Approach to Organising the Literature Review

The research questions identified in the first chapter aim to design a pricing mechanism for a vertically separated railway system. The research questions have the following four features, which will be used as criteria to scope the literature that need to be reviewed.

~~1)~~ The research questions are revenue management issues

The key decision to be made in the research is how an IM should charge a FOC. The proposed research question has the typical feature that a revenue management problem should have. Therefore, in the literature review, revenue management literature will be covered.

2) The research questions involve the conflicting interests of the different stakeholders in the rail freight industry.

To decide the pricing mechanism, both the IM's and the FOC's decision making processes need to be considered. There are conflicting interests between the IM and the FOC. The IM's decision determines the best price tariff for train itineraries to make the maximum profits from the FOC; whereas the FOC's decision making target is also to maximise its own profits while having to pay the IM's charge. To consider the conflicting interest, game theory needs to be applied to model their interactions. Further, as the decisions involve the freight service network consisting of correlated train itineraries, the research can be categorised as network revenue management game. Therefore, one stream of literature to be reviewed will be network revenue management and the related game theory.

3) The research questions also involve network design

Network design is a traditional issue in the field of operations research. A FOC's decision in a vertically separated railway system is in the nature of a network design problem. The FOC's decision-making is to design a freight service network based on shippers' demands and the IM's price tariff. Hence, the literature related to network design particularly in the rail freight industry will be reviewed.

4) The research questions involve a supply chain consisting of an IM, a FOC and end customers

The network revenue management game is carried out on a supply chain involving the IM, the FOC, and shippers. Therefore, the literature on supply chain management, in particular, pricing and revenue management for supply chains will be reviewed.

To sum up, the research questions to be considered is a revenue management problem that involves a network, as well as multiple stakeholders with conflicting interests, thus it can be categorised as network revenue management game. Network revenue management game is characterised by dependent demands and multiple stakeholders. To conduct a thorough literature review, the other types of revenue management related to network revenue management game will be considered in the study. More specifically, the literature to be reviewed will include:

1) traditional revenue management that involves a single stakeholder and independent demands (homogenous products)
2) revenue management in a supply chain that involves multiple stakeholders and independent demands
3) network revenue management that involves a single stakeholder but dependent demands (heterogenous products), e.g., seat control in airline industry
4) network revenue management game that involves multiple stakeholders and dependant demands

An overview of the streams of literature to be reviewed is shown in the following Table 5

	Independent Demands	**Dependent Demands**
One stakeholder	Traditional Revenue Management	Network Revenue Management
Multiple stakeholders (Game theoretical model)	Revenue Management on Supply Chain	Network Revenue Management Game

Table 5 An Overview of the Streams of Literature

In additional to the above four streams of literature relating to the different aspects of revenue management, the literature on rail freight service network design will also be reviewed as it is the key issue for a FOC.

In what follows, the context in the railway industry that motivates the research will be briefly discussed; then the five streams of literature discussed above will be reviewed.

2.2 Background

Freight transportation is a vital part in the development of an economy. To a large extent, the efficiency of freight transportation affects the operational efficiency of many direct and indirect stakeholders in the other industries across the chains of productivity and circulation. Also, freight transportation costs form a significant part of the prices of the products required by raw material providers, manufacturers, and end customers on supply chains.

It is believed that, apart from the costs of material, labour and finance, economic development is also at the cost of degradation of the environment (Azomahou, Laisney, & Nguyen Van, 2006); however there are choices which can reduce these negative effects (Saboori, Sulaiman, & Mohd, 2012; Azomahou et al., 2006; Bickford et al., 2014). Certainly, the rail freight industry also has to comply with the economic rules. As an essential part of economic development, transportation does inevitably generate some adverse consequences, greenhouse gas emission is one of these negative aspects. Statistics data in Table 6 shows that in 2015 domestic transportation was responsible for 24% of greenhouse gas emission and negatively affected the air quality especially at the roadside. The UK set a target to reduce greenhouse gas emissions by at least 80% by 2050 (from 1990 baseline). To minimise the negativities, in 2008, the Climate Change Act 2008 was implemented (Legislation.gov.uk, 2008). This is the first legal document established to binding climate change targets. In order to achieve the emission targets, the UK government has put in a lot of efforts, including providing grants to the related research investigating and finding mechanisms to reduce emissions generated from the burning of fossil fuel; encouraging people to use non-motorised transportation modes such as cycling, walking or the use of electric-powered vehicles, etc. Among these, there is an important policy relating to the freight industry termed ‘modal shift’. This policy encourages freight volume shifting from road to rail (and waterways transport). The reason for this is that lorries are responsible for more than 33% of the UK’s transport related CO_2 emission (Department for Transport (DfT), 2018). This policy aims at using cleaner transport modes instead of lorries. Due to its unique environment-friendly advantage, rail freight is a preferable substitution mode to road freight transport. It is a more efficient way to move the same amount of cargoes at a lower environmental cost to society.

From the commercial point of view, especially in an open market environment, the freight rate is a key element that will affect the shippers’ decision making on modal selection and result in (positive or negative) modal shift among the different transportation modes. Therefore, to realise the shift from road to rail, a reasonable and competitive freight rate is crucial. According to a quotation from a representative truck company and a rail freight operator, to deliver a 40 feet standard container weighing 250,000kg from Felixstowe to Southampton in June 2019, the average cost using truck transport is £526 while the cost is £446 using the rail freight service. This clearly shows that the rail transport is cheaper than a truck by 15.2%. This research will contribute to this aspect by investigating the pricing problem of the rail freight service in the UK where a vertical separation governance structure is being adopted.

			Million tonnes of carbon dioxide equivalent/percentage													
By source			**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	***Percent of all domestic in 2015***
Domestic transport	**Road transport**	Cars and taxis	77.0	77.4	77.4	76.8	77.1	74.8	72.5	70.2	69.5	69.2	68.0	68.3	69.1	***14***
		Heavy goods vehicles	20.2	20.6	20.8	20.8	21.0	19.5	18.0	18.6	18.1	18.2	18.3	18.8	19.6	***4***
		Light vans	15.0	15.4	15.8	16.3	16.8	16.0	15.8	16.1	16.2	16.4	16.6	17.3	18.0	***4***
		Buses and coaches	4.9	4.6	4.6	4.7	4.8	4.2	4.2	4.3	4.0	3.8	3.8	3.8	3.7	***1***
		Motorcycles and mopeds	0.8	0.7	0.7	0.7	0.7	0.6	0.6	0.6	0.5	0.5	0.5	0.5	0.5	-
		Other road transport emissions	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.5	0.5	0.5	0.5	-
		Total	**118.2**	**119.2**	**119.9**	**119.9**	**121.0**	**115.8**	**111.6**	**110.3**	**108.8**	**108.6**	**107.8**	**109.3**	**111.5**	***22***
	Other transport	Rail	1.8	1.9	1.9	1.9	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.9	-
		Domestic aviation	2.2	2.3	2.6	2.5	2.3	2.2	1.9	1.8	1.7	1.6	1.7	1.5	1.5	-
		Domestic shipping	2.5	2.6	2.7	2.6	2.6	2.8	2.6	2.6	2.5	2.4	2.3	2.4	2.5	***1***
		Other	3.6	3.6	3.4	4.0	4.3	3.8	3.5	3.4	3.3	3.0	2.8	2.6	2.6	***1***
		Total	**10.2**	**10.4**	**10.5**	**11.0**	**11.3**	**10.8**	**10.0**	**9.8**	**9.4**	**9.1**	**8.8**	**8.5**	**8.5**	***2***
Total domestic transport			**128.5**	**129.6**	**130.4**	**130.9**	**132.3**	**126.6**	**121.6**	**120.1**	**118.3**	**117.7**	**116.5**	**117.8**	**120.0**	***24***
Net domestic emissions all sources			**698.3**	**694.3**	**685.8**	**679.9**	**667.7**	**647.2**	**590.1**	**605.9**	**557.6**	**575.2**	**558.3**	**515.1**	**495.7**	***100***

Table 6 Greenhouse Gas emissions by transport mode, United Kingdom: 2003 to 2015
Source: (Department for Transport (DfT), 2018)

2.3 Traditional Pricing and Revenue Management

Revenue is the total income generated by a business or an organisation over a period of time from selling goods or providing services to its customers without deducting any cost or expenses. The commercial objective of any business or company is to make profits or at least cover the costs incurred in the operation process. Profits can be calculated by taking costs away from revenue, which may be profitable, break even or negative depending on the operation in the particular time period (Talluri, 2008). It can be used as a parameter to assess the operational result of an enterprise. Revenue Management (RM) is the strategy or technique aiming to improve the profit by adjusting product availability, pricing strategy and distribution based on the prediction of customer behaviour, forecast and analysis of the key market parameters (Rodríguez-Algeciras and Talón-Ballestero, 2017). It originated in the airline industry in the 1970s. Pricing strategy is one of the most important elements in revenue management of any business especially when substitutable products are available in the market or competitors exist, as the price can affect the choice of the customer.

In the section, the focus will be on the traditional pricing and revenue management where a single stakeholder considers how to maximise its revenue for homogeneous products (independent demands) associated with a single price. In the domain of traditional pricing and revenue management, the mathematical function depicting the relationship between historical price and the corresponding demand needs to be collected first. Generally, demand will fall with increasing price. By analysing the historical data, the parameter of the demand function can be determined. For example, the collected historical price and demand data for product XYZ is listed in

Table *7*. The price-demand function can be demonstrated by : $d = 20000 - 2000p$, shown as Figure 11.

	Price									
	1	2	3	4	5	6	7	8	9	10
Demand	18000	16000	14000	12000	10000	8000	6000	4000	2000	0

Table 7 Historical Data of Price and Demand

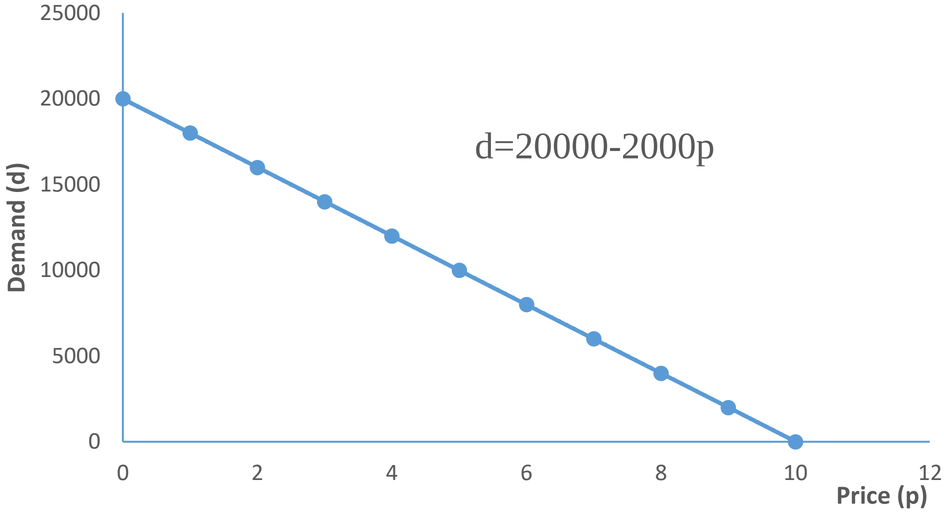

Figure 11 The Price-Demand Function

For the above demand function, the revenue S can be calculated by multiplying the price with the demand, i.e., $S = (20000 - 2000p) * p$. In the example, the revenue can be visualised in Figure 12.

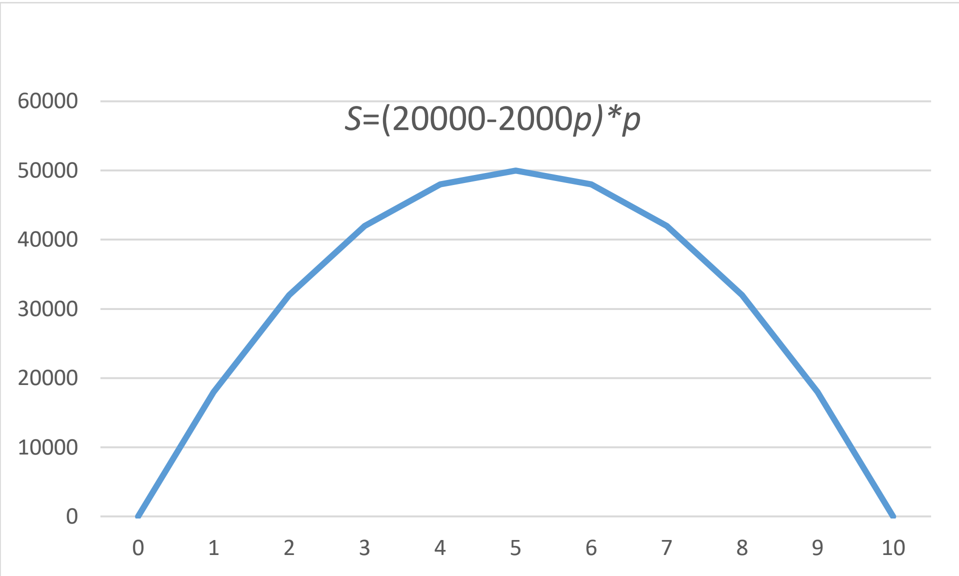

Figure 12 The Price-Revenue Function

It is easy to identify the optimal price that can maximise the revenue by computing the first order derivative of the price-revenue function (Chopra and Meindl, 2016).

The above example is a very simple and traditional pricing problem which has been discussed in many revenue management textbooks (McMahon-Beattie, 2004); (Ian Yeoman; Una

McMahonBeattie, 2010); (Talluri, 2008). However, the problem may become very complex when demands are stochastic and decisions are made over multiple stages, which will lead to stochastic dynamic programming (Birge and Louveaux, 2011)(Dimitri P. Bertsekas, 2018). Further, the pricing problem may be coupled with other management issues, e.g. inventory control and product innovation. A large number of studies have been carried out for this type of revenue management that may involve stochastic dynamic programming or stochastic linear programming and the research results have been summarised in many textbooks (Talluri, 2008); (McMahon-Beattie, 2004)(Ian Yeoman; Una McMahonBeattie, 2010).

2.4 Revenue Management in Supply Chains

Traditional revenue management only considers a single stakeholder and independent demands (homogenous products). Over past decades, research has been carried out to extend traditional revenue management to multiple stakeholders (Cao, Wan and Lai, 2013). This generally leads to pricing and revenue management on supply chains, e.g., between a retailer, a wholesaler, a supplier, and end customers. This type of research requires the application of game theory to model the conflicting interest of different stakeholders (Vafa Arani, Rabbani and Rafiei, 2016a).

In what follows, the nexus between a vertically separated rail freight system and supply chain will be analysed. It will be shown that a vertically separated rail freight system can be considered as a three-tier service supply chain. Following this idea, the literature in relation to supply chain pricing, revenue management and coordination will be examined.

2.4.1 The nexus between supply chain management and vertically separated rail system

In a vertically separated rail freight system, there exists three different stakeholders: the IM, the FOC and end customers (shippers or consignees). An IM provides all the rail freight infrastructure and offers itineraries to the FOC. A FOC pays the IM for accessing the freight transport capacity/itineraries, operates freight service on the purchased itineraries and provides a cargo transport service following the requirement from the shippers. The relationship between the IM, the FOC and shippers can be treated as a three-echelon service supply chain. The IM can be treated as a supplier who provides train itineraries to the FOC; and the FOC as a retailer who uses purchased itineraries to provide freight service to the shippers. The ‘products’ sold across the three-echelon service supply chain are a freight service network that comprise interconnected train itineraries.

Supply Chain Management (SCM) concept was first introduced when the managers realised that optimising the way of operating supply chain activities can not only reduce the costs but also increase their competitiveness by providing customised service (Lawrence v. Snyder, 2011a). Because of its significant impact on the profit of each stakeholder on supply chain (Vafa Arani, Rabbani and Rafiei, 2016b), SCM is an active research area and attracting more and more scholars to work on it.

As shown in Figure 13, a supply chain starts from suppliers and ends with customers. The figure demonstrates all the possible processes to fulfil customer demand. The performance of each stakeholder involved in the supply chain can influence the operation of the entire system. By analysing the figure, it can be found that there are four different types of connection/relationship between the supplier and the customers (Chopra and Meindl, 2016):

(1) Supplier-Manufacturer- Distributor-Retailer-Customer;

(2) Supplier- Distributor-Retailer- Customer;

(3) Supplier-Retailer- Customer;

(4) Supplier- Customer.

A vertically separated railway system is type (3) supply chain. It is worth mentioning that a vertically integrated railway system may be categorised as type (4) supply chain.

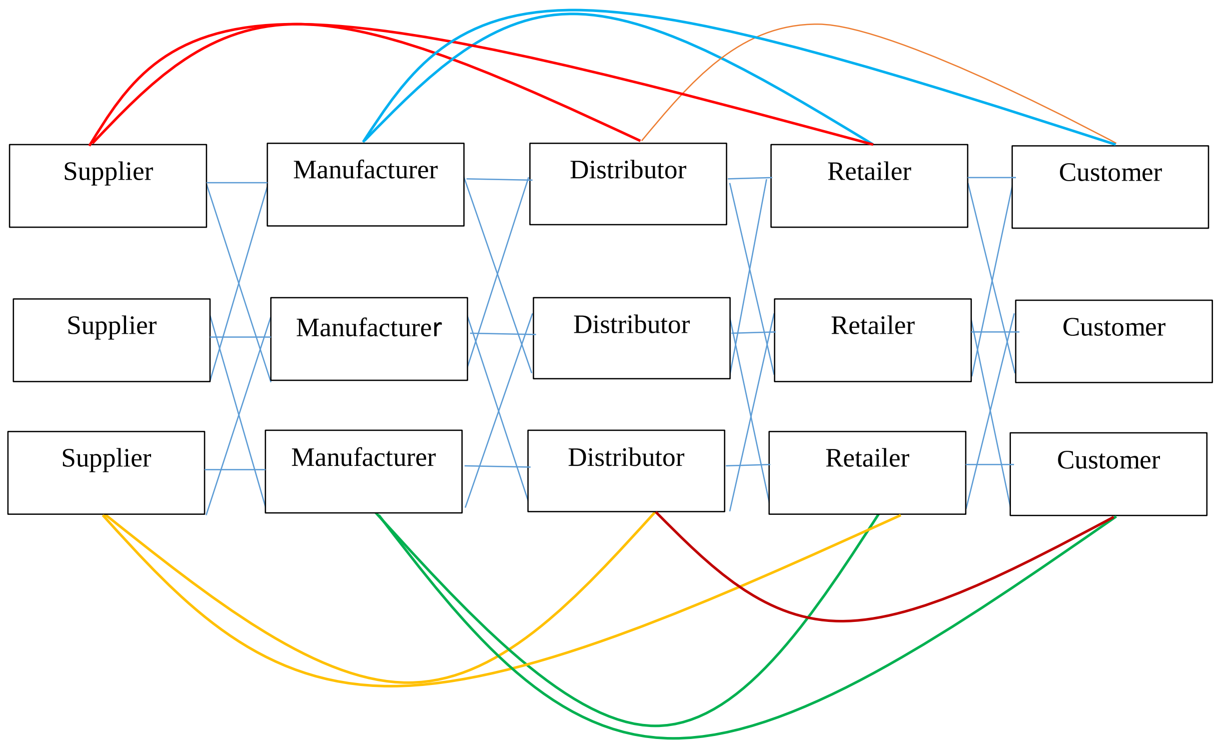

Figure 13 Supply Chain Stages
Source: (Chopra and Meindl, 2016)

Intensive research fruits have emerged in the literature relating to supply chain management in recent decades. Research has been carried out for different aspects of supply chain management, i.e., supply chain design (Goetschalckx, Huang and Mital, 2013), inventory management problem (Dillon, Oliveira and Abbasi, 2017), environment impact or green supply chain management (Vanalle *et al.*, 2017), risk management (Song and Zhuang, 2017), supply chain efficiency (JIA and JIA, 2017); (Biswal, Jenamani and Kumar, 2018), and supply chain contract design (Cachon and Lariviere, 2005); (Seifert, Zequeira and Liao, 2012)(Zhao *et al.*, 2014)(Xu, Cheng and Sun, 2015).

In the following, research on revenue management and pricing for supply chain will be reviewed.

2.4.2 Pricing and revenue management in the supply chain

In recent decades, supply chain coordination via pricing has attracted more attention. Researchers tried to improve the supply chain performance by designing a more efficient pricing strategy. Moon, Yao and Park (2011) investigated the pricing negotiation in contract design between a seller and a buyer under uncertain demand. Zheng and Negenborn (2015) extended (Moon, Yao and Park, 2011)'s study by considering the buyer's fixed and elastic demand. (Kuo and Huang, 2012) used finite dynamic programming to investigate pricing negotiation issues for one retailer selling two generations of products with limited inventories in a supply chain. (Chen, Federgruen and Zheng, 2000) considered joint pricing and replenishment for a two-echelon supply chain with deterministic demands. Chung, Talluri and Narasimhan (2011) studied the price markdown issue for a supply chain that included a supplier, a manufacturer, and a retailer. Matsui (2017) investigated the timing for applying wholesale and retail prices. Elmaghraby and Keskinocak (2003) provided an overview of current practices in dynamic pricing with the presence of inventory considerations.

More and more industrial practitioners and academic researchers have realised that a coordinated supply chain is ideal for the relevant supply chain stakeholders, although conflict between them always exists as the primary concern (Ellegaard & Andersen 2015). The view relating to supply chain coordination significantly affects inter-organizational cooperation in the buyer-supplier relationships (Kyu et al. 2010). Xiao and Qi (2008) studied the coordination of a supply chain with one manufacturer and two retailers where the production cost of the manufacturer was disrupted. Two pricing strategies were investigated as coordination mechanisms: an all-unit quantity discount and an incremental quantity discount. Vafa Arani, Rabbani, & Rafiei (2016) presented a coordination mechanism for a two-echelon supply chain using dynamic game theory. The mechanism was evaluated against two scenarios depending on the player who dominates the market and makes the first decision: retailer-led and manufacturer-led supply chain. In the retailer-led model, the retailer was deemed to be the leader of the supply chain and made decisions on the order quantity and the price they can afford, and then passed the information to the manufacturer. The manufacturer then decided his own production quantity based on the information specified in the contract with the retailer. While in the supplier-led scenario, the decision sequence was reversed. The authors modelled the quantitative relationship such as the order quantity of the retailer, production quantity of the manufacturer and the realised demand for different scenarios. Similarly, Yao, Leung, & Lai (2008) presented a Stackelberg (leader–follower) game which involves one manufacturer and two retailers who were competitors to each other. Similar research on supply chain cooperation can also be found in the studies conducted by He & Zhao (2012) , Venegas & Ventura (2018), Seo, Dinwoodie, & Roe (2016), Hou *et al.* (2017), Venegas and Ventura (2018). Qiu and Lee (2019) studied the revenue management problem for rail which involves intermodal transport.

To maximise the profit of all stakeholders, the pricing problem in a dry port system was modelled as a Stackelburg game. The dry port was the leader of the game and the shippers were the followers. The demands considered by Qiu and Lee (2019) were independent.

Although a vertically separated railway freight system can be treated as a three-echelon supply chain, the research findings in the literature related to supply chain pricing and revenue management, such as (Heydari, Govindan and Aslani, 2018); (Huang, Yang and Zhang, 2012); (Seifert, Zequeira and Liao, 2012), cannot be applied to this thesis due to the network effect of the rail freight service. Importantly, the existing supply chain pricing literature focuses on homogenous products that do not have correlations between each other. In other words, only a single price needs to be optimised for homogeneous products in the existing study. This is a research gap as the itineraries that make up a freight service network are heterogeneous since each itinerary has different train arrival and departure time, origin and destination stations, and operational costs. Therefore, in this study, the price for each itinerary needs to be optimised. To address the pricing for heterogeneous products with correlation, another stream of literature – network revenue management needs to be examined to attempt to close this research gap.

2.5 Network Revenue Management

In recent years, network revenue management has become an active research field which is drawing increasing attention from academia because it can deal with correlated (dependant) demands. Before network revenue management was introduced, the studies in revenue management were based on the assumption that the demand for one product does not affect the availability of the other products (Strauss, Klein and Steinhardt, 2018). Network revenue management is the technique dealing with dependent demands. The research questions in this dissertation in fact involve dependent demands as the itineraries on railway network are subject to capacities associated with sections and at stations.

The research on network revenue management started in the airline industry. The airline industry is also a successful example where revenue management research is being actively implemented. Belobaba (1989) studied the seat inventory control in airlines by applying a probabilistic decision model to maximise total flight revenue. Curry (1990) combined two network revenue management approaches: marginal seat revenue and mathematical programming, and promisingly the new method has the merits of the two approaches. Brumelle and McGill (1993) proposed to use fix-limit booking control for nested fair class. Ryzin and McGill (2000) proposed an adaptive algorithm to control seat booking without the need of forecasting and optimisation. An early review of network revenue management in the airline industry was conducted by Barnhart, Belobaba and Odoni (2003) as part of an extensive review on the application of operation research in the field. The authors reviewed airline revenue management from two aspects: overbooking and leg-based and network-based seat inventory management. In order to reduce the revenue losses associated with no-shows, the airlines allowed the seat reservations to exceed the aircraft capacity. This is termed overbooking. It has been proven that effective overbooking can generate positive impact on airline revenue management. Leg-based and network-based seat inventory management are two techniques of airline revenue management. Leg-based management method focuses on maximising revenue by adjusting seats in different booking (fare) class on each future flight-leg departure. As this method can only optimise the revenue on one leg and cannot guarantee the optimisation on the network, the network-based seat inventory control was introduced and applied particularly in the largest and advanced airlines. It aims at maximising the revenue of the entire network. This method is proven to be especially useful when a large proportion of passenger itineraries involve multiple flight legs and connections at the connecting hubs. After the review, further studies have been carried out. Bertsimas and de Boer (2005) investigated the booking limits in airline revenue management using a simulation method. A stochastic gradient algorithm combined with approximate dynamic programming algorithm is proposed in this research to improve the airline revenue. A case study based on the proposed approach showed that applying the booking limit had a positive impact on revenue improvement. Huang and Lin (2014) adopted a simulation method to improve the computation of bid price control. (Birbil *et al.* (2014) proposed a decomposition method for calculating the large-scale revenue management problem. Chatwin (2014) proved the optimal condition for fix-limit booking control. Hosseinalifam, Marcotte and Savard (2016) improved the estimation of time-dependant bid prices while considering customers' choices. Comprehensive literature reviews on the application of network revenue management in the transport industry was conducted by Ryzin and Mcgill (2000). Ryzin and Mcgill (2000) reviewed the research on transportation revenue management since 1959, specifically, focusing on the forecasting, overbooking, seat inventory control, and pricing problems.

In the limited studies relating to rail network revenue management, the majority focus on rail passenger transportation where seat control is a focus, e.g.,(Ben-khedher *et al.*, 1998; Ciancimino *et al.*, 1999; Kraft, Srikar and Phillips, 2000; Xiaoqiang, Lang and Jin, 2017). In the studies relating to the pricing problem in rail freight, it is common that only a single stakeholder is considered. For instance, Kraft (2002) adopted a bid price approach to schedule railway shipment delivery times for a railway operation company. Li and Tayur (2005) developed a medium-term pricing model for an inter-model transportation operator. Gorman (2001) and Gorman (2005) investigated the pricing problems for a railway company, Burlington Northern and Santa Fe Railway (BNSF), in the USA. Bharill and Rangaraj (2008), Hetrakul and Cirillo (2014) investigated the application of revenue management principles in railway passenger transportation. Kapetanović, Bojović and Milenković (2018) applied the booking limits and bid price policies in rail freight transportation aiming to maximise the revenue. (Bilegan and Brotcorne (2015) focused on the maximisation of revenue for a rail freight transportation company or intermodal freight company. The authors developed a load acceptance system to decide whether or not to accept the transportation request aiming to maximise final profit.

2.6 Network Revenue Management Game

The studies discussed in the above section were carried out for a single stakeholder who is normally a transport operator. However, in practice, there is normally more than one stakeholder who gets involved in the transport system. To model the conflicting interests of multiple stakeholders, game theory needs to be applied to extend the research of network revenue management. Based on the game theory application, the existing literature in network revenue management game can be classified into two groups: Nash equilibrium and Stackelberg equilibrium.

2.6.1 Nash Equilibrium

The competition between airlines has been considered in network revenue management game which normally leads to the Nash equilibrium. In these games, the decisions are made by the relevant competitors simultaneously. Netessine & Shumsky (2005) examined the seat inventory control problem for two airlines competing for passengers on the same flight leg which was defined as the horizontal competition scenario. Li et al. (2016) extended Netessine & Shumsky (2005)'s work, and considered callable products that grant airlines the rights to cancel passengers' bookings. The concept of callable products is first introduced by Guillermo Gallego, S. G. Kou (2008). It refers to units of capacity sold to self-selected low-fare customers who agree that the airline can "call" the capacity at a pre-specified recall price. The method is always being used together with other mechanisms. Grauberger & Kimms (2014) proposed a heuristic method to solve Nash equilibrium in non-zero-sum games for airline alliance. The developed model improved the model presented by Jiang & Pang (2011) by overcoming the shortcomings in modelling the capacities of aircraft, the random demand and the decision variables. Jiang &

Pang (2011) set these parameters to be continuous non-negative numbers which is not the case when using this mechanism to optimise the booking problem for the airline companies. Both Grauberger & Kimms (2014) and Jiang & Pang (2011) analysed the competition game between the players. In their studies, each player made decisions on the basis of the predicted reaction of the other competitor. W. Grauberger and Kimms (2016) investigated joint price and quantity competitions between airlines and developed a heuristics method to solve the game. Waldemar Grauberger and Kimms (2016) has investigated the solution to Nash equilibrium considering horizontal and vertical competitions. Grauberger and Kimms (2018) proposed a mixed-integer model to compute a pure Nash equilibrium.

Apart from obtaining the solution to the Nash equilibrium, collaboration mechanisms have also been developed for competition. For example, the cooperation in the airport-airline was analysed by Saraswati & Hanaoka (2014). Asgari et al. (2013) investigated the competition and cooperation strategies amongst ports and shipping companies. Saeed (2013) explored the cooperation between freight players in different transportation modes.

2.6.2 Stackelberg Equilibrium

Stackelberg Equilibrium has not been found in air transportation, but it has been applied in a few studies in road transport. Labbé, Marcotte and Savard (1998) developed a bilevel model to optimise highway tolls on a multi-commodity transportation network. Brotcorne *et al.* (2000) developed bilevel programs for a freight tariff-setting problem. There were two main assumptions in their research: the first one was that the customer demands were known and fixed; and the second assumption was that the leader was not a dominant player in the market. Brotcorne *et al.* (2008) simultaneously considered the network design and pricing problem. Mixed-integer bi-level program method was applied to develop mathematical models for a carrier (leader) and a shipper (follower). The leader's model aimed to determine a network as well as a tariff to charge the follower; and the follower decided how to meet customer transportation demands using the network based on the tariff decided by the leader. The authors proposed an iterative algorithm based on a Lagrangian relaxation framework (Lawrence v. Snyder, 2011a).

Stackelberg Equilibrium has also been applied in the railway freight industry. However, the conflicting interests of multiple stakeholders and the application of game theory in rail network revenue has not received much attention although it has been suggested that cooperation between multiple stakeholders can improve the profits (T. Li *et al.*, 2016). An early study involving both multiple stakeholder and track pricing was carried out by Harker and Hong (1994). They used pricing as a lever to coordinate the train scheduling between different divisions in a railway company. Variational inequalities were used to model the game. Crevier et al. (2012) considered the joint optimisation of pricing and capacity for rail freight using a bi-level model. Both Harker and Hong (1994) and Crevier et al. (2012) made pricing decisions with regard to capacities which can be modelled as continuous variables, and the calculation of duals in their models have been used. This research is different from the previous studies in the following aspects: 1) the bi-level model in the study has binary integer variables in the lower level problem, this makes the existing duals based solution method invalid for this problem; 2) this research conducts a further study on the solution for system optimisation for the network revenue game.

The IM-FOC game considered in the study can be applied to oligopoly market structure where markets are only dominated by a small number of firms. The common gaming models used in investigating oligopoly market in the existing literature are Cournot and Bertrard game (Naimzada, Tramontana and Milano-bicocca, 2012). Cournot game is used to determine the Nash Equilibrium in a duopoly market where the companies produce homogeneous goods and compete in output. Bertrand derives a Nash equilibrium where the companies compete in price (Tremblay and Tremblay, 2011). Both Cournot and Bertrard games assume that the players in the games make a decision simultaneously whereas the IMs and the FOCs make decisions sequentially. Further, the two games cannot be applied in this study, as the products in this study are the train itineraries. They are heterogeneous products which have different properties including original station, destination station, departure time, arrival time, etc.

2.7 Network Design

As discussed earlier, the FOC's decision is to design a freight service network. Therefore, the relevant literature in relation to rail network design is also covered.

Rail network design is a traditional research topic in operations research and transportation science. Similar to the literature in network revenue management, rail network design in the published studies was mainly conducted for a single stakeholder, e.g. Infrastructure Manager. Crainic & Rousseau (1986) developed a modelling framework to optimise the network design process for a multimode, multi-commodity freight transportation problem and also provided an algorithm to solve the model. Crainic (2000) provided an overview of the models relating to service network design. Pazour et al. (2010) presented models from a policy maker's point of view for high-speed rail freight distribution network design in the US. Lulli et al. (2011) proposed a customised mathematical model to design the Italian rail service network. Lin et al. (2012) formulated models to optimise the freight train connection problem in a large-scale railway network in China. A simulated annealing algorithm was applied in the optimisation process. Murali et al. (2016) developed a decision tool for train planners to select the best route in terms of travel time. Capacity limits for the train movement was considered. This tool used integer programming to model the capacity of the network and a genetic algorithm was used to solve the model. It can be observed that there are very limited studies considering the conflicting interests of multiple stakeholders in network design.

2.8 Research Gaps

Based on the current state of the art and the above comprehensive literature review in this research field, research gaps were identified.

Armstrong & Meissner (2010) observed a lack of studies in rail network revenue management after reviewing 18 relevant papers on both passenger and freight rail revenue management. They found that the existing research covers the issues including capacity allocation, service differentiation, and booking horizon, but importantly network revenue management in rail freight industry has been overlooked, this is a clear research gap. To further clarify the contributions that this research will make to the existing body of literature, a summary of the most relevant research is given in Table 8. When analysing the literature, the following aspects were considered: 1) if the proposed model was deterministic or stochastic; 2) if the number of stakeholders was one or more than one where game theory was applied; 3) if the model involved integer variables; 4) if the topological structure of network was considered.

Exsiting studies	***Deterministic Or Stochastic***	***Stakeholder Number***	***Integer Or Continuous***	***Network considered***
Crainic and Rousseau, 1986	D	1	I	√
Pazour, Meller and Pohl, 2010	D	1	I	√
Crevier, Cordeau and Gilles Savard, 2012	D	Multiple	C	√
Crainic, 2000	D	1	I	√

Lin et al., *2012*	D	1	I	√
Lulli, Pietropaoli and Ricciardi, 2011	D	1	I	√
Marcotte, Savard and Zhu, 2009	D	Multiple	C	√
Murali et al. 2016)	D	1	I	√
Harker and Hong, 1994	D	2	C	X
This study	S	Multiple	I	√

Table 8 A summary of the relevant literature

Based on Table 8 and the above literature review, the following research gaps are identified.

GAP 1: There are very limited studies on the network revenue management game in the rail freight industry. Only two studies have been found in the extensive literature search: Harker and Hong (1994) and Crevier et al. (2012). However, both studies only model the capacities that a FOC should purchase as a continuous decision variable, which does not capture the network design issue faced by the FOC.

More specifically, Harker and Hong (1994) and Crevier et al. (2012) considered how many wagons a FOC should purchase between OD pairs. This arrangement made it possible to model the IM-FOC game using continuous models such as variational equations or continuous bilevel linear programming. The deficiencies of their approaches was that their models are not in line with industrial practice. In the real-world system, the IM normally sells the freight service to FOCs on the basis of itinerary rather than individual wagons; the wagons associated with a particular train itinerary have different prices, and thus are heterogeneous products. In the study, bilevel integer linear programming will be used to overcome the deficiencies of the two earlier models.

GAP 2: To obtain a solution to the Stackelberg Equilibrium for the network revenue management game in the railway freight industry, a bilevel linear programming model with the lower level being a network design problem needs to be solved; however, the existing literature does not provide such an algorithm.

A similar study was carried out by (Brotcorne et al., 2008). In their bilevel model, pricing and network design that involves binary decision variables that are all on the upper level, and all the decision variables in the lower level are continuous. This arrangement enabled them to convert low level problems to constraints for the upper level by using the duals of the lower level model. However, the model developed in this thesis has binary integer decision variables in the lower level that does not have a dual problem. Therefore, we cannot use a Brotcorne et al. (2008) solution method. Another similar study was conducted by Crevier et al. (2012). However, the Crevier et al. (2012) study also has no integer decision variables in the lower level problem, and thus provided no solution to the problem to be discussed in this dissertation.

GAP 3: There is the need to develop a solution method to identify the global optimal solution for the Stackelberg game for network revenue management in the rail freight industry. The literature reviews mainly focused on how to obtain the solution to the Stackelberg Equilibrium, but there are no studies conducted to obtain the solution leading to the global optimisation of the Stackelberg game.

