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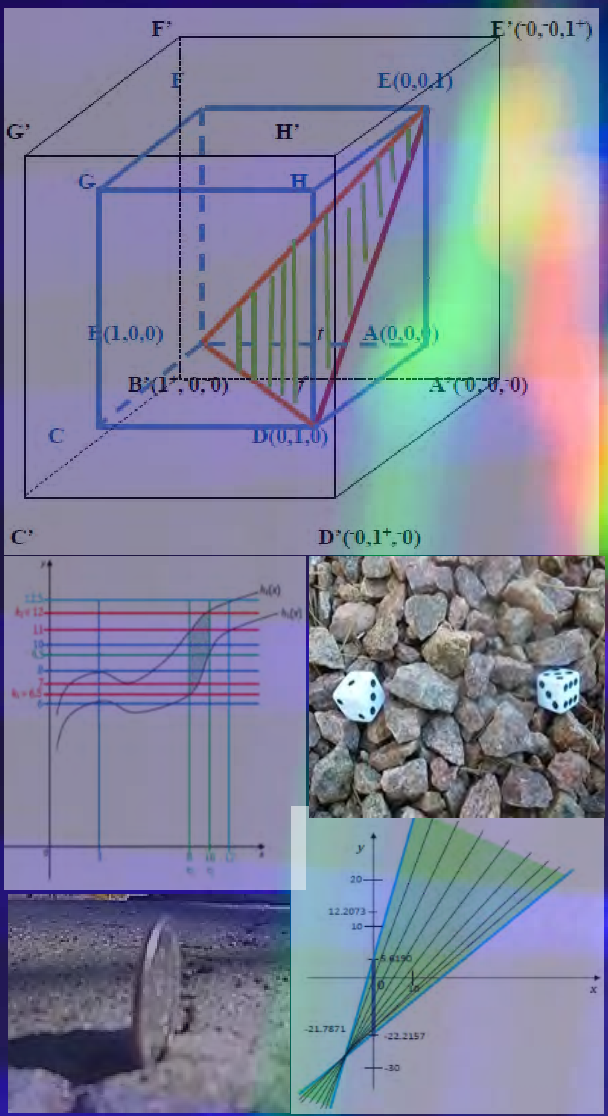

<A> <neutA> <antiA>

Florentin Smarandache , Mohamed Abdel-Basset
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University of New Mexico

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*"*Neutrosophic Sets and Systems*"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Google Dictionaries have translated the neologisms "**neutrosophy**" (1) and"**neutrosophic**" (2), coined in 1995 for the first time, into about 100 languages.
FOLDOC Dictionary of Computing (**1, 2**), Webster Dictionary (**1, 2**), Wordnik (**1**),Dictionary.com, The Free Dictionary (**1**), Wiktionary (**2**), YourDictionary (**1, 2**),OneLook Dictionary (**1, 2**), Dictionary / Thesaurus (**1**), Online Medical Dictionary (**1,2**), Encyclopedia (**1, 2**), Chinese Fanyi Baidu Dictionary (**2**), Chinese Youdao Dictionary (**2**) etc. have included these scientific neologisms.

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March 20, 2019

Prof. Florentin Smarandache
Univ New Mexico, Gallup Campus

Dear Prof. Florentin Smarandache,

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Sincerely,

Marian Hollingsworth

Marian Hollingsworth
Director, Publisher Relations

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De-Neutrosophication Technique of Pentagonal Neutrosophic Number and Application in Minimal Spanning Tree

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Abstract: In this current era, neutrosophic set theory is a crucial topic to demonstrate the ambiguous information due to existence of three disjunctive components appears in it and it provides a wide range of applications in distinct fields for the researchers. Generally, neutrosophic sets is the extended version of crisp set, fuzzy set and intuitionistic fuzzy sets to focus on the uncertain, hesitant and ambiguous datas of a real life mathematical problem. Demonstration of pentagonal neutrosophic number and its classification in different aspect is focused in this research article. Manifestation of de-neutrosophication technique of linear pentagonal neutrosophic number using removal area method has been developed here which has a remakable impact in crispfication of pentagonal neutrosophic number. Afterthat, utilizing this invented result, a minimal spanning tree problem has been solved in pentagonal neutrosophic environment. Comparision analysis is done with the other established method in this article and this noble design will be benificial for the researchers in neutrosophic domain in future.