In the existing literature, the Stackelberg model aims to identify the best strategy for the leader to optimise their profits, i.e., the optimal solution to Stackelberg equilibrium. These studies always assume that the leader is a selfish decision-maker. However, in the IM-FOC game considered in the study, the IM is a representative of government, therefore, there is the chance that the IM's decision target may be to optimise the social benefit, i.e., the total profits of the system. However, the existing literature does not consider how to design the best strategy for the leader who aims to maximise the profits of all the stakeholders in the game. There is a research vacuum for this aspect.

GAP 4: Contract or mechanism design for the network revenue management game in the rail freight industry is required in both practice and theory. There are studies on mechanism design in the context of supply chain revenue management where demands are independent and the leader-follower game is applied; and there are also studies on mechanism design for the Nash game for the competitions in network revenue game. However, no studies have been found for the coordination mechanism for the Stackelberg game in network revenue management.

The existing studies in the supply chain contract design mainly focus on homogeneous products with a single price. Therefore, the designed contract only involves a single price. As a contrast, there are many itineraries, each associated with a single price, in the study. Since the FOC's decisions are to choose which of these heterogeneous products to purchase, binary decisions variables have to been used to model the behaviours. Therefore, the previous approach used for contract design, particularly the first order and the second order optimality used in news vendor game, is not applicable to the thesis. There is another body of literature focusing on the mechanism design for Nash game, which is also not applicable to this study.

GAP 5: Practitioners lack the knowledge on how to manage a vertically separated rail freight system. The interviews with relevant stakeholders in the rail freight industry have indicated that the current pricing mechanism is not ideal, and that they have no clue on how to manage their revenue effectively. Further information with regard to the interviews will be discussed in the following chapter.

Since the vertical separation governance structure was introduced to the rail freight industry, how an IM should charge FOCs became a major decision making problem. In practice, an IM lacks the knowledge and tools to identify the best price/strategy for the new system. More specifically, an IM does not know what prices should be set to maximise their own profits; and what prices should be set to maximise the profits for the whole freight system. For example, up to now, the UK rail freight system, which is one of the earliest countries adopting the vertical separation governance structure, has only experienced price changes six times. The limited number of pricing processes has not provided enough experience for the UK to manage the prices of using rail infrastructure.

Further, it may take several years for an IM and FOCs to set a new price tariff in the UK freight industry as there are a large number of consultations to be conducted. From the perspective of practitioners, it would be more desirable if the price agreement between an IM and FOCs can be set up in a more timely manner.

2.9 Summary of the Chapter

This chapter analysed the features of the research questions and scoped the literature review accordingly. Research gaps were identified based on five streams of literature reviewed including: traditional pricing and revenue management, revenue management in supply chains, network revenue management, network revenue management game and network design.

To solve the research questions and fill in the gaps, research framework and methodology to be applied in this thesis will be discussed in the next chapter.

Chapter 3 Research Framework and Research Methodology

This chapter will provide an overview of the research framework adopted in the study, it will also introduce the main research methods selected and applied.

3.1 Research Framework

To address the three research questions introduced in Chapter 1, the following research framework with five steps was developed as shown in Figure 14

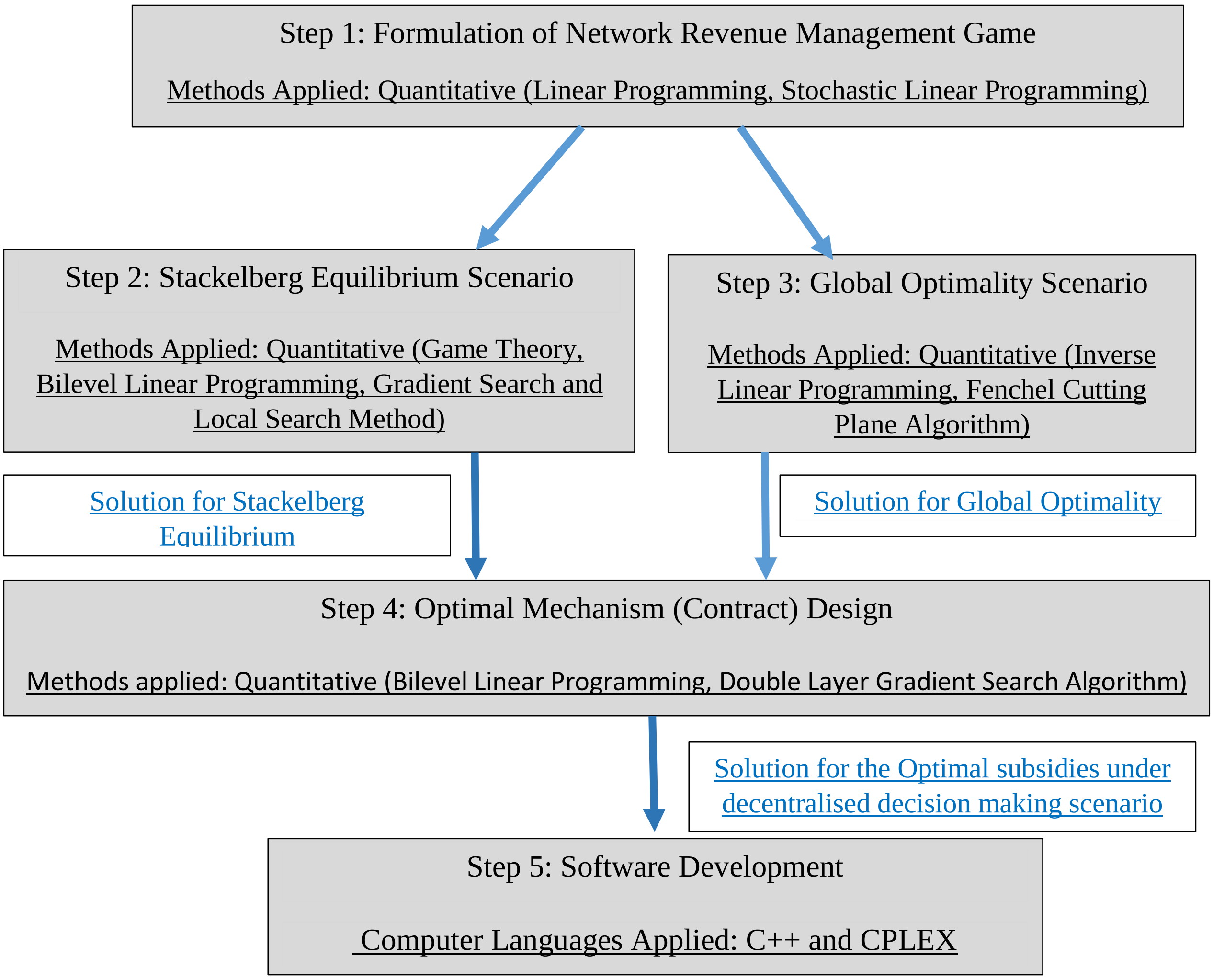

Figure 14 Describing the Research Framework

Step 1: Formulation of Network Revenue Management Game

The proposed question will be mathematically formulated, which is the basis of the research. Stackelberg (Leader/Follower) game theory (to be discussed in detail in section 3.2.6) will be employed to model the relationship between an IM and a FOC. They are deemed as two players in a Leader-Follower game. Figure 15 shows the interaction between the IM and the FOC. In the UK, Network Rail, the IM, represents the Leader in the game who is responsible for determining the price tariff for train itineraries in the rail freight service network. A freight operating company (FOC), e.g. DB Cargo, acting as the Follower, will make its decisions on the purchase of train itineraries based on the prices decided by the Leader and end customer's (shipper's) demands. Based on the FOC's itinerary purchasing plan, the IM can calculate its costs and profits. It should be pointed out that the activities in the two phases are conducted repetitively. In other words, the IM will repetitively adjust the prices based on the decision made by the FOC in the previous round.

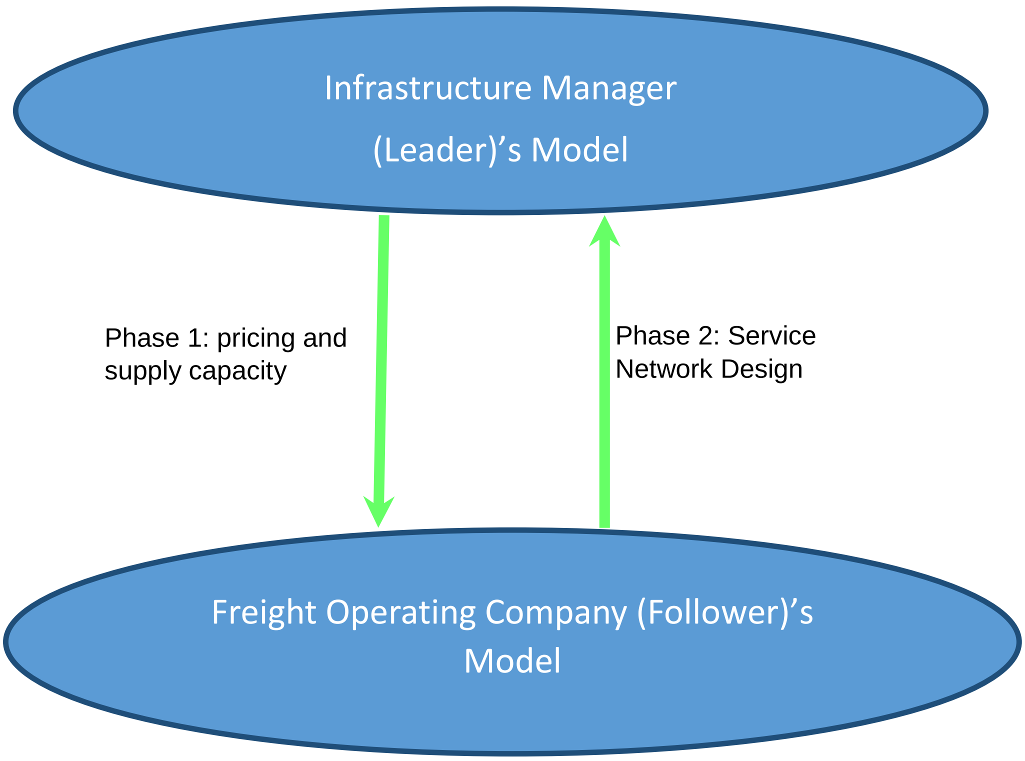

Figure 15 Gaming Process of the IM's and the FOC's Decision Making

In contrast to the traditional Stackelberg game which normally does not involve networks and assumes homogeneous product associated with a single price, the problem to be investigated in this research is based on the rail freight service network consisting of multiple itineraries with multiple prices. Therefore, the question to be investigated is consequently more challenging. Due to the network effect, integer decision variables must be used in the models, which make it very difficult to solve the game.

Mixed integer linear programming is employed to develop the IM's model and two-stage stochastic linear programming is adopted to model the operation of the freight service operator. The FOC's model is a two stage stochastic programming model where the uncertainty of the shipper's demands will be considered. In the first stage of the FOC's model, the decision centres on the rail freight route selection/ network design. It will model the relationship between capacity acquisition cost and freight service network design. In the second stage, the operational performance of the corresponding network design plan will be evaluated. This stage will involve the modelling of operational issues such as fulfilment of customer demands and compute the revenue and the costs corresponding to the selected operation plan and the designed service network.

Customer demand is one of the main parameters in the FOC's model. Because of the dynamics of the economic environment, this parameter has a high level of uncertainty which cannot be neglected when making decisions on itineraries purchasing plan. Therefore, it is necessary to consider customer demand as a stochastic variable to reflect its uncertain characteristics. Sample Average Approximation (SAA) method will be employed to accommodate the stochastic factor (Dantzig and Thapa, 2003).

Step 2: Solution of the Stackelberg Equilibrium of the Game

To address the first research question in the study, i.e., identifying the prices at Stackelberg Equilibrium, a bilevel linear programming model with binary decision variables in the lower level is developed. This is different from the existing studies related to Stackelberg game model where the first-order or second-order derivatives of a nonlinear continuous functions are the main solution approaches. The adoption of bilevel linear programming in the thesis is due to the heterogeneous train itineraries.

Bilevel programming is an optimisation technique, it is a branch of hierarchical mathematical optimisation. In a bilevel programming problem, the upper level decision making process integrates the reaction of the lower level to its decisions. Once the upper level decision variables are determined, the follower solves an optimisation problem at the lower level, taking as exogenous the leader's variables. Such an interaction between the upper level model and lower level model in a bilevel model can exactly reflect the relationship between the IM's and FOC's decision making. It is an appropriate method to be applied for finding the Stackelberg Equilibrium solution of the game. Detailed introduction to this method is in section 3.2.6.

The existing solution methods for the bilevel programming model requires conversion of the lower level problem into a dual problem (Dantzig and Thapa, 2003), which cannot be used for the bi-level model developed in the dissertation due to these binary variables. Therefore, a specialised solution method based on gradient search will be developed for the proposed bilevel programming model.

Step 3: Solution of the Global Optimality of the Game

The second research question is to identify the solutions leading to global optimality of the Stackelberg game. In this step, a solution for the IM-FOC game when the two stakeholders have perfect cooperation is first identified. In order to make this solution optimal to the scenario when the IM and the FOC make decisions independently, an inverse linear programming method is applied. Detailed information for this method will be provided in section 3.2.5. Due to the integer variables involved, a Fenchel cutting plane method (Boyd, 1994) is used to solve the inverse integer programming model. Stochastic programming is applied to handle the uncertain demand.

Step 4: Optimal Mechanism (Contract) Design

The various contracts/mechanisms that may coordinate the freight service network supply chain will be explored. The 'coordinate' means that the contracts/mechanism can lead to global optimality and ensure the IM and the FOC are better off than the current industrial practice, i.e., the IM's unilaterally designing the prices without collaboration with the FOC. A government subsidy based mechanism is designed to coordinate the rail freight system. A double-layer gradient search enhanced by local search method is developed to find out the optimal value of the subsidy rate.

Step 5: Software Development using C++ and IBM CPLEX

Finally, in this research, the algorithms and mathematical models will be implemented in a software package tool. The software tools will be developed using C++ and CPLEX. The following will explain the main reasons for choosing the computer language and tool.

There are many choices of computer languages for developing the software package. To choose a suitable language is a crucial step for the particular task. In the field of computer science, there are some studies that have been conducted to compare the computational performance of different languages. In these studies, the same algorithm will be coded using all tested languages. The key criterion for the comparison is the computational speed and time required for a particular task. (Aruoba and Fernández-Villaverde, 2015) summarised the speed of several languages in Table 9.

Average and relative run time (Seconds).

	Mac			Windows		
Language	Version/Compiler	Time	Rel. Time	Version/Compiler	Time	Rel. Time
C++	GCC-4.9.0	0.73	1.00	Visual C++ 2010	0.76	1.00
	Intel C++ 14.0.3	1.00	1.38	Intel C++ 14.0.2	0.90	1.19
	Clang 5.1	1.00	1.38	GCC-4.8.2	1.73	2.29
Fortran	GCC-4.9.0	0.76	1.05	GCC-4.8.1	1.73	2.29
	Intel Fortran 14.0.3	0.95	1.30	Intel Fortran 14.0.2	0.81	1.07
Java	JDK8u5	1.95	2.69	JDK8u5	1.59	2.10
Julia	0.3.7	1.91	2.62	0.3.7	1.80	2.37
Matlab	2014a	7.91	10.88	2014a	6.74	8.92
Python	Pypy 2.2.1	31.90	43.86	Pypy 2.2.1	34.14	45.16
	CPython 2.7.6	195.87	269.31	CPython 2.7.4	117.40	155.31
R	3.1.1, compiled	204.34	280.90	3.1.1, compiled	184.16	243.63
	3.1.1, script	345.55	475.10	3.1.1, script	371.40	491.33
Mathematica	9.0, base	588.57	809.22	9.0, base	473.34	626.19
Matlab, Mex	2014a	1.19	1.64	2014a	0.98	1.29
Rcpp	3.1.1	2.66	3.66	3.1.1	4.09	5.41
Python	Numba 0.13	1.18	1.62	Numba 0.13	1.19	1.57
	Cython	1.03	1.41	Cython	1.88	2.49
Mathematica	9.0, idiomatic	1.67	2.29	9.0, idiomatic	2.22	2.93

Table 9 Table showing the speed of several languages for the same algorithm

As the software to be developed will mainly be used to do scientific calculation in this study. A large amount of calculations will be needed in the case study and data analysis process. Therefore, run time will be an important criterion for selecting language. If the computer language is not efficient enough, the computational time required will be significant. From Table 9, it can be found, compared with other computer languages, C++ has very good performance in terms of computational time.

Meanwhile, in the software, it is necessary to call IBM CPLEX library function to calculate the model. CPLEX will provide basic functions to compute Mixed Integer Linear Programming. CPLEX library functions has the best compatibility with C/C++ although it also supports JAVA, PYTHON, MATLAB and C. This is because CPLEX was designed for C/C++ originally and developed later to connect with other languages.

In light of the above discussions, C++ and CPLEX were chosen as the computer language and tool to develop the software package tool to implement the algorithms and models in this research.

3.2 Research Methods to be Applied

To address the research questions, the following methods will be employed at different stages of this research:

- Game theory
- Linear programming
- Stochastic linear programming
- Sample Average Approximation
- Inverse Mixed Integer Linear Programming
- Bilevel linear programming
- Gradient search method

In what follows, an introduction to these methods will be presented.

3.2.1 Game theory

Game theory studies the decision strategies of multiple players. Mathematical models including the objectives and constraints of different parties involved in the decision making environment are normally developed to capture the relationship between the multiple decision-makers that may have both conflicts and cooperation (Drew Fudenberg and Tirole, 1991) . Game theory has now been applied in economics, political science, psychology, logistics, computer science, as well as biology, to study the conflict and cooperation between two or several decision-makers. The earliest researchers who studied game theory in economics were Cournot (1838), Bertrand (1883) and Edgeworth (1925). The book *Theory of Games and Economic Behaviour* written by John von Neumann and Oskar Morgenstern (1944) is known as the first book which introduced the idea of a general theory of games in the economic field. The authors applied game theory to analyse most economic problems in the book. They introduced the definition of the min-max solution as well as the zero-sum games. In these games, the players have conflicting interests and have no commonality. They also proved that the min-max solution always exists in the zero-sum games which involve two players. In two-player zero-sum games, the sum of the total utilities is a fixed number (constant) and the amount one player wins is exactly the same as the others lose. The political election problem is a typical example of zero-sum game. The total number of votes is constant and if one candidate has more votes, it means the others will have to lose by the same number of votes.

John Forbes Nash Jr. also made a significant contribution to game theory by introducing the concept of "Nash Equilibrium" in 1950 to study the equilibria. It was then developed for specific models by (Cournot, 1838), (Bertrand, 1883) and Drew Fudenberg and Tirole (1991). In the Nash Equilibrium, there are two assumptions: the first one is that each player has full knowledge of the strategy of his opponents, this means each player can correctly predict the competitor's response to their strategy. The second one is that each player tries to maximise their payoff based on their estimation of their competitor's reaction. Nash Equilibrium is a strategic combination in which each player's strategy is the best response to the other's. In other words, under such a situation, each player achieves the maximum payoff he/she could have in the game. This is a different type of game and also a non-zero-sum game.

To understand the insight of game theory, a simple example in practice will be given. Suppose that a manufacturer needs to determine a price between a range (between a high and a low price) for its products. When making this decision, the manufacturer needs to consider production cost that includes the raw material costs, labour cost etc. as well as the prices of other manufacturers' same or substitutable products. A simple approach to design the pricing strategy is to choose a price similar to the competitors' prices for the same or substitutable products. An improved method is to apply game theory to consider the other competitions' behaviours, i.e., how they may change their prices in response to the manufacture's price. To apply game theory, the manufacturer needs to mathematically capture: 1) how customer demand will respond to the change of prices based on their knowledge of the market environment; and 2) how each individual competitor will adjust their price in response to the prices designed by the manufacture and the other competitors. Based on these mathematical models, a Nash Equilibrium status may be identified and the prices at Nash Equilibrium can be obtained.

In this research, the IM and the FOC are two stakeholders involved in rail freight system. They are independent decision makers with each having their own operational objectives. They have a conflict of interest in setting the freight rate because, in general, the higher rate can increase the IM's profit but inevitably increase the FOC's costs. Also, the IM's decision on price can influence the FOC's decision on purchasing train itineraries. The FOC's decision on itinerary purchase will affect the IM's profits, and how the FOC will fulfil the customer orders. For each individual itinerary, the IM may set a tariff which is the highest price that the FOC can afford. If the prices are higher than this, the FOC will stop buying any itineraries. There exists a price that can lead to an Equilibrium status where the IM and the FOC can achieve their own best possible benefits when they make decisions independently. The above process between the IM and the FOC can be categorised as a Stackelberg game, and the equilibrium for the game will be the Stackelberg Equilibrium. As the total profits of the system will change with the IM's price tariff, the IM-FOC game is a non-zero-sum Stackelberg game.

When applying game theory to identify the prices at equilibrium, two independent optimisation models will be developed: the IM's model and the FOC's model. The IM's model aims to firstly determine a track access price tariff; the FOC's model aims to determine an itinerary purchasing plan based on the prices determined by the IM and end customers' orders. In the game, the two models will be applied repetitively. At the beginning of the game, e.g., time period 0, the IM's model will be applied to generate a price tariff for using train itineraries; based on this price tariff and the end customers' orders, the FOC's model will be applied afterwards to decide the optimal purchasing plan for train itineraries. Then the IM will make adjustment to its price tariff in the hope that a more profitalbe response from the FOC can be received to increase its profits. The above process may repeat many times until an equilibrium status can be achieved.

3.2.2 Linear programming

Linear programming will be used to develop the IM's and the FOC's models. Linear programming was first introduced by *George B. Dantzigg* and his colleges in 1947 as a method to obtain the best solution for a particular goal (Dantzig, George B., 1997a). *George B. Dantzig* is believed to be the pioneer in linear programming research and he has made significant contribution to this field. He published more than four hundred papers, books and reports in the research of this field (Dantzig, George B., 1997). Dantzig, George B. (1997) defined linear programming as the problems which aim to determine the maximum or minimum of a linear objective function. The variables in the problem are subject to linear equality and inequality constraints. The general Linear Programming Problem can be formulated as

$$\max c^T x = Z$$

s.t.

$$Ax \leq b$$
$$x \in R^+$$

Where the linear objective function of the model is to find out the maximal value of Z. A is a $m \times n$ matrix; c is a $1 \times n$ vector, b is a $m \times 1$ vector. All the elements in matrix A, b, c are rational. R^+ is nonnegative n-dimentional real vector. $x$is $n \times 1$ continuous non-negtive decision varialbes and all x are constrained by :$Ax \le b$.

After several decades' development, linear programming theory has been applied widely and extended to the other mathematical programming fields including integer programming, stochastic programming and nonlinear programming. One of the major extensions that has close connection to this study is Mixed Integer Linear Program (MILP). Like a LP problem, a MILP problem has a linear objective function, has bounds and linear constraints, but no nonlinear constraints; particularly, some components of decision variable set are integer values. A standard formulation of a Mixed Integer Linear Programme problem (MILP) is:

$$\mathrm{Z} = max(c^T x + d^T y)$$

s.t.

$$Ax + hy \le b$$
$$x \in Z_+^n, \mathrm{y} \in R_+^p$$

Where the linear objective function of the model is to find out the maximal value of Z. x is $n \times 1$ non-negtive integer decision variables, and thus belongs to Z_+^n ; y $p \times 1$ non-negative continuous decision variables, $\mathrm{y} \in R_+^p$; h $m \times p$ matrix; d $1 \times p$ matrix.

Many researchers extended linear programming to mathematical optimisation theory (Lenstra, Shmoys, & Tardos, 1990; Ben-Tal, Goryashko, Guslitzer, & Nemirovski, 2004; X. Chen, Sun, & Xu, 2018). Now linear programming has become a cornerstone for operations research (Charnes, Cooper, & Rhodes, 1978; Crowder, Johnson, & Padberg, 1983; Desrochers Martin, Jacques Desroseris 1991; Mathur & Puri, 1995), management science (Fisher & Fisher, 1981; Elmaghraby & Keskinocak, 2003), economics (Ghodsypour & O'Brien, 1998; Ghodsypour & O'Brien, 2001)and computer science(Yannakakis, 1991). Linear Programming is also used in combinatorial optimisation problems and network flow maximisation problems. Linear programming has been used widely in industrial practices, e.g., petroleum industry throughout the whole process, from oil extraction, refining, blending to distribution, transportation, supply chain management, food processing industry, iron and steel industry, metalworking industry etc. (Dantzig, George B., 1997). Linear programming provides a way to model the problems in practice mathematically. Linear programming models can be solved through some algorithms more efficiently than non-linear programming. The output of a linear programming model can inform what is the best decision to be taken and what are the best results corresponding to the decision.

3.2.3 *Two-Stage stochastic programming*

Stochastic programming is a mathematic optimisation method dealing with the probability and stochastic factors. A stochastic programming problem normally contains a set of variables that include one or more random variables. The earliest research on stochastic programming appeared in the papers by George Dantzig, Martin Beale and Ferguson in 1955 and 1956. The research at this early stage in this field was in isolation and its importance was not fully recognised, which resulted in the very slow development of stochastic programming and things did not improve until late 1980s when computer science technology became mature.

One of the most common stochastic programming is termed two-stage program with fixed recourse by Dantzig (1955). It can be formulated as

$$
\begin{aligned}
& \min z = f^T X + E_{\omega \in \Omega}[q(\omega)^T Y(\omega)] \\
\text{s.t.} \quad & AX = b \\
& C(\omega)X + Dy(\omega) = k(\omega) \\
& X \geq 0, Y(\omega) \geq 0
\end{aligned}
$$

In the first stage, the decision variable is X, and the coefficient is a vector f. X is the first stage decision that needs to be made first before the realisation of any decision variables. Hence, X is independent of the random events. In the second stage, a set of random events, denoted by Ω, may happen, and ω is one of the random events. $C(\omega)$, D, $k(\omega)$ are the coefficients; $y(\omega)$ is the decision variable, which are all dependent on a realisation of stochastic variable. The target of the second stage is to minimise the expected costs relating to $Y(\omega)$ for a given X.

If the number of random events is finite, i.e., $\{\omega_1, \dots, \omega_k, \dots \omega_{|\Omega|}\}$, the corresponding probabilities for the random events are $\{p_1, \dots, p_k, \dots p_{|\Omega|}\}$, the above two-stage program can be re-written as,

$$
\begin{aligned}
& \min z = f^T X + \sum_{k=1}^{|\Omega|} p_k Q(X, \Delta(\omega_k)) \\
\text{s.t.} \quad & AX = b \\
& C(\omega)X + Dy(\omega) = k(\omega) \\
& X \geq 0, y(\omega) \geq 0
\end{aligned}
$$

Where, $Q(X, \Delta(\omega_k)) = \min_y \{q(\omega_k)^T y | Dy \geq k(\omega) - C(\omega_k)X\}$;

$\Delta(\omega_k) = \{q(\omega_k), C(\omega_k), k(\omega_k)\}$.

The majority of published research in stochastic linear programming involves two-stage problems as solving the two-stage stochastic programming is relatively easy. This technique has been widely applied in optimisation problems(Fangruo CHen, 1999). For example, Chen *et al.* (2015) applied this method to design the supply chain for Original Equipment Manufacturers (OEMs) considering the uncertainty of the value of remanufacturing. Restrepo, Gendron and Rousseau (2017) formulated a multi-activity scheduling problem considering the uncertain demand as a discontinuous two-stage stochastic linear model. In their study, a multi-cut L-shaped method was developed to solve the model, and the computional results showed that the developed models and solution method can help to reduce staff costs.

3.2.4 Sample average approximation (SAA)

It is a big challenge to solve the aforementioned stochastic programming problem, due to the inherent complexity and large computational requirement (Emelogu *et al.*, 2016). For example, consider a road network with 200 links, and each link may have five states with each representing a certain level of traffic congestion. Each state may happen with a certain probability. Therefore, the total scenarios to be considered will be 5^{200}, i.e., $|\Omega| = 5^{200}$. This will lead to a large number of constraints and scenario-dependent variables, which may be easily beyond the capacity of an advanced modern computer.

Sample Average Approximation (SAA) approach was proposed to calculate the near-optimal solutions for these large-scale stochastic optimisation problems with uncertain parameter(s). SAA is a sample-based approach. The idea of SAA is that a number of sample scenarios, e.g., $(\xi_1, \xi_2, \dots, \xi_n)$ are generated according to the distribution functions of random variables. The expected value of the objective function such as $E_{\omega \in \Omega}[q(\omega)^T y(\omega)]$ and $\sum_{k=1}^{|\Omega|} p_k Q(X, \Delta(\omega_k))$ is approximated by the sample average function $\frac{1}{N} \sum_{i=1}^{n} Q(X, \Delta(\xi_n))$. An additional merit of SAA is that it can even approximate the expected value of a function that does not have a closed form.

Many studies with regard to SAA have been carried out. Some scholars studied this approach theoretically, e.g., Ermoliev and Norkin (2013), Kleywegt, Shapiro and Homem-de-Mello (2002). Some other scholars applied SAA into different industry practice. Dong, Lee and Song

(2015) studied the service capacity planning and dynamic container routing problem in the shipping network in which customer demand is an uncertain parameter. The researchers applied the SAA method to deal with the uncertainty of customer demand. Wang and Meng (2017) explored the intermodal freight transportation network design problem in their research and proposed a sampling-based heuristic policy in which they applied SAA approach. Wu and Sioshansi (2017) employed the SAA method to solve a two-stage stochastic model to manage the electric vehicle charging problem. This method was also used in the location-allocation problem (Amiri-Aref, Klibi and Babai, 2018). The demand in the study was a stochastic parameter in a multi-period location-inventory optimisation problem in a multi-echelon supply chain network.

In literature, the SAA method can be categorized into two groups: (1) SAA with fixed sample size; (2) SAA with variable sample size. In this research, the freight demand of the FOCs from the shippers, measured by the number of orders and wagons required in each order, are all random. Further, the number of orders that need to be served by a FOC is large. Due to these features, SAA with fixed sample size is an appropriate approach for this study.

The FOC's model will be formulated as a two-stage stochastic programming model with recourse. The input data for the FOC's model is the IM's price and the random customer demands. SAA method will be adopted to handle the random customer demands, i.e., customer demands will be sampled, and then fed into the FOC's model.

3.2.5 Inverse mixed integer linear programming (InvMILP)

In this study, the global optimality of the above Stackelberg game will be also investigated. To determine the optimal prices that can lead to the global optimality of the game, Inverse Mixed Integer Linear Programming will be applied in this research. This method has been widely applied in the optimisation field (Chow and Recker, 2012; You, S.I.a , Chow, J.Y.J.b, Ritchie, 2016; Tayyebi and Aman, 2016)

Similar to the other optimisation problems, an MILP also includes a set of parameters and a set of variables; and can generate different optimal solutions (output) when a different parameter set (input) is given. The Inverse Linear Programming Problem (InvMILP) aims to identify a set of cost coefficients in objective function that can convert a known feasible solution to an optimal one.

Consider the following standard MILP problem,

$$z = \min_{x \in \mathcal{P}} c^T X$$

s.t.

$$AX = b$$
$$X \in \mathbb{Z}^r \times \mathbb{R}^{n-r},\ \mathrm{c} \in \mathbb{R}^n$$

Mathematically, InvMILP aims to identify a vector of cost coefficients, *d*, in the objective function, which can make a known feasible solution x^0 optimal. *d* is normally obtained by minimally adjusting a known cost coefficient *c*. As *d* might be not unique, i.e., there might be many possible sets of vectors that can make x^0 be the optimal solution, some criterion will be needed to choose a vector *d*. The commonly adopted criterion is some selected L_p norm, i.e., $||\mathrm{c} - d||_{\mathrm{p}} = \left[\sum_{j \in J} |d_j - c_j|^p\right]^{\frac{1}{p}}$, where p is an integer ranging from 1 to $+\infty$. In this research, L_1 norm will be used for the sake of simplification. Hence, the Inverse Mixed Integer Linear Programme (InvMILP) can be formulated as the following:

$$\min_{d} |c - d|$$

Subject to:

$$d^T x^0 \leq d^T x$$
$$Ax = b$$
$$x \in \mathbb{Z}^r \times \mathbb{R}^{n-r}$$

where, x^0 is a feasible solution for the original problem, but is the optimal solution corresponding to the new cost coefficient vector *d*.

There are some applications of the Inverse Linear Programme in the existing literature. For example, Bitran *et al.* (1981) proposed to use Langragain techniques to solve an inverse optimisation problem, and applied the proposed method in a plant location problem. (Albert Tarantola (2005) applied the inverse linear programming method in geophysical research. In recent years, this method attracted more researchers, e.g.,Yang & Zhang (1999);Wang (2009); Zhang et al. (2011); Bulut & Ralphs (2013);Ralphs & Bulut (2014) and its application is not just limited to mathematics and geophysical sciences, it has also been applied in many other different fields (Mostafaee, Hladík and Černý, 2016). (Burton and Toint (1994) and Toint (2013) explored inverse shortest paths problem which aimed to recover the arc costs based on the given information about the shortest paths in a graph. Some other researchers expanded this theory and developed different algorithms. Ahuja and Orlin (2001) studied the algorithm for inverse optimization problem and proved that the inverse problem of a linear programming problem is also a linear programming problem under L_1 and L_∞ norm. They claimed that the inverse problem of the optimisation problem can be simplified when a particular condition was satisfied. Wang (2009) developed a cutting plane algorithm for the inverse MILP problem. Zhang and Xu (2010) investigated the inverse optimization for linearly constrained convex separable programming problems and explored the application practically, such as using this method to control quality in production systems and to manage the production capacity planning problem. Bulut and Ralphs (2013) systemically discussed the mathematical property and complexity of inverse mixed integer linear programme and proposed an algorithm to identify the optimal solution of InvMILP. Tayyebi & Aman (2016) proposed a binary search technique based algorithm to solve inverse linear programming problem. The authors also did a case study for an algorithm for the inverse minimum cost flow problem which showed a positive result. Most recently, You, S.I.a , Chow, J.Y.J.b, Ritchie (2016) applied this method in vehicle route problems to find the optimal route solution. The above successful examples of the application of inverse linear programming indicate that InvMILP is an appropriate research method and can be employed to solve the problem proposed in the thesis.

In this thesis, when the IM and the FOC are deemed as a single organisation under perfect collaboration, there will be no charges between each other, and the freight system can achieve system optimality, i.e., the sum of the IM's and the FOC's profits is maximised. By solving the model under prefect collaboration, the best itinerary purchasing plan and order fulfilment plan can be obtained. By setting an appropriate price tariff, the optimal solution for the case under prefect collaboration can also be the optimal solution leading to system optimality for the aforementioned Stackelberg game in a vertically separated railway system. From the mathematical point of view, there might be more than one price vector which can satisfy the aforementioned requirement. In this study, L_1 norm will be followed, thus the price vector that is closest to a known price vector will be selected. Using L_1 norm can minimise the disturbance to the freight market and keep the market stable.

3.2.6 Bilevel linear programming

In game theory, it is common to consider two different decision-making approaches: centralised and decentralised. In the centralised decision-making system, a central but sometimes virtual planner who can access the global information and makes decisions for all the stakeholders. The centralised decision-making mechanism represents an ideal situation as there is no central planner in reality for the majority of games. As a contrast, in a decentralised decision-making system, each stakeholder may have their own decision-making objective and make decisions independently.

In this study, the bilevel linear programming method will be adopted to identify the prices at equilibrium when the IM and the FOC make decisions independently in a decentralised decision-making system. The bilevel linear programming method is an approach to model a two-players non-cooperative game.

Bilevel linear programming problem (BLPP) is a type of hierarchical mathematical optimisation. In BLPP, there exists two decision-makers known as "the leader" and "the follower" who make upper level and lower level decisions, respectively. In game theory, BLPP is introduced as a method to solve Stackelberg game (Amirtaheri *et al.*, 2017). In such a game, one player acts as the leader in the problem and makes decisions first, while the other, as the follower, will determine its decision strategy according to the decision made by the leader. The leader and follower have their own decision variable sets and the corresponding constraints sets and aim to optimise their own objective functions independently. It should be noted that, although the leader makes a decision first, the leader's performance, e.g., costs or profits, are often affected by the lower level decision.

A standard formulation of BLPP can be written as:

$$
\begin{array}{ll}
 & \min f(x,y) \\
\text{s.t.} & B(x,y) \leq 0 \\
 & x \in X \\
 & \min g(x,y) \\
\text{s.t.} & K(x,y) \leq 0 \\
 & y \in Y
\end{array}
$$

Note: one or both "min" operators may be replaced by "max" operator.

In the above formulation, x is the set of decision variables of the leader (upper level), $f(\cdot)$ is the leader's objective function, $B(x,y)$ denotes the leader's constraints. For the follower (lower level), y is the set of decision variables, $g(\cdot)$ is its objective function and $K(x,y)$ are the constraints.

Bilevel programming has many applications. For example, it was applied in strategic planning and pursuit-evasion games in which the two player's objectives are opposed (Dempe *et al.*, 2015). These models are in the form of max-min formulations. Many other applications of bilevel programming in literature deal with policies/regulations for governments and industries (Li, Ukkusuri and Fan, 2018; Feng et al., 2018). In recent years, bilevel programming approach was applied in many other fields to facilitate decision making. Amirtaheri et al. (2017) used this method to investigate a production-distribution supply chain problem. Calvete et al. (2014) studied the planning process of a decentralised distribution network using a bilevel optimization approach. In the study, the effect between decisions made at the distribution stage and the manufacturing stage were considered. Labbé (2011) modelled the pricing optimisation problems using bilevel programming. In the study, linear prices and network effects have been considered.

3.2.7 Gradient search approach

In this study, a special bilevel programming model will be developed. In the bilevel model, there will be binary integer variables in the lower level. Solving this type of bilevel model is not reported in the existing literature. A special solution algorithm for this unique bilevel programming model will be developed in the study. In developing the solution algorithm, gradient search will be employed.

Gradient search is an optimisation method using a first-order iterative approach to find the minimum or maximum value of a function. Gradient descent or gradient ascent may be used depending on whether the optimisation problem is to find the minimum or maximum value of a function. At the beginning of the algorithm, a start searching point (initial value) needs to be selected and the step length needs to be defined. The gradient descent will be used to search the minimum value of a function along the negative direction of gradient. As a contrast, gradient ascent will be used for searching the maximum value of a function along the positive direction of gradient (Alzaman, Zhang and Diabat, 2018).

Suppose that a function $F(x)$ with multiple variables is differentiable at an arbitrary point x, then the gradient of $F(x)$ at x is $\nabla \mathrm{F}(x)$. Let k denote the length of search step, which is a positive scalar. To calculate the minimum value of $F(x)$, the negative of the gradient, i.e., gradient descent, should be used to determine searching step. The searching point should be updated as

$$x^{n+1} = x^n - k * \nabla \mathrm{F}(x^n)$$

To calculate the maximum value of $F(x)$, the negative of the gradient, gradient ascent, should be used. Hence, the searching point should be updated as

$$x^{n+1} = x^n + k * \nabla \mathrm{F}(x^n)$$

A gradient search based algorithm normally follows the following procedure: at the beginning, an initial point of $F(x)$, x^0, is chosen as a start point of the search. Then, following the above two equations to update the searching point depending on whether it is a maximisation or minimisation problem. For a minimisation problem, the search stops when $F(x^{n+1}) \geq \mathrm{F}(x^n)$; and for a maximisation problem, the search stops when $F(x^{n+1}) \leq \mathrm{F}(x^n)$

In recent years, this gradient search method was applied widely, especially, in the research of machine learning. In this dissertation, gradient ascent will be applied to find out the maximum profits for the relevant stakeholders in the aforementioned Stackelberg game.

3.3 Summary of the Chapter

This chapter provided an overview of the research framework. Also, the research methods to be applied in the thesis to answer the identified research questions were provided which are the knowledge basis of the research.

Chapter 4 Rail Freight Charging Regime in the UK and Data Collection

In this chapter, the governance structure of the UK rail freight system will be explained in detail. The information will provide the basis to develop mathematical models in the subsequent chapters. Data collected from industry will be presented, which will then be used to test the mathematical models and the corresponding solution algorithms.

4.1 Governance Structure of the UK Rail Freight

The rail freight industry in the UK originated in the early 19th century after the first steam locomotive was built, long before the railway network was used for providing regular passenger services. Similar to many other countries, the UK railway has experienced reforms, e.g. nationalisation, privatisation, and innovations such as electrification, over the centuries. One of the major reforms that happened in the UK was that the UK government changed the governance structure of their railway system from vertical integration to vertical separation in the 1990s in the hope that the railway system could become more efficient and competitive.

In the current UK rail freight industry, a vertically separated governance system characterised by the separation of railway ownership from the operations is adopted. Under this governance system, the owner of the railway system does not provide transport services to shippers directly. Instead, it provides itineraries to some licensed freight operating companies (FOCs) who provide freight transport services to shippers. Shippers need to pay the railway owner for using the railway infrastructure including tracks, signals, electricity, bridges, tunnels, etc. Currently, the rail freight service is provided by competing FOCs, and the infrastructure is managed by a single Infrastructure Manager (IM), Network Rail. The three stakeholders in the rail freight industry form a three echelon supply chain where the IM is the supplier of the rail freight service supply chain, the FOC is the retailer and provides freight service to the end customer. In contrast to a traditional supply chain, the products in the chain are the itineraries where each itinerary has different properties including original station, destination station, departure time, arriving time, passing stations etc. The most important property of the itineraries is the network effect.

Compared to the vertically separated railway governance structure in other countries, the UK railway industry has a unique feature. In the UK, both the FOCs and the IM are all regulated by the Office of Rail and Road (ORR) who is responsible for the development of the railway industry by setting strategy and overseeing its delivery. It operates within the framework set by UK and EU legislation, e.g. DIRECTIVE 2012/34/EU, Commission Implementing Regulation (EU) 2015/909 , and is accountable through Parliament and the courts.

Under such a governance structure, after many years of development, the railway industry has become an important economic sector in the UK. Figure 16 presents the contribution of the rail industry to the UK economy (Office of Rail and Road, 2019a) and the income of the rail industry in 2016 is shown in Table 10.

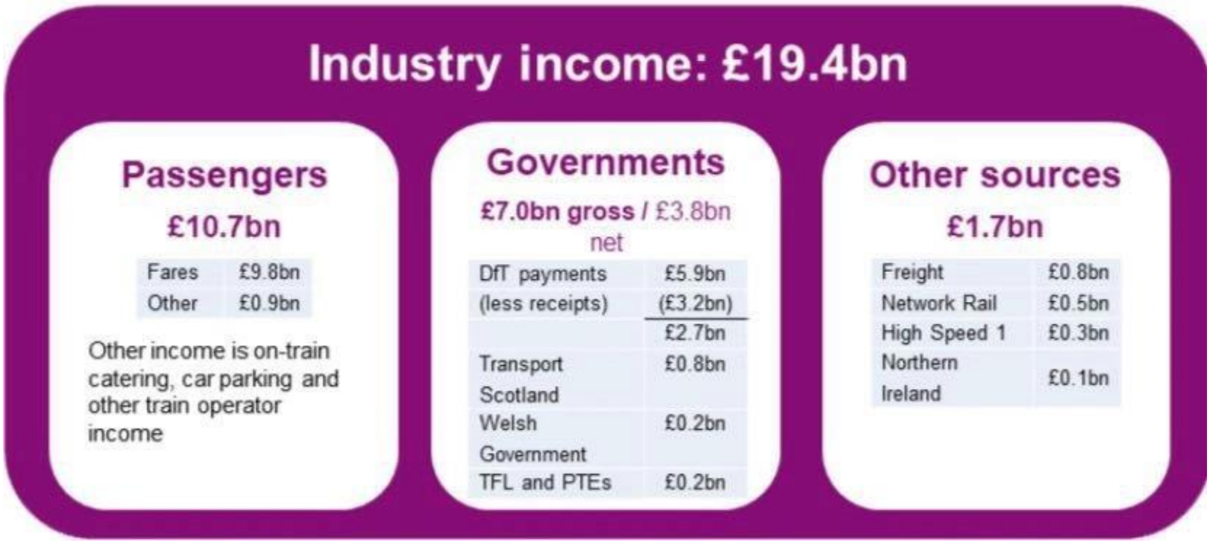

Figure 16 Industry Income in 2017-2018
Source: (Office of Rail and Road, 2019a)

Values in 2016	Railway system (A)	Rail supply sector (B)	Station retailers and their supply chain (C)	Total of all rail-related industries (A)-(C)	Total induced impacts (D)	Total of all rail-related impacts (A)-(D)
Output or sales (£ billion)	**22.9**	**31.6**	**4.0**	**58.5**	**17.6**	**76.1**
GVA (£ billion)	**11.3**	**15.1**	**1.5**	**27.9**	**8.5**	**36.4**
Employment (thousands)	**114.5**	**248.9**	**41.6**	**405.0**	**192.1**	**597.1**
Tax revenues (£ billion)	**3.6**	**4.3**	**0.6**	**8.4**	**2.6**	**11.0**

Table 10 Overview of the UK Rail Related industries' economic footprint
Source: (Oxford Economics, 2003)

4.2 The Current Charging Regime in the UK Rail Freight Industry

Under the governance of vertical separation, Network Rail needs to charge the FOCs to collect revenue and recover its costs (although sometimes not-fully) incurred in providing the freight infrastructure to the freight operating companies. This arrangement is reasonable as it follows the principle that "users pay for the costs". Network Rail and Office of Rail and Road have put a lot of effort into setting up the cost-reflective tariff (Office of Rail and Road (ORR), 2015).

The tariff that the IM adopts to charge FOCs is generally reviewed every 5 years; this is termed a Control Period (CP). The only exception is CP2, which was 3 years long due to historical reasons related to the predecessor of Network Rail - Railtrack. To match with the start and end date of a financial year, each Control Period begins on 1st April and ends on 31st March. The current control period is CP 6 which started on 1st April 2019 and will end in 2024. This reviewing mechanism has been implemented in the UK rail freight industry since 1996. Since 1996, there were 5 CPs, which are: Control Period 1 (CP1): 1996-2001; Control Period 2 (CP2): 2001-2004; Control Period 3 (CP3): 2004–2009; Control Period 4 (CP4): 2009–2014; Control Period 5 (CP5): 2014–2019. The planned control periods are: Control Period 7 (CP7): 2024-2029; Control Period 8 (CP8): 2029-2034 (Network Rail, 2017b)

The most recent track access charges for Control Period 6 (CP6) were published on 3 December 2018 and the price tariff was in line with ORR's Final Determination valid from 1 April 2019 to 31 March 2024.

The new price tariff for freight operations in CP6 includes the following elements:

• Variable Usage Charge (VUC);

• Freight Specific Charge (FSC);

• Open Access Infrastructure Cost Charge (ICC);

• Electrification Asset Usage Charge (EAUC);

• Fixed Track Access Charges (FTAC)

The approach Network Rail used to recalibrating each charge for CP6 is explained in (Network Rail, 2017) in details. The official document produced by Network Rail in 2017 is very lengthy. The calculations methods to determine each charge element are very similar to the previous ones that Network Rail have developed during the time period from CP1 to CP5. However, these calculation methods have been updated to reflect the changing industry condition in recent years. The key points for recalibrating charges in CP6 will be summarised as follows. It should be noted that the updated principles and methods for each charge element have been audited by third party companies.

- The current structure of the UVC remain unchanged for CP6. However, the charge rates have been updated to reflect the new industrial changes, e.g., Network Rail's costs and patterns of traffic flows on the network (Office of Rail and Road, 2019). The calculation method is audited by ARUP.

- The CP6 EAUC rates follows the same methodology in CP5 but based on the latest forecasts of traffic volumes. Steer independently audited the EAUC recalibration.

- In calculating EAUC, comparing to that in CP5, ORR approved the removal of the meter tolerance factor from the Traction Electricity Rules.

- The FTAC is determined based on the methodology revised by Network Rail in May 2018.

The following is the key milestones for setting up the charges for CP6 (Network Rail, 2017b):

Autumn 2016 – Network Rail published its initial industry plans.

Spring 2017 – ORR published advice to ministers and set out requirements for Network Rail's Strategic Business Plan.

Summer 2017 – Department for Transport (DfT) and Transport Scotland published their high level output specifications (HLOS) and statements of funds available (SOFA).

Winter 2018 – Network Rail published its strategic business plan (Network Rail, 2018a).

Summer 2018 – ORR published its draft determination.

Autumn 2018 – ORR published its final determination.

April 2019 – CP6 started.

It is clear that the consultation for CP 6 started in 2016 and lasts for at least 2 years before a new control period starts. The 2-year timespan might result in the collected information being outdated when they are used to determine the track access tariff. If the IM's tariff is not designed appropriately due to the outdated information, the freight rate that the FOCs charge the shippers will be also unreasonable.

The review in each CP includes track usage tariff, which is very important information for Network Rail, the owner of most of the rail infrastructure in the UK, to plan their financial flows for the coming years. The review result provides the detailed charging rate for different categories, for example, rail freight related charges including capacity charge, track usage charge and traction electricity consumption charge. More specifically, for freight train, the track usage tariff obtained from the review includes the following elements:

1) Variable Usage Charge (£/kgtm)
   - Freight Variable Usage Charge rates
   - Freight Variable Usage Charge default rates
2) Electrification Asset Usage Charge (£ per electrified kgtm)
3) Freight Specific Charge (£/kgtm)
   - Freight Coal ESI and Iron Ore Freight Specific Charge rates
   - Freight Biomass Freight Specific Charge rates
   - Freight Spent Nuclear Fuel Freight Specific Charge rates

Note: kgtm means kilogram per train mile.

Currently only Variable Usage Charge and Electrification Asset Usage Charge are applied to freight trains transporting general cargos (Network Rail, 2018c). There are no fixed charges applied to freight trains, and fixed charges are only applied to the franchised passenger operators (Network Rail, 2018b). In fact, as occurs in railway industries in other countries, the fixed costs do not change with traffic volume and account for a significant proportion of the cost of the railway network. In practice, the total fixed costs incurred by Network Rail are huge. For example, in 2013, in England & Wales and Scotland, the amounts were £2,379,350,841, £1,759,663,413 and £619,687,428 respectively (Swattridge, 2013).

It has been noticed that "There is currently a relatively low level of understanding around the drivers of Network Rail's fixed costs" (Office of Rail and Road (ORR), 2016). To ensure a healthy and sustainable development of the rail sector, the freight operators need to at least pay for the cost generated for using the network. As ORR's Director of Markets and Economics, Cathryn Ross, said “Under the current regime, freight companies only pay a small proportion of the costs they create using the network for freight industry” (Office of Rail and Road (ORR), 2016). Network Rail employed an independent costing expert, Brockley Consulting, to review the cost allocation approach and suggest improvements. In contrast to the existing charging regime, the report prepared by Brockley Consulting proposed to also apply a fixed cost to freight operators (Mantzos, 2017). On 22nd September 2017, Network Rail had a consultation with regard to the allocation of fixed costs to freight operators in Control Period 6 (CP6)(Network Rail, 2018a). The consultation was to check the stakeholders’ views on the new cost allocation approach. There are a lot of arguments and disagreements among the stakeholders, for and against the cost spread method. All of these actions indicate that authorities are trying to improve the charging regime in the hope that the principle “charges to better reflect costs” can be followed.

In contrast to road transport that is normally operated in an open market with few regulations, rail freight services are operated within a fully regulated market environment. Notably, Network Rail still relies on government grant (subsidiary) to sustain their company. In the current freight charging regime, there are three main sources for Network Rail to recover their total costs for operating the network, and the government grant is one of them: -

(1) charges to the freight service operating companies;

(2) the government network grant;

(3) other sources of income (Network Rail, 2017b).

As the public funds are used to subsidise the activities of Network Rail, it is very important for the UK government and the public to understand the cost structure of Network Rail and reduce the contribution of public fund as much as possible. However, in practice, both the track access charging and Network Rail’s cost structures are very complex, and it is very difficult for the UK government and the public to ascertain Network Rail’s real costs. Further, as the government aims to use public funds to cover Network Rail’s loss, Network Rail may have the incentive to exaggerate their costs. Therefore, increasing transparency of Network Rail’s costs and simplification of the charging scheme is of interest to the UK government and the general public.

In the UK rail freight industry practice, there is no adequate collaboration between Network Rail and the FOCs. Network Rail is the only infrastructure manager, and is in a leading position to set up the track access prices. As the monopolist in the market, the IM makes decisions on the track access tariff independently without collaboration with FOCs. Although the charges suggested by the IM need to be approved by the ORR, and the ORR may apply some caps on the profit the IM can make, the IM can still generate any profit by exaggerating the costs. The FOCs need to make a decision based on the track access tariff as well as historical customer demands to purchase train itinerates. Hence, the FOCs' profits are subject to the IM's track access tariff. There is the possibility that the IM makes the decision to maximise its own profits without caring about FOCs' profits. Modern game theory indicates that, in many fields, collaboration between stakeholders often leads to better outcome than non-collaboration (Fudenberg and Tirole, 1991). It remains unclear whether or not both the FOCs and the IM can be better off when some sort of collaboration mechanism is introduced.

To sum up, the current charging regime has a few drawbacks:

1) The review on the track access tariff normally takes 2 years. Therefore, it is very likely that a tariff may not catch up with a swift change in the freight market.
2) The FOCs do not pay fixed charges for accessing railway infrastructure whereas the fixed cost is a large portion of costs incurred by the IM.
3) Since the IM receives the subsidy from the UK government, it is important to ascertain its costs and revenue generated from charging the FOCs as the information is useful for the UK government to determine the amount of public funds to be released to the IM. A way forward might be to increase the transparency of the IM's spending (costs) as well as simplify the track access charging regime.
4) There is lack of cooperation between the IM and FOCs in designing the track access tariff.

These drawbacks in a vertically separated governance system have been recognised by the UK government. The UK government has recognised that it is necessary to renovate the charging regime that the IM adopts to charge the FOCs. As stated in "The Rail Freight Strategy" designed by the Government, the new pricing mechanism/strategy is required to meet the current and future requirements of rail freight and generate the positive impact of track access charge on the freight industry (Department for Transport (DfT), 2016b).

This research is one of the efforts to improve understanding of the charging regime. The focus of this study is to introduce a collaboration mechanism to coordinate the decision making between an IM and a FOC using Stackelberg game theory. This new mechanism will enable both of the two parties to be better off than making the decision independently. In designing the mechanism, the fixed charge will be considered, which is consistent with the new methodology proposed by Network Rail published on May 2018 (Network Rail, 2018a) . The models and the solution algorithms will be implemented in a software tool, which can facilitate fast decision-making.

4.3 Data Collection

In the following chapters, mathematical models and algorithms will be developed. The best way to validate the developed models and algorithms is to apply them using real data collected from the freight industry in the UK. In order to collect data from Network Rail and the freight operating companies, various approaches were adopted on numerous occasions.

There are two mathematical models developed in this study including the IM's model and the FOC's model. In order to validate the two models, relevant data from the industry is required. Data collection was conducted as an important part of this research. To collect data, two methods, questionnaire and interview, were applied. The two methods are used simultaneously in data collection process rather than being applied independently.

At the beginning of the study, based on the literature review and documents published on the websites of Office of Rail and Road, Department of Transport, Network Rail and operating companies, the primary mathematical models were developed for the IM and the FOC respectively. The cost structures of the models included fixed costs, variable costs. Questionnaires were designed based on the data requirements of the two models. To validate the model structures and collection of the data set required by the model, interviews with industry practitioners were also conducted. Several interviews involved the experts from Network Rail, Rail Freight Group, Consultants, Chartered Institute of Logistics and Transportation (CILT). The detailed information about each interview will be provided in the following section. By analysing the information collected from the interviews, it was found that collecting detailed cost elements from Network Rail and freight operators were unrealistic. Therefore, the structure of fixed cost and variable costs were adjusted to an aggregated value rather than different categories, the models for the IM and the FOC were changed according to this adjustment. The questionnaire was designed for collecting the data set based on the structure of the models accordingly.

To collect data from Network Rail and operating companies to validate the IM's model and the FOC's model respectively, two different questionnaires were developed. Background of the study was provided in the first part of each questionnaire so that the participants can have a good understanding of the data collection requirement. Each questionnaire was designed according to the model structure respectively. For the IM's questionnaire, apart from investigating the current pricing methodology and principle, the related data required in the model were listed and explained including the cost, price, freight capacity information. For the FOC's questionnaire, the interviewees are required to provide the price they currently offered to the shipper, typical customer demands over 3 days, average cost per wagon per train mile, estimated profits for meeting the provided customer demand for each itinerary. Open questions were also designed in the questionnaires which contains personal information about the interviewees.

The questionnaires were distributed on various occasions directly or indirectly to the rail freight practitioners. Interviews with the practitioners based on the designed questionnaires were also implemented. The total number of interviewees was 24. The total number of questionnaires distributed was 45. The returned questionnaires with completed answers excluded open questions were treated as valid returned questionnaires. Valid questionnaires returned were from 2 companies. In the valid returned questionnaires, the participants took a case from their business as an example and provided the required data information including costs and price.

Due to the commercial confidentiality, the profit information was unable to be collected. To estimate the profit that a freight operating company may earn, the FOCs' average profits information, which is open to the public, is used. This method makes it reasonable to estimate the profit information of the freight operators.

By going through this data collection process, the key data information required to validate the developed models was thus obtained.

Data collection process also includes:
Firstly, attempts were made to obtain data from my industry supervisor based in Network Rail.

- The data collection issue was initially raised at the first External Supervisory Team Meeting held on 16th November 2015 to *Dr. Meena Dasigi*, my external supervisor from Network Rail. She introduced the state of art of the current rail freight practice, but no data information was provided after the meeting and this issue was raised again in the iCASE meeting on 1st Sep 2016 held at Imperial College, London, organized by Dr Dasigi's supervision, but still no data was provided.

- Ms. Nadia Hoodbhoy, the current Principal Engineer (Acting) has replaced Dr. Meena Dasigi as the industrial supervisor following Dr Dasigi's retirement in February 2017. The data collection questionnaire was presented to her in the first External Supervisory Team Meeting held in March 2017 in Network Rail Office, London, and with her help, an interview with Mr. Guy Bates, Head of Freight Development at Network Rail on 8th June 2017 was carried out. At this very informative meeting, a broader understanding of the rail freight industry in the UK as well as the operational process of rail freight transportation was introduced. He suggested abandoning the very detailed cost structure in the models, "as the costs are composed of thousands of pieces from here and there, and it will cost you the rest of your life to get it!"

Secondly, the supervisory team helped to approach their industry network for help.

- Since January 2017, Mr. Phil Mortimer, from freight logistics consultancy helped to collect data for this research. He arranged a meeting with Mrs. Maggie Simpson from the Rail Freight Group (RFG) on 13th March 2017. Maggie kindly provided details of her contact in Office of Rail and Road (ORR);
- In February 2017, the supervisory team organised a meeting with Mr. Paul Davison, a Principal Consultant at AECOM to see if they had any data available from previous projects. They kindly gave some suggestions in relation to the data collection questionnaire;
- Effort had also been put into participating in the CILT Rail Freight Forum held on 14th March 2017 at Newcastle University. During the meeting, the data questionnaire was discussed with Mr. Julian Worth, Chair of CILT Rail Freight Forum and he invited the meeting participants, particularly those from rail freight industry to help with the data requirement.
- Freight Operating Companies were contacted.
- I participated in the CILT Forum held on 26th November 2018. With kind help from Mr. Julian Worth Chair of the Forum, some shippers agreed to provide the price information they paid to the FOC with the signing of a confidentiality agreement.

Thirdly, published documents on the Network Rail website as well as the ORR website were investigated. Some pricing information relating to freight train operation is open to the public and provide a good level of details. Especially, the profit percentage of the FOC, percentage of each type of cost in the IM’s total cost are published.

After analysing and integrating the collected information, the IM’s and FOC’s models were developed. The structures of the two mathematical models have been validated through interviews with freight industry practitioners. Information collected indicate that, in practice, the fixed costs and variable costs for the IM to provide itineraries to the FOC may include thousands of cost elements which make it impossible to collect each of them individually. This is the same for the cost elements for FOC. Therefore, it is advised by the industry experts that it is reasonable to use aggregated values to denote the fixed costs and variable costs in the models. The advice given by the experts in the UK rail freight industry has been followed in designing both the IM’s and FOC’s models.

The datasets used for validating the models were designed based on the freight service prices that the FOC charge shippers. According to the price information provided to shippers and the average profit percentage of the FOC obtained from the published financial document, the fees that the FOCs pay to access the IM’s itineraries can be determined. As currently Network Rail is supposed to be non-profitable for operating the freight system, the fee they received from the FOC will be close to the cost for providing the itinerary. Moreover, the published statics data indicates that the IM’s fixed cost accounts for 70 percent of the total cost. This was applied as the principle for seperating the IM’s cost into fixed cost and operational variable cost.

Following the above steps, two hypothetical cases have been designed to test the models and algorithms developed in the study. These datasets were not provided by the FOCs and Network Rail directly, but these datasets are consistent with the relevant statistics published by the relevant stakeholders as well as my interviews with rail industry representatives.

4.4 Data Set 1

In this case, a freight service between two stations (station A and B) is considered. There are three train itineraries operating on the line every day. The input data for the experiment are:

- There are 3 available trains from Station A to Station B and the customer demand in 3 days is:

{8,6,6,7,6,7}

{9,7,4,7,6,8}

{7,1,2,4,5,5}

- The FOC's revenue for different order in different trains are

{15,9,6,6,8,9},

{10,12,10,8,9,8},

{8,9,7,9,10,15}.

Demands and revenue information are shown in Table 11.

	Orders (Number of freight wagons, Revenue in Thousand Pounds)					
Day 1	8,15	6,9	6,6	7,6	6,8	7,9
Day 2	9,10	7,12	4,10	7,8	6,9	8,9
Day 3	7,8	1,9	2,7	4,9	5,10	5,15

Table 11 Information of Customer Demand and Revenue

- The IM's fixed operational cost is £2,000 per freight wagon.
- The capacity for all the trains is 30 wagons.
- The IM's variable cost £ 10 per wagon;
- The FOC's variable cost £ 10 per wagon;
- The distance between A and B is 100 miles.

4.5 Data Set 2

In this case, a subnetwork shown in Figure 17 chosen from the national railway network is selected as the freight network to be studied. The network is composed of four stations including Mossend, DRIFT (Daventry International Rail Freight terminal), Southampton, and Felixstowe. Six itineraries operating in two directions are considered. They are: (1) Mossend-DRIFT-Southampton; (2) Mossend-DRIFT-Felixstowe; (3) Southampton- DRIFT-Felixstowe; (4) Southampton- DRIFT-Mossend; (5) Felixstowe-DRIFT-Mossend; (6) Felixstowe-DRIFT - Southampton. For each path, there are three itineraries available for sale. It is assumed that the FOC makes decision based on three days' operation data. All lines include the same node: DRIFT. Therefore, the capacity on any individual line will be constrained by the total handling capacity of station DRIFT. The problem is to investigate the IM's pricing strategy for these lines in this subnetwork.

To test the effectiveness and validity of the research outcomes, the following data from the two parties, the IM and the FOC is needed:

- The available freight capacity on the line;
- The customer demand for the 3 days;
- The fixed operational cost per unit per train mile of the IM and the FOC;
- The total variable cost per unit per train mile of the IM and the FOC;
- The FOC's revenue for serving different orders using different trains;
- The allowed maximum number of wagons for all the trains run on the line;
- The distance between the original station and the destination station.

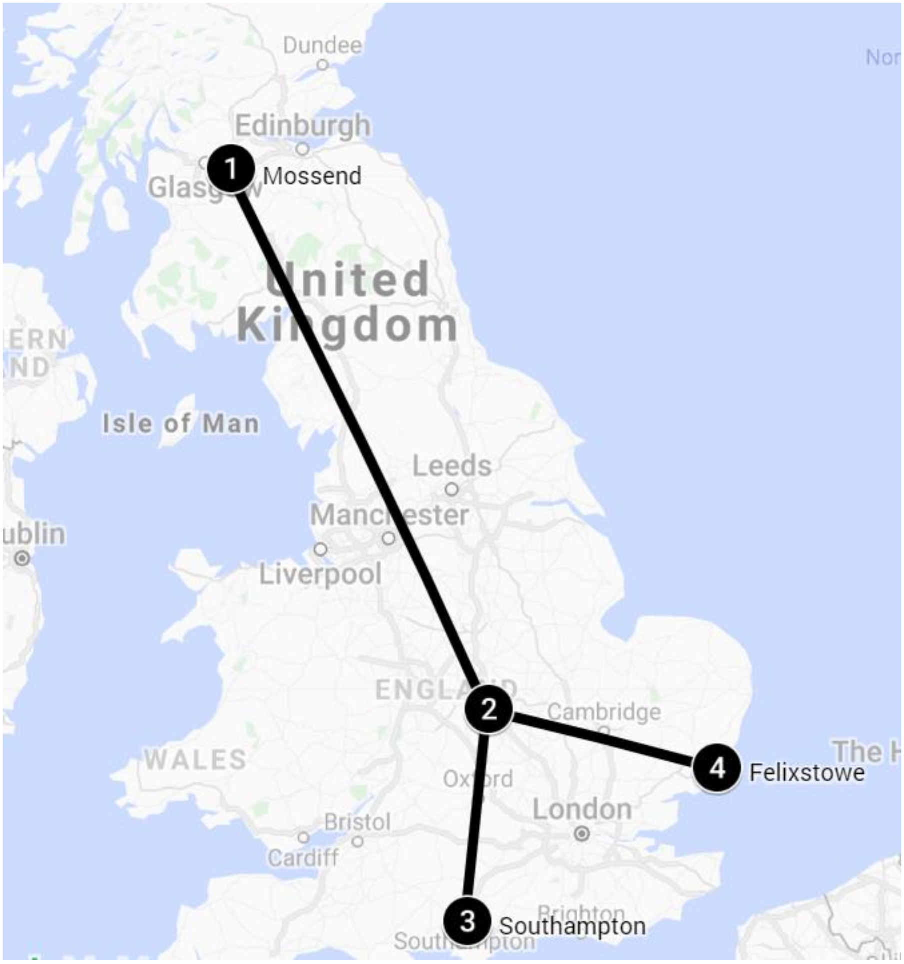

Figure 17 A four-node railway network in the UK

The railway lines in case 2 are mainly used for transporting containers. Therefore, the amount of orders and the relevant costs are measured in "containers" (termed Units in UK industrial practice). In the following, the data used in the case are presented, i.e., the orders received, the price the FOC charge shippers, and the other data such as costs and capacity.

(1) Mossend-DRIFT-Southampton

- The customer demands in 3 days are:

  On day 1, there are six orders and the number of units required by the six order orders are 10, 4, 10, 8, 5, 9 respectively.

  On day 2, there are six orders and the number of units required by the six order orders are 11,12,9,7,10,4 respectively.

  On day 3, there are six orders and the number of units required by the six order orders are 10,6,6,4,12,11 respectively.

  The customer demand vector can be written in the following:

  {10, 4, 10, 8, 5, 9},

  {11,12,9,7,10,4},

  {10,6,6,4,12,11}.

- The IM's fixed operational cost between Mossend and Southampton is £0.05 per unit per train mile.
- The FOC charges to the shippers for different orders using different trains:

  Train 1: The price the FOC charges to the shipper is £583 per unit for serving order 1. Similarly, the FOC charge £587 per unit for order 2, £572 per unit for order 3, £576 per unit for order 4, £577 per unit for order 5 and £586 per unit for order 6.

  Train 2: The price the FOC charges to the shipper is £586 per unit for serving order 1. Similarly, the FOC charge £572 per unit for order 2, £582 per unit for order 3, £583 per unit for order 4, £583 per unit for order 5 and £574 per unit for order 6.

  Train 3: The price the FOC charges to the shipper is £583 per unit for serving order 1. Similarly, the FOC charge £578 per unit for order 2, £585 per unit for order 3, £585 per unit for order 4, £588 per unit for order 5 and £591 per unit for order 6.

  The following is the price vector for the three trains:

  {583,587,572,576,577,586}

  {586,572,582,583,583,574}

  {583,578,585,585,588,591}

- According to physical constraints on the railway lines, the allowed maximum number of units for all the trains running between Mossend and Southampton is 32.
- The total variable cost for the IM on this line per unit per train mile is £0.05;
- The FOC's variable cost is £1 per unit per train mile;

- The distance between Mossend and Southampton is 470 train miles.

(2) Mossend-DRIFT-Felixstowe;

- The customer demands in 3 days are:

  On day 1, there are six orders and the number of units required by the six order orders are 8, 6, 10, 6, 8, 9,12 respectively.

  On day 2, there are six orders and the number of units required by the six order orders are 12,10,12,9,12,6 respectively.

  On day 3, there are six orders and the number of units required by the six order orders are 13,9,5,9,15,10 respectively.

  The customer demand can be written in the following vector:

  {8, 6, 10, 6, 8, 9,12},

  {12,10,12,9,12,6},

  {13,9,5,9,15,10}.

- The IM's fixed operational cost between Mossend and Felixstowe is £0.05 per unit per train mile;
- The FOC's price for different orders in different trains:

  Train 1: The price the FOC charges to the shipper is £590 per unit for serving order 1. Similarly, the FOC charge £587 per unit for order 2, £582 per unit for order 3, £588 per unit for order 4, £588 per unit for order 5 and £584 per unit for order 6.

  Train 2: The price the FOC charges to the shipper is £592 per unit for serving order 1. Similarly, the FOC charge £592 per unit for order 2, £592 per unit for order 3, £591 per unit for order 4, £600 per unit for order 5 and £580 per unit for order 6.

  Train 3: The price the FOC charges to the shipper is £590 per unit for serving order 1. Similarly, the FOC charge £580 per unit for order 2, £597 per unit for order 3, £598 per unit for order 4, £591 per unit for order 5 and £600 per unit for order 6.

  The following is the price vector for the three trains:

  {590,587,582,588,588,584}

  {592,592,592,591,600,580}

  {590,580,597,598,591,600}

- According to physical constraints on the railway lines, the allowed maximum number of units for all the trains running between Mossend and Felixstowe is 32;
- The total variable cost for the IM on this line per unit per train mile is £0.05;
- The FOC's variable cost is £1 per unit per train mile;
- The distance between Mossend and Felixstowe is 490 train miles.

(3) Southampton- DRIFT-Felixstowe

- The customer demands in 3 days are:

  On day 1, there are six orders and the number of units required by the six order orders are 8, 6, 10, 6, 7, 12 respectively.

  On day 2, there are six orders and the number of units required by the six order orders are 10,12,8,8,9,6 respectively.

  On day 3, there are six orders and the number of units required by the six order orders are 10,8,6,6,14,15 respectively.

  The customer demand can be written in the following vector:

  {8,6,10,6,7,12},

  {10,12,8,8,9,6},

  {10,8,6,6,14,15}.

- The IM's fixed operational cost between Southampton and Southampton is £0.11 per unit per train mile;
- The FOC's price for different orders in different trains:

  Train 1: The price the FOC charges to the shipper is £391 per unit for serving order 1. Similarly, the FOC charge £386 per unit for order 2, £387 per unit for order 3, £384 per unit for order 4, £388 per unit for order 5 and £388 per unit for order 6.

  Train 2: The price the FOC charges to the shipper is £386 per unit for serving order 1. Similarly, the FOC charge £388 per unit for order 2, £386 per unit for order 3, £391 per unit for order 4, £385 per unit for order 5 and £385 per unit for order 6.

  Train 3: The price the FOC charges to the shipper is £390 per unit for serving order 1. Similarly, the FOC charge £385 per unit for order 2, £386 per unit for order 3, £385 per unit for order 4, £386 per unit for order 5 and £391 per unit for order 6.

  The following is the price vector for the three trains:

  {391,386,387,384,388,388},

  {386,388,386,391,385,385},

  {390,385,386,385,386,391}.

- According to physical constraints on the railway lines, the allowed maximum number of units for all the trains running between Southampton and Felixstowe is 32;
- The total variable cost for the IM on this line per unit per train mile is £0.05;
- The FOC's variable cost is £1 per unit per train mile;
- The distance between Southampton and Felixstowe is 300 train miles.

(4) Southampton- DRIFT-Mossend

- The customer demands in 3 days are:

  On day 1, there are six orders and the number of units required by the six order orders are 10,6,8,5,8,7 respectively.

  On day 2, there are six orders and the number of units required by the six order orders are 12,5,6,5,6,10 respectively.

  On day 3, there are six orders and the number of units required by the six order orders are 4,8,7,6,15,12 respectively.

  The customer demand can be written in the following vector:

  {10,6,8,5,8,7},

  {12,5,6,5,6,10},

  {4,8,7,6,15,12}.

- The IM's fixed operational cost between Southampton and Southampton is £0.05 per unit per train mile;
- The FOC's price for different orders in different trains:

  Train 1: The price the FOC charges to the shipper is £580 per unit for serving order 1. Similarly, the FOC charge £577 per unit for order 2, £582 per unit for order 3, £578 per unit for order 4, £578 per unit for order 5 and £584 per unit for order 6.

  Train 2: The price the FOC charges to the shipper is £582 per unit for serving order 1. Similarly, the FOC charge £572 per unit for order 2, £572 per unit for order 3, £591 per unit for order 4, £588 per unit for order 5 and £570 per unit for order 6.

  Train 3: The price the FOC charges to the shipper is £581 per unit for serving order 1. Similarly, the FOC charge £570 per unit for order 2, £587 per unit for order 3, £588 per unit for order 4, £581 per unit for order 5 and £589 per unit for order 6.

  The following is the price vector for the three trains:

  {580,577,582,578,578,584}

  {582,572,572,591,588,570}

  {581,570,587,588,581,589}

- According to physical constraints on the railway lines, the allowed maximum number of units for all the trains running between Southampton and Felixstowe is 32;
- The total variable cost for the IM on this line per unit per train mile is £0.05;
- The FOC's variable cost is £1 per unit per train mile;
- The distance between Southampton and Felixstowe is 470 train miles.

(5) Felixstowe-DRIFT-Mossend

- The customer demands in 3 days are:

  On day 1, there are six orders and the number of units required by the six order orders are 8,6,10,6,8,9 respectively.

  On day 2, there are six orders and the number of units required by the six order orders are 12,10,12,9,12,6 respectively.

  On day 3, there are six orders and the number of units required by the six order orders are 13,9,5,9,15,10 respectively.

  The customer demand can be written in the following vector:

  {8,6,10,6,8,9},

  {12,10,12,9,12,6},

  {13,9,5,9,15,10}.
- The IM's fixed operational cost between Southampton and Southampton is £0.05 per unit per train mile;
- The FOC's price for different orders in different trains:

  Train 1: The price the FOC charges to the shipper is £591 per unit for serving order 1. Similarly, the FOC charge £582 per unit for order 2, £587 per unit for order 3, £585 per unit for order 4, £585 per unit for order 5 and £589 per unit for order 6.