1. Introduction

Currently, one of the eminent experimental studies of this era is on the subject of unpredictability and indeterminateness. On this aspect, Conception of Fuzzy set [1] has come up with an efficient way to work on. The theory of uncertainty plays an important role to deal with different issues relating to structure modelling in engineering domain, to do statistical calculation, in the field of social science and in any sort of real life problems relating to decision making and networking. After the invention of fuzzy set theory, researchers from several fields developed triangular [2, 3], trapezoidal [4], pentagonal [5] fuzzy number and its applications in various field of research. Professor Atanassov [6] put forward the concept of intuitionistic fuzzy sets where he considered both the idea of membership and non-membership functions. Later, in 2007 Liu F [7], merged the idea of triangular fuzzy set and intuitionistic set and created triangular intuitionistic fuzzy set. Further, Ye [8] familiarized with a basic concept on trapezoidal intuitionistic fuzzy set which includes both the truthiness and falseness membership function which are trapezoidal number in nature. Disjunctive interesting models in science and technology are developed day by day due to the invention of uncertainty theory.

In year 1995, Smarandache proposed the concept of neutrosophic sets, which was published in 1998 [9], comprised of three distinct logical components: i) truthfulness, ii) skepticism, iii) falsity. Due to the presence of hesitation component this theory gave a high impact in different kind of research domain. Further, Wang et al. [10] proposed single valued neutrosophic sets; Ye [11] formulated the concept of simplified Neutrosophic Sets, and Peng et. al. [12, 13] introduced some ideas on novel operations and aggregation operators. Recently, the concept of several forms of triangular and trapezoidal neutrosophic numbers having membership functions that are dependent or independent was manifested by Chakraborty et.al [14, 15]. In 2015, R. Helen [16] manifested the idea of pentagonal fuzzy number and A.Vigin [17] utilized it in neural network. T.Pathinathan [18] provided with the conception of reverse order triangular, trapezoidal and pentagonal fuzzy number. Several researches on neutrosophic arena were published in different fields like multi criteria decision making [19-26], graph theory [27-31], optimization techniques [32, 33] etc. Recently (2019), Chakraborty A [34] manifested the concept of pentagonal neutrosophic number and its classification component wise and applied it in solving a transportation problem in neutrosophic domain. Demonstration of pentagonal neutrosophic fuzzy number and its de-Neutrosophication value using removal area technique has been developed in this article, moreover it is applied on graph theory problem to evaluate the minimal spanning tree.

In this current epoch, neutrosophic set theory is applied in different sections of graph theory to evaluate the minimum path. Minimal spanning tree is one of the extremely vital concepts in the field of graph theory. Single valued neutrosophic minimal spanning tree and clustering method associated with it was originated by Ye [35]. Mandal & Basu [36] introduced similarity measure in optimum spanning tree problems related with neutrosophic arena. Mullai et. al [37] formulated minimum spanning tree problem in bipolar neutrosophic domain. Further, Broumi et.al [38, 39] manifested the concept of shortest path problem in neutrosophic graphs. Later, Broumi et.al [40] generated the perception of decision-making problem with the help of interval valued neutrosophic number and Kandasamy [41] developed double-valued neutrosophic sets and their application in minimum spanning tree problems. Currently, Broumi et.al [42] formulated neutrosophic shortest path for solving Dijkstra algorithm in graph theory. A few published articles [43-50] are addressed here related with neutrosophic domain which plays an important role in uncertainty research arena. Recently, in 2017 F. Smarandache developed a concept namely Plithogenic set, which has a great impact in current research arena and its is applied in hospital care system [51], IoT based problem [52], multi criteria decision making problem [53], cancer related problems [54], fractal programming problem [55], hybrid MCDM problem [56,57] and forecasting problems [58] etc.