  Train 2: The price the FOC charges to the shipper is £593 per unit for serving order 1. Similarly, the FOC charge £594 per unit for order 2, £594 per unit for order 3, £590 per unit for order 4, £600 per unit for order 5 and £85 per unit for order 6.

  Train 3: The price the FOC charges to the shipper is £593 per unit for serving order 1. Similarly, the FOC charge £582 per unit for order 2, £592 per unit for order 3, £591 per unit for order 4, £593 per unit for order 5 and £602 per unit for order 6.

  The following is the price vector for the three trains:

  {591,582,587,585,585,589}

  {593,594,591,594,590,600}

  {593,582,592,591,593,602}
- According to physical constraints on the railway lines, the allowed maximum number of units for all the trains running between Southampton and Felixstowe is 32;
- The total variable cost for the IM on this line per unit per train mile is £0.05;
- The FOC's variable cost is £1 per unit per train mile;
- The distance between Southampton and Felixstowe is 490 train miles.

(6) Felixstowe-DRIFT - Southampton

- The customer demands in 3 days are:

On day 1, there are six orders and the number of units required by the six order orders are 5, 8, 10, 4, 8, 14 respectively.

On day 2, there are six orders and the number of units required by the six order orders are 12,16,6,6,8,6 respectively.

On day 3, there are six orders and the number of units required by the six order orders are 12,8,8,10,10,12 respectively.

The customer demand can be written in the following vector:

{5, 8, 10, 4, 8, 14}

{12,16,6,6,8,6}

{12,8,8,10,10,12}

- The IM's fixed operational cost between Southampton and Southampton is £0.12 per unit per train mile;
- The FOC's price for different orders in different trains:

  Train 1: The price the FOC charges to the shipper is £390 per unit for serving order 1. Similarly, the FOC charge £388 per unit for order 2, £384 per unit for order 3, £385 per unit for order 4, £382 per unit for order 5 and £386 per unit for order 6.

  Train 2: The price the FOC charges to the shipper is £389 per unit for serving order 1. Similarly, the FOC charge £384 per unit for order 2, £389 per unit for order 3, £394 per unit for order 4, £382 per unit for order 5 and £384 per unit for order 6.

  Train 3: The price the FOC charges to the shipper is £392 per unit for serving order 1. Similarly, the FOC charge £395 per unit for order 2, £388 per unit for order 3, £383 per unit for order 4, £388 per unit for order 5 and £393 per unit for order 6.

  The following is the price vector for the three trains:

  {390,388,384,385,382,386}

  {389,384,389,394,382,384}

  {392,395,388,383,388,393}
- According to physical constraints on the railway lines, the allowed maximum number of units for all the trains running between Southampton and Felixstowe is 32;
- The total variable cost for the IM on this line per unit per train mile is £0.05;
- The FOC's variabl cost is £1 per unit per train mile;
- The distance between Southampton and Felixstowe is 300 train miles.

The data sets in Case 2 are summarised in the following Table 12,Table 13, Table 14.

Felixstowe to Southampton						
Day 1	8	6	6	7	6	7
Day 2	9	7	6	7	6	8
Day 3	7	8	6	9	10	9
Southampton to Mossend						
Day 1	10	6	8	5	8	7
Day 2	12	5	6	5	6	10
Day 3	4	8	7	6	15	12
Felixstowe to Mossend						
Day 1	8	4	10	7	6	9
Day 2	12	15	6	7	8	4
Day 3	6	8	12	4	5	10
Mossend to Southampton						
Day 1	10	4	10	8	5	9
Day 2	11	12	9	7	10	4
Day 3	10	6	6	4	12	11
Mossend to Felixstowe						
Day 1	8	6	10	6	8	9
Day 2	12	10	12	9	12	6
Day 3	13	9	5	9	15	10
Southampton to Felixstowe						
Day 1	8	6	10	6	7	12
Day 2	10	12	8	8	9	6
Day 3	10	8	6	6	14	15

Table 12 Customer Order (units)

Felixstowe to Southampton						
Day 1	390	388	384	385	382	386
Day 2	389	384	389	394	382	384
Day 3	392	395	388	383	388	393
Southampton to Mossend						
Day 1	580	577	582	578	578	584
Day 2	582	572	572	591	588	570
Day 3	581	570	587	588	581	589
Felixstowe to Mossend						
Day 1	591	582	587	585	585	589
Day 2	593	594	591	594	590	600
Day 3	593	582	592	591	593	602
Mossend to Southampton						
Day 1	583	587	572	576	577	586
Day 2	586	572	582	583	583	574
Day 3	583	578	585	585	588	591
Mossend to Felixstowe						
Day 1	590	587	582	588	588	584
Day 2	592	592	592	591	600	580

Day 3	590	580	597	598	591	600
			Southampton to Felixstowe			
Day 1	391	386	387	384	388	388
Day 2	386	388	386	391	385	385
Day 3	390	385	386	385	386	391

Table 13 Unit revenue for each order (£ / unit)

	Path 1	Path 2	Path 3	Path 4	Path 5	Path 6
The IM's fixed operational cost (£/ unit/mile)	0.11	0.12	0.05	0.05	0.05	0.05
Capacity (Units or Wagons)	34/17	32/16	32/16	32/16	32/16	32/16
the IM's variable costs (£/unit/mile)	0.05	0.05	0.05	0.05	0.05	0.05
The FOC'S variable costs (£/unit/mile)	1	1	1	1	1	1
Mileage (miles)	300	470	490	470	490	300

Table 14 The other data

Note: Path 1: Felixstowe to Southampton; Path 2: Southampton to Mossend; Path 3: Felixstowe to Mossend; Path 4: Mossend to Southampton; Path 5: Mossend to Felixstowe; Path 6: Southampton to Felixstowe

The above two data sets will be used to verify the models and algorithms together with the software tool to be developed in the following chapters.

4.6 Summary of the Chapter

This chapter reviewed the current charging regime of UK freight industry and identified the weaknesses of the existing mechanism. Two data sets were prepared for validating the models and algorithms to be developed in the following chapters. The data collection process was introduced in detail.

Chapter 5 Stackelberg Equilibrium of the IM-FOC Game

Based on the industrial practice discussed in Chapter 4, this chapter will investigate the scenario whereby the IM makes the decision on the design of track access tariff independently to maximise its own profits without cooperation with the FOC. This scenario reflects exactly what the IM is doing in practice and is modelled as a Stackelberg game. A bilevel optimisation model was developed for the scenario and a gradient search method was designed to solve the bilevel model. The solution obtained was the optimal track access tariff that can maximise the IM's profit unilaterally.

5.1 Introduction

To avoid monopolies and increase competition in the rail freight market and to eventually improve the efficiency and service quality of the railway network (Nash et al., 2013; Laroche & Guihéry, 2013; Alexandersson & Rigas, 2013), vertical separation of railway infrastructure ownership from operation was implemented in the UK and some other western countries such as Sweden, The Netherlands, Romania and Germany (Nash, J. E. Nilsson and Link, 2013). Vertical separation involves two types of independent companies: Infrastructure Manager (IM) which provides track, signalling, bridges, tunnels and stations; and Freight Operating Company (FOC) which operates freight service using the infrastructure provided by the IM. In the UK, Network Rail is the representative of IM, and they charge the FOCs for using the railway infrastructure they provide.

Before vertical separation was adopted in the railway industry, the pricing problem did not exist as there was only a single company that had all the functions that IMs and the FOCs provide. Under the new arrangement, the pricing problem, i.e. setting up the tariff for train itineraries, has become a major issue. An IM's tariff significantly affects not only the IM and FOCs' profitability but also the utilisation rate of the railway system, which should be maximised from the perspective of government and society.

It is common in practice that an IM may take advantage of its leader position in designing the tariff to unilaterally maximise its' profit without caring about the FOC's profit. According to my industrial visits to the UK rail freight industry, to prevent an IM from doing this, the UK government applies a certain cap on the IM's profit. As a countermeasure, the IM often chooses to exaggerate its' costs, and attempts to gain additional profits. In such a battle between the government and the IM, the government is often not in a good position since it has great difficulty in ascertaining the IM's genuine costs due to the complexity of railway system operations. Hence, the current pricing mechanism is not as effective as expected. There is a need to design a better strategy to improve the pricing process for the rail freight transport system adopting a vertical separation governance structure.

In recognition of this issue, this chapter aims to propose novel mathematical models to capture the complicated relationships between the IM and the FOC.The best prices that an IM can charge to maximise its own profits unilaterally without cooperation with FOCs are determined. This is also what an IM is trying to do in practice, but currently they do not have a rigorous mathematical tool to determine the prices of their train itineraries. They largely rely on their intuitive experience or manually change their prices repeatedly and choose the one that leads to maximum profits, which is in nature a trial and error approach.

When developing the model to investigate the above issues, a stylised railway system that adopts the vertical separation governance structure is considered as a three-echelon service network based supply chain comprising an IM, a FOC, and end customers. The whole pricing process between the IM and the FOC is considered as a dynamic Stackelberg (leader-follower) game (Stackelberg, 2011). As the leader, the IM's decision is the price tariff for accessing their network consisting of all the itineraries; and the follower, the FOC needs to design a service network based on the IM's tariff and the shippers' orders, which is a typical network design problem.

It is a novel idea in rail freight industry to apply Stackelberg (leader-follower) game to investigate the interaction between the IM and the FOC and obtain the optimal solution for the IM-FOC game. In previous studies (REFS), the Stackelberg game was seldom used in the network revenue management game, and hence the IM's and the FOC's sequential decisions were not considered in the existing models. By applying this Stackelberg game model, the market position of the IM and the FOC is demonstrated appropriately. The IM is treated as the leader in the game and always has the priority to make decisions. This complies with the IM's monopolist position in the current practice.

Further, when developing the mathematical models for the IM-FOC game, the network effect was considered to reflect the rail freight service property. This has not been considered in the existing studies. In the developed Stackelberg models, itinerary with different properties, e.g. original station, destination station, depart time, arrival time, route, etc. were treated as heterogeneous products in the freight service supply chain. A Stackelberg game considering heterogeneous products is rare in the existing study. More specifically, the study has used binary decision variables to model the FOC's decisions on which itineraries should be purchased. In contrast to the common differentiable Stackelberg game, this leads to a new type of Stackelberg game, which is more challenging to solve than the existing Stackelberg game in the literature. Therefore, the study will have to develop novel solution methods.

A novel game theoretical modelling framework is to be used to answer the following research questions:

What are the optimal prices of the IM-FOC game at the Stackelberg Equilibrium? The solution at Stackelberg Equilibrium specifies the prices that the IM can charge to maximise its profits unilaterally without any cooperation with the FOC.

The unique feature of the problem to be investigated is that the 'product' sold across the supply chain is a freight service network consisting of multiple itineraries, which makes it different from the traditional Stackelberg game that normally does not involve networks and assumes homogeneous product and a single price, and consequently the problem investigated here is more challenging. Due to the network effect, integer decision variables have to be used in the models, which make it very difficult to solve the game and subsequently address the two aforementioned research questions. The approach selected in this study to address the research question was to develop a bilevel programming model with binary decision variables in the lower level. The existing bilevel solution methods that involve converting the lower level problem to its dual problem cannot be used due to these binary variables. Therefore, a specialised solution method for the proposed bilevel programming model needed to be developed. To this end, a solution method based on stochastic gradient search and local search has been developed. It is worth mentioning that the uncertainty in customer demands in the gaming model is also considered. Stochastic programming is applied to handle the uncertain demand.

The contributions of this chapter to knowledge are:

1. A novel stochastic pricing problem for a three-tier service network supply chain is studied. The products are heterogeneous and have network effects. The problem was formulated as a Stackelberg game that included an IM's model and a FOC's model.
2. A bilevel programming model was developed to find the solution of the game at equilibrium. Due to the binary integer decision variables in the lower level model, a specifically designed gradient search based algorithm enhanced by local search was developed to solve the model.

5.2 Problem description

A stylised railway system considered in this research adopted a vertical separation management structure as implemented in many western countries such as the UK, Sweden, Netherlands and Germany. For example, in the UK, Network Rail, as the only IM, operates all the track, signalling, bridges, tunnels and stations but not the rolling stock. The IM is the monopolist infrastructure provider who provides paths for freight and passenger transportation service operators. To gain the right to access the paths provided, a FOC needs to pay the IM for using paths at certain periods based on a tariff agreed earlier. After gaining the access to tracks, the FOC may buy or rent different types of locomotives and wagons, then start to provide freight transport service for shippers. Shippers need to pay a certain amount of fees to the FOC for using the service according to agreed transportation rates. The IM, the FOC, and shippers form a three-tier service supply chain.

As shown in Figure 18, the above process was executed repetitively, and eventually reached an equilibrium status. Once the IM makes a change to the track access tariff, the FOC will make a response by changing its itinerary purchasing plan, i.e. designing a new freight service network. Based on the new service network, the FOC will need to decide a new plan to fulfil customer orders. According to the FOC's itinerary purchasing plan and customer orders fulfilment plan, the IM can calculate its' profit. Based on the profit generated, the IM may adjust its tariff once again aiming at further improving its profit in the next round. However, after some rounds, the IM may find the maximum profit and the corresponding tariff and cannot find other plans that can lead to higher profit. This situation will be an equilibrium status of the Stackelberg game.

In practice, the IM may adopt a trial-and-error method to identify the maximum profit in the above game, and thus the solution quality is not guaranteed. In this chapter, a rigorous mathematical model will be developed to overcome the deficiency.

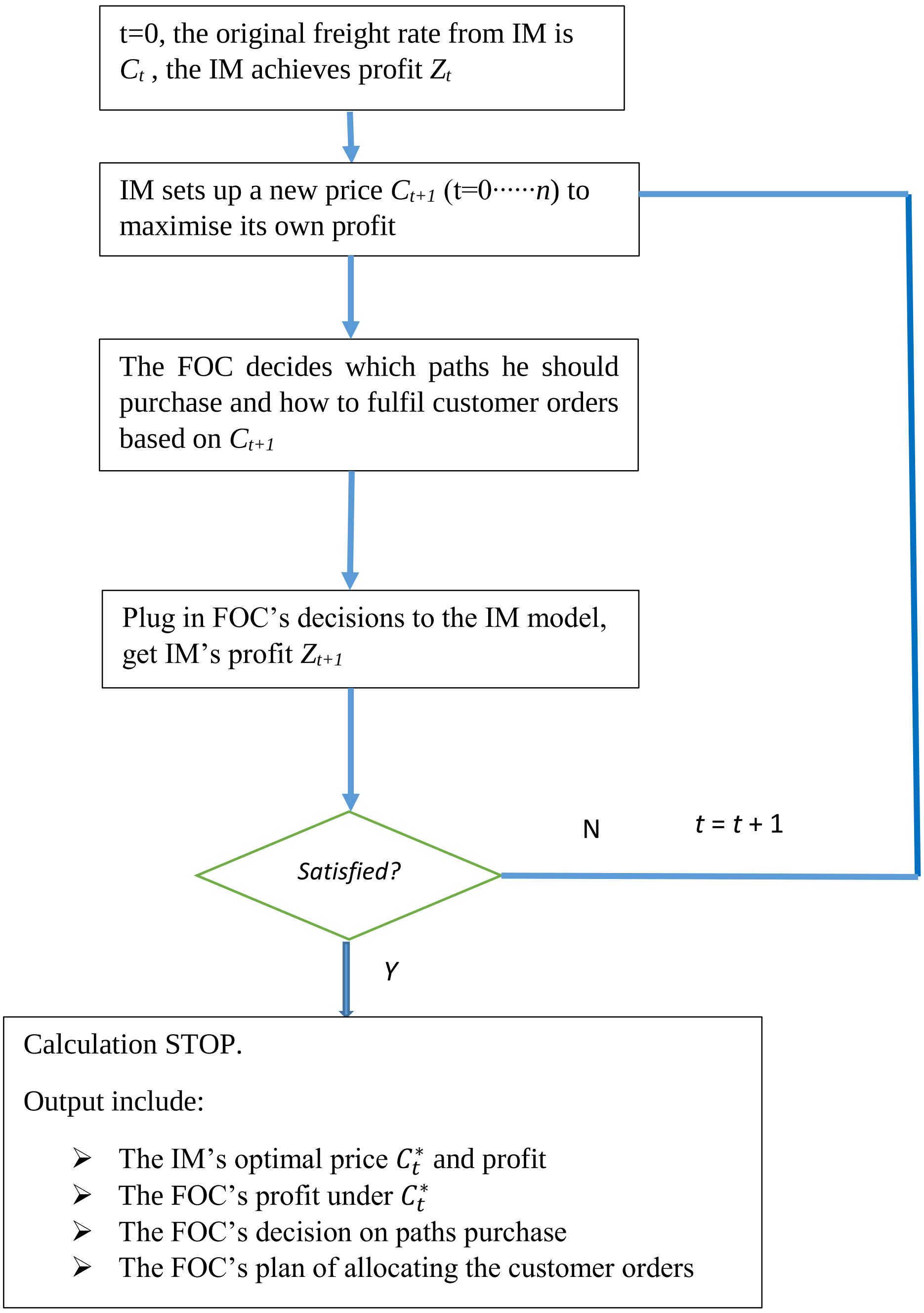

Figure 18 The Optimization Process When the IM Making Decisions Independently

5.3 Assumptions

Before formulating the problem, the assumptions to be adopted are as follows:

Assumption 1: The capacity required by passenger services has been pre-determined.

It should be noted, although the IM normally provides capacity for both freight and passenger transportation service, passenger trains normally have priority in capacity/paths acquisition over freight trains as they are much more profitable. As a result, freight trains can only be allocated to the remaining itineraries after the allocation of passenger trains. Following the industrial practice, it is assumed in the study that all passenger trains' capacity requirements have already been satisfied prior to making the decisions for a freight service.

Assumption 2: The pricing information for end customers is known.

Rail freight transport is not the only choice available to the shippers, and the shippers may use the other transportation modes such as road if a FOC charges high transportation rates which they cannot bear. Due to the competition with other transportation modes, the price the FOC can charge the shippers is relatively stable. Therefore, in this study, it is assumed that the price the FOC charges the shippers is known information.

An analysis of the statistical data from the government (see Figure 19) suggests that within the UK, 76% domestic freight was moved by road in 2015 while rail was 9% and water was 15% (Department for Transport, 2016). This clearly indicates that road transportation plays a dominant position in the freight market. The freight rate is mostly determined by road, the freight market leader. In this research, the freight market price is assumed to be stable. In this context, the price the FOC offers to their customers can be assumed to be fixed.

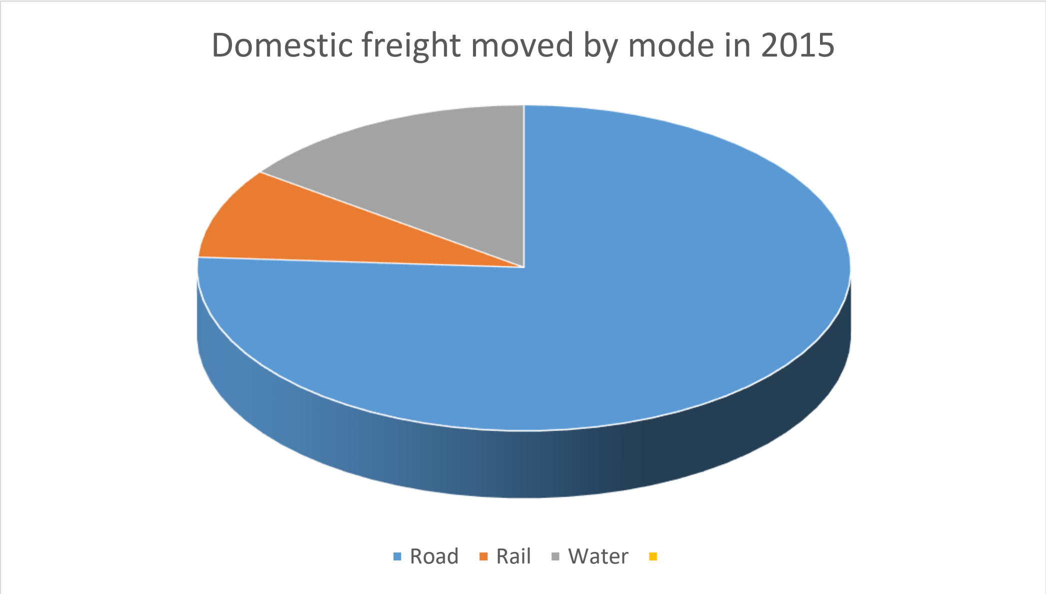

Figure 19 Showing Domestic freight in the UK moved by mode in 2015
Source: (Department for Transport (DfT), 2016a)

Assumption 3: Only direct transportation services are considered

In many western countries where vertically separated operation mode was adopted, e.g., the UK, due to the decline of the rail freight industry before the 1980s, large marshalling or classification yards have been closed and dismantled, e.g., Tinsley Marshalling Yard in Sheffield (Woodburn, 2001). Nowadays there are no hump facilities, and there are only very limited flat-shunting facilities left in the UK. The percentage of wagon transhipments is very low. Therefore, in the study wagon transhipments were not considered.

The pricing process is considered as a dynamic Stackelberg (Leader-Follower) game that involves an IM and a FOC as the decision makers. The game can be divided into two phases. The gaming process details are provided and their formulations are given as follows.
In the following section, the notations used in the study will be defined first. Based on the notations and the text-based problem description in 5.2, mathematical formulation for the problem will be given.

5.4 Notations

Set:

$\mathcal{P}$	a set of routes each linking a pair of origin and destination stations
$\mathcal{I}_n$	a set of itineraries (or trains) on route *n*
$\mathcal{D}$	a set of periods over the planning horizon
$\mathcal{J}$	a set of jobs (customer orders)
Ω	a set of sample processes of customer demands
$\mathcal{L}$	a set of sections on the rail network
$\mathcal{K}$	a set of train stations

Index:

j	a transportation task from customers
n	a route between a pair of origin and destination train stations
i	an itinerary (a train); an itinerary is equivalent to a train;
d	a day (or a period)
ξ	a sample process of customer demands in the planning horizon
k	a station on the network
l	a section between two consecutive stations

Parameters:

T_{ndj}	the volume in wagons required for serving transport task j on day d on route n

r_{ndj} the revenue a FOC can obtain for transporting one wagon for fulfilling task j on day d on route n

C_{ni} the maximum number of wagons that train i can carry on route n

$O(\cdot)$ an indicator function which indicates the origin of a job or a train

$D(\cdot)$ an indicator function which indicates the destination of a job or a train

a_{ni} fixed operational cost of train i on route n

V_n IM's variable cost per tain mile per wagon on route n

S_n travel distance between the pair of origin and destination station in route n

Q^l The maximum number of trains that can be accommodated at section l per day

Q^k the handling capacity in wagons at station k

V_n' FOC's variable operational cost on route n per train mile per wagon

δ_{ni}^l binary input data. 1 indicates that section l is used by train i on route n; otherwise, 0

γ_{ni}^k binary input data. 1 indicates that station k is visited by train i on route n; otherwise, 0

N the number of samples in Ω

Decision variables:

f_{ni} binary variable. 1 if train i on route n is purchased; otherwise, 0;

$x_{ndij}(\xi)$ binary variable dependent of a sample process of demand ξ. 1 if a job j is served by train i on day d on route n; otherwise, 0

p_{ni} the price charged by the IM for running train i on route n

5.5 Mathematical Models

From discussions on current structure of rail freight rates in section 4.2, together with the information collected from the UK freight industry practice, the mathematical models for the stakeholders of the rail freight system will be developed in this section, taking into account the application of fixed costs to freight transport.

In this research, the IM and the FOC are all deemed to be typical commercial organisations whose operational target is to maximise its profit.

5.5.1 *Phase 1: Leader (the IM)'s decision*

In phase 1, as the Leader, the IM first designs a tariff in which the price for using each itinerary at each time period is specified. As the IM is a monopolist who owns almost all the railway infrastructure, theoretically the IM can charge any price it wishes. However, there are some practical constraints. If the IM charges low prices, its profit may be low or they may even incur a loss from selling the railway itineraries; if the prices are too high, the number of itineraries they can sell to the FOC may be low, and the revenue they can gain may be low as well.

The IM's objective function is to maximise its expected profit, which can be formulated as follows,

$$\text{Max } Z(p,f,x) = \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} p_{ni} \cdot f_{ni} - \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} a_{ni} \cdot f_{ni} - \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x_{ndij}(\xi) \cdot V \cdot S_n \tag{1}$$

The first term on the right-hand-side (RHS) of Eq (1) represents the revenue generated by selling itineraries to the FOC. The second term represents the total fixed cost of the sold itineraries. The third term is the IM's total expected variable costs, which reflect the track wear arising from transporting shippers' cargoes. This cost is proportional to the amount of customer demands served, the average variable cost per wagon per train-mile *V*, and the mileage in the network between the O-D pair in route *n*, S_n.

The price tariff is the IM's decision. However, the maximisation of the IM's profit depends on f_{ni} , i.e, the FOC's itinerary purchasing plan, and the order fulfilment plan $x_{ndij}(\xi)$, which are decisions made by the FOC at the second phase.

5.5.2 *Phase 2: Follower (the FOC)'s decision*

The FOC needs to design a service network as well as a customer order fulfilment plan to maximise its profit. More specifically, given the IM's tariff, the operating cost for each itinerary, and the unit revenue generated for fulling demands, the FOC needs to make decisions on which itineraries it should purchase; how to fulfil customer demands, i.e., which customer orders should be accepted or rejected; and which train should serve which order.

The Phase 2 gaming process is considered as a two-stage stochastic programming model. The first stage is to purchase itineraries from the IM; and the second stage is to fulfil stochastic customer demands using purchased itineraries. The stochastic factor considered in the model is customer demand. Sample Average Approximation method (Kleywegt, Shapiro and Homem-de-Mello, 2002) is applied to handle the uncertain factor. The objective function of Phase 2 decision making is to maximise the expected profits of the FOC as formulated in Eq. (2)

$$\text{Max } Y(f,\, x) = -\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p_{ni}\cdot f_{ni} + \frac{1}{N}F(x,\xi) \tag{2}$$

The first term on the right-hand-side (RHS) of Eq. (2) represents the acquisition cost of itineraries from the IM, which is also the fees the FOC needs to pay the IM for track access. The first term corresponds to the costs incurred in the first stage decision making. The second term represents the expected revenue from freight service operation, which is associated with the second stage decision making.

The details of the FOC's activities at the aforementioned two stages are described as follows.

Stage 1: Itinerary Purchasing

In the first stage, the FOC needs to decide which itinerary they should purchase. Binary variables are used to denote the decision. A binary variable takes 1 when the FOC chooses to buy an itinerary; otherwise 0. When providing train itineraries to the FOC, the IM is constrained by the capacity at a section and the handling capacity at a station. Hence, we have

$$\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} f_{ni}\cdot\delta_{ni}^{l} \le Q^{l} \quad \forall\, l\in\mathcal{L} \tag{3}$$

$$\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} f_{ni}\cdot\gamma_{ni}^{k}\cdot C_{ni} \le Q^{k} \quad \forall\, k\in\mathcal{K} \tag{4}$$

Stage 2: Operations of Freight Service

At the second stage, the FOC makes decisions on the operational activities according to the purchasing plan or the designed service network decided in the first stage. The FOC's objective is to maximise its profits that is equal to the income minus the costs.

The FOC's income is generated from providing freight service to the shippers. It is directly determined by the number of orders (wagons) / customer demand served on the railway itineraries (paths) that the FOC purchased from the IM.

After analysing the existing charging structure and the potential changes to the cost calculation methodology, the FOC's cost components are considered:

1) The fee that the FOC pays for access to the railway track, which is also the IM's charge;

2) The variable operational costs, which is related to the number of wagons used and the distance that the wagons travel on the network;

At present, two pieces of software are used in UK freight practice in measuring the variable costs. One is called Vampire which is used to simulate the track damage caused by providing the freight service. The software can build the train's dynamic model and study the response and reaction between the vehicles and the tracks. RFCpro is another piece of software used to calculate the Suspension Factors which are used in determining the variable track usage charge of freight trains (Rhodes and Ling, 2012) (DfT, 2017).

For the other related costs, information from interviews with people from Network Rail's Freight Department suggest that the freight transportation costs structure is extremely complex consisting of thousands of factors. Collection of detailed data of every part of the costs is not realistic. This makes it impossible to obtain the exact costs. Under such an industrial fact, it is reasonable to set up the freight rate to be a lump sum as per line for one commodity.

As to the operators' model, it is assumed that the freight customers' demand in the rail network is uncertain which is treated as a stochastic factor in the model. To cope with this stochastic factor, Sample Average Approximation (SAA) approach is applied.

Following the above considerations, the profit obtained from running the freight service can be formulated in the following equation.

$$F(x,\xi) = \sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi)\cdot r_{ndj}\cdot T_{ndj} - \sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi)\cdot V'\cdot S_n\cdot T_{ndj} \tag{5}$$

The first item in Eq. (5) is the revenue generated from order fulfilment; and the second item is the FOC's operational costs including fuel or energy consumption cost, crew cost and all the other costs related to travel distance and loading statues (the number of laden wagons).

By plugging Eq. (5) into Eq. (2), the complete objective function of the FOC can be obtained.

$$\text{Max } Y(f,x) = -\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p_{ni}\cdot f_{ni} + \frac{1}{N}\left[\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi)\cdot r_{ndj}\cdot T_{ndj} - \sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi)\cdot V'\cdot S_n\cdot T_{ndj}\right] \tag{6}$$

In the second stage, the FOC needs to consider how to satisfy customer demands subject to the purchased service network. The FOC's constraints are formulated as follows.

Constraint 1:

$$x_{ndij} \leq f_{ni} \quad \forall n, d, i, k \tag{7}$$

The transportation task *j* on day *d* on route *n* can only be allocated to train *i* when train *i* has been purchased.

Constraint 2:

$$\sum_{n \in P} \sum_{i \in I_n} x_{ndij} \leq 1 \; \forall n, d, j \tag{8}$$

No more than 1 train will be needed to serve job *j*.

Constraint 3:

$$\sum_{j \in \mathcal{J}} x_{ndij} \cdot T_{ndj} \leq C_{ni} \; \forall n, d, i \tag{9}$$

This is the capacity constraint. The total amount of wagons required by all tasks allocated to train *i* cannot exceed the capacity of train *i* on route *n*.

Constraint 4:

$$x_{ndij} = 0 \quad \forall \{i, j \mid O(j) \neq O(i), D(j) \neq D(i)\}, \forall n, d \tag{10}$$

Order *j* cannot be allocated to train *i* when *j* cannot be covered by train *i* geographically.

Constraint 5:

$$x_{ndij} \in \{0,1\}, f_{ni} \in \{0,1\} \quad \forall \; n, i$$

5.6 Equilibrium solution of the IM – FOC game

The dynamic gaming process between the IM and the FOC as discussed above can be formulated as a bilevel optimisation model. However, solving the bilevel optimisation model is not straightforward due to the integer variables in the lower level. In this section, the explicit form of the bilevel optimisation model is given and then a special solution method is developed for it.

5.6.1 Bi-level optimisation model for the IM – FOC game

Based on the Eq. (1) – (10), the complete bi-level model below can be obtained:

Upper Level:

$$\text{Max } Z(p, f, x) = \sum_{n \in \mathcal{P}} \sum_{i \in \mathcal{I}_n} p_{ni} \cdot f_{ni} - \sum_{n \in \mathcal{P}} \sum_{i \in \mathcal{I}_n} a_{ni} \cdot f_{ni} - \frac{1}{N} \sum_{\xi \in \Omega} \sum_{n \in \mathcal{P}} \sum_{i \in \mathcal{I}_n} \sum_{d \in \mathcal{D}} \sum_{j \in \mathcal{J}} T_{ndj} \cdot x_{ndij}(\xi) \cdot V \cdot S_n \tag{11}$$

Subject to*:* $p_{ni} \geq 0 \;\; \forall \; n, i$

For a given p_{ni}, solves

Lower Level:

Max $Y(f, x) = -\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p_{ni} \cdot f_{ni} + \frac{1}{N}(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi) \cdot r_{ndj} \cdot T_{ndj} - \sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi) \cdot V' \cdot S_n \cdot T_{ndj})$

Subject to:

$$x_{ndij} \le f_{ni} \qquad \forall n, d, i, j$$

$$\sum_{n\in P}\sum_{i\in I_n} x_{ndij} \le 1 \qquad \forall n, d, j$$

$$\sum_{j\in\mathcal{J}} x_{ndij} \cdot T_{ndj} \le C_{ndi} \qquad \forall n, d, i$$

$$x_{ij} = 0 \qquad \forall \{i, j \mid O(j) \neq O(i), D(j) \neq D(i)\}$$

$$x_{ndij} \in \{0,1\} \qquad \forall n, d, i, j$$

$$f_{ni} \in \{0,1\} \qquad \forall n, i$$

The upper level of this bilevel model denotes the decision making of the IM. The objective is to maximise its total profit. As to any given price, the IM will have a different profit. The decision variable of the IM is the price which can lead to a maximal profit.

The lower level of the bi-level model presents the FOC's profit optimisation process. For any price given by the IM, based on its own costs and demand from the customer, the FOC will determine how to maximise its profit by making decisions on path purchasing and the customer orders allocating to the purchased trains.

All the variables in the lower level of the above bilevel model are binary integer variables, therefore the lower level model has no dual form and cannot be solved easily. An Approximate Gradient Search based method is developed to solve the model.

5.6.2 Approximate gradient search based algorithm

As the binary integer variables are contained within the FOC's model and there is complex interaction between the upper and lower level models, the gradient of the IM's profit function can only be approximately estimated. Let $\boldsymbol{C^k} = \left(c_1^k, c_2^k, \cdots, c_i^k, \cdots c_{|\mathcal{P}|}^k\right)$ denote the price vector, $\boldsymbol{Z^k}$ the IM's profit; $\frac{dZ_i^k}{dc_i^k}$ the change of the IM's profit with regard to a small change of c_i^k, at the k^{th} iteration. The gradient of the IM's function at $\boldsymbol{C^k}$, $grad(\boldsymbol{C_k})$, at the k^{th} iteration can be defined as,

$$grad(\boldsymbol{C^k}) = \frac{d\boldsymbol{Z^k}}{d\boldsymbol{C^k}} = \left(\frac{dZ_1^k}{dc_1^k}, \frac{dZ_2^k}{dc_2^k}, \cdots, \frac{dZ_n^k}{dc_n^k}, \cdots \frac{dZ_{|\mathcal{P}|}^k}{dc_{|\mathcal{P}|}^k}\right) \tag{12}$$

To compute $grad(\boldsymbol{C^k})$, it is essential to compute $\frac{dZ_n^k}{dc_n^k}$, which is an element in the $grad(\boldsymbol{C^k})$. Let $Z_n^{k'}$ denote the IM's profit when the price vector is set as $\boldsymbol{C_k^{i'}} = (c_1, c_2, \cdots, c_i + \Delta, \cdots c_n)$, where Δ is a non-negative small number, we can have

$$\frac{dZ_n^k}{dc_n^k} = \frac{Z_n^{k'} - Z_n^k}{\Delta} \tag{13}$$

Z_n^k and $Z_n^{k'}$ can be obtained by solving the lower level model in Eq. (11) for the given price vectors $\boldsymbol{C_k^i}$ and $\boldsymbol{C_k^{i'}}$, respectively. The detailed steps for calculating Z_n^k or $Z_n^{k'}$ are described below.

Algorithm 5.1: Computing the IM's profits for a given price vector

Step 1: substitute the given price vector into the lower level model in Eq. (11) - the FOC's model;

Step 2: solve the FOC's model using standard integer programming method, and obtain the optimal decisions the FOC will make, $x_{ndij}^*(\xi)$ and f_{ni}^*;

Step 3: substitute $x_{ndij}^*(\xi)$, f_{ni}^* , and the price vector into the objective function of the upper level model, and the profit of IM, $Z(p, f, x)$, can be obtained.

At some point, the IM's profit under a new price vector may be no better than the previous optimal profit. Before stopping the algorithm, a local search procedure will be performed. A positive amount will be added on the price vector to generate a new one, i.e., $C_{k+1} = C_k + S$, where *S* is the length of search step. If there is no improvement after searching a certain number of times, the algorithm will stop. The local search method is helpful to avoid the calculation stopping at a local optimal price (profit).

The algorithm that combines Gradient Search and Local Search is described as follows.

Algorithm 5.2: A Gradient Search based Algorithm for Solving Integer Bi-level Optimisation Model

Step1: Initialisation.

Set a counter M to record the times of local search attempts, $M = 0$;

a counter *k* to record the number of steps the algorithm runs, *k* = 0

the initial price vector 0, i.e., $\boldsymbol{C^0} = \boldsymbol{0}$;

the Optimal_Profit_So_Far = $-Z$ (*Z* is a very big nonnegative number),

the Optimal_Price = $\boldsymbol{C^0}$;

step length = L, (L is a pre-defined nonnegative number)

the maximum number of local search attempts = A (A is pre-defined number)

Step 2: if M < A continue; otherwise, stop.

Step 3: Computing Z^k with $\boldsymbol{C^k}$ using Algorithm 5.1.

Step 3.1 if Z^k > Optimal_Profit_So_Far; continue; otherwise, go to **Step 3.2**

Optimal_Price $= \boldsymbol{C^k}$; Optimal_Profit_So_Far = Z^k; $k = k + 1$

Calculate gradient of $\boldsymbol{C^k}$ using Eq. (13) and Algorithm 1,

$\boldsymbol{C^{k+1}} = \boldsymbol{C^k} + L * grad(\boldsymbol{C^k})$

Step 3.2: Local Search

$\boldsymbol{C^{k+1}} = \boldsymbol{C^k} + S;$ (S is a predefined step); $M = M + 1$;

Step 4: Go to Step 2.

In the first step of the algorithm, some data is initialised: counter M which records the times of local search attempts, counter k recording the number of steps of calculation are set to be 0, initial optimal price vector $\boldsymbol{C^0}$ is set to be **0**. Current optimal profit of the IM is assumed to be a very big negative number -Z. Step length for the local search in the algorithm is defined as a nonnegative number and the allowed local search attempts is given as integer A.

Step 2: Check the current local search attempts M, if it is less than the predefined allowed attempts A, go to Step 3. Otherwise, calculation stop.

Step 3: Calculate the IM's profit Z^k when price vector is $\boldsymbol{C^k}$.

If Z^k is larger than the current optimal profit of IM, substitute the current optimal profit with Z^k, substitute the current optimal price with $\boldsymbol{C^k}$. Increase the value of counter k to k+1.

Calculate gradient of $\boldsymbol{C^k}$ using Eq. (13) and Algorithm 5.1. Generate new price vector with formulation $\boldsymbol{C^{k+1}} = \boldsymbol{C^k} + L * grad(\boldsymbol{C^k})$. Go to step 2.

If Z^k is less than the current optimal profit of IM, implement local search, generate new price vector using: $\boldsymbol{C^{k+1}} = \boldsymbol{C^k} + S;$ (S is a predefined step); $M = M + 1$; and go to step 2.

The output of the algorithm is the optimal price which can lead a maximal profit of the IM.

5.7 Numerical Examples

In this section two case studies are provided. The first case is hypothetical, which has one service route with three trains (itineraries). The second case considers a network with four freight rail stations, multiple service routes and multiple itineraries in the UK. The purpose is to illustrate the effectiveness of the models and the algorithms.

5.7.1 Solutions to the IM-FOC game at Stackelberg Equilibrium in case 1

Firstly, feed the first data set provided in Chapter 4 into the developed bilevel programming programme in Eq (11) to calculate the non-cooperative price which enables the IM to maximise its price unilaterally. The calculation results are shown in Table 15.

	Initial Price	Optimal price	the IM's profit	FOC's profit	System profit
Optimal System	(55,55,55)	***(245,245,245)***	***162***	***0.33***	***162.33***

Table 15 The IM's and The FOC's Profits at Equilibrium (Thousand Pounds)

The initial price is set to be (55,55,55), the programme stops when it shows the optimal price is (245,245,245), the IM's profit is 162 and the FOC's profit is 0.33. The calculation result indicates that the IM's profit is relatively high whereas the FOC's profit is close to zero. Further increasing the optimal price, the FOC will choose to not to operate any line, and the IM's profit will go down. Therefore, the current price (245,245,245) is the best price the IM can charge the FOC in the sense that the IM's profit is maximised unilaterally. It also indicates that, under the price (245,245,245), the game will be at Stackelberg equilibrium.

5.7.2 Solutions to the IM-FOC game at Stackelberg Equilibrium in case 2

By plugging the second data set in Chapter 4 into the developed models and applying the designed algorithms to solve the models, the solutions for equilibrium are obtained as shown in Table 16. It can be observed the best profits that the IM can achieve without collaboration is £6264, the FOC's profit is £3008.33, which corresponds to a system profit of £9272.33.

Path	Price (£)	System profit(£)	IM's profit(£)	FOC's profit(£)
MOSSEND to Southampton	3766, 3206, 3206	9272.33	6264	3008.33
MOSSEND to Felixstowe	2425, 2425, 2425			
Southampton to Felixstowe	1899, 1899,1899			
Felixstowe to MOSSEND	2466,2466, 2466			
Southampton to MOSSEND	3532,3788,6420			
Felixstowe to Southampton	1902,1902,1902			
MOSSEND to Southampton	3766, 3206, 3206			
MOSSEND to Felixstowe	2425, 2425, 2425			

Table 16 Stackelberg Equilibrium Scenario

5.8 Summary of the Chapter

This chapter investigates the scenario when the IM sets up the freight tariff aiming to maximise its own profit without any consideration of the FOC's profit. The prices the IM should charge the FOC to achieve the optimal profit were determined. The profit of the FOC and the entire freight system under this optimal price set were also provided by the designed algorithm. It is clearly identified that the IM's profit was maximised under the optimal price, but it is not clear whether the system was maximised or not. This question will be answered in the next chapter.

Chapter 6 Global Optimisation of the IM-FOC Game

This chapter firstly focused on a scenario where the IM and the FOC have perfect cooperation under centralised decision making, which is different from the non-collaboration scenario discussed in Chapter 5. The solution to the IM-FOC game under perfect collaboration will lead to the global optimal system profit. Afterwards, the scenario in practice under decentralised decision making was investigated. An Inverse Mixed Integer Linear Programming (InvMILP) model and a Fenchel Cutting Plane solution algorithm were proposed to identify the optimal prices that enable the scenario under which decentralised decision-making had the same performance as the centralised decision-making mode.

6.1 Introduction

In Chapter 5, a stylised freight railway system under vertical separation governance structure was considered. The scenario where the IM made decisions on the track access prices unilaterally without cooperation with the FOC was discussed. The interaction between the FOC and the IM was analysed through a proposed bilevel linear programming model. The outputs of the algorithm included: the highest profits the IM can achieve unilaterally and the optimal non-operative price, the FOC's profit under the price. The research question answered in Chapter 5 was "What are the optimal prices of the IM-FOC game at the Stackelberg equilibrium?" The solution at Stackelberg equilibrium specified the prices that the IM can charge to maximise its profits unilaterally without any cooperation with the FOC.

In this Chapter, the research question to be answered is: -

What are the optimal prices that can maximise the overall profits of the whole rail freight system (supply chain) and lead to global optimality under vertical separation?

The global optimal solution of the game will be compared with the solution at Stackelberg equilibrium obtained in the current industrial practice as discussed in Chapter 5. The comparison of the results from the two different scenarios will demonstrate the importance of the collaboration between the stakeholders and inspire the stakeholders to follow a better alternative, cooperative and innovative pricing strategy. The new strategy will help to build up a new relationship between different stakeholders.

In fact, in the UK, many practitioners from the rail freight industry have realised the necessity of changing the relationship among the stakeholders. People working in the rail industry noticed that the relationship between Network Rail and the ORR as well as Network Rail and the FOC still have room for improvement. For example, Network Rail's Chairman Sir Peter Hendy thought the relationship between Network Rail and the ORR needed to be "reset". HS1 chief executive Dame Colette Bowe believes that a close examination of the ORR's role and responsibilities is needed in reviewing the planning of multi-billion pound enhancement schemes. Importantly, interviews with people from Network Rail indicate that insufficient cooperation between Network Rail and the FOCs has led to a large amount of unnecessary costs and generated negative impacts on the performance of the rail freight system.

In this chapter, whether cooperation between stakeholders generates more profits than non-cooperation will be investigated. If cooperation outperforms non-cooperation, then what are the optimal cooperative prices that the IM should charge the FOC? To achieve the target, an ideal full collaboration case where the IM and the FOC are treated as a single company will be investigated. The ideal collaboration case is actually a centralised decision-making system with a virtual central planner. This case can identify the highest level of profits that the IM-FOC game can obtain. Then, an inverse programming method is applied to identify the optimal prices under a decentralised decision-making system that is being adopted in industrial practice. The prices determined by the inverse programming model can lead to the same performance as the ideal cooperation scenario. Due to the integer variables involved in the inverse programming model, a Fenchel cutting plane method (Boyd, 1994) is used to solve the inverse integer programming model. It is worth mentioning that the uncertainty in customer demands in the gaming model is also considered. Stochastic programming is applied to handle the uncertain demand.

The contributions of the chapter to knowledge are:

1) A new pricing strategy that can achieve the global optimality is identified.
2) A Fenchel cutting plane based solution method is developed to solve an inverse integer linear programming model.

In the remainder of the Chapter, the following definitions will be used frequently.

Definition 1: *System profit* is defined as the sum of the IM's and the FOC's profits

For the IM-FOC game, the system profits with regard to a given track access tariff needs to be calculated in two phases. In the first phase, the given track access tariff needs to be fed into the

FOC's model in Chapter 5. By solving the FOC's model using standard MILP solver such as CPLEX, the FOC's profit as well as its optimal itinerary purchasing plan and order fulfil plan can be obtained. By plugging the optimal purchasing plan, order fulfil plan and the given track access tariff into the IM's model, the IM's profit can be obtained. By totalling the IM's profit and the FOC's profit, the corresponding system profit can be obtained. The calculation process is visualised in Figure 20 *Chart showing the method of calculating the system profit*.

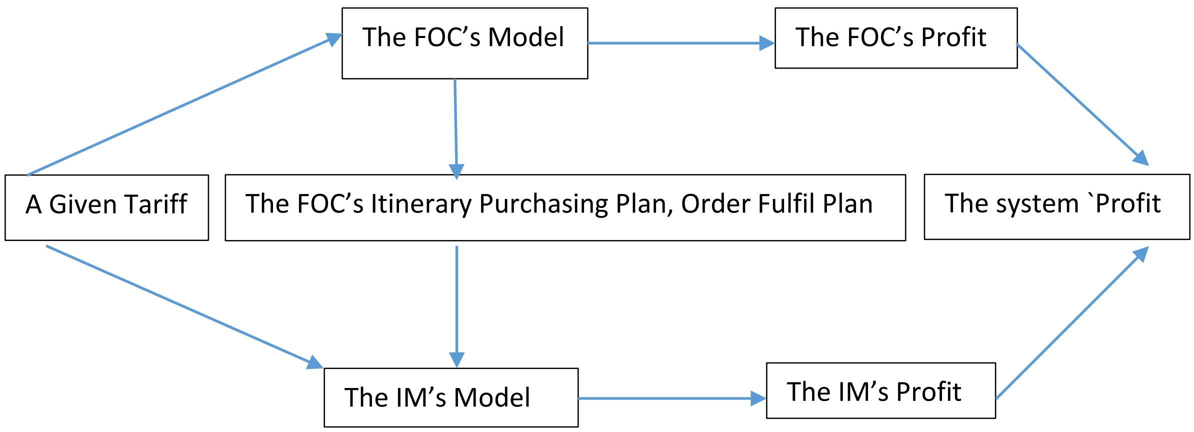

Figure 20 Chart showing the method of calculating the system profit

In this section, a full cooperation scenario where the IM and the FOC are treated as a virtual single company or there exists a virtual central planner for the whole supply chain will be investigated. In this case, a single MILP model will be developed for the two stakeholders, and the objective function of the MILP model is the system profit. Therefore, by solving the MILP model, the optimal system profit can be obtained for the full cooperation scenario.

Definition 2: *Global optimality* is defined as the situation where the system profit is maximised. In Chapter 5, the system profits at Stackelberg equilibrium without cooperation has been obtained. However, it remains unclear whether the profits are globally optimal. In this chapter, efforts will be made to identify the optimal system profit under cooperation and non-cooperation.

Definition 3: *Centralised decision-making* is a decision-making mode in which a virtual central planner makes decision for all the relevant stakeholders aiming at the maximisation of system profits.

In game theory, centralised decision making also leads to full cooperation between different stakeholders (Drew Fudenberg and Tirole, 1991).

Definition 4: *Decentralised decision-making* is a decision-making mode whereby each stakeholder makes the decision independently to maximise their own profits.

Decentralised decision-making is very common in the real world applications. A decentralised decision-making system may or may not have cooperation between stakeholders. Normally many applications involving game theory aim to design cooperation mechanisms for decentralised decision-making systems. The designed cooperation mechanisms should have the same performance as the corresponding centralised decision-making systems (Drew Fudenberg and Tirole, 1991).

In what follows, the scenario with perfect collaboration under centralised decision-making mode will be discussed first; then Inverse MILP will be employed to determine the optimal prices for decentralised decision-making mode and the determined prices will have the same performance as the centralised decision-making mode, i.e. achieve global optimality.

6.2 Perfect Collaboration Under Centralised Decision Making

In Chapter 5, an algorithm was designed to obtain the optimal non-cooperative price as well as the maximum profits the IM can obtain unilaterally. However, the price that is optimal for the IM may not be optimal for the entire freight service supply chain. In this section, the way to achieve global optimality is explored, i.e., maximise the system profit of the whole supply chain. This section considers an ideal situation where the IM and the FOC is deemed as a single virtual organisation, i.e., having perfect collaboration.

Proposition*:* A freight system will achieve global optimality when the IM and the FOC have perfect cooperation under centralised decision-making mode.

Proof:

Under the decentralised decision-making situation where the IM and the FOC are two independent commercial organisations. The IM tends to make profits for providing itineraries to the FOC, hence the prices that the IM charge the FOC include both costs and profits. The prices will be higher than that under global optimality scenario where the two players belong to one single organisation. The reason for this is that when the IM and the FOC are two different departments in the same company, the IM does not aim to make profits by providing the freight capacity to the FOC. Hence, the IM can accept a zero-profit situation, i.e., charge the FOC the price that only recovers the IM's cost. In doing so, the FOC's costs for purchasing the itineraries is minimised.

From the perspective of the FOC, the price they can offer to the shipper / freight customers is largely dependent on the cost for providing the freight service. The cost comprises the fee they pay for the IM, the target profit and the operational costs associated with customer demand fulfilment. As the operational costs result from serving customer orders and the profit ~~is~~ substantially fixed, the shippers' price is largely determined by the fee the FOC pays to the IM. Under the perfect collaboration scenario, the fee the FOC pays to the IM is lowest, which will lead to the minimum prices they can offer to end customers. The minimum shippers' price can further attract maximum customer demand which can subsequently lead to the maximum profit of the total system. Therefore, it can be concluded that under perfect collaboration (global optimality) scenario, the freight system can achieve the maximum profit.

For the non-ideal situation, e.g. the current practice when the IM and the FOC are operating as two separate organisations who make decisions independently, the system profit under perfect collaboration scenario is the maximum profit the system can obtain. Setting up the appropriate prices to achieve this maximum sum profit and sharing the profits between the IM and the FOC will lead to better profits for the two stakeholders.

The perfect collaboration will lead to the following standard Integer Programming model.

$$\text{Max} \quad Z = -\sum_n \sum_i a_{ni} \cdot f_{ni} + \frac{1}{N}\Big(\sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{J}} x_{ndij}(\xi) \cdot r_{ndj} \cdot T_{ndj} - \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x_{ndij}(\xi) \cdot V \cdot S_n - \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{d\in\mathcal{D}} \sum_{i\in\mathcal{I}_n} \sum_{j\in\mathcal{J}} x_{ndij}(\xi) \cdot V' \cdot S_n \cdot T_{ndj}\Big)$$

$$\begin{aligned}
& x_{ndij} \le f_{ni} && \forall n, d, i, j \\
& \sum_{n\in P} \sum_{i\in I_n} x_{ndij} \le 1 && \forall n, d, j \\
& \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} f_{ni} \cdot \delta_{ni}^{l} \le Q^{l} && \forall\, l \in \mathcal{L} \\
& \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} f_{ni} \cdot \gamma_{ni}^{k} \cdot C_{ni} \le Q^{k} && \forall\, k \in \mathcal{K} \\
& \sum_{j\in\mathcal{J}} x_{ndij} \cdot T_{ndj} \le C_{ni} && \forall n, d, i \\
& x_{ij} = 0 && \forall\, \{i, j \mid O(j) \ne O(i), D(j) \ne D(i)\} \\
& x_{ndij} \in \{0,1\} && \forall n, d, i, j \\
& f_{ni} \in \{0,1\} && \forall\, n, i
\end{aligned} \tag{14}$$

The above model for perfect cooperation is a combination of the IM's and the FOC's model discussed in Chapter 5. The objective function is the system profit of the IM-FOC game. It should be pointed out that, in the objective function, there are no charges between the IM and the FOC as the two companies are treated as the same virtual company. Therefore, the item $\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p_{ni}\cdot f_{ni}$, which appeared in the IM's and the FOC's model has been removed. The constrains in the perfect cooperation model are exactly the same as those in the FOC's model.

The optimal value of the objective function of the above perfect cooperation model will be the optimal system profit. However, the system optimal profit is obtained under a centralised decision-making mode, which does not exist in practice in the UK rail freight industry. In what follows, the optimal price under the decentralised decision-making model will be identified, which can also lead to system optimality.

6.3 The Optimal Prices Under Decentralised Decision Making

The perfect collaboration case described above does not exist in a vertically separated railway management system. This section considers how a vertically separated railway system should set prices which can make the system profits the same as in the ideal case.

In practice, the IM needs to charge the FOC to recover its costs as well as generate profit. In what follows, the optimal price the IM should charge the FOC to achieve system optimisation under decentralised decision making will be identified. The technique to be adopted is Inverse Linear Programming (ILP) (Toint 2013; Tayyebi & Aman 2016; You, Chow & Ritchie, 2016). The idea of ILP is to make a known feasible solution be the optimal one by changing the coefficients of objective function appropriately. The classical solution method (Ahuja & Orlin, 2001) that uses the dual form of the lower model cannot be applied into our problem. This is because the proposed problem has binary variables in the lower level model and has no duals. In what follows, the formulation of inverse programming model is introduced and then Fenchel cutting plane algorithm (Boyd, 1994) is developed to solve the game.

By solving the perfect collaboration model in Eq. (14) discussed in section 6.2, the FOC's purchasing plan $f^*_{ni}(\forall n,i)$ and the optimal order fulfilment plan $x^*_{ndij}(\forall\, n,d,i,j)$ were obtained. The problem of inverse programming is, for a given set of $f^*_{ni}(\forall n,i)$ and

$x^*_{ndij}(\forall\, n,d,i,j)$ obtained in Eq (14), how to determine the prices $p_{ni}(\forall n,i)$ which can ensure that the FOC will make the same decision under the governance structure of vertical separation.

To simplify the narrative, let P_k denote the FOC's model at the k^{th} iteration, Inv_P$_k$ denotes a specific inverse form of the model at the k^{th} iteration. There might be many different sets of $p_{ni}(\forall n,i)$ which can make the given set of $f^*_{ni}(\forall n,i)$ and $x^*_{ndij}(\forall\, n,d,i,j)$ obtained in the model described in Eq. (14) the optimal solution for the FOC's model. The L1 norm is used to limit the choices. By complying with the L1 norm, only a single set $p_{ni}(\forall n,i)$ will be selected. Let $\boldsymbol{c}=\{c_{ni}|\forall n,i\}$ denote an known initial price vector; $\boldsymbol{d}=\{d_{ni}|\forall n,i\}$ the price vector that can make a feasible set of price, $\boldsymbol{x^0}$, to be optimal as discussed above; $\boldsymbol{x^{(k)}}$ another feasible solution of prices at the k^{th} iteration. The problem Inv_P$_k$ can be formulated as,

(Inv_P$_k$)

$$\mathrm{Z}(\boldsymbol{d}) = min|\mathbf{c}-\mathbf{d}| \qquad (15)$$
$$d^T x^0 \le d^T x^{(k)}$$

In Eq. (15), the L1 norm can be linearized by introducing a positive auxiliary vector $\boldsymbol{\theta}=\{\theta_{ni}\ge 0|\forall n,i\}$. By substituting x^0 with $f^*_{ni}(\forall n,i)$ and $x^*_{ndij}(\forall\, n,d,i,j)$ obtained in the model described in Eq. (1), Eq.(2) can be re-formulated as,

(Inv_P$_k$)

$$\mathrm{Z}\big(\boldsymbol{d}^{(\boldsymbol{k})}\big) = \min \textstyle\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\theta^{(k)}_{ni}$$

Subject to

$$c_{ni}-\mathrm{d}^{(k)}_{\mathrm{ni}} \le \theta^{(k)}_{ni} \qquad \forall n,i$$
$$\mathrm{d}^{(k)}_{\mathrm{ni}}-\mathrm{c}_{ni} \le \theta^{(k)}_{ni} \qquad \forall n,i$$
$$\textstyle\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\mathrm{d}^{(k)}_{\mathrm{ni}}\cdot f^*_{ni}-\frac{1}{N}\big(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}}x^*_{ndij}(\xi)\cdot r_{ndj}\cdot T_{ndj}-\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}}x^*_{ndij}(\xi)\cdot V'\cdot S_n\cdot T_{ndj}\big) \le \qquad (16)$$
$$\textstyle\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\mathrm{d}^{(k)}_{\mathrm{ni}}\cdot f^{(k)}_{ni}-\frac{1}{N}\big(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}}x^{(k)}_{ndij}\cdot r_{ndj}\cdot T_{ndj}-\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}}x^{(k)}_{ndij}(\xi)\cdot V'\cdot S_n\cdot T_{ndj}\big)$$

Note in the above formulation, the variables associated with the superscript (k) will be updated at each iteration.

Based on the formulation of inverse linear programming, an algorithm is developed to obtain the optimal price which can lead to the global optimality.

Algorithm 6.1: A cutting plane algorithm for inverse MILP

Step 1: Initialisation. Set a counter of steps, k, and let $k \leftarrow 0$; and $\boldsymbol{d}^{(k)} \leftarrow \boldsymbol{c}$,

Step 2: Substitute the price vector $\boldsymbol{d}^{(k)}$ into $P_{K,}$ solve the FOC's model P_K, and obtain its optimal solution $f_{ni}^{(k)}(\forall n, i)$ and $x_{ndij}^{(k)}(\xi)(\forall\, n, d, i, j, \xi)$.

Add the following cut to inv_PK, $\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\mathrm{d}_{\mathrm{ni}}^{(k)}\cdot f_{ni}^{*} - \frac{1}{N}(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{*}(\xi)\cdot r_{ndj}\cdot T_{ndj} - \sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{*}(\xi)\cdot V'\cdot S_n\cdot T_{ndj}) \le \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\mathrm{d}_{\mathrm{ni}}^{(k)}\cdot f_{ni}^{(k)} - \frac{1}{N}(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{(k)}\cdot r_{ndj}\cdot T_{ndj} - \sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{(k)}(\xi)\cdot V'\cdot S_n\cdot T_{ndj})$

Solve inv_PK, and obtain the optimal solution $\boldsymbol{d}^{*}$, $\boldsymbol{d}^{k} \leftarrow \boldsymbol{d}^{*}$;

$k \leftarrow k+1$;

Step 3: if $\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\mathrm{d}_{\mathrm{ni}}^{(k)}\cdot f_{ni}^{*} - \frac{1}{N}(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{*}(\xi)\cdot r_{ndj}\cdot T_{ndj} - \sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{*}(\xi)\cdot V'\cdot S_n\cdot T_{ndj}) \le \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\mathrm{d}_{\mathrm{ni}}^{(k)}\cdot f_{ni}^{(k)} - \frac{1}{N}(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{(k)}\cdot r_{ndj}\cdot T_{ndj} - \sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}^{(k)}(\xi)\cdot V'\cdot S_n\cdot T_{ndj})$, go to Step 2; otherwise, Stop.

In the above algorithm, $\boldsymbol{c}$ theoretically can be set as any value. However, in our experiment a zero vector for $\boldsymbol{c}$ was set. This means we seek a price vector that is close to 0 rather than any other possible vectors. This complies with the idea of providing the minimal price to the end customer to ensure the competitiveness of rail freight in the freight market.

6.4 Numerical Example

In this section, numerical experiments were carried out for: 1) the perfect cooperation model under centralised decision-making; 2) the Inverse Linear Programming model for decentralised decision-making. The two datasets provided in Chapter 4 are used to conduct these experiments. The perfect cooperation model was solved using the Branch and Bound algorithm built in IBM CPLEX. The Fenchel cutting plane method was solved using C++ and CPLEX.

The calculation results are shown in Table 17, Table 18 for case 1 and case 2 respectively.

Case 1:

Scenario	Optimal Price per train	the IM's profit	The FOC's Profit	System profit	Purchasing plan
Perfect collaboration	N/A	N/A	N/A	221.3	1,1,0
Global optimality	58,58,130	7	214.3	221.3	1,1,0

Table 17 Stakeholders' Profit Level Under Different Price
*Note that the prices and profits in the table are measured in thousand pounds.

In Table 17, it can be observed, when the price is set as £58 thousand, £58 thousand, £130 thousand for each train respectively, the corresponding system profit is the same as that under perfect collaboration. However, when global optimality is achieved, the profit that the IM can obtain is only £7 thousand, which is much lower than £162 thousand obtained in the equilibrium status of the game where the IM makes decisions unilaterally to maximise its profits. Interestingly, it can also be found that the system profit at equilibrium, £162.33 thousand, is lower than £221.3 thousand which is the maximum profit the entire supply chain can obtain under global optimisation scenario. The difference between the two system profits is £221.3 thousand - £162.3 thousand = £58.97 thousand. This indicates that, if global optimality is achieved, there is the chance that both the IM and the FOC can be better off and a win-win situation can be achieved.

In this experiment the convergence of Fenchel cutting plane based algorithm is also observed. It has been found, that there is only a need to generate two cuts as shown in Table 18 to reach the optimal point.

	Optimal Price per train	The IM's profit	The FOC's Profit	System profit	Purchasing plan
Cut 1	120,130,130	37	152.3	189.3	1,0,0
Cut 2	120,58,130	-25	214.3	189.3	0,1,0

Table 18 Convergence of Fenchel Cut Based Solution Algorithm for Inverse Programming Game

By applying Algorithm 3 (discussed in section 6.3), Cut 1 is generated first. Compared to the scenario under perfect collaboration, it is easy to find that system profit is less than the targeted perfect collaboration case. This indicates that the algorithm has not converged yet. Cut 2 then needs to be generated. However, it also cannot make the system profit equal to that under perfect collaboration. After adding Cut 2 into the model described by Eq. (4) in Chapter 5, the algorithm converged, and the optimal solution in the second row in Table 17 is obtained.

Case 2:

By plugging data into the developed models and applying our algorithms to solve the models, the solutions for global optimality are obtained as shown in Table 19. It can be observed that if the IM chooses to implement global optimality, the best profit that it can achieve drops from £6264 to £1368, which corresponds to a system profit increase from £9272.33 to £10878. This indicates that if the IM collaborates with the FOC, they can generate more profits, which will provide a chance for both of them to be better off than Stackelberg equilibrium.

Path	Price (£)	System profit(£)	IM's profit (£)	FOCs' profit(£)
MOSSEND to Southampton	1859.67,1859.67,2150	10878	1368	9510
MOSSEND to Felixstowe	2050, 2200, 2200			
Southampton to Felixstowe	1400, 1600, 1600			
Felixstowe to MOSSEND	1505.7,1505.7,2550			
Southampton to MOSSEND	1629,1629,2200			
Felixstowe to Southampton	1400,1600,1600			
MOSSEND to Southampton	1859.67,1859.67,2150			

MOSSEND to Felixstowe	2050, 2200, 2200

Table 19 Global optimisation scenario

6.5 Managerial Insights and Policy Recommendation

Vertical separation has been adopted in western countries such as the UK, Sweden and Germany as a way to eliminate the monopolist position of the railway infrastructure owner. The vertical separation has been practiced for a couple of decades, and it has been found useful in not disadvantage the FOC. However, the infrastructure manager may take advantage of its leader position in designing the track access price tariff. As indicated by the solutions at equilibrium discussed above, the infrastructure manager can maximise its own profits, and the FOC can only obtain very minor profits which sometimes may be close to 0. According to our interviews with the railway industry in the UK, the government has a review procedure regarding the IM's income and cost before providing national grant. They try to impose a cap for the profits the IM can obtain. However, as a counter measure, the IM often chooses to exaggerate their costs and thus to disguise their real profits. As the IM's costs may be built in many business activities, it is a very challenging task for the government to verify whether the IM's claimed costs are genuine or not.

This study has demonstrated that there exists global optimality for the game. Achieving global optimality will be beneficial for both the IM and the FOC as the profits the whole transport service supply chain can obtain under global optimality is higher than in case~~s~~ where the IM makes decision~~s~~ independently and selfishly. However, to achieve global optimality, the IM needs to shift its goal of decision making from unilaterally maximising its profits to maximising the profits of the entire supply chain.

In summary, the managerial insights we can obtain from the cases are: under the governance structure of vertical separation, the IM and the FOC need to expand the profit pie, and then share the pie appropriately, which will create a win-win situation for both parties.

6.6 Summary of the Chapter

This chapter investigated the scenario where the IM and the FOC have perfect cooperation under centralised decision-making and proved the solution to the IM-FOC game under perfect collaboration will lead to the global optimal system profit. To also achieve global optimality in practice under decentralised decision-making, an Inverse Mixed Integer Linear Programming

(InvMILP) model and a Fenchel Cutting Plane algorithm were proposed. The proposed method can identify the optimal prices that enable the scenario under which decentralised decision making also has the same system performance as the centralised decision-making mode. Two numerical examples validated the developed algorithm~~,~~ and revealed that the IM needs to shift its goal of decision making from unilaterally maximising its profits to maximising the profits of the entire supply chain.

Chapter 7 Subsidy Contract Design to Coordinate the Freight System

Under the Stackelberg equilibrium of the IM-FOC game investigated in Chapter 5, the IM makes decisions on the freight tariff only considering its' own benefits without caring about the other stakeholder's benefits. By taking the advantage of its leadership position in the game, the IM can obtain its maximum non-cooperative profits unilaterally. In Chapter 6, an ideal scenario where the IM and the FOC have full cooperation was explored. With reference to the ideal scenario, the track access price tariff which can lead the freight system under decentralised decision making mode to achieve global optimality was determined. The research methods adopted were the Inverse Mixed Integer Linear Programming model and the Fenchel cutting plane method. The comparison of the results obtained in Chapter 5 and Chapter 6 indicates that the cooperation between the IM and the FOC can lead to a better result for both players in the game compared to the non-cooperation case.

In this chapter, the alternative mechanisms, e.g., supply chain contracts, which may lead to system optimality of the game will be explored. In particular, the discussion will focus on how to utilise the government subsidy to achieve system optimality.

7.1 Introduction to Mechanism Design

The IM, the FOC and the end customers form a three-echelon supply chain. To serve the end customers' transport demands, the IM sells freight transport capacity on the railway network as a supplier; the FOC provides the transport service to the end customers using purchased train itineraries as a retailer. The products on the three-echelon supply chain are the freight service network.

As the independent commercial bodies in the freight transport market, the operational goals of the IM and the FOC are not to maximise the profits of the entire supply chain but their own individual profits (Pfeiffer, 2016). The IM tries to charge a higher price to the FOC while the FOC always hopes to pay less for accessing the freight capacity. Therefore, they have conflicting interests. When no cooperation mechanisms are applied, which is also what happens in practice, both the players make their decisions independently. In this situation, as the leader of the business, the IM tries to set up the price for any itinerary as high as possible to maximise its' profits, which will have the effect of increasing the FOC's total cost for providing freight services to the shippers. This will reduce the number of profitable orders that the FOC can serve and consequently the total revenue of the whole system. The non-cooperative decision making process will lead to a suboptimal solution for the supply chain system (Lawrence v. Snyder, 2011a). To avoid this suboptimal situation and achieve a global optimal solution, a contract mechanism should be applied to coordinate the supplier (the IM) and the retailer (the FOC) in the rail freight system.

In this Chapter, a number of mechanism (contracts) that may lead to system optimality will be explored. These mechanisms have intensively investigated supply chain coordination and proved to be an efficient process to achieve global optimality of the supply chain. Due to the network effect of a railway freight service, it remains unclear how these commonly used contracts in supply chain management perform in a network revenue management game. Therefore, the research question in this Chapter was defined as:

What is the best supply chain contract that can lead to system optimality of the IM-FOC game?

The contribution of the Chapter to knowledge is that the performance of traditional supply chain contracts will be evaluated in the context of a network revenue management game. The research in the chapter will ascertain how well traditional supply chain contracts perform in network revenue game and provide alternative ways to achieve global optimality of the IM-FOC game.

In the remainder of this chapter, an overview of the contracts commonly used in the supply chain management will be provided first; and then a subsidy contract will be selected and formulated; a Double-Layer-Gradient-Search Algorithm is proposed to determine the optimal subsidy. Numerical experimentation is provided at the end of the chapter to validate the designed models and algorithms.

7.2 An Overview of Supply Chain Contracts

In the SCM context, the decisions made by each of the involved parties affects all stakeholders' as well as the entire supply chain's profit (Chakraborty *et al.*, 2018). Conflicting interests of the parties always leads to a suboptimal supply chain (SC) as the partners make decisions in isolation. It has been proven that cooperation and coordination between the players can improve the efficiencies of entire supply chain (Chakraborty, Chauhan and Vidyarthi, 2015). Supply chain coordination can be achieved by the application of a *contract mechanism* between the supplier and retailer (Bolton, P., Dewatripont, M., & Campbell, 2005). A supply chain contract is an important element in supply chain management because it can significantly improve the profits of the whole chain (Hou *et al.*, 2017). In order to coordinate a supply chain, choosing a proper form of supply contract is one of the crucial steps. There have been many studies focusing on contract design in the supply chain since the mid-1990s (Zhuo, Shao and Yang, 2018; Cachon and Lariviere, 2002; Katok and Wu, 2009; Wang and Liu, 2014; Wang *et al.*, 2018).

In the literature, the main contract models that were studied intensively include: wholesale price contract, buyback contract, revenue-sharing contract, quantity flexibility contract and subsidy contract. Detailed properties of the existing contracts in supply chain context are available in the textbook "Fundamentals of Supply Chain Theory" (Lawrence v. Snyder, 2011). These contracts also are widely discussed in the academic literature. For example, Cai *et al.*, (2017) summarised supply chain contract when they designed subsidy contracts for Vendor Managed Inventory system on supply chain. Katok and Wu (2009) analysed the performance of the wholesale contract, buyback contract and revenue-sharing contract mechanisms in a laboratory investigation. In the following, a brief introduction to each contract will be provided.

7.2.1 Wholesale Price Contract

The wholesale price contract has a very simple structure. Under such a contract, the retailer pays a wholesale price per unit to the supplier and sells the products to end customers. The profit of the supplier P_S , the profit of the retailer P_r and the profit of the entire supply chain P_a are all the function of the wholesale price *w* and the order quantity Q.

$$P_S = F(w, Q_s), P_r = F(w, Q_r),\ P_a = F(w, Q_a)$$

If there exists a wholesale price *w*, and an identical order quantity for each stake holder, i.e., $Q^* = Q_S = Q_r = Q_a$, which can maximise the profits for the supplier, the retailer and the supply chain system, P_S^*, P_r^* , P_a^*, respectively, then the supply chain is coordinated by the wholesale price. However, in such a situation, it has been proven that the profit of the supplier is non-positive (Lawrence v. Snyder, 2011). As there is not a wholesale price *w* which can make $Q_S = Q_r = Q_a$ and ensure that the supplier and the retailer earn positive profits, the wholesale price contract is known as non-coordination contract (Lawrence v. Snyder, 2011). The other reason for the wholesale price contract being unable to coordinate a supply chain is that there exists double marginalization (Cai *et al.*, 2017).

7.2.2 Buyback Contract

In a buyback contract, the retailer pays a unit price w to the supplier for obtaining a certain amount of credit. At the end of the contract period, the supplier pays a fee of q per unit to the retailer for unsold products. The retailer does not need to return the product to the supplier physically but obtains a credit instead. The buyback contract can avoid the product being sold for a very low price by the retailer. Under a buyback contract, the retailer's risk on overage is reduced and shared by the supplier. This mechanism also makes the retailer avoid losing profit for being out of stock. The supply chain can therefore arrive at a higher profit and may lead to the coordination of a supply chain. The unit purchasing price w is a function of buyback price q, $w(q)$. It is proven that by applying a buyback contract, the players' profit can be improved or at least remain unchanged by setting up the proper contract parameters, q and $w(q)$ (Lawrence v. Snyder, 2011). In practice, the buyback price is affected by the players' positions in the respective market, i.e., who is the leader in the market. The supplier oriented market or the retailer oriented market may have different contract parameters. Generally, the buyback price is determined through negotiation between the two players. In literature, Zhao *et al.*, (2014) explored buy back contract application in a supplier retailer supply chain where the price-dependent demand is stochastic.

7.2.3 Revenue Sharing Contract

Revenue Sharing (RS) Contract/ Mechanism is another type of supply chain contract used as a method to coordinate the decentralised supply chain system. The revenue-sharing contract is an extension of the wholesale price contract. In general, under the revenue-sharing contract, the retailer needs to pay the wholesale price for every unit of product plus an agreed reward to the supplier where the reward here can be a certain percentage of revenue or net profit of the retailer as negotiated by the two partners. Revenue Sharing (RS) is normally applied to the supply chains with uncertain demands (Cachon and Lariviere, 2005; Giannoccaro and Pontrandolfo, 2004). Applying revenue sharing mechanism with optimised contract parameters can improve the profit of each involved stakeholder in the chain and achieve a win-win situation.

RS mechanism can be applied in a broader context, e.g., a company's employees, companies in alliance (Sarah Miller, 2015). This mechanism can even be applied in different departments of the government, for example, the government of the United States has applied this revenue sharing mechanism in their taxation system between 1972 and 1986 (James M. Cannon, 1986). The RS mechanism has also been applied in railway operation in the UK. In the Periodic Review 2013 (PR13), ORR published the Route-level Efficiency Benefit Sharing (REBS) mechanism baselines which was introduced by Network Rail (Network Rail, 2014). Under this mechanism, participating operators can share a certain percentage of surplus from Network Rail if any; and have to contribute to covering a percentage of Network Rail's loss conversely for each route. This method can be understood as FOC sharing the profit/loss of IMs based on an agreed rate basis.