1.1 Motivation

The invention of uncertainity theory plays a vital role in formulation of real-life scientific mathematical model, structural modelling in engineering domain, multi criteria oriented medical diagonosis problem etc. Recently, a question will arrise if someone choose pentagonal neutrosophic number in any field of research then what will be the crispification value of this said number? How can we convert a pentagonal neutrosophic number equivalent to a crisp number in logical and scientific way? How can we generated some motivating approach in de-neutrosophication technique? Again, The concept of minimal spanning tree is a very well known concept in mathematics field. Now, generally we considered crisp numbers in place of weight in a spanning tree problem. But, suppose the exact value of the weights are unknown to us and decision maker's mind is in dilema in case of putting the exact weights. Thus, it is a conception of neutrosophic number which contains truth, falsity and hesitation components. Here we consider pentagonal neutrosophic numbers to allocate the weights of a spanning tree problem. Now, question will arise at once how can we tackle this problem in neutrosophic environment? From this aspect we shall try to built up this article.

The following table discusses the measurement of uncertaininty, vagueness and hesitation of four disjunctive types of Minimal Spanning Tree including crisp environment, fuzzy environment, intuitionistic fuzzy environment and pentagonal neutrosophic environment.

Edge Parameters in case of Minimal Spanning Tree Problem	Measurement of Uncertainty	Measurement of Hesitation	Measurement of Vagueness
Crisp Number	×	×	×
Crisp Interval Valued Number	×	×	×
Fuzzy Number	Can Determine	×	×
Interval Valued Fuzzy Number	Can Determine	×	×
Intuitionistic Fuzzy Number	Can Determine	×	Can Determine
Interval Valued Intuitionistic Fuzzy Number	Can Determine	×	Can Determine
Pentagonal Neutrosophic Number	Can Determine	Can Determine	Can Determine

From the above table, it is observed that only pentagonal neutrosophic environment can tackle the impreciseness, hesitation and truthiness in a membership function of a uncertain number, which is more reliable, logical and realistic for a decision maker. Thus, we consider our minimal spanning tree model in neutrosophic arena and all the edges of the graph as pentagonal neutrosophic number all the graph.

Advantage and Restrictions of disjunctive categories of set

The below table will shows us the advantage and restrictions of different kind of parameters in our real life mathematical problems.

Disjunctive Categories of Set/Number	Advantages	Restrictions
Crisp Number	Determine the accurate value of a realistic problem perfectly.	Cannot determine the uncertainty information of a realistic problem.
Fuzzy Number	Can describe the uncertainty information of a realistic problem.	Cannot describe the hesitation & falsity information of a realistic problem.
Intuitionistic Fuzzy Number	Can determine the uncertainty &falsity information of a realistic problem.	Cannot determine the hesitation information of a realistic problem.
Pythagorean Fuzzy Number	Can deal with the uncertainty & falsity information of a realistic problem.	Cannot deal with the hesitation information of a realistic problem.
Neutrosophic Fuzzy Number	Can describe the uncertainty, falsity & hesitation information of a realistic problem.	Cannot describe the incomplete weight information of a realistic problem.

1.2 Contribution

In this research article, researchers are primarily focused on pentagonal neutrosophic fuzzy number and its properties. A very engrossing question will arises among the researchers from all around the world that how a neutrosophic number can be transformed into a crisp number? From the last century, researchers are tried to develop lots of new methods associated with the de-Neutrosophication technique for crispification. Here, we generate the idea of crispification of pentagonal neutrosophic fuzzy number is enlarged using removal area skill. Nowadays, researchers are giving their attention to solve the problem of minimal spanning tree in neutrosophic arena. By utilizing the idea of newly generated de-Neutrosophication skill on pentagonal neutrosophic number field, we can able to tackle the problems on minimal spanning tree. Lastly, comparison analysis is done with the established methods to show the importance of this algorithm.

1.3 Novelties

Several research articles had already published in different journals on neutrosophic arena. Researches from different domain applied this concept in distinct areas also. The conception of pentagonal neutrosophic number is totally new in research domain. Thus it can be extended into different fields and can be applied into various research arenas. However a few numbers of articles has been developed in pentagonal neutrosophic environment till now. Thus, our motivation and target is to try to sketch out some unpublished points that are described below.

- Formulation of linear pentagonal neutrosophic number and its classification.
- De-Neutrosophication technique of linear pentagonal neutrosophic number.
- Application in minimal spanning tree problem.

1.4 Structure of the paper

In this reseach article section 1 contains introduction and litrature survey of neutrosophic number, section 2 covers mathematical preliminaries, section 3 admits a de-neutrosophication technique of of linear pentagonal neutrosophic fuzzy number, section 4 covers minimal spanning tree problem in neutrosophic environment, section 5 shows comparison table and lastly section 6 contains the conclusion part of the total reseach work.