In the academic community, the RS contract has been intensively studied. Avinadav, Chernonog and Perlman (2015) provided an extensive review of the application of the revenue sharing contract using mobile Apps. Chakraborty, Chauhan and Vidyarthi (2015) compared wholesale price and revenue sharing contracts in product selling process. (Saraswati and Hanaoka, 2014) employed multi-airport multi-airline non-cooperative games within a network model to analyse the cooperation between the airport-airline in a revenue sharing context. (Wang and Liu, 2014) integrated the fairness preference theory with the traditional principal–agent model to discuss the revenue sharing ratio in public private partnership (PPP) projects. Xie *et al.* (2018) designed a contract that combined a revenue-sharing contract with a cost-sharing contract for a dual-channel closed-loop supply chain and then applied Stackelberg game theory to investigate this coordination mechanism. Li, Zhu and Huang (2009) considered a consignment contract with a revenue sharing contract for a supply chain with one manufacturer and one retailer. A Nash bargaining game theoretical model was developed to realise profit sharing and achieve the cooperation between the two players.

7.2.4 Quantity Flexibility Contract

A quantity flexibility contract can be viewed as a special format of a buy back contract. The difference between the two contracts is that the quantity flexibility contract requires the supplier to "recycle" the unsold units from the retailer; whereas, in the buyback contract, the supplier will only pay a percentage of the original purchasing price to buy-back the unsold products. Li *et al.* (2016) discussed the application of a quantity flexibility contract between a cosmetic manufacture and retailer. In the study, the cosmetic retailer is allowed to adjust the purchasing quantity of a new type of cosmetic product based on the actual customer demand and inventory level. The impact of applying flexible contracts on the performance of both retailer and supplier was investigated by Cai *et al.*(2015).

7.2.5 Subsidy Contract

A subsidy contract was first proposed by Cai *et al.* (2017) to study the coordination of a vendor-managed inventory (VMI) supply chain with service level sensitive customers. In the study, the retailer provides a subsidy to the supplier. There were three variations of the subsidy contract considered in the study in terms of the subsidy scope and level. Numerical examples showed that the proposed contracts can improve the supply chain collaboration and performance. In some other cases, the subsidy contract has been adopted to promote the implementation of a project or the cooperation between partners. For example, Yi and Li (2018) investigated the cooperation of the supplier and the retailer in a supply chain for energy saving and emission reduction using a Stackelberg game model. A subsidy contract was proposed in the research as the government provides subsidies to the energy saving product and imposes tax on the carbon emitted. The results show that the subsidy can promote the cooperation between the upstream and downstream companies in the supply chain.

7.3 Rationales for the Adoption of the Subsidy Contract

Freight capacity, which is also the product of the freight service supply chain in this research, is perishable, which means that the freight capacity is no longer available after a certain point in time. However, in quantity flexibility contracts and buyback contracts, an essential requirement is that the supplier needs to "recycle" the unused products in some way. This makes the two contracts too impractical to be implemented for the IM-FOC game. As the intention of applying any mechanism is to coordinate the IM and the FOC, the possibility of applying the wholesale contract is also excluded because it is a non-coordinating contract. The revenue sharing contract has been applied in rail freight operation in the UK (Network Rail, 2014) as introduced in section 7.2.3. However, in the existing revenue sharing contract, there is no consideration of the government subsidy which is one of the sources of income for Network Rail. According to (Office of Rail and Road, 2019a), the UK government's net subsidy was £3.8 billion in 2017-18, which accounts for 30.6% of total income of the railway industry. This research will try to introduce a novel mechanism to coordinate the stakeholders in rail freight system using the government subsidy as a lever.

As the rail freight transportation system in this research is composed of the IM and the FOC, all the income generated for the system will be from end customers. Generally, the system profit is determined by how many profitable customers' demands the system can serve. From the prospective of the FOC, a particular shipping order is profitable only when the revenue generated by serving this order is more than the costs incurred in the operation process. If the FOC's costs for serving the order are reduced, the number of profitable orders will be increased. As a consequence, the FOC can then serve more orders and generate higher benefits. This means that the system's profits could be improved if the FOC's cost for serving an order is reduced. It is clear that reducing the FOC's cost for accessing the rail freight network will have such an effect.

The FOC's costs can be reduced if the IM reduces its charges, i.e., setting up a track assessing tariffs lower than the optimal price at Stackelberg equilibrium. In doing so, the FOC may choose to purchase more train itineraries, service more customer orders and consequently increase the system profits. However, the IM's profit under the reduced price tariff will be lower than that at Stackelberg equilibrium. Hence, the IM will have no motivation to choose the lower track assessing tariff. In order to encourage the IM to choose the lower traffic accessing tariff, reduce the freight charges to the FOC and subsequently increase the system profit, the Government could consider providing a subsidy (allowance) to the IM. The subsidy can be paid back later from the increased profit of the freight transport system. This will avoid the Government incurring extra financial burden.

In light of the above discussion, the subsidy mechanism is designed as follows: the government provides a certain amount of subsidy to the IM. The IM follows the bilevel model presented in Chapter 5 to determine the optimal prices to charge the FOC. At the end of the contract period, the IM and the FOC needs to pay the subsidy back to the government.

If the initial amount of subsidy is set appropriately, the above subsidy contract policy will lead to system optimality of the game. In the IM-FOC game, different amounts of government subsidy may lead the IM to design different optimal track access price tariffs, which may result in the FOC's options for optimal purchasing plans and consequently different system profits. The target of the subsidy mechanism is to identify the optimal subsidy that can lead to the maximum system profit.

In the following, the subsidy contract will be mathematically formulated. A solution algorithm to identify the optimal subsidy will be designed. Detailed discussion will be provided in the following sections of this chapter.

7.4 Formulation of Subsidy Contract

In the section, the IM's and the FOC's decision making models under the subsidy contract will be formulated respectively. The notations to be used in the formulation is given as follows.

7.4.1 Notations

w	The subsidy per wagon provided by the government to the IM
Δ	A non-negative small number
T	A counter indicating the times of local search attempts
Z	A very big non-negative number
$grad(w^k)$	The gradient of the subsidy w at iteration k
S^k	The system profit when subsidy is w^k
$S^{k'}$	The system profit when subsidy is $w^{k'}$
A	A predefined integer, indicate the allowed maximum local search times
L	A predefined nonnegative number, which is the step length of inner gradient search
r	r is a predefined number denoting the value of adjustment for each local search iteration
AE	The total subsidy provided by the government

7.4.2 Problem Description and formulation

As shown in Figure 21 , under the subsidy contract, the first step of the gaming process is that the government decides a subsidy rate, *w*, for each customer order to be served; the second step is that the IM makes decisions on the track access tariff based on the subsidy rate *w*; the third step is that the FOC uses its model to decide the itinerary purchasing plan and customer order fulfilment plan. The FOC's decision including the itinerary purchasing plan and the customer order fulfilment plan will be fed into the IM's model as input data, and then the IM can compute its profits. The IM may adjust its tariff in the next round decision making in the hope its profits can be improved. The gaming process is very similar to that identified previously in Figure 18 of Chapter 5, they only differ with respect to the subsidy element.

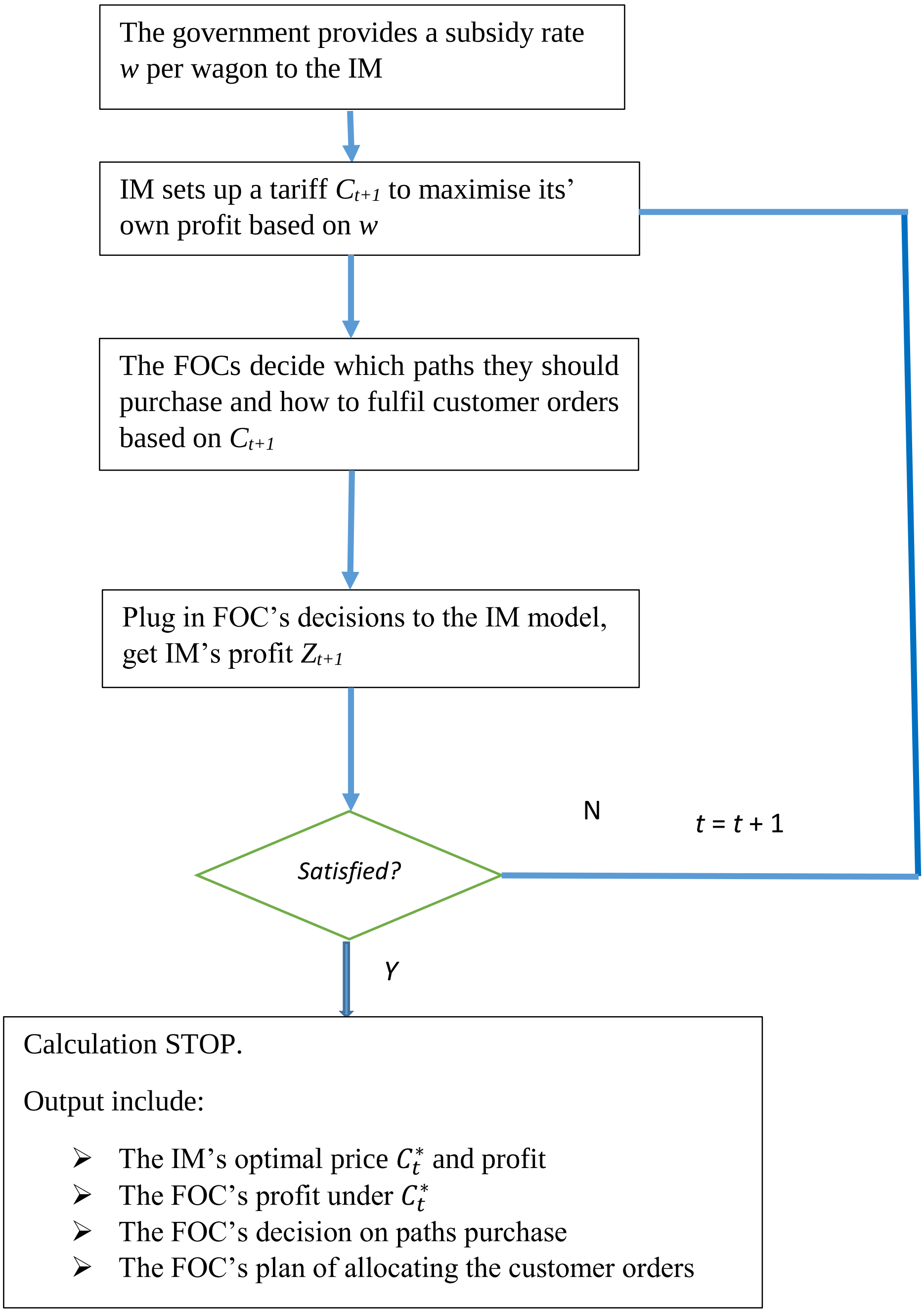

Figure 21 The Optimization Process When The IM Making Decisions Independently

In the aforementioned gaming process, the IM's and the FOC's models need to run repetitively. A bilevel model was developed to model the interaction between the IM and the FOC

<u>Upper Level</u>:

$$\text{Max } Z(p,f,x) = \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p_{ni}\cdot f_{ni} - \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} a_{ni}\cdot f_{ni} - \frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x_{ndij}(\xi)\cdot V\cdot S_n + \frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x_{ndij}(\xi)\cdot w \tag{17}$$

Subject to: $p_{ni} \geq 0 \;\; \forall\, n, i$

For a given p_{ni}, solves

Lower Level:

Max $Y(f, x) = -\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p_{ni}\cdot f_{ni} +$

$\frac{1}{N}(\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi)\cdot r_{ndj}\cdot T_{ndj} -$

$\sum_{n\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{i\in\mathcal{I}_n}\sum_{j\in\mathcal{J}} x_{ndij}(\xi)\cdot V'\cdot S_n\cdot T_{ndj})$

Subject to:

$x_{ndij} \le f_{ni} \quad \forall n, d, i, j$

$\sum_{n\in P}\sum_{i\in I_n} x_{ndij} \le 1 \quad \forall n, d, j$

$\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} f_{ni}\cdot\delta_{ni}^{l} \le Q^{l} \quad \forall\, l\in\mathcal{L}$

$\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} f_{ni}\cdot\gamma_{ni}^{k}\cdot C_{ni} \le Q^{k} \quad \forall\, k\in\mathcal{K}$

$\sum_{j\in\mathcal{J}} x_{ndij}\cdot T_{ndj} \le C_{ni} \quad \forall n, d, i$

$x_{ij} = 0 \quad \forall\, \{i, j \mid O(j) \ne O(i), D(j) \ne D(i)\}$

$x_{ndij} \in \{0,1\} \quad \forall n, d, i, j$

$f_{ni} \in \{0,1\} \quad \forall\, n, i$

The above formulation is similar to the bilevel model in Chapter 5, and the difference only lies in the IM's objective function. The first term on the right-hand-side (RHS) of Eq (17) represents the revenue generated by selling the train itineraries in the railway network to the operator. The second term represents the total fixed operational cost of the sold itineraries. The third term is of the IM's total variable cost. The fourth item is the subsidy provided by the government per number of wagons delivered on the sold train itineraries during the given time period.

Proposition 1: The IM will not set up a price tariff that can lead to the global optimality if the subsidy rate of the contract is smaller than a lower bond *w*,

$$w = \frac{B}{\frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x^{*}_{ndij}(\xi)'}$$

where, $B = \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p^{*}{}_{ni}\cdot f^{*}{}_{ni} - \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} a_{ni}\cdot f^{*}{}_{ni} -$

$\frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x^{*}_{ndij}(\xi)\cdot V\cdot S_n - \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p^{*}_{ni}{}'\cdot f^{*}_{ni}{}' +$

$\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} a_{ni}\cdot f^{*}_{ni}{}' + \frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x^{*}_{ndij}(\xi)'\cdot V\cdot S_n$

Proof*:*

Without applying the subsidy contract, the IM's profit can be calculated by:

$$Z(p^*, f^*, x^*) = \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} {p^*}_{ni} \cdot {f^*}_{ni} - \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} a_{ni} \cdot {f^*}_{ni} - \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi) \cdot V \cdot S_n$$

Where p^*, f^*, x^* are the optimal solution at the Stakelberge Equilibrium.

If the proposed contract is accepted, the IM's profit would be:

$$Z'(p^{*\prime}, f^{*\prime}, x^{*\prime}) = \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} p^*_{ni}{}' \cdot f^*_{ni}{}' - \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} a_{ni} \cdot f^*_{ni}{}' - \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi)' \cdot V \cdot S_n + \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi)' \cdot \mathrm{w}$$

And $p^{*\prime}, f^{*\prime}, x^{*\prime}$ are the optimal solution of the IM-FOC game with applying the subsidy contract.

To stimulate the IM to accept this contract, their profit after applying this mechanism must be guaranteed better than or at least no less than the profit in the scenario without any contract. That is:

$$Z'(p^{*\prime}, f^{*\prime}, x^{*\prime}) \geq Z(p^*, f^*, x^*)$$

$$\sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} p^*_{ni}{}' \cdot f^*_{ni}{}' - \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} a_{ni} \cdot f^*_{ni}{}' - \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi)' \cdot V \cdot S_n + \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi)' \cdot \mathrm{w} \geq \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} {p^*}_{ni} \cdot {f^*}_{ni} - \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} a_{ni} \cdot {f^*}_{ni} - \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi) \cdot V \cdot S_n$$

Then,

$$w \geq \frac{B}{\frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi)'}$$

where, $B = \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} {p^*}_{ni} \cdot {f^*}_{ni} - \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} a_{ni} \cdot {f^*}_{ni} - \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi) \cdot V \cdot S_n - \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} p^*_{ni}{}' \cdot f^*_{ni}{}' + \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} a_{ni} \cdot f^*_{ni}{}' + \frac{1}{N} \sum_{\xi\in\Omega} \sum_{n\in\mathcal{P}} \sum_{i\in\mathcal{I}_n} \sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{J}} T_{ndj} \cdot x^*_{ndij}(\xi)' \cdot V \cdot S_n$

Therefore, in the designed contract, the subsidy rate, *w*, is equal to or bigger than the above inequation RHS value is the primary requirement.

Proposition 2: The system profit cannot exceed an upper bound no matter how the government increases the subsidy rate for the IM.

Proof: For any given railway line, the total number of wagons that can be served by the FOC is constrained by the available section and station capacity and the allowed maximum number of wagons for each train. The maximum profit of the freight system is the total profit earned by serving orders subject to all available capacities. Therefore, this maximum system profit is also bounded. The charging rate which can attract most profitable customer orders is the best price for the system; and the subsidy rate that leads to the IM setting up the price as the aforementioned charging rate is the optimal subsidy. A higher subsidy may lead to lower price from the IM, but if no more orders can be accommodated in the freight network, no extra profit can be generated for the system. Therefore, the optimal subsidy is also bounded.

The key step in the subsidy contract design is to find the optimal subsidy that can maximise the system profit. Under the subsidy contract, the system profit is calculated by adding the profit of the IM and the profit of the FOC but taking away the total subsidy provided by the government.

7.4.3 A Double-Layer-Gradient-Search Solution Algorithm

As previously discussed, the subsidy contract is designed to use the government financial support to encourage the IM to reduce their charges to the FOC. In doing so, the FOC can secure more customer orders and achieve the system optimality of the game. In this section, an algorithm that can identify the optimal subsidy leading to the system optimality of the game was developed.

In the proposed algorithm, the gradient search method will be applied twice. The outer level gradient search method is used to identify the government's optimal subsidy per wagon. The identified optimal subsidy can lead to the freight transport system achieving global optimality. The inner level gradient search is used to calculate the bilevel model for a given subsidy. The inner level gradient search is very similar to the gradient search method developed to solve the bilevel model in Chapter 5. Similar to the discussion in 5.2, to avoid the local optimisation, the local search method is combined with the gradient search method in the double-layer-gradient search algorithm.

Let w^k denote the subsidy per wagon the IM receives from the government; S^k the freight system profit at the Stackelberg equilibrium corresponding to the subsidy per wagon w^k from the government; $\frac{dS^k}{dw^k}$ the change of the system profit with regard to a small change of w^k , at the k^{th} iteration. The gradient of the system profit at w^k, $grad(w_k)$, at the k^{th} iteration can be defined as,

$$grad(w^k) = \frac{dS^k}{dw^k} \tag{18}$$

To compute $grad(w^k)$, let $S^{k'}$ denote the system profit corresponding to another given subsidy per wagon $w^{k'} = w^k + \Delta$ from the government, where Δ is a non-negative small number, we can have

$$\frac{dS^k}{dw^k} = \frac{S^{k'} - S^k}{\Delta} \tag{19}$$

For the given subsidy w^k and $w^{k'}$, the optimal profit S^k and $S^{k'}$ can be obtained by the following algorithm.

Algorithm 7.1 Computing the freight system profits for a given subsidy.

Step 1: for a given *w*, apply gradient search algorithm (as stated in 5.2) to calculate $p^*_{ni}, x^*_{ndij}(\xi)$ and f^*_{ni}, the profit of the FOC, $Y^*(f,x)$

Step 2: substitute $x^*_{ndij}(\xi)$, f^*_{ni} , the subsidy *w* , and the price vector into the objective function of the upper level model, and the profit of IM, $Z^*(p,f,x)$, can be obtained;

Step 3: the total profit of the freight system ***S*** equals to the profit of the FOC, $Y^*(f,x)$ plus the profit of IM, $Z^*(p,f,x)$ minus the total subsidy provided by the government **AE =** $\frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x^*_{ndij}(\xi)\cdot w.$

To identify the optimal subsidy *w**, the following double-layer gradient search based algorithm is proposed.

Algorithm 7.2 Double-layer Gradient Search Algorithm to Find the Optimal Subsidy

Step1: Initialization

Set a counter *T* to record the times of local search attempts, $T = 0$;

a counter *k* to record the number of steps the algorithm runs, *k* = 0

the initial subsidy 0, i.e., $\boldsymbol{w^0} = \mathbf{0}$;

the Optimal_System-Profit_So_Far = $-Z$ (Z is a very big nonnegative number),

the Optimal_Subsidy = $\boldsymbol{w^0}$;

step length = L, (L is a pre-defined nonnegative number)

the maximum number of local search attempts = A (A is pre-defined number)

Step 2: if T < A continue; otherwise, stop.

Step 3: Computing S^k with $\boldsymbol{w^k}$ using Algorithm 7.1.

Step 4.1 if S^k > Optimal_System-Profit_So_Far; continue; otherwise, go to **Step 4.2**

Optimal_Subsidy $= \boldsymbol{w^k}$; Optimal_System-Profit_So_Far = S^k; *k* = *k* + 1;

gradient of $\boldsymbol{w^k}$, $grad(\boldsymbol{w^k})$, using Eq. (19) and Algorithm 7.1;

$\boldsymbol{w^{k+1}} = \boldsymbol{w^k} + L * grad(\boldsymbol{w^k})$; *T* = 0

Step 4.2: Local Search

$\boldsymbol{w^{k+1}} = \boldsymbol{w^k} + r$; (*r* is a predefined step); *T* = *T* + 1;

Step 5: Go to Step 2.

In Algorithm 7.2, Step 1 is to initialise the calculation, the counter *T* is used to record the times of local search attempts, counter *k* the number of steps the algorithm has run so far. Both *T* and *k* are initialized to be 0, the subsidy *w* is set to be 0. The current optimal system profit is set to be a very big negative number to ensure the calculation start. The current optimal subsidy is set to be 0; the step length is set to be *L*, (L is a pre-defined nonnegative number) and the maximum number of local search attempts is set to be A (A is pre-defined number).

Step 2 is to check if the times of local search attempts is more than the pre-defined allowed maximum number. If it does, the algorithm stops; otherwise, go to step 3.

Step 3 is to apply Algorithm 7.1 to calculate system profit S^k when subsidy is set to be w^k.

At Step 4.1, if S^k is more than the current optimal system profit, the current subsidy w^k is set to be the current optimal subsidy and the current system profit is set to be the current optimal system profit; update k with $k+1$; calculate $grad(\boldsymbol{w^k})$. To calculate $grad(\boldsymbol{w^k})$, inner level gradient search algorithm is applied to calculate the IM's optimal profit when subsidy $\boldsymbol{w^k}$ per wagon is applied. Update $\boldsymbol{w^{k+1}}\ with\ \boldsymbol{w^k} + L * grad(\boldsymbol{w^k})$, and reset T = 0.

At Step 4.2, local search will be applied. The subsidy will be updated as $\boldsymbol{w^{k+1}} = \boldsymbol{w^k} + S$ (S is a predefined step). After each local search is completed, the counter will be updated, T = T + 1.

At Step 4, the algorithm will go back to Step 2 to check if the algorithm should be terminated.

The outputs of the double-layer gradient search algorithm include the optimal value of subsidy per wagon and the freight system profit, the IM's profit and the FOC's profit under this subsidy.

7.5 Numerical Example

A software tool using C++ and CPLEX is developed to compute the optimal solution for the problem. Feeding the data sets in Chapter 4 into the developed models and algorithms, the results for case 1 and 2 were obtained and shown in Table 20 and Table 21.

7.5.1 Case 1

In this case, when the government provides a £ 22 subsidy per wagon to the IM,

- The freight system profit can arrive at £221.33 thousand which is the same as its maximal profit under global optimisation scenario and better than equilibrium scenario (£162.33 thousand);
- The IM's profit is £162 thousand which is the same as its maximal profit under equilibrium scenario;
- The FOC's profit is £59.33 thousand. It is significantly higher than the equilibrium scenario when the IM and the FOC are non-cooperate (£0.33 thousand).

	Subsidy (£)	Price (thousand £)	Profit of the system (thousand £)	Profit of the IM (thousand £)	Profit of the FOCs (thousand £)
Subsidy Contract	22	58,58,130	221.33	162	59.33

Table 20 The Freight System Applying Subsidy Mechanism

7.5.2 Case 2

After plugging the data into the developed programme, the calculation results are shown in Table 21. When the IM receives a £4 subsidy per wagon from the government,

- The system profit will be £10612, which is close to the profits under perfect cooperation scenario (£11147.67) and higher than that under equilibrium scenario (£9272.33);
- The IM's maximum profit is £6264 which is the same as its maximum profit under equilibrium scenario;
- The FOC's profit is £4348. It is significantly higher than the equilibrium scenario when the IM and the FOC are non-cooperative (£3008.33).

Path	Subsidy	System Profit	The IM's Profit	The FOC's profit
MOSSEND to Southampton	4	10612	6264	4348
MOSSEND to Felixstowe	4			
Southampton to Felixstowe	4			
Southampton to MOSSEND	4			
Felixstowe to MOSSEND	4			
Felixstowe to Southampton	4			

Table 21 Optimal Profit of Each Stakeholder under the Subsidy Contract

In the experiment, the freight system under the subsidy contract did not obtain the exactly same profit as that in the full cooperation scenario. This is because the proposed gradient search combined local search method is an approximate solution approach due to the integer variables involved. The calculation accuracy may also be constrained by the searching step length. It is possible to reduce the profit differences under the two scenarios by reducing the searching step length, however, this may result in long computational time.

7.6 Summary of the Chapter

A subsidy contract has been proposed as the mechanism to coordinate the relationship between the IM and the FOC. In contrast to the existing subsidy contract in the literature, the subsidy in this proposed mechanism provided by the government will be used as a motivating factor and will be paid back. This ensures that the mechanism will not impose extra financial burden on the government.

The profits of the IM, the FOC and the entire freight system obtained under subsidy contract was compared with that at Stackelberg game in Chapter 5 as well as that in perfect collaboration scenario discussed in Chapter 6. The comparison result indicates that subsidy provided by the government can stimulate the IM to set up a lower rate in comparison to the non-cooperative scenario discussed in Chapter 5. As the lower rate can attract more customer demands and lead to an increased number of lines purchased by the FOC, it will consequentially increase the profits of the IM, the FOC and the freight system. The numerical test proves and demonstrates that the proposed subsidy contract is an efficient mechanism to coordinate the freight system.

Chapter 8 Conclusions

This chapter presents an overview of this research and summarises how the research questions have been addressed and what research findings have been obtained. Contributions to knowledge and recommendations for future work are also given in this chapter.

8.1 Conclusions

This thesis started with an introduction to the unique characteristics of rail freight in terms of cost-efficiency, environmental impact, reliability and effect on road congestion. Afterwards, the role of rail in the multimodal freight transportation system and the state of art in the UK rail freight industry were analysed. Following these discussions, the weaknesses existing in current charging process that motivate this research were identified. Three research questions were formulated: -

1) *What are the optimal prices of the IM-FOC game at the Stackelberg equilibrium? The solution at Stackelberg equilibrium specifies the prices that the IM can charge to maximise its profits unilaterally without any cooperation with the FOC.*
2) *What are the optimal prices that can maximise the overall profits of the whole rail system (supply chain) and lead to system optimality under vertical separation operation structure?*
3) *What is the best contract that should be adopted to coordinate the whole rail freight system?*

Based on the analysis of the research questions, the scope of literature review was identified and conducted in Chapter 2. The intensive literature review revealed a number of research gaps.

1) There is very limited study on the network revenue management game in the rail freight industry. The most relevant research treated the FOC's decision making for itinerary purchasing as continuous variables which is not consistent with practice in reality.
2) To solve the IM-FOC game at the Stackelberg Equilibrium and find the optimal solutions, the existing method in literature cannot be applied directly due to the need for the binary variables. A novel solution algorithm is needed to solve this problem.

3) In literature there is no research foci on the global optimal solution for the Stackelberg game for network revenue management in the rail freight industry.
4) Although mechanism design for supply chain revenue management where demands are independent in the leader-follower game and also for the competitions in network revenue game in the Nash game can be found in literature, the coordination mechanism for Stackelberg game in network revenue management is missing.
5) Importantly, the industry interview indicated that there is a requirement for improving the charging regime in the UK freight transport practice.

Before addressing the aforementioned research questions and filling the identified research gaps, the following research methods to be applied into the study were reviewed in Chapter 3:

- Linear Programming: to be applied to develop the IM's model;
- Stochastic Programming: to be applied in developing the FOC's model where the customer demand was the stochastic factor;
- Game Theory: to be applied to demonstrate the interaction between the IM and the FOC;
- Inverse Optimisation: This optimisation technique was used to determine the optimal price which can make the decentralised freight system achieve the same profit as that under the scenario where the IM and the FOC have full cooperation;
- Gradient Search: This method was used to find the optimal subsidy rate which can lead to the coordination between the IM and the FOC under decentralised decision making scenario.

In Chapter 4, the pricing charging regime of the UK rail freight system has been reviewed. This review provided a basis for the model development. Also, two datasets have been collected, which have been used for testing the various models and solution algorithms.

In Chapter 5, the first research question reflecting the current industrial practice where the IM and the FOC make decisions independently without cooperation was investigated. In this dissertation, this question has been approached rigorously using a Stackelberg game theoretical method. To determine the optimal track access tariff at Stackelberg equilibrium, a bilevel linear programming model and a gradient search based solution algorithm was proposed. The outputs of the developed model and the solution algorithm include important information at Stackelberg equilibrium: the IM's optimal non-cooperative price tariff for track access, the corresponding profits of the IM, the FOC and the freight system under this price, the FOC's train itinerary purchasing plan, and the FOC's plan for fulfilling customer demand.

The second research question aims to reveal whether it is worthwhile to introduce cooperation between the IM and the FOC. To answer this question, an ideal scenario with full cooperation under a centralised decision making system was investigated in Chapter 6 and formulated as a Mixed Integer Linear Programming model. By solving the model using standard commercial software such as IBM CPLEX, the maximum system profit can be obtained. Further, an Inverse Mixed Integer Linear Programming model and a Fenchel Cutting Plane based algorithm were developed to determine the optimal track access prices under a decentralised decision making process, which can also lead to the same system profit as that obtained in the ideal full cooperation case. The experimental results indicate that the optimal track access prices under a decentralised decision making process outperforms the prices at Stackelberg equilibrium. In other words, the numerical experiment results support that cooperation between the IM and the FOC can improve the total profit of the freight system.

The third research question aims to design a mechanism to promote the cooperation between the IM and the FOC, and guide the IM to set up a track access tariff that can lead to the system optimisation. In Chapter 7, after reviewing the existing mechanisms used in supply chain coordination, a subsidy contract was selected. This contract is particularly useful for the scenario where the freight industries in many countries are operating at a loss and are highly reliant on the government subsidy. A double layer gradient search algorithm has been developed to identify the optimal subsidy rate that leads system optimisation. The numerical experiment indicates that the subsidy contract and the double layer gradient search algorithm are useful to achieve global optimality and the profits obtained under the optimised subsidy contract is on a par with that under a full collaboration case.

In this study, managerial insights in relation to the profits for the IM, the FOC, and the system, have been obtained through numerical experiments.

System Profit

If the IM tries to set up the track access tariff to maximise its profit independently without cooperation with the FOC, the system profit of the freight system will be very low. As a contrast, under full collaboration scenario, the freight system profit is maximised and much higher than that in the non-cooperation case. It has been found that, by applying an optimised subsidy contract, the obtained freight system profit is very close to the maximised profit in the full collaboration case, and significantly higher than the profit under the Stackelberg equilibrium scenario.

The IM's profit

When the IM makes decisions independently without cooperation with the FOC, the IM can maximise its profit when the game reaches Stackelberg equilibrium. Under the full cooperation scenario, the IM's profit is lower than the maximum profit at Stachelberg equilibrium. By applying the proposed subsidy contract, the IM still has the same maximal profit as that under the Stackelberg equilibrium scenario since its loss from reducing track accessing tariff can be compensated by the government subsidy.

The FOC's profit

In the Stackelberg Equilibrium scenario, when the IM makes decisions on freight tariff independently without cooperation with the FOC, the FOC will normally have a very low profit. When the IM and the FOC have full cooperation, the FOC's profit is significantly increased and much higher than that under Stackelberg equilibrium scenario. By providing government subsidy to the IM according to the wagons delivered in the freight network, the FOC's profit is improved as well compared with that under Stackelberg Equilibrium scenario.

Based on the discussion and calculation in this thesis it can be concluded that the price set up by the IM is the most important factor that can influence the profit of the IM, the FOCs as well as the freight system. Although being the monopolist of the freight system, the IM should not make decisions independently without cooperation with the FOC as this will lead to low profits for both the stakeholders. By applying the appropriate contract mechanism, the IM's, the FOC's, and the system profit can be improved, and the freight system can be coordinated.

This study also explored some propositions which is summarised in Table 22.

Chapter 6	**Proposition*:*** A freight system will achieve global optimality when the IM and the FOC have perfect cooperation under centralised decision-making mode.
Chapter 7	**Proposition 1:** The IM will not set up a price tariff that can lead to the global optimality if the subsidy rate of the contract is smaller than a lower bond *w*, $w = \frac{B}{\frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x^{*}_{ndij}(\xi)'}$ where, $B = \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p^{*}{}_{ni}\cdot f^{*}{}_{ni} - \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} a_{ni}\cdot f^{*}{}_{ni} -$ $\frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x^{*}_{ndij}(\xi)\cdot V\cdot S_n -$ $\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} p^{*}_{ni}{}'\cdot f^{*}_{ni}{}' + \sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n} a_{ni}\cdot f^{*}_{ni}{}' +$ $\frac{1}{N}\sum_{\xi\in\Omega}\sum_{n\in\mathcal{P}}\sum_{i\in\mathcal{I}_n}\sum_{d\in\mathcal{D}}\sum_{j\in\mathcal{J}} T_{ndj}\cdot x^{*}_{ndij}(\xi)'\cdot V\cdot S_n$
	Proposition 2: The system profit cannot exceed an upper bound no matter how the government increases the subsidy rate for the IM.

Table 22 Summary of propositions explored in this study

8.2 Contributions

The thesis has contributed to the knowledge in the following aspects:

1. A novel stochastic pricing problem for a three-echelon service network supply chain is studied. The 'products' considered in the supply chain are heterogeneous and have network effects. The problem is formulated as a Stackelberg game that includes an IM's model and a FOC's model. The IM's decision is to determine track accessing prices, and the FOC's decision is to design a freight service network and customer fulfilment plan.

In the existing literature, the pricing problem on supply chain normally considers homogeneous products with a single price, and there are no correlations between these homogenous products. As a contrast, in this study, each train itinerary needs to be considered as a unique product since each train itinerary is associated with a unique combination of Origin and Destination stations and departure and arrival times; also train itineraries are correlated as they are all constrained by the capacities associated with stations and rail segments.

The existing literature relating to network revenue game mainly consider competitions between different competitors, which leads to Nash equilibrium. As a contrast, in the study, as an IM and a FOC makes decisions sequentially, Stackelberg game theory needs to be applied to model their interactions.

There are only two previous studies, (Harker and Hong,1994) and (Crevier et al., 2012) where Stackelberg equilibrium was considered in the network revenue game. However, the two studies only considered selling a percentage of capacity on a given rail line, modelled as continuous variables, and hence the follower's decision is not a typical network design issue. Further, the two studies also have not modelled individual customer orders, and thus the decisions as to whether to accept or reject a customer order and which train an individual customer order should be allocated to are not included in their models. The FOC's model developed in this thesis has overcome the aforementioned drawbacks, and thus is a more accurate representation of industrial practice.

2. A bilevel programming model with the binary integer decision variables in the lower level is developed to find the solution of the game at Stackelberg equilibrium. As the existing method cannot solve the problem, a unique specifically designed gradient search based algorithm enhanced by local search is developed to solve the model. This method contributes to the literature to solve the special bilevel programming model that contains binary integer decision variables in the lower level.

As the FOC's decisions include network work design and an order fulfilment plan as discussed above, binary integer decision variables have been widely used in the lower level of the bilevel programming model. There have previously been no solution algorithms proposed to solve bilevel programming model with integer decision variables in the lower level. This study uniquely proposed the gradient search based algorithm enhanced by a local search to fill this gap in the literature.

3. This study is the first use of the applied inverse linear programming and Fenchel cutting plane method to identify the exact global optimal solution of Stackelberg game in network revenue management. A new pricing strategy that can achieve the global optimality of the game is identified. The solution approach is to combine inverse integer

linear programming model and a Fenchel cutting plane method to solve the Stackelberg game, which has not been reported in the previous literature.

The global optimality for the Stackelberg game of network revenue management has not been discussed in the literature. The only two published papers (Harker and Hong, 1994; Crevier et al., 2012) dealing with Stackelberg game of network revenue management only consider the determination of equilibria, and there was no discussion about the global optimality of the game even considering the game formulated in a simplified format without considering many operational details.

4. An efficient subsidy mechanism (contract) that can coordinate the IM-FOC game has been proposed for the first time in this study. An algorithm has also been developed to determine the parameters in the mechanism.

As described above, the previous studies relating to the Stackelberg game of network revenue management only considered the equilibria of the game, and there was no discussion about how to design the mechanism for the game.

In this study, a subsidy contract has been designed for the game. The subsidy contract is in line with rail freight industrial practice in the UK because 30.6% of total income of the railway industry comes from the government subsidy. This research is the first study aiming to help the government to decide the optimal amount of subsidy that can lead to global optimisation of the rail freight system.