2. Mathematical Preliminaries

Definition 2.1: Fuzzy Set: [1] A set $\tilde{C}$, is denoted as $\tilde{C} = \{(x, \mu_{\tilde{C}}(x)) : x \in X, \mu_{\tilde{C}}(X) \in [0,1]\}$ and is generally represented by $\left(x, \mu_{\tilde{C}}(x)\right)$, where $x \in$ the crisp set X and $\mu_{\tilde{C}}(X) \in$ the interval $[0,1]$, then set $\tilde{C}$ is called an intuitionistic fuzzy set.

Definition 2.2: Intuitionistic Fuzzy Set (IFS): A set [6] $\tilde{P}$, is defined as $\tilde{P} = \{\langle x; [\tau(x), \varphi(x)]\rangle : x \in X\}$, where $\tau(x): X \to [0,1]$ is named as the truth membership function which indicate the degree of assurance, $\varphi(x): X \to [0,1]$ is named the falsity membership and $\tau(x), \varphi(x)$ satisfies the following the relation

$0 \le \tau(x) + \varphi(x) \le 1.$

Definition 2.3: Neutrosophic Set: [9] A set $\widetilde{nA}$ is called a neutrosophic set if $\widetilde{nA} = \left\{\langle x; \left[\rho_{\widetilde{nA}}(x), \sigma_{\widetilde{nA}}(x), \omega_{\widetilde{nA}}(x)\right]\rangle : x \in X\right\}$, where $\rho_{\widetilde{nA}}(x): X \to [0,1]$ is said to be the truth membership function, $\sigma_{\widetilde{nA}}(x): X \to [0,1]$ is said to be the indeterminacy membership function and $\omega_{\widetilde{nA}}(x): X \to [0,1]$ is said to be the falsity membership function.

$\rho_{\widetilde{nA}}(x), \sigma_{\widetilde{nA}}(x) \& \omega_{\widetilde{nA}}(x)$ exhibits the following relation:

$-0 \le \rho_{\widetilde{nA}}(x) + \sigma_{\widetilde{nA}}(x) + \omega_{\widetilde{nA}}(x) \le 3+$

Definition 2.4: Single-Valued Neutrosophic Set: A Neutrosophic set $\widetilde{nA}$ in the definition 2.1 is said to be a Single-Valued Neutrosophic Set $(\widetilde{SnA})$ if x is a single-valued independent variable. $\widetilde{SnA} = \left\{\langle x; \left[\alpha_{\widetilde{SnA}}(x), \beta_{\widetilde{SnA}}(x), \gamma_{\widetilde{SnA}}(x)\right]\rangle : x \in X\right\}$, where $\alpha_{\widetilde{SnA}}(x), \beta_{\widetilde{SnA}}(x) \& \gamma_{\widetilde{SnA}}(x)$ denoted the concept of trueness, indeterminacy and falsity memberships function respectively.

If there exist three points $p_0, q_0 \& r_0$, for which $\alpha_{\widetilde{SnA}}(p_0) = 1, \beta_{\widetilde{SnA}}(q_0) = 1 \& \gamma_{\widetilde{SnA}}(r_0) = 1$, then the $\widetilde{SnA}$ is called neut-normal.

$\widetilde{SnS}$ is called neut-convex, which follows the relation:

$\alpha_{\widetilde{SnA}}\langle \delta p_1 + (1-\delta)p_2\rangle \ge min\langle \alpha_{\widetilde{SnA}}(p_1), \alpha_{\widetilde{SnA}}(p_2)\rangle$

$\beta_{\widetilde{SnA}}\langle \delta p_1 + (1-\delta)p_2\rangle \le max\langle \beta_{\widetilde{SnA}}(p_1), \beta_{\widetilde{SnA}}(p_2)\rangle$

$\gamma_{\widetilde{SnA}}\langle \delta p_1 + (1-\delta)p_2\rangle \le max\langle \gamma_{\widetilde{SnA}}(p_1), \gamma_{\widetilde{SnA}}(p_2)\rangle$

where $p_1 \& p_2 \in \mathbb{R}$ and $\delta \in [0,1]$

Definition 2.5: Single-Valued Pentagonal Neutrosophic Number: A Single-Valued Pentagonal Neutrosophic Number $(\tilde{S})$ is defined and described as $\tilde{s} = \langle[(g^1, h^1, i^1, j^1, k^1); \rho], [(g^2, h^2, i^2, j^2, k^2); \sigma], [(g^3, h^3, i^3, j^3, k^3); \omega]\rangle$, where $\rho, \sigma, \omega \in [0,1]$. The truth membership function $(\theta_{\tilde{S}}):\mathbb{R}\rightarrow[0,\rho]$, the indeterminacy membership function $(\phi_{\tilde{S}}):\mathbb{R}\rightarrow[\sigma,1]$ and the falsity membership function $(\psi_{\tilde{S}}):\mathbb{R}\rightarrow[\omega,1]$ are given as:

$$\theta_{\tilde{S}}(x)=\begin{cases}\theta_{\widetilde{Sl1}}(x) & g^1\le x<h^1\\ \theta_{\widetilde{Sl2}}(x) & h^1\le x<i^1\\ \rho & x=i^1\\ \theta_{\widetilde{Sr2}}(x) & i^1\le x<j^1\\ \theta_{\widetilde{Sr1}}(x) & j^1\le x<k^1\\ 0 & otherwise\end{cases},\quad \phi_{\tilde{S}}(x)=\begin{cases}\phi_{\widetilde{Sl1}}(x) & g^2\le x<h^2\\ \phi_{\widetilde{Sl2}}(x) & h^2\le x<i^2\\ \sigma & x=i^2\\ \phi_{\widetilde{Sr2}}(x) & i^2\le x<j^2\\ \phi_{\widetilde{Sr1}}(x) & j^2\le x<k^2\\ 1 & otherwise\end{cases}$$

$$\psi_{\tilde{S}}(x)=\begin{cases}\psi_{\widetilde{Sl1}}(x) & g^3\le x<h^3\\ \psi_{\widetilde{Sl2}}(x) & h^3\le x<i^3\\ \omega & x=i^3\\ \psi_{\widetilde{Sr2}}(x) & i^3\le x<j^3\\ \psi_{\widetilde{Sr1}}(x) & j^3\le x<k^3\\ 1 & otherwise\end{cases}$$

3. De-Neutrosophication of a Linear Neutrosophic Pentagonal Number

On development of the de-Neutrosophication technique, results can be generated into a crisp number according to the results of pentagonal neutrosophic number and its membership functions. Researchers from all around the globe are concerned to know what shall be the crisp value associating the pentagonal neutrosophic number having membership function? By the passing days, they have continuously developed some convenient means to change a fuzzy number to a crisp number and some of these approaches are discussed below:

1. BADD (basic defuzzification distributions)
2. BOA (bisector of area)
3. CDD (constraint decision defuzzification)
4. COA (center of area)
5. COG (center of gravity)
6. ECOA (extended center of area)
7. EQM (extended quality method)
8. FCD (fuzzy clustering defuzzification), etc.

On this pentagonal neutrosophic arena, researches had an ambiguity in finding the suitable method of changing the pentagonal neutrosophic number to a crisp number. There are three distinct membership functions present in pentagonal neutrosophic number. To transform a neutrosophic number to a crisp number, "removal area method" is proposed on this article.

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Suppose, we consider a linear pentagonal neutrosophic number as follows

$$\tilde{A}_{Bineu} = (i_1, i_2, i_3, i_4, i_5; j_1, j_2, j_3, j_4, j_5; k_1, k_2, k_3, k_4, k_5)$$