5. Managerial insights have been obtained from the provided analysis and case study. It has been found that: (i) an IM's selfish pursuit to profit maximisation without cooperation with the FOC would lead to a poor system performance; (ii) there exists a pricing strategy that can lead to global optimality. Thus, the best strategy for an IM is to charge the prices leading to global optimality first and then share the profits with the FOC afterwards. In short, it is recommended that an IM should expand the profit pie, and then share part of the pie with the FOC. Under this new strategy, both parties can be better off. (iii) a subsidy contract can lead to system optimality if the amount of subsidy can be set appropriately.

For example, in the case 2, under Stackelberg Equilibrium scenario, the IM's profit is £6264, the FOC's profit is £3008.33, the freight system profit is £9272.33; under full cooperation scenario, the IM's profit is £1368, the FOC's profit is £9510, the profit of the freight system is £10878; by applying subsidy mechanism, the IM's profit is £6264, the FOC's profit is £4348, the profit of the freight system is £10612.

Compared with Stackelberg Equilibrium scenario, which represents the current freight practice, under the full cooperation scenario, although the IM's profit decreases from £6264 to £1368, the system profit increases from £9272.33 to £10878, which is an increase of 17.3% and the FOC's profit increases from £3008.33 to £9510 which is an increase of 216%. As the IM's profit under full operation is lower than that under Stackelberg equilibrium, the IM may have no incentive to implement full operation. However, since the FOC's profit is increased, if the FOC agreed to share part of its profit with the IM, this will give the IM motivation to follow the prices leading to global optimisation. By applying subsidy mechanism and setting up the subsidy rate as £4 per delivered wagon to the IM, the IM's profit is £6264, the FOC's profit is £4348 and the system profit is £10612. The IM obtains the same profit as under Stackelberg Equilibrium scenario, and the system profit increases from £9272.33 to £10612, which is an increase of 14.4% and the FOC's profit increases from £3008.33 to £4348 which is an increase of 44.5%. Under the subsidy contract, both the IM and the FOC will be better off than Stackelberg equilibria. Also the subsidy contract in the decentralised freight system can generate the profits similar to that in the centralised freight system, hence it can coordinate the freight system.

To apply the designed mechanism into industry practice, it is necessary that the three stakeholders, the IM, the FOC and the government gain benefits or at least will not lose profit from it. The calculated results of the two numerical experiments indicate that the proposed government subsidy based mechanism can maximise the IM's profit and the entire freight system profit at the same time. This means the IM will not incur any profit loss when applying the mechanism. In the meantime, the FOC's profit level is higher than that under Stackelberg Equilibrium scenario which represents the current operation practice. In other words, the FOC will have a better profit after applying the mechanism, hence a FOC may be willing to adopt such a mechanism in practice. As the mechanism is uniquely designed to pay back the total subsidy to the government, this can avoid the extra financial burden to the government for introducing the mechanism to the industry. Moreover, the social facility in terms of entire freight system profit will be improved for the mechanism application.

Although the dissertation considered the UK railway industry as the context of the numerical examples, the proposed mechanism has the potential to be applied in the vertically separated railway industry in the other countries. The reason for this is that in designing the mechanism, the IM and the FOC were only considered as independent commercial organisations in the game who follow the pure market rules. The government, the subsidy provider, is not required to invest in the industry as the total subsidy will be paid back at the end. Therefore, theoretically, any vertically separated operating railway industry can adopt the proposed government subsidy mechanism.

The study considered the vertically separated railway freight industry as a three-tier service supply chain which is a novel position from which to investigate the relationship between the IM, the FOC and the end customers. The conflicting interests were taken into account when designing the pricing strategy of the itineraries by applying game theory. The developed bilevel models considered the uncertainty of customer demand and involve integer variables in the lower level. By applying the novel solution algorithms provided in this study to solve the proposed models, the key parameters for the stakeholders, e.g. the optimal pricing strategy for the IM, best network design and customer demand fulfil plan for the FOC, the best subsidy rate the government should apply to the IM, can be determined.

8.3 Recommendations for Further Study

In this research, one FOC has been considered. In the UK rail industry practice, there exists more than one freight operating company, for example, DB Cargo, Freight Liner, DRS etc. For a future study, two or more FOCs could be considered in the game. With different FOCs operating on the same freight network, there will be competition between the companies for accessing the freight capacity as well as serving customer demand. In this situation, the interaction between the IM and each FOC will be considered as Stackelberg game and the interaction between each pair of FOCs will be a Nash game. The whole system is the combination of a set of Stackelberg games and Nash game. It will be a very challenging task to solve the problem that involves the two different types of games.

This study assumed that the freight charges applied to shippers are known information, which are not decision variables. In practice, truck transport and rail freight transport always compete with each other. Price may be an important weapon for the two transport modes to win the competition. Evaluating the impact of the competition between the two freight transportation modes in the IM-FOC game can be another further study. To develop models for this scenario when considering the freight price offered to the shippers as a variable which changes with freight market competition, the customer demand of the FOC that is very sensitive to the price needs to be interpreted as a function of the price.

In this study, only the subsidy contract has been evaluated. However, there might be other contract mechanisms that may also be able to coordinate the freight service supply chain. Potentially, these unidentified contracts may have a simpler format or better performance than the subsidy contract proposed in this study. Therefore, designing and analysing more contracts may be a future research direction where the existing contract formats can be the basis of the mechanism.

Further research can also be the empirical analysis of the application of the optimal pricing strategy obtained from the inverse programming model or the subsidy contract in practice. This type of research may be the collection of the attitudes of different stakeholders towards the implementation of these research outcomes; or the evaluation of financial performance of the rail freight system after the implementation of the research findings.

Appendices A: Questionnaire for Data Collection (FOCs Related)

Background

We would like to collect the following information and data as a part of PhD project entitled "Network Revenue Management Game in the Rail Freight Industry". This research is sponsored by EPSRC and Network Rail. It will focus on the design of an optimal pricing mechanism between the IM (Infrastructure Manager) and the FOCs (Freight Operating Companies) in the rail freight industry.

An early consultation with rail freight industry experts and literature on the rail freight service price is set, suggests that there are weaknesses and ambiguity in the current pricing process. At present, the price is set by the Infrastructure Manager then applied, following approval by Office of Rail and Road (ORR). FOCs are just consulted in setting the price. This current approach was the impetus to set up this novel research which aims to provide a method for involving the IM and the FOCs actively in the pricing process, in order to maximize profits for both.

To achieve this, the research adopts an innovative methodology to design a new pricing mechanism for IMs/FOCs which centres on the coordination of the two parties in pricing design and improve the relationship management between them. The mechanism (contract) can design the optimal price which will create a higher volume of freight service and profit level, as well as share profit equitably between the partners.

Two mathematical models will be developed, to be used by the IM and the FOCs respectively, in the decision-making process. Also the interaction between the two models will be studied, using leader/follower gaming theory. Two numerical cases will demonstrate the process and software will be developed to complete the calculation. The study will attempt to prove that the suggested mechanism can lead to improved profitability for all players.

To enable this research, we are seeking your help to provide the following information and data. All information will be kept strictly confidential. Thank you very much for your cooperation.

Information and data needed

1. How do you decide if you should buy the route according to the prices set by the IM? What are the factors you would consider in this process? Do you use any formula to calculate your revenue, cost and profit? If yes, what is it?

2. In a typical 3 days' period, what volume / number of wagons do you receive from the customers (i.e. shippers/ consignees)? Please provide details of the number of customers and the respective volumes/ wagons for each of the 3 days. If necessary, please attach detailed information. Alternatively, can you please provide the daily average customers' demand and its fluctuation scope?

3. What is the average cost per wagon per train mile for laden wagons and empty wagons? If there is no direct information for this, can you please provide information for average renting fee per wagon/ locomotive, average driver salary and working hours/month

4. What is the average fixed operational cost per train on this line?

5. What is the estimated profit that you can earn from meeting the demands mentioned in Q2? (if it is difficult to provide this information, can you please identify the average profit percentage roughly)

6. What is the storage cost per wagon per day in the rail freight yard/hub?

About You

To enable statistical analysis, please answer the following questions about yourself.

Name

Name of your company

Which of the following categories best describes your role?

- CEO
- Senior Management
- Middle Management
- Operational
- Administration
- Other (please specify)

Which of the following best describes your business?

- Freight operator
- Infrastructure manager
- Logistics service provider
- Passenger operator
- rail freight customer
- Consultant
- Other (please specify)

Gender

Length of service in the rail sector

- 0-5 years
- 5-10 years
- 10+ years

In which area of the UK do you work? If outside UK, in which country do you work?

What is the highest degree or level of schooling you have completed?

Are you a member of any professional bodies? Please detail any associated qualifications.

Appendices B: Questionnaire for Data Collection (IM Related)

Background

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An early consultation with rail freight industry experts and literature on the rail freight service price is set, suggests that there are weaknesses and ambiguity in the current pricing process. At present, the price is set by the Infrastructure Manager then applied, following approval by Office of Rail and Road (ORR). FOCs are just consulted in setting the price. This current approach was the impetus to set up this novel research which aims to provide a method for involving the IM and the FOCs actively in the pricing process, in order to maximize profits for both.

To achieve this, the research adopts an innovative methodology to design a new pricing mechanism for IMs/FOCs which centres on the coordination of the two parties in pricing design and improve the relationship management between them. The mechanism (contract) can design the optimal price which will create a higher volume of freight service and profit level, as well as share profit equitably between the partners.

Two mathematical models will be developed, to be used by the IM and the FOCs respectively, in the decision-making process. Also the interaction between the two models will be studied, using leader/follower gaming theory. Two numerical cases will demonstrate the process and software will be developed to complete the calculation. The study will attempt to prove that the suggested mechanism can lead to improved profitability for all players.

To enable this research, we are seeking your help to provide the following information and data. All information will be kept strictly confidential. Thank you very much for your cooperation.

Information and data needed:

1. Can you please briefly introduce the current pricing process? For NR, what is the principle when set up the rate?

2. I can find some charging information related with freight trains from NR website, there are some different categories: 1) Freight Variable Usage Charge default rates or Freight variable usage charge rates (is calculated by VTISM); 2) Freight Electrification Asset Usage Charge rates 3) Charter Slot Charge rates 4) Freight Traction Electricity Modelled Consumption Rates 5) capacity charge rates

   I understand that all above charges are the price should be applied to the FOCs in terms of different cases, to determine the integrated price for FOCs, is it just the sum of different charging categories? (Are they the price structure?)

   Do you take into account the FSOs' benefit? Are the prices being not only cover cost itself? If the price structure for a specific train (e.g. electric traction trains in combined used line) is: 1+2+4+5+6, How to classify the cost among the price? How about the cost structure?

3. Is there any bidding procedure to acquire freight capacity?
   - Yes
   - No

   If yes, can you please briefly illustrate this process? How long before they will bid for the capacity? How long is the fixed term for this price（is that 5 years which is the same as control period）?

4. For the purpose of consulting a case study in detail, is it possible to get such cost and price information for small part of the railway network? If yes, please select a railway segment in the UK freight network for which you can provide the most information. Please specify the names of the origin and destination stations.

5. On the selected line, from Original station to Destination, a) how many freight trains can be allocated (i.e. maximum capacity of rail freight trains on the line), b) what is the number of trains on average currently being operated and c) what is the distance between Original station and Destination? d) What is the maximum number of wagons permitted per train on this line?

6. Can you please give an example for a particular train which carry one single commodity, what is the cost (other than charge rates published on NR website) per wagon per train mile? (Laden and empty, if applicable)

About You

To enable statistical analysis, please answer the following questions about yourself.

Name

Name of your company

Which of the following categories best describes your role?

- o CEO
- o Senior Management
- o Middle Management
- o Operational
- o Administration
- o Other (please specify)

Gender

Length of service in the rail sector

- o 0-5 years
- o 5-10 years
- o 10+ years

In which area of the UK do you work? If outside UK, in which country do you work?

What is the highest degree or level of schooling you have completed?

Are you a member of any professional bodies? Please detail any associated qualifications.

Appendices C: Programme for Stackelberg Equilibrium Scenario

```
#include "ilcplex/ilocplex.h"
ILOSTLBEGIN
#include <vector>
	using std::vector;
static void
	usage (const char *progname),
	populatebyrow     (IloModel model, IloNumVarArray var, IloRangeArray con),
	populatebycolumn  (IloModel model, IloNumVarArray var, IloRangeArray con),
	populatebynonzero (IloModel model, IloNumVarArray var, IloRangeArray con);
int
	main (int argc, char **argv)
{
	IloEnv   env;
	try {
		IloModel model(env);
		IloObjective FSOobj,FSOobj1,FSOobj2;
		IloInt d,i,j,t;
		IloInt Ndays=3;
		IloInt Ntrains=3;
		IloInt Norders=6;
		double a=27;//fixed operational cost
		IloNum p[3]={ 2050, 2200, 2000 };//initial price per train

		IloNum r[3][6]=          //profit per railcar
		{
			{ 101,92,97,95,95,99 },
			{ 103,104,101,104,100,110 },
			{ 103,92,102,101,103,112 }
		};
		IloNum V=0.05; //variable cost per railcar per mile
		IloNum S=490; //distance between teo stations
		double grad[3];
		double stepL=1;
		double optimalprice[3]
		IloBoolVarArray  f(env,Ntrains);
		IloArray<IloArray<IloBoolVarArray> >   xdij(env);
		IloInt T[3][6] =
		{
		   { 8,6,10,6,8,9},
		   {12,10,12,9,12,6},
		   {13,9,5,9,15,10}
		};
		IloInt C[3][3] =
		{
			{ 32,32,32 },
			{ 32,32,32 },
			{ 32,32,32 }
```

```
	};
	double optimalprofit = -1000000;
	double ST = 0;
	while (ST < 5)
	{
		//calculate IM profit at initial price
	//solve FSO model
		IloModel model1(env);
		IloBoolVarArray  k(env, Ntrains);
		IloArray<IloArray<IloBoolVarArray> >   ydij(env);
		IloCplex cplex(env);
		//creat ydij
		for (d = 0; d < Ntrains; d++)
		{
			IloArray<IloBoolVarArray> tempmatrix(env);
			for (i = 0; i < Ntrains; i++)
			{
				IloBoolVarArray tempbool(env, Norders);
				for (j = 0; j < Norders; j++)
				{
					std::ostringstream ossX;
					ossX << "ydij" << "(" << d << ")" << "(" << i <<
")" << "(" << j << ")";

					tempbool[j].setName(ossX.str().c_str());
				}
				tempmatrix.add(tempbool);
			}
			ydij.add(tempmatrix);
		}
		//creat k
		for (i = 0; i < Ntrains; i++)
		{
			std::ostringstream ossX;
			ossX << "k" << "(" << i << ")";
			k[i].setName(ossX.str().c_str());
		}
		//generate objective function
		//revenue
		IloExpr obj(env);
		for (d = 0; d < Ndays; d++)
		{
			for (j = 0; j < Norders; j++)
			{
				IloExpr temExp(env);
				for (i = 0; i < Ntrains; i++)
				{
					temExp = temExp + ydij[d][i][j] * T[d][j];

				}
				obj += temExp*r[d][j];
			}
		}
```

```
obj = obj / Ndays;
// train purchasing costs
IloExpr E(env);
for (i = 0; i < Ntrains; i++)
{
	E += p[i] * k[i];
}
obj = obj - E;
IloConstraintArray cc(env);
//generate constraints:ydij<=k
for (i = 0; i < Ntrains; i++)
{
	for (d = 0; d < Ndays; d++)
	{
		for (j = 0; j < Norders; j++)
		{
			cc.add(ydij[d][i][j] <= k[i]);
		}
	}
}
//generate constraints: 2
for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{
		IloExpr objf(env);
		for (i = 0; i < Ntrains; i++)
		{

			objf = objf + ydij[d][i][j];
		}
		cc.add(objf <= 1);
	}
}
//generate constraints 3
for (d = 0; d < Ndays; d++)
{
	for (i = 0; i < Ntrains; i++)
	{
		IloExpr objf1(env);
		for (j = 0; j < Norders; j++)
		{
			objf1 = objf1 + ydij[d][i][j] * T[d][j];
		}
		cc.add(objf1 <= C[d][i]);
	}
}
//add constraints to model1
model1.add(cc);
//add objilt1 to model1
model1.add(FSOobj = IloMaximize(env, obj));
//objilt.end();
```

```
			cplex.extract(model1);
			cplex.exportModel("ilt1.lp");
			cplex.solve();F
			env.out() << "   Profit of FSO when price is "<<   p[0] <<endl <<p[1]
<<endl <<p[2]<<endl << "     =    " << cplex.getObjValue() << endl;
			cplex.writeSolution("FSO_Initial.sos");
			vector<vector<vector<bool>>> Fvec; //SOLUTION for FSO model-
ydij;
			for (d = 0; d < Ndays; d++)
			{
				vector<vector<bool>> Fivec;

				for (i = 0; i < Ntrains; i++)
				{
					vector<bool> Ftvec;
					for (j = 0; j < Norders; j++)
					{
						Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
					}
					Fivec.push_back(Ftvec);
				}
				Fvec.push_back(Fivec);
			}
			vector<bool> Fvecf; //solution for FSO model-ki;
			for (i = 0; i < Ntrains; i++)
			{
				Fvecf.push_back(cplex.getValue(k[i]));
			}
			for (i = 0; i < Ntrains; i++)
			{
				cout << Fvecf[i] << endl;
			}
			model1.remove(FSOobj);
			obj.clear();
			//IM profit
			double IMformer=0;
			double IMr=0;
			double IMfixedc=0;
			double IMvar = 0;
			//	double IMvarc;
			for (i = 0; i < Ntrains; i++)
			{
				IMr += p[i] * Fvecf[i];
			}
			for (i = 0; i < Ntrains; i++)
			{
				IMfixedc += a*Fvecf[i];
			}
			for (d = 0; d<Ndays; d++)
			{
				for (j = 0; j<Norders; j++)
				{
```

```
				double temp = 0;
				for (i = 0; i<Ntrains; i++)
				{
					temp = temp + Fvec[d][i][j] * T[d][j]
				}
				IMvar = IMvar + temp*V*S;
			}
		}
		IMformer = IMr - IMfixedc-IMvar;
		env.out() << "IM profit is     " << IMformer << endl;
		Fvec.clear();
		Fvecf.clear();
			re:
		if (IMformer > optimalprofit)
		{
			for (i = 0; i < Ntrains; i++)
			{
				optimalprice[i] = p[i];

				cout << "optimal price is :   "<< optimalprice[i]<<endl;
			}

			optimalprofit = IMformer;
			cout << "optimal profit of IM is " << optimalprofit << endl;
			// calculate gradient of initial price
			double s = 0.5;
			//calculate grad[0]
			p[0] = p[0] + s;
			IloExpr obj1(env);
			for (d = 0; d < Ndays; d++)
			{
				for (j = 0; j < Norders; j++)
				{
					IloExpr temExp(env);
					for (i = 0; i < Ntrains; i++)
					{
						temExp = temExp + ydij[d][i][j] *
T[d][j];

					}
					obj1 += temExp*r[d][j];
				}
			}
			obj1 = obj1 / Ndays;
			// train purchasing costs
			IloExpr EE(env);
			for (i = 0; i < Ntrains; i++)
			{
				EE = p[i] * k[i] + EE;
			}
			obj1 = obj1 - EE;
			model1.add(cc);
			model1.add(FSOobj = IloMaximize(env, obj1));
```

```
//objilt.end();
cplex.extract(model1);
cplex.exportModel("ilt2.lp");
cplex.solve();
cplex.writeSolution("FSO_0.sos");
for (d = 0; d < Ndays; d++)
{
	vector<vector<bool>> Fivec;
	for (i = 0; i < Ntrains; i++)
	{
	vector<bool> Ftvec;
		for (j = 0; j < Norders; j++)
		{

Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
		}
		Fivec.push_back(Ftvec);
	}
	Fvec.push_back(Fivec);
}
for (i = 0; i < Ntrains; i++)
{
	Fvecf.push_back(cplex.getValue(k[i]));
}
for (i = 0; i < Ntrains; i++)
{
	cout << Fvecf[i] << endl;
}
double IMr01=0;
for (i = 0; i < Ntrains; i++)
{
	IMr01 += p[i] * Fvecf[i];

}
//cout << "IMr01=" << IMr01<<endl;
double IMfixedc01=0;
for (i = 0; i < Ntrains; i++)
{
	IMfixedc01 += a*Fvecf[i];
}
//	cout << "IMfixedc01=" << IMfixedc01<<endl;
double IMvar01 = 0;
for (d = 0; d<Ndays; d++)
{
	//cout << "j=" << j << endl;
	//cout << "Norders" << Norders<<endl;
	for (j = 0; j<Norders; j++)
	{
		double temp = 0;
		for (i = 0; i<Ntrains; i++)
		{
		//	cout << Fvec[d][i][j] << endl;
```

```
//		cout << T[d][j] << endl;
			temp = temp + Fvec[d][i][j] * T[d][j];
		}
		IMvar01 = IMvar01 + temp*V*S;
	}
}
double IMcurrent01=0;
IMcurrent01 = IMr01 - IMfixedc01-IMvar01;
cout << "current IM profit when p[0]=p[0]+s is" << IMcurrent01<<
endl;
double Prodiff=0;
Prodiff = IMcurrent01 - IMformer;
cout << " profit difference when p[0]=p[0]+s   is:" << Prodiff
<< endl;

grad[0] = Prodiff / s;
Fvec.clear();
Fvecf.clear();
model1.remove(FSOobj);
obj1.clear();
cout << "grad[0]=" << grad[0] << endl;
//calculate grad[1]
double IMr02=0;
double IMfixedc02=0;
double IMvar02 = 0;
p[0] = p[0] - s;
p[1] = p[1] + s;
cout << "p[0]=" << p[0] << endl;
cout << "p[1]=" << p[1] << endl;
cout << "IM former when p[1]=p[1]+s is" << IMformer <<
endl;
IloExpr obj2(env);
for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{
		IloExpr temExp(env);
		for (i = 0; i < Ntrains; i++)
		{
			temExp = temExp + ydij[d][i][j] *
T[d][j];

		}
		obj2 += temExp*r[d][j];
	}
}
obj2 = obj2 / Ndays;
// train purchasing costs
IloExpr EE1(env);

for (i = 0; i < Ntrains; i++)
{
	EE1 = p[i] * k[i] + EE1;
```

```
}
obj2 = obj2 - EE1;
model1.add(cc);
model1.add(FSOobj = IloMaximize(env, obj2));
//objilt.end();
cplex.extract(model1);
cplex.exportModel("ilt2.lp");
cplex.solve();
env.out() << "   Profit of FSO when p[1]=p[1]+s   = " << cplex.getObjValue() <<
endl;
cplex.writeSolution("FSO_1.sos");
for (d = 0; d < Ndays; d++)
{
	vector<vector<bool>> Fivec;

	for (i = 0; i < Ntrains; i++)
	{
		vector<bool> Ftvec;
		for (j = 0; j < Norders; j++)
		{
		Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
		}
		Fivec.push_back(Ftvec);
	}
	Fvec.push_back(Fivec);
}
for (i = 0; i < Ntrains; i++)
{
	Fvecf.push_back(cplex.getValue(k[i]));

}
for (i = 0; i < Ntrains; i++)
{
	Fvecf.push_back(cplex.getValue(k[i]));
	cout << "purchasing plan is " << Fvecf[i] << endl;
}
for (i = 0; i < Ntrains; i++)
{
	IMr02 += p[i] * Fvecf[i];
}
for (i = 0; i < Ntrains; i++)
{
	IMfixedc02 += a*Fvecf[i];
}
	for (d = 0; d<Ndays; d++)
{
	for (j = 0; j<Norders; j++)
	{
		double temp = 0;
		for (i = 0; i<Ntrains; i++)
		{
			temp = temp + Fvec[d][i][j] * T[d][j];
```

```
			}
			IMvar02 = IMvar02 + temp*V*S;
		}
	}
	double IMcurrent02=0;
	IMcurrent02 = IMr02 - IMfixedc02-IMvar02;
	Prodiff = IMcurrent02 - IMformer;
	cout << " profit difference when p[1]=p[1]+s   is:" << Prodiff
<< endl;
	grad[1] = Prodiff / s;
	Fvec.clear();
	Fvecf.clear();
	model1.remove(FSOobj);
	obj1.clear();
	cout << "grad[1]=" << grad[1] << endl;
	//calculte grad[2]
	double IMr03=0;
	double IMfixedc03=0;
	double IMvar03 = 0;
	double IMcurrent03=0;
	p[1] = p[1] - s;
	p[2] = p[2] + s;
	cout << "p[0]=" << p[0] << endl;
	cout << "p[1]=" << p[1] << endl;
	cout << "p[2]=" << p[2] << endl;
	IloExpr obj3(env);
	for (d = 0; d < Ndays; d++)
	{
		for (j = 0; j < Norders; j++)
		{
			IloExpr temExp(env);
			for (i = 0; i < Ntrains; i++)
			{
				temExp = temExp + ydij[d][i][j] *
T[d][j];
			}
			obj3 += temExp*r[d][j];
		}
	}
	obj3 = obj3 / Ndays;
	// train purchasing costs
	IloExpr EE2(env);
	for (i = 0; i < Ntrains; i++)
	{
		EE2 = p[i] * k[i] + EE2;
	}
	obj3 = obj3 - EE2;
	model1.add(cc);
	model1.add(FSOobj = IloMaximize(env, obj3));
	//objilt.end();
	cplex.extract(model1);
	cplex.exportModel("ilt3.lp");
```

```
				cplex.solve();
			env.out() << "   Profit of FSO when p[2]=p[2]+s   = " << cplex.getObjValue()
<< endl;
				cplex.writeSolution("FSO_2.sos");
				for (d = 0; d < Ndays; d++)
				{
					vector<vector<bool>> Fivec;
					for (i = 0; i < Ntrains; i++)
					{
						vector<bool> Ftvec;
						for (j = 0; j < Norders; j++)
						{
						Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
						}
						Fivec.push_back(Ftvec);
					}
					Fvec.push_back(Fivec);
				}
				for (i = 0; i < Ntrains; i++)
				{
					Fvecf.push_back(cplex.getValue(k[i]));
				}
				for (i = 0; i < Ntrains; i++)
				{
					Fvecf.push_back(cplex.getValue(k[i]));
					cout << "purchasing plan is " << Fvecf[i] << endl;
				}
				for (i = 0; i < Ntrains; i++)
				{
					IMr03 += p[i] * Fvecf[i];
				}
				for (i = 0; i < Ntrains; i++)
				{
					IMfixedc03 += a*Fvecf[i];
				}
				for (d = 0; d<Ndays; d++)
				{
					for (j = 0; j<Norders; j++)
					{
						double temp = 0;
						for (i = 0; i<Ntrains; i++)
						{
						temp = temp + Fvec[d][i][j] * T[d][j];
						}
						IMvar03 = IMvar03 + temp*V*S;
					}
				}
				IMcurrent03 = IMr03 - IMfixedc03-IMvar03;
				cout << "IM profit when p[2]=p[2]+s is" << IMcurrent03 <<
endl;
				Prodiff = IMcurrent03 - IMformer;
```

```
cout << " profit difference when p[2]=p[2]+s   is:" << Prodiff
<< endl;
grad[2] = Prodiff / s;
Fvec.clear();
Fvecf.clear();
model1.remove(FSOobj);
obj3.clear();

p[2] = p[2] - s;
cout << "gradient at this point:" << endl;
cout << "grad[0]=" << grad[0] << endl;
cout << "grad[1]=" << grad[1] << endl;
cout << "grad[2]=" << grad[2] << endl;
//new price
for (i = 0; i < 3; i++)
{
	p[i] = p[i] + grad[i] * stepL;
	cout << "p" << i << "=" << p[i] << endl;
}
//IM profit at new price
double IMlformer=0, IMlcurrent=0;
double IMrl=0;
double IMfixedcl=0;
double IMvarc1 = 0;
//solve FSO modle to get purchaing plan
//revenue
IloExpr objL(env);
for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{
		IloExpr temExp(env);
		for (i = 0; i < Ntrains; i++)
		{
		temExp = temExp + ydij[d][i][j] * T[d][j];
		}
		objL += temExp*r[d][j];
	}
}
objL = objL / Ndays;

// train purchasing costs
IloExpr EL(env);
for (i = 0; i < Ntrains; i++)
{
	EL += p[i] * k[i];
}
objL = objL - EL;
IloConstraintArray cL(env);
//generate constraints:ydij<=k
for (i = 0; i < Ntrains; i++)
{
```

```
for (d = 0; d < Ndays; d++)
	{
		for (j = 0; j < Norders; j++)
		{
			cL.add(ydij[d][i][j] <= k[i]);
		}
	}
}
//generate constraints: 2
for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{ IloExpr objf(env);
		for (i = 0; i < Ntrains; i++)
		{
			objf = objf + ydij[d][i][j];
		}
		cL.add(objf <= 1);
	}
}
//generate constraints 3
for (d = 0; d < Ndays; d++)
{
	for (i = 0; i < Ntrains; i++)
	{
		IloExpr objf1(env);
		for (j = 0; j < Norders; j++)
		{
			objf1 = objf1 + ydij[d][i][j] * T[d][j];
		}
		cL.add(objf1 <= C[d][i]);
	}
}
//add constraints to model1
model1.add(cL);
//add objilt1 to model1
model1.add(FSOobj1 = IloMaximize(env, objL));
//objilt.end();
cplex.extract(model1);
cplex.exportModel("ilt.lp");
cplex.solve();
env.out() << "   Profit of FSO when price = " << "(" << p[0] << "," << p[1] << "," << p[2] <<
")" << "  is" << cplex.getObjValue() << endl;
cplex.writeSolution("FSO.sos");
for (d = 0; d < Ndays; d++)
{
	vector<vector<bool>> Fivec;
	for (i = 0; i < Ntrains; i++)
	{
		vector<bool> Ftvec;
		for (j = 0; j < Norders; j++)
		{
```

```
Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
                    }
                    Fivec.push_back(Ftvec);
                }
                Fvec.push_back(Fivec);
            }
            for (i = 0; i < Ntrains; i++)
            {
                Fvecf.push_back(cplex.getValue(k[i]));
                cout << "purchasing plan is " << Fvecf[i] << endl;
            }
            model1.remove(FSOobj1);
            objL.clear();
            //IM profit
            for (i = 0; i < Ntrains; i++)
            {
                IMrl += p[i] * Fvecf[i];
            }
            for (i = 0; i < Ntrains; i++)
            {
                IMfixedcl += a*Fvecf[i];
            }
            for (d = 0; d<Ndays; d++)
            {
                for (j = 0; j<Norders; j++)
                {
                    double temp = 0;
                    for (i = 0; i<Ntrains; i++)
                    {


                        temp = temp + Fvec[d][i][j] * T[d][j];
                    }
                    IMvarc1 = IMvarc1 + temp*V*S;
                }
            }
            IMlcurrent = IMrl - IMfixedcl-IMvarc1;
            env.out() << "current IM profit  is   " << IMlcurrent << endl;
            Fvec.clear();
            Fvecf.clear();

//          cout << "optimalprofit=  " << optimalprofit << endl;
            IMformer = IMlcurrent;
            cout << "IMformer=" << IMformer << endl;
            ST = 0;
            goto re;
        }
        else
        {
            double D = 1;
            for (i = 0; i < 3; i++)
```

```
                        {
                                p[i] = p[i]-grad[i] * stepL + D;
                                cout << "p" << i << "=" << p[i] << endl;
                        }
                        ST++;
                }
                cout << "optimal profit of IM is    " << optimalprofit;
        }

        env.out() <<"press any key to continue..."<<endl;
        getchar();
        throw 20;
    } //end of try
    catch(int e)
    {
        cout << "An exception occurred. Exception Nr. " << e << '\n';
    }


    env.end();

    return 0;
}

// END main
```

Appendices D: Programme for Global Optimisation Scenario

```
#include "ilcplex/ilocplex.h"
ILOSTLBEGIN
#include <vector>

using std::vector;

static void
usage(const char *progname),
populatebyrow(IloModel model, IloNumVarArray var, IloRangeArray con),
populatebycolumn(IloModel model, IloNumVarArray var, IloRangeArray con),
populatebynonzero(IloModel model, IloNumVarArray var, IloRangeArray con);

int
main(int argc, char **argv)
{
        IloEnv   env;
        try {
                IloModel model(env);
                IloObjective FSOobj;
                IloInt d, i, j, t;
                IloInt Ndays = 3;
                IloInt Ntrains = 3;
                  IloInt Norders = 6;
                double a = 27;//fixed operational cost
                IloNum p[3] = { 2050, 2200, 2000 };//price per train

                IloNum r[3][6] =          //profit per railcar
                {
                        { 101,92,97,95,95,99 },
                        { 103,104,101,104,100,110 },
                        { 103,92,102,101,103,112 }

                };

                IloNum V = 0.05; //variable cost per railcar per mile
                IloNum S = 490; //distance between two stations
                IloBoolVarArray  f(env, Ntrains);
                IloArray<IloArray<IloBoolVarArray> >   xdij(env);
                IloInt T[3][6] =
                {
                { 8,6,10,6,8,9 },
                { 12,10,12,9,12,6 },
                { 13,9,5,9,15,10 }

                };
                IloInt C[3][3] =
                {
                        { 32,32,32 },
                        { 32,32,32 },
                        { 32,32,32 }
```

```
	};



	for (d = 0; d<Ntrains; d++)
	{
		IloArray<IloBoolVarArray> tempmatrix(env);
		for (i = 0; i<Ntrains; i++)
		{
			IloBoolVarArray tempbool(env, Norders);
			for (j = 0; j<Norders; j++)
			{
			std::ostringstream ossX;
			ossX << "x" << "(" << d << ")" << "(" << i << ")" << "(" << j
<< ")";

			tempbool[j].setName(ossX.str().c_str());
			}

			tempmatrix.add(tempbool);
		}
		xdij.add(tempmatrix);
	}

	for (i = 0; i<Ntrains; i++)
	{
		std::ostringstream ossX;
		ossX << "f" << "(" << i << ")";
		f[i].setName(ossX.str().c_str());
	}

	//solve model for system profit and calculate x0

	//generate objective function

	IloExpr obj(env);

	//revenue

	for (d = 0; d<Ndays; d++)
	{
		for (j = 0; j<Norders; j++)
		{
			IloExpr tempExp(env);
			for (i = 0; i<Ntrains; i++)
			{

				tempExp += xdij[d][i][j] * T[d][j];

			}
			obj += tempExp*r[d][j];
		}
```

```
}
obj = obj * 1 / Ndays;
// fixed operational costs
IloExpr A(env);
for (i = 0; i<Ntrains; i++)
{
	A += a*f[i];
}
obj = A - obj;
// add in variable cost related with distance
IloExpr B(env);
for (d = 0; d<Ndays; d++)
{
	for (j = 0; j<Norders; j++)
	{
		IloExpr tempexpr(env);
		for (i = 0; i<Ntrains; i++)
		{

			tempexpr += xdij[d][i][j] * T[d][j];

		}
		obj += tempexpr*V*S;
	}
}

//generate constraints:xdij<=fi

for (i = 0; i<Ntrains; i++)

{
	for (d = 0; d<Ndays; d++)
	{

		for (j = 0; j<Norders; j++)
		{
			model.add(xdij[d][i][j] <= f[i]);

		}
	}
}
//generate constraints: 2

for (d = 0; d<Ndays; d++)
{

	for (j = 0; j<Norders; j++)
	{
		IloExpr obj2(env);
		for (i = 0; i<Ntrains; i++)
		{
```

```
                obj2 = obj2 + xdij[d][i][j];

            }
            model.add(obj2 <= 1);
        }
}

//generate constraints 3

for (d = 0; d<Ndays; d++)
{
        for (i = 0; i<Ntrains; i++)
        {
                IloExpr obj3(env);
                for (j = 0; j<Norders; j++)
                {
                        obj3 = obj3 + xdij[d][i][j] * T[d][j];
                }
                model.add(obj3 <= C[d][i]);
        }
}
//add obj to model
model.add(IloMinimize(env, obj));
obj.end();
IloCplex cplex(env);
cplex.extract(model);
cplex.exportModel("sumprofit.lp");
cplex.solve();
double Maxsysprofit = 0;
Maxsysprofit = cplex.getObjValue();

//env.out() <<cplex.getValue(xdij[0][0][0])<<endl;
env.out() << "Maxsysprofit = " << cplex.getObjValue();

cplex.writeSolution("SystemOptimal.sos");

vector<vector<vector<bool>>> vec;
for (d = 0; d<Ndays; d++)
{
        vector<vector<bool>> ivec;

        for (i = 0; i<Ntrains; i++)
        {
                vector<bool> tvec;
                for (j = 0; j<Norders; j++)
                {
                        tvec.push_back(cplex.getValue(xdij[d][i][j]));

                }
                ivec.push_back(tvec);
        }
        vec.push_back(ivec);
```

```
		}

		vector<int> vecf;
		for (i = 0; i<Ntrains; i++)
		{
			vecf.push_back(cplex.getValue(f[i]));

		}


		//solve original problem (FSO model )and calculate x1
		IloModel model1(env);
		IloBoolVarArray  k(env, Ntrains);
		IloArray<IloArray<IloBoolVarArray> >   ydij(env);
		//creat ydij
		for (d = 0; d<Ntrains; d++)
		{
			IloArray<IloBoolVarArray> tempmatrix(env);
			for (i = 0; i<Ntrains; i++)
			{
				IloBoolVarArray tempbool(env, Norders);
				for (j = 0; j<Norders; j++)
				{
					std::ostringstream ossX;
					ossX << "ydij" << "(" << d << ")" << "(" << i << ")" <<
"(" << j << ")";

					tempbool[j].setName(ossX.str().c_str());
				}

				tempmatrix.add(tempbool);
			}
			ydij.add(tempmatrix);
		}

		//creat k
		for (i = 0; i<Ntrains; i++)
		{
			std::ostringstream ossX;
			ossX << "k" << "(" << i << ")";
			k[i].setName(ossX.str().c_str());
		}

		//generate objective function

		//revenue
		IloExpr objilt(env);
		for (d = 0; d<Ndays; d++)
		{
			for (j = 0; j<Norders; j++)
			{
				IloExpr temExp(env);
```

```
			for (i = 0; i<Ntrains; i++)
			{

				temExp = temExp + ydij[d][i][j] * T[d][j];

			}
			objilt += temExp*r[d][j];
		}
	}
	objilt = objilt / Ndays;

	// train purchasing costs
	IloExpr E(env);

	for (i = 0; i<Ntrains; i++)
	{
		E += p[i] * k[i];
	}

	objilt = E - objilt;

	//generate constraints:ydij<=k


	for (i = 0; i<Ntrains; i++)

	{
		for (d = 0; d<Ndays; d++)
		{

			for (j = 0; j<Norders; j++)
			{
				model1.add(ydij[d][i][j] <= k[i]);

			}
		}
	}
	//generate constraints: 2

	for (d = 0; d<Ndays; d++)
	{

		for (j = 0; j<Norders; j++)
		{
			IloExpr objf(env);

			for (i = 0; i<Ntrains; i++)
			{

				objf = objf + ydij[d][i][j];

			}
```

```
			model1.add(objf <= 1);
		}
	}

	//generate constraints 3

	for (d = 0; d<Ndays; d++)
	{
		for (i = 0; i<Ntrains; i++)
		{
			IloExpr objf1(env);
			for (j = 0; j<Norders; j++)
			{
				objf1 = objf1 + ydij[d][i][j] * T[d][j];
			}

			model1.add(objf1 <= C[d][i]);
		}
	}

	//add objilt1 to model1
	model1.add(FSOobj = IloMinimize(env, objilt));
	//objilt.end();
	cplex.extract(model1);
	cplex.exportModel("ilt1.lp");
	cplex.solve();
	env.out() << "   Profit of FSO at ilt1(dx1)= " << cplex.getObjValue() << endl;
	cplex.writeSolution("FSO_Initial.sos");
	double F1;
	F1 = cplex.getObjValue();
	vector<vector<vector<bool>>> Fvec; //SOLUTION for FSO model-ydij;
	for (d = 0; d<Ndays; d++)
	{
		vector<vector<bool>> Fivec;

		for (i = 0; i<Ntrains; i++)
		{
			vector<bool> Ftvec;
			for (j = 0; j<Norders; j++)
			{
				Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
			}
			Fivec.push_back(Ftvec);
		}
		Fvec.push_back(Fivec);

	}

	vector<bool> Fvecf; //solution for FSO model-ki;
	for (i = 0; i<Ntrains; i++)
	{
		Fvecf.push_back(cplex.getValue(k[i]));
```

```
}

//calculate d1*x0
IloNum FSOprofit;
for (d = 0; d<Ndays; d++)
{
	for (j = 0; j<Norders; j++)
	{
		double temExpre = 0;
		for (i = 0; i<Ntrains; i++)
		{
			temExpre = temExpre + vec[d][i][j] * T[d][j];

		}
		FSOprofit += temExpre*r[d][j];
	}
}
FSOprofit = FSOprofit / Ndays;
IloNum FSOprofit1;

for (i = 0; i<Ntrains; i++)
{

	FSOprofit1 += p[i] * vecf[i];
}

FSOprofit = FSOprofit1 - FSOprofit;
env.out() << "   FSOprofit(dx0)= " << FSOprofit << endl;

IloNum W;
W = FSOprofit - F1;
env.out() << "   W= " << W << endl;
int X = 0;
IloConstraintArray cc(env);
//solve inverse problem
IloModel modelinv(env);
IloExpr objinv(env);
IloExpr objinv1(env);
IloNumVarArray newprice(env, Ntrains, 0.0, IloInfinity, ILOFLOAT);
IloNumVarArray Q(env, Ntrains, 0.0, IloInfinity, ILOFLOAT);
vector<double> optpricevec;
double FSOFinalpro = 0;
for (i = 0; i<Ntrains; i++)
{
	std::ostringstream ossX;
	ossX << "Q" << "(" << i << ")";
	Q[i].setName(ossX.str().c_str());
}

for (i = 0; i<Ntrains; i++)
{
```

```
        std::ostringstream ossX;
        ossX << "d" << "(" << i << ")";
        newprice[i].setName(ossX.str().c_str());
}

//add objective function
for (i = 0; i<Ntrains; i++)
{

        objinv += Q[i];
}

modelinv.add(IloMinimize(env, objinv));

//add constraint(newprice[i]-p<=Q[i])

for (i = 0; i<Ntrains; i++)
{
        cc.add((newprice[i] - p[i]) <= Q[i]);
}

//add constraint(p-newprice[i]<=Q[i])

for (i = 0; i<Ntrains; i++)
{
        cc.add((p[i] - newprice[i]) <= Q[i]);
}

while (W>0)//
{
        optpricevec.clear();

        //calculate newprice*x0

        double targetprofit1 = 0;

        for (d = 0; d<Ndays; d++)
        {

                for (j = 0; j<Norders; j++)
                {
                        double pro = 0;

                        for (i = 0; i<Ntrains; i++)
                        {
                                pro = pro + vec[d][i][j] * T[d][j];

                        }
                        targetprofit1 += pro*r[d][j];
                }
        }
        targetprofit1 = targetprofit1 / Ndays;
```

```
//cout<<"targetprofit1=     "<<targetprofit1<<endl;

IloExpr targetprofit(env);

for (i = 0; i<Ntrains; i++)
{

        targetprofit += newprice[i] * vecf[i];
}

targetprofit = targetprofit - targetprofit1;
//cout<<"targetprofit=     "<<targetprofit<<endl;

//calculate newprice*x1

double newprofit1 = 0;
for (d = 0; d<Ndays; d++)
{

        for (j = 0; j<Norders; j++)
        {
                double pro = 0;

                for (i = 0; i<Ntrains; i++)
                {
                        //cout<<"ydij=    "<<Fvec[d][i][j]<<endl;
                        pro = pro + Fvec[d][i][j] * T[d][j];

                }
                newprofit1 += pro*r[d][j];
        }
}
newprofit1 = newprofit1 / Ndays;

//cout << "newprofit1=    "<<newprofit1;
IloExpr newprofit(env);

for (i = 0; i<Ntrains; i++)
{
        //      cout<<"k=    "<<Fvecf[i]<<endl;
        newprofit += newprice[i] * Fvecf[i];
}

newprofit = newprofit - newprofit1;
cout << "newprofit=     " << newprofit << endl;
//add constraint(dx0<dx1)

cc.add(targetprofit <= newprofit);

modelinv.add(cc);
objinv.end();
```

```
cplex.extract(modelinv);

std::ostringstream INVname;
INVname << "INV" << X << ".lp";

cplex.exportModel(INVname.str().c_str());

cplex.solve();

env.out() << "  inverse problem objective function " <<
cplex.getObjValue() << endl;

vector<double> newpricevec;
for (i = 0; i<Ntrains; i++)
{
	newpricevec.push_back(cplex.getValue(newprice[i]));
}


for (i = 0; i<Ntrains; i++)
{
	optpricevec.push_back(cplex.getValue(newprice[i]));
}

model1.remove(FSOobj);
objilt.clear();

//solve original problem (FSO model) with price=newprice;

//generate new objective function for fso model

//revenue

for (d = 0; d<Ndays; d++)
{

	for (j = 0; j<Norders; j++)
	{
		IloExpr pro(env);

		for (i = 0; i<Ntrains; i++)
		{
			pro = pro + ydij[d][i][j] * T[d][j];

		}
		objilt += pro*r[d][j];
	}
}
objilt = 1.0*objilt / Ndays;


// train purchasing costs
```

```
			E.clear();
			for (i = 0; i<Ntrains; i++)
			{
		cout << "newpice     " << i << "              =" << cplex.getValue(newprice[i]) <<
endl;

				E += cplex.getValue(newprice[i])*k[i];
			}

			objilt = E - objilt;

			//add objilt1 to model1
			model1.add(FSOobj = IloMinimize(env, objilt));
			objilt.clear();
			//IloCplex cplex1(env);

			cplex.extract(model1);

			std::ostringstream FSOname;
			FSOname << "FSO" << X << ".lp";
			cplex.exportModel(FSOname.str().c_str());
			cplex.solve();
			//SOLUTION for new FSO model-ydij;
			Fvec.clear();
			for (d = 0; d<Ndays; d++)
			{
				vector<vector<bool>> Fivec;

				for (i = 0; i<Ntrains; i++)
				{
					vector<bool> Ftvec;
					for (j = 0; j<Norders; j++)
					{
						Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
					}
					Fivec.push_back(Ftvec);
				}
				Fvec.push_back(Fivec);

			}

					//solution for new FSO model-ki;
			Fvecf.clear();
			for (i = 0; i<Ntrains; i++)
			{
				Fvecf.push_back(cplex.getValue(k[i]));

			}

			env.out() << "   Profit of FSO at ilt2 (dx1)" << "= " <<
cplex.getObjValue() << endl;
			std::ostringstream FSOSoluion;
```

```
FSOSoluion << "FSO" << X << ".sos";
cplex.writeSolution(FSOSoluion.str().c_str());

//calculate new W

//dx0
double NFSOprofit = 0;
for (d = 0; d<Ndays; d++)
{

	for (j = 0; j<Norders; j++)
	{
		double temp = 0;
		for (i = 0; i<Ntrains; i++)
		{
			//cout<<"vec ========
"<<vec[d][i][j]<<endl;

			temp = temp + vec[d][i][j] * T[d][j];
			//cout<<" temp"<<temp<<endl;

		}
		NFSOprofit = NFSOprofit + temp*r[d][j];
	}
}

//cout<<Ndays<<endl;
NFSOprofit = NFSOprofit / Ndays;


double n = 0;

for (i = 0; i<Ntrains; i++)
{
	n = n + newpricevec[i] * vecf[i];
}
//env.out() <<"n====="<<n<<endl;
NFSOprofit = n - NFSOprofit;
//	env.out() << "dx0 under new price is :"<<NFSOprofit<<endl;


//	cout<<"F1="<<F1<<endl;
cout << cplex.getObjValue() << endl;

FSOFinalpro = cplex.getObjValue();

W = NFSOprofit - cplex.getObjValue();


env.out() << "w=========" << W << endl;

model1.remove(objilt);
```

```
		objilt.clear();
		X++;
		env.out() << "X========" << X << endl;

	}//endwhile

	double IMprofit1 = 0;

	for (d = 0; d<Ndays; d++)
	{

		for (j = 0; j<Norders; j++)
		{
			double temp = 0;
			for (i = 0; i<Ntrains; i++)
			{


				temp = temp + Fvec[d][i][j] * T[d][j];


			}
			IMprofit1 = IMprofit1 + temp*V*S;
		}
	}

	double IMprofit2 = 0;
	for (i = 0; i<Ntrains; i++)
	{

		IMprofit2 = IMprofit2 - a*Fvecf[i] + optpricevec[i] * Fvecf[i];

	}

	double IMprofit = 0;
	IMprofit = -IMprofit1 + IMprofit2;

	cout << "IM profit under optimal price is :   " << IMprofit << endl;

	double Systemprofit = 0;
	Systemprofit = IMprofit - FSOFinalpro;
	cout << "System profit under optimal price is :   " << Systemprofit << endl;

	env.out() << "  Maximum system Profit = " << -Maxsysprofit << endl;

	double difference = 0;

	difference = Systemprofit + Maxsysprofit;

	cout << "difference between Maxmum system profit and system profit under
optimal price is :   " << difference << endl;
```

```
        env.out() << "press any key to continue..." << endl;
        getchar();

    } //end of try

    catch (IloException&e) {
        cerr << "Concert exception caught: " << e << endl;
    }
    catch (...) {
        cerr << "Unknown exception caught" << endl;
    }


    env.end();

    return 0;
}

// END main
```

Appendices E: Programme for Mechanism Design

```
#include "ilcplex/ilocplex.h"
#include <iostream>
#include <fstream>
#include <stdlib.h>
ILOSTLBEGIN
#include <vector>

        using std::vector;

static void
        usage (const char *progname),
        populatebyrow     (IloModel model, IloNumVarArray var, IloRangeArray con),
        populatebycolumn  (IloModel model, IloNumVarArray var, IloRangeArray con),
        populatebynonzero (IloModel model, IloNumVarArray var, IloRangeArray con);

double Sysoptimalpro(double RS, double Price[]);


                IloObjective FSOobj,FSOobj1,FSOobj2;
                IloInt d,i,j,t;
                IloInt Ndays=3;
                IloInt Ntrains=3;
                IloInt Norders=6;
                double a=27;


                IloNum r[3][6]=
                {
                        { 101,92,97,95,95,99 },
                        { 103,104,101,104,100,110 },
                        { 103,92,102,101,103,112 }
                };

                double V=0.05;
                double S=490;
                double grad[3];
                double stepL=1;

                IloInt T[3][6]=
                {
                        { 8,6,10,6,8,9 },
                        { 12,10,12,9,12,6 },
                        { 13,9,5,9,15,10 }
                };

                IloInt C[3] = { 32,32,32 };
                        ofstream myfile;

                int main()
```

```
		{
			myfile.open("rev.csv");

			double p[3] = { 1600,1600,2100 };//initial price per train

			try {
				double RS = 0;
				double OPRS = 0;
				double systemoptimalprofit = 0;
				double systemoptimalprofit1 = 0;
				systemoptimalprofit = Sysoptimalpro(RS, p);
				cout << "systemoptimalprofit=" << systemoptimalprofit<<endl;
				double q =1;
				RS = RS + q;
				double pp[3] = { 1600,1600,2100 };
				systemoptimalprofit1 = Sysoptimalpro(RS, pp);
				cout << "systemoptimalprofit1=" << systemoptimalprofit1 <<
endl;
				double D = 0;
				D = systemoptimalprofit1 - systemoptimalprofit;
				cout << "D=  " << D<< endl;
				double gradR = 0;
				gradR = D / q;

				cout << "gradR=    " <<gradR << endl;
				int N = 0;
				int T = 0;

			//	vector<double >
	myfile << "N=" << N << "," << "allowrance=" << RS << "," << "gradR=" <<
gradR<<"," << endl;
				while (N < 50)

				{
				 if (gradR > 0.01)
					{
						RS = RS + gradR*stepL;
						N = 0;
						OPRS = RS;
						T++;
						cout << "RS=" << RS << endl;
					}

					else if (gradR<0.01 && T<=0)
					{
						RS = RS + 2;

				cout << "RS " << RS << endl;
				cout << " The optimal system profit is :" <<
systemoptimalprofit;

						N++;
```

```
                    }
                    else if (gradR < 0.01 &&T>0)
                    {
                        cout << "The optimal RS=  " << OPRS << endl;
                        break;
                    }
                    cout << "N=" << N << endl;
                    cout << "T=" << T << endl;


                        double Nsystemoptimalprofit = 0;
                        double Nsystemoptimalprofit1 = 0;
                        double np[3] = { 1600,1600,2100 };
                    Nsystemoptimalprofit = Sysoptimalpro(RS, np);
                    RS = RS + q;
                    cout << "RS= " << RS << endl;
                    double npp[3] = { 1600,1600,2100 };
                    Nsystemoptimalprofit1 = Sysoptimalpro(RS, npp);
                    D = Nsystemoptimalprofit1 - Nsystemoptimalprofit;
                    gradR = D / q;

                    myfile << "N=" << N << "," << "Ratio=" << RS << ","
<< "gradR=" << gradR << "systemoptimalprofit="<< Nsystemoptimalprofit<<endl;

                }
                myfile.close();

            }

        catch (int e)
        {
            cout << "An exception occurred. Exception Nr. " << e << '\n';
        }

        getchar();


        return 0;
}

    double Sysoptimalpro(double RS, double p[3])

        {

        IloEnv   env;

                double optimalshiftR = 0;
                double systemprofit = 0;

                double optimalprice[3] = { 0,0,0 };
                double optimalprofit = -100000;
                double OPFOCprofit = 0;
```

```
double FOCprofit = 0;
double ST = 0;
vector<bool>OPf;

while (ST < 20)
{
	IloModel model1(env);
	IloBoolVarArray  k(env, Ntrains);
	IloArray<IloArray<IloBoolVarArray> >   ydij(env);
	IloCplex cplex(env);
	for (d = 0; d < Ntrains; d++)
	{
		IloArray<IloBoolVarArray> tempmatrix(env);
		for (i = 0; i < Ntrains; i++)
		{

			IloBoolVarArray tempbool(env,
Norders);

			for (j = 0; j < Norders; j++)
			{
		std::ostringstream ossX;
	ossX << "ydij" << "(" << d << ")" << "(" << i << ")" << "(" << j << ")";

	tempbool[j].setName(ossX.str().c_str());
			}

			tempmatrix.add(tempbool);
		}
		ydij.add(tempmatrix);
	}

	//creat k
	for (i = 0; i < Ntrains; i++)
	{
		std::ostringstream ossX;
		ossX << "k" << "(" << i << ")";
		k[i].setName(ossX.str().c_str());
	}

	//generate objective function

	//revenue
	IloExpr obj(env);
	for (d = 0; d < Ndays; d++)
	{
		for (j = 0; j < Norders; j++)
		{
			IloExpr temExp(env);
			for (i = 0; i < Ntrains; i++)
			{

		temExp = temExp + ydij[d][i][j] * T[d][j];
```

```
            }
            obj += temExp*r[d][j];
        }
}
obj = obj / Ndays;

// train purchasing costs
IloExpr E(env);

for (i = 0; i < Ntrains; i++)
{
        E += p[i] * k[i];
}

obj = obj - E;

IloConstraintArray cc(env);

//generate constraints:ydij<=k

for (i = 0; i < Ntrains; i++)

{
        for (d = 0; d < Ndays; d++)
        {

            for (j = 0; j < Norders; j++)
            {
                cc.add(ydij[d][i][j] <= k[i]);

            }
        }
}
//generate constraints: 2

for (d = 0; d < Ndays; d++)
{

        for (j = 0; j < Norders; j++)
        {
            IloExpr objf(env);

            for (i = 0; i < Ntrains; i++)
            {

                objf = objf + ydij[d][i][j];

            }
            cc.add(objf <= 1);
        }
}
```

```
//generate constraints 3

for (d = 0; d < Ndays; d++)
{
	for (i = 0; i < Ntrains; i++)
	{
		IloExpr objf1(env);
		for (j = 0; j < Norders; j++)
		{
			objf1 = objf1 + ydij[d][i][j] *
T[d][j];
		}

		cc.add(objf1 <= C[i]);
	}
}



model1.add(cc);
model1.add(FSOobj = IloMaximize(env, obj));

cplex.extract(model1);
cplex.exportModel("ilt1.lp");

cplex.solve();
FOCprofit = cplex.getObjValue();

env.out() << "   Profit of FSO when price is " << p[0] << endl << p[1] << endl << p[2] <<
endl << "     =    " << FOCprofit << endl;

cplex.writeSolution("FSO_Initial.sos");


	vector<vector<vector<bool> > > Fvec; //SOLUTION for FSO model-ydij;

for (d = 0; d < Ndays; d++)
{
	vector<vector<bool>> Fivec;

	for (i = 0; i < Ntrains; i++)
	{
		vector<bool> Ftvec;
		for (j = 0; j < Norders; j++)
		{

	Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
		}
		Fivec.push_back(Ftvec);
	}
	Fvec.push_back(Fivec);
```

```
}

for (d = 0; d < Ndays; d++)
{

    for (i = 0; i < Ntrains; i++)
    {

        for (j = 0; j < Norders; j++)
        {
            cout << "ydij (" << d << i << j <<
" )  =   " << Fvec[d][i][j] << endl;
        }

    }

}

vector<bool>FvecfL;
vector<bool> Fvecf; //solution for FSO model-ki;
for (i = 0; i < Ntrains; i++)
{

    Fvecf.push_back(cplex.getValue(k[i]));

}

for (i = 0; i < Ntrains; i++)
{

    cout << Fvecf[i] << endl;

}

for (i = 0; i < Ntrains; i++)
{

    FvecfL.push_back(Fvecf[i]);

}
for (i = 0; i < Ntrains; i++)
{

    cout << FvecfL[i] << endl;
```

```
}

double shiftrevenue = 0;

for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{

		for (i = 0; i < Ntrains; i++)
		{

			shiftrevenue += Fvec[d][i][j] *
T[d][j];

		}

	}
}
shiftrevenue = RS*shiftrevenue / Ndays;
model1.remove(FSOobj);
obj.clear();




//IM profit

double IMformer = 0;
double IMr = 0;
double IMfixedc = 0;
double IMvarc = 0;
//	double IMvarc;
for (i = 0; i < Ntrains; i++)
{
	IMr += p[i] * Fvecf[i];
}

for (i = 0; i < Ntrains; i++)
{
	IMfixedc += a*Fvecf[i];
}

for (d = 0; d<Ndays; d++)
{

	for (j = 0; j<Norders; j++)
	{

		for (i = 0; i<Ntrains; i++)
		{
```

```
					IMvarc = IMvarc + Fvec[d][i][j] * T[d][j];


						}

					}
				}

				IMvarc = IMvarc*V*S;

				IMformer = IMr - IMfixedc - IMvarc + shiftrevenue;

				env.out() << "IM profit is     " << IMformer << endl;

				Fvec.clear();
				Fvecf.clear();

				double allowrance = 0;

			re:
				if (IMformer > optimalprofit)
				{
					OPf.clear();
					for (i = 0; i < Ntrains; i++)
					{

						OPf.push_back(FvecfL[i]);
						cout << OPf[i] << endl;

					}

					OPFOCprofit = FOCprofit;

					allowrance = shiftrevenue;


					for (i = 0; i < Ntrains; i++)
					{
						optimalprice[i] = p[i];

						cout << "optimal price is :   " << 
optimalprice[i] << endl;
					}



				optimalprofit = IMformer;
			cout << "optimal profit of IM is " << optimalprofit << endl;

			cout << "FOC profit under optimal price =  " << OPFOCprofit 
<< endl;
```

```
systemprofit = OPFOCprofit + optimalprofit - allowrance;
cout << "systemprofit is " << systemprofit << endl;

optimalshiftR = shiftrevenue;
cout << "optimal shift revenue is  " << optimalshiftR << endl;
// calculate gradient of initial price

        double s = 2;
                //calculate grad[0]
                p[0] = p[0] + s;

                IloExpr obj1(env);
                for (d = 0; d < Ndays; d++)
                {
                        for (j = 0; j < Norders; j++)
                        {
                                IloExpr temExp(env);
                                for (i = 0; i < Ntrains; i++)
                                {

                                        temExp = temExp +
ydij[d][i][j] * T[d][j];


                                }
                                obj1 += temExp*r[d][j];
                        }
                }
                obj1 = obj1 / Ndays;

                // train purchasing costs
                IloExpr EE(env);

                for (i = 0; i < Ntrains; i++)
                {
                        EE = p[i] * k[i] + EE;
                }

                obj1 = obj1 - EE;

                model1.add(cc);
                model1.add(FSOobj = IloMaximize(env, obj1));
                //objilt.end();

                cplex.extract(model1);
                cplex.exportModel("ilt2.lp");

                cplex.solve();

        cplex.writeSolution("FSO_0.sos");

                for (d = 0; d < Ndays; d++)
                {
```

```
vector<vector<bool>> Fivec;

for (i = 0; i < Ntrains; i++)
{
	vector<bool> Ftvec;
	for (j = 0; j < Norders; j++)
	{

Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
	}
	Fivec.push_back(Ftvec);
}
Fvec.push_back(Fivec);

}

for (i = 0; i < Ntrains; i++)
{
	Fvecf.push_back(cplex.getValue(k[i]));

}

for (i = 0; i < Ntrains; i++)
{

	cout << Fvecf[i] << endl;

}


double shiftrevenue01 = 0;

for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{

		for (i = 0; i < Ntrains; i++)
		{

shiftrevenue01 += Fvec[d][i][j] * T[d][j];

		}

	}
}
shiftrevenue01 = RS*shiftrevenue01 / Ndays;

double IMr01 = 0;
for (i = 0; i < Ntrains; i++)
{
	IMr01 += p[i] * Fvecf[i];
```

```
}

//cout << "IMr01=" << IMr01<<endl;

double IMfixedc01 = 0;
for (i = 0; i < Ntrains; i++)
{
	IMfixedc01 += a*Fvecf[i];
}
//	cout << "IMfixedc01=" <<
IMfixedc01<<endl;
double IMvarc01 = 0;

for (d = 0; d<Ndays; d++)
{

	for (i = 0; i<Ntrains; i++)
	{

		for (j = 0; j < Norders; j++)

		{

	IMvarc01 += Fvec[d][i][j] * T[d][j];

		}

	}
}

IMvarc01 = IMvarc01*V*S;

double IMcurrent01 = 0;
IMcurrent01 = IMr01 - IMfixedc01 - IMvarc01 + shiftrevenue01;

cout << "current IM profit when p[0]=p[0]+s is" << IMcurrent01 <<
endl;

double Prodiff = 0;
Prodiff = IMcurrent01 - IMformer;
cout << " profit difference when p[0]=p[0]+s   is:" << Prodiff << endl;

grad[0] = Prodiff / s;
Fvec.clear();
Fvecf.clear();
model1.remove(FSOobj);
obj1.clear();


cout << "grad[0]=" << grad[0] << endl;
```

```
//calculate grad[1]
double IMr02 = 0;
double IMfixedc02 = 0;
double IMvarc02 = 0;

p[0] = p[0] - s;
p[1] = p[1] + s;

cout << "p[0]=" << p[0] << endl;
cout << "p[1]=" << p[1] << endl;
cout << "IM former when p[1]=p[1]+s is" << IMformer <<
endl;
IloExpr obj2(env);
for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{
		IloExpr temExp(env);
		for (i = 0; i < Ntrains; i++)
		{

temExp = temExp + ydij[d][i][j] * T[d][j];

		}
		obj2 += temExp*r[d][j];
	}
}
obj2 = obj2 / Ndays;

// train purchasing costs
IloExpr EE1(env);

for (i = 0; i < Ntrains; i++)
{
	EE1 = p[i] * k[i] + EE1;
}

obj2 = obj2 - EE1;

model1.add(cc);
model1.add(FSOobj = IloMaximize(env, obj2));
//objilt.end();
cplex.extract(model1);
cplex.exportModel("ilt2.lp");

cplex.solve();

env.out() << "   Profit of FSO when p[1]=p[1]+s   = " << cplex.getObjValue()
<< endl;

cplex.writeSolution("FSO_1.sos");
```

```
for (d = 0; d < Ndays; d++)
{
	vector<vector<bool>> Fivec;

	for (i = 0; i < Ntrains; i++)
	{
		vector<bool> Ftvec;
		for (j = 0; j < Norders; j++)
		{

Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
		}
		Fivec.push_back(Ftvec);
	}
	Fvec.push_back(Fivec);

}

for (i = 0; i < Ntrains; i++)
{
	Fvecf.push_back(cplex.getValue(k[i]));

}

for (i = 0; i < Ntrains; i++)
{
	Fvecf.push_back(cplex.getValue(k[i]));
cout << "purchasing plan is " << Fvecf[i] << endl;
}

double shiftrevenue02 = 0;

for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{

		for (i = 0; i < Ntrains; i++)
		{

shiftrevenue02 += Fvec[d][i][j] * T[d][j];

		}

	}
}
shiftrevenue02 = RS*shiftrevenue02 / Ndays;


for (i = 0; i < Ntrains; i++)
{
	IMr02 += p[i] * Fvecf[i];
```

```
}

for (i = 0; i < Ntrains; i++)
{
        IMfixedc02 += a*Fvecf[i];
}

for (d = 0; d<Ndays; d++)
{

        for (j = 0; j<Norders; j++)
        {

                for (i = 0; i<Ntrains; i++)
                {

IMvarc02 = IMvarc02 + Fvec[d][i][j] * T[d][j];

                }

        }
}
IMvarc02 = IMvarc02*V*S;

double IMcurrent02 = 0;
double Prodiff02 = 0;
IMcurrent02 = IMr02 - IMfixedc02 - IMvarc02 + shiftrevenue02;
Prodiff02 = IMcurrent02 - IMformer;
cout << " profit difference when p[1]=p[1]+s   is:" << Prodiff02 <<
endl;

grad[1] = Prodiff02 / s;
Fvec.clear();
Fvecf.clear();
model1.remove(FSOobj);
obj1.clear();
cout << "grad[1]=" << grad[1] << endl;
//calculate grad[2]

double IMr03 = 0;
double IMfixedc03 = 0;
double IMvarc03 = 0;
double IMcurrent03 = 0;
double Prodiff03 = 0;
p[1] = p[1] - s;
p[2] = p[2] + s;

cout << "p[0]=" << p[0] << endl;
cout << "p[1]=" << p[1] << endl;
cout << "p[2]=" << p[2] << endl;

IloExpr obj3(env);
```

```
for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{
		IloExpr temExp(env);
		for (i = 0; i < Ntrains; i++)
		{

temExp = temExp + ydij[d][i][j] * T[d][j];

		}
		obj3 += temExp*r[d][j];
	}
}
obj3 = obj3 / Ndays;

// train purchasing costs
IloExpr EE2(env);

for (i = 0; i < Ntrains; i++)
{
	EE2 = p[i] * k[i] + EE2;
}

obj3 = obj3 - EE2;



model1.add(cc);
model1.add(FSOobj = IloMaximize(env, obj3));
//objilt.end();


cplex.extract(model1);
cplex.exportModel("ilt3.lp");

cplex.solve();
env.out() << "   Profit of FSO when p[2]=p[2]+s   = " << cplex.getObjValue() << endl;

cplex.writeSolution("FSO_2.sos");

for (d = 0; d < Ndays; d++)
{
	vector<vector<bool>> Fivec;

	for (i = 0; i < Ntrains; i++)
	{
		vector<bool> Ftvec;
		for (j = 0; j < Norders; j++)
		{
```

```
Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
					}
					Fivec.push_back(Ftvec);
				}
				Fvec.push_back(Fivec);

			}

			for (i = 0; i < Ntrains; i++)
			{
				Fvecf.push_back(cplex.getValue(k[i]));

			}

			for (i = 0; i < Ntrains; i++)
			{
				Fvecf.push_back(cplex.getValue(k[i]));
			cout << "purchasing plan is " << Fvecf[i] <<
endl;

			}

			double shiftrevenue03 = 0;

			for (d = 0; d < Ndays; d++)
			{
				for (j = 0; j < Norders; j++)
				{

					for (i = 0; i < Ntrains; i++)
					{

			shiftrevenue03 += Fvec[d][i][j] * T[d][j];

					}

				}
			}
			shiftrevenue03 = RS*shiftrevenue03 / Ndays;

			cout << "shiftrevenue is " << shiftrevenue;

			for (i = 0; i < Ntrains; i++)
			{
				IMr03 += p[i] * Fvecf[i];
			}

			for (i = 0; i < Ntrains; i++)
			{
				IMfixedc03 += a*Fvecf[i];
			}
```

```
for (d = 0; d<Ndays; d++)
{

	for (j = 0; j<Norders; j++)
	{

		for (i = 0; i<Ntrains; i++)
		{

IMvarc03 = IMvarc03 + Fvec[d][i][j] * T[d][j];

		}

	}
}

IMvarc03 = IMvarc03*V*S;
IMcurrent03 = IMr03 - IMfixedc03 - IMvarc03 + shiftrevenue03;

cout << "IM profit when p[2]=p[2]+s is" << IMcurrent03 << endl;

Prodiff03 = IMcurrent03 - IMformer;

cout << " profit difference when p[2]=p[2]+s   is:" << Prodiff03 <<
endl;

grad[2] = Prodiff03 / s;
Fvec.clear();
Fvecf.clear();
model1.remove(FSOobj);
obj3.clear();

p[2] = p[2] - s;
cout << "gradient at this point:" << endl;

cout << "grad[0]=" << grad[0] << endl;
cout << "grad[1]=" << grad[1] << endl;
cout << "grad[2]=" << grad[2] << endl;

//new price

for (i = 0; i < 3; i++)
{
	p[i] = p[i] + grad[i] * stepL;

	cout << "p" << i << "=" << p[i] << endl;

}

//IM profit at new price
```

k[i]);

```
double IMlformer = 0, IMlcurrent = 0;
double IMrl = 0;
double IMfixedcl = 0;
double IMvarcl = 0;


IloExpr objL(env);
for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{
		IloExpr temExp(env);
		for (i = 0; i < Ntrains; i++)
		{

temExp = temExp + ydij[d][i][j] * T[d][j];

		}
		objL += temExp*r[d][j];
	}
}
objL = objL / Ndays;

// train purchasing costs
IloExpr EL(env);

for (i = 0; i < Ntrains; i++)
{
	EL += p[i] * k[i];
}

objL = objL - EL;

IloConstraintArray cL(env);

//generate constraints:ydij<=k

for (i = 0; i < Ntrains; i++)

{
	for (d = 0; d < Ndays; d++)
	{

		for (j = 0; j < Norders; j++)
		{
			cL.add(ydij[d][i][j] <=
k[i]);


		}
	}
}
//generate constraints: 2
```

```
for (d = 0; d < Ndays; d++)
{

	for (j = 0; j < Norders; j++)
	{
		IloExpr objf(env);

		for (i = 0; i < Ntrains; i++)
		{

			objf = objf + ydij[d][i][j];

		}
		cL.add(objf <= 1);
	}
}


//generate constraints 3

for (d = 0; d < Ndays; d++)
{
	for (i = 0; i < Ntrains; i++)
	{
		IloExpr objf1(env);
		for (j = 0; j < Norders; j++)
		{
	objf1 = objf1 + ydij[d][i][j] * T[d][j];
}

		cL.add(objf1 <= C[i]);
	}
}


//add constraints to model1
model1.add(cL);


//add objilt1 to model1

model1.add(FSOobj1 = IloMaximize(env,
objL));

//objilt.end();

cplex.extract(model1);
cplex.exportModel("ilt.lp");

cplex.solve();
env.out() << "   Profit of FSO when price = " <<
"(" << p[0] << "," << p[1] << "," << p[2] << ")" << "  is" << cplex.getObjValue() << endl;
```

```
FOCprofit = cplex.getObjValue();
//      cout << " FOC PROFIT =" <<
FOCprofit;

cplex.writeSolution("FSO.sos");

for (d = 0; d < Ndays; d++)
{
	vector<vector<bool>> Fivec;

	for (i = 0; i < Ntrains; i++)
	{
		vector<bool> Ftvec;
		for (j = 0; j < Norders; j++)
		{

	Ftvec.push_back(cplex.getValue(ydij[d][i][j]));
		}
		Fivec.push_back(Ftvec);
	}
	Fvec.push_back(Fivec);

}



for (i = 0; i < Ntrains; i++)
{
	Fvecf.push_back(cplex.getValue(k[i]));
//cout << "purchasing plan is " << Fvecf[i] << endl;
}


double shiftrevenueL = 0;

for (d = 0; d < Ndays; d++)
{
	for (j = 0; j < Norders; j++)
	{

		for (i = 0; i < Ntrains; i++)
		{

shiftrevenueL += Fvec[d][i][j] * T[d][j];
	}

	}
}
shiftrevenueL = RS* shiftrevenueL / Ndays;

//shiftrevenue = shiftrevenueL;

model1.remove(FSOobj1);
```

```
objL.clear();

//IM profit
for (i = 0; i < Ntrains; i++)
{
	IMrl += p[i] * Fvecf[i];
}

for (i = 0; i < Ntrains; i++)
{
	IMfixedcl += a*Fvecf[i];
}

for (d = 0; d<Ndays; d++)
{

	for (j = 0; j<Norders; j++)
	{

		for (i = 0; i<Ntrains; i++)
		{

IMvarcl = IMvarcl + Fvec[d][i][j] * T[d][j];

		}

	}
}

IMvarcl = IMvarcl*V*S;

IMlcurrent = IMrl - IMfixedcl - IMvarcl +
shiftrevenueL;
env.out() << "current IM profit  is   " << IMlcurrent <<
endl;

for (i = 0; i < Ntrains; i++)
{
FvecfL.push_back(Fvecf[i]);
}
Fvec.clear();
Fvecf.clear();

IMformer = IMlcurrent;
shiftrevenue = shiftrevenueL;

cout << "IMformer=" << IMformer << endl;
ST = 0;
goto re;
}
else
{
```

```
                        double D = 1;

                        for (i = 0; i < 3; i++)
                        {
                                p[i] = p[i] + D;

                        cout << "p" << i << "=" << p[i] << endl;

                        }
                        ST++;
                }

        }
        //

        cout << "optimal profit of IM is    " << optimalprofit << endl;

        for (i = 0; i < Ntrains; i++)
        {

                cout << "purchasing plan is " << OPf[i] << endl;

                cout << "optimal price is :   " << optimalprice[i] <<
endl;
        }
        cout << "FOC profit:" << OPFOCprofit << endl;
        cout << "optimal shift revenue is  " << optimalshiftR << endl;


        cout << "the total system profit ==  " << systemprofit << endl;
        cout << "ST=" << ST << endl;

        //env.out() ;

    env.end();

return systemprofit;
}
```

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