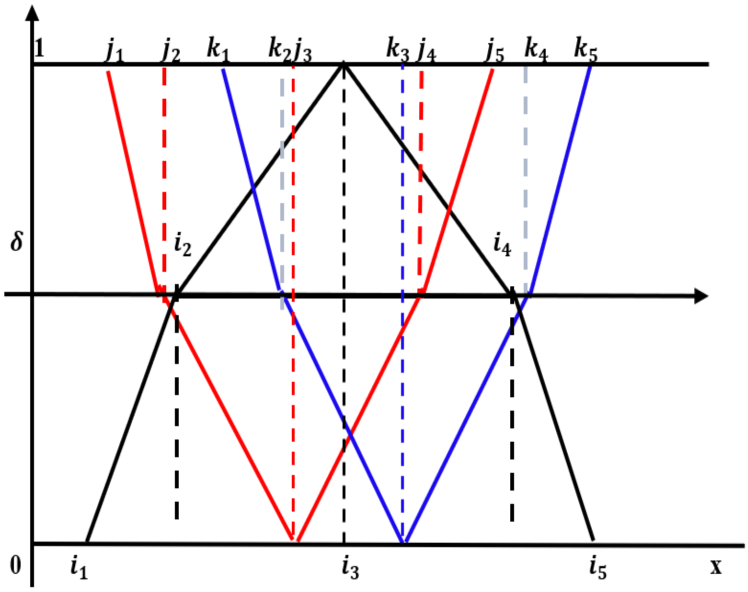

Figure 3.1: Graphical representation of Linear Pentagonal Neutrosophic Number.

Description of above figure: On the above figure we focused on the graphical presentation of linear pentagonal neutrosophic number. The black lined pentagonal represent the truth membership function. Red lined pentagonal represents the falsity membership function and blue lined pentagonal shows indefiniteness membership function of the number. In this, τ follows the relation $0 \leq \tau \leq 1$. The pentagonal number can be altered to triangular neutrosophic number if $\tau = 0$ *or* 1.

Let us assume a real number $s \in R$ and a fuzzy number $\breve{P}$ for black line specified pentagons, area of the left side distribution of $\breve{P}$ w.r.t s, is $A_{Neu\,l}(\breve{P}, s)$ that indicates the zone fenced by s and the left side of the fuzzy number $\breve{P}$. Proceeding in this way, the right zone area of $\breve{P}$ w.r.t s is $A_{Neu\,r}(\breve{P}, s)$. Considering a real number $s \in R$ along with the fuzzy number $\breve{Q}$ for the left most top and inverted pentagon, then area of lest side of $\breve{Q}$ wrt s is $A_{Neu\,l}(\breve{Q}, s)$is described as the area bounded by l and the left portion of the fuzzy number $\breve{Q}$. For the second time, the area of right side of Q wr.t l is $A_{Neu\,r}(\breve{Q}, s)$, A fuzzy number $\breve{R}$ for the right most top and inverted pentagon, then left side removal of $\breve{R}$ w.r.t s is $A_{Neu\,l}(\breve{R}, s)$ is described by the area bounded by s and the left side of the fuzzy number $\breve{R}$. similarly, the right portion removal of $\breve{R}$ w.r.t s is $A_{Neu\,r}(\breve{R}, s)$.

Mean is described as $A_{Neu}(\breve{P}, s) = \frac{A_{Neu\,l}(\breve{P}, s) + A_{Neu\,r}(\breve{P}, s)}{2}$, $A_{Neu}(\breve{Q}, s) = \frac{A_{Neu\,l}(\breve{Q}, s) + A_{Neu\,r}(\breve{Q}, s)}{2}$,

$A_{Neu}(\breve{R}, s) = \frac{A_{Neu\,l}(\breve{R}, s) + A_{Neu\,r}(\breve{R}, s)}{2}$

Then, we quantified the de-neutrosophication value of a linear pentagonal neutrosophic number as,

$$A_{Neu}(\widetilde{D_{Pen}}, s) = \frac{A_{Neu}(\breve{P}, s) + A_{Neu}(\breve{Q}, s) + A_{Neu}(\breve{R}, s)}{3}$$

For $s = 0$,

$$A_{Neu}(\check{P},0) = \frac{A_{Neu\,l}(\check{P},0)+A_{Neu\,r}(\check{P},0)}{2},\ A_{Neu}(\check{Q},0) = \frac{A_{Neu\,l}(\check{Q},0)+A_{Neu\,r}(\check{Q},0)}{2},\ A_{Neu}(\check{R},0) = \frac{A_{Neu\,l}(\check{R},0)+A_{Neu\,r}(\check{R},0)}{2}$$

Thus, $A_{Neu}(\widetilde{D_{Pen}},0) = \frac{A_{Neu}(\check{P},0)+A_{Neu}(\check{Q},0)+A_{Neu}(\check{R},0)}{3}$

Here, we take $\check{X} = (i_1, i_2, i_3, i_4, i_5), \check{Y} = (j_1, j_2, j_3, j_4, j_5), \check{Z} = (k_1, k_2, k_3, k_4, k_5)$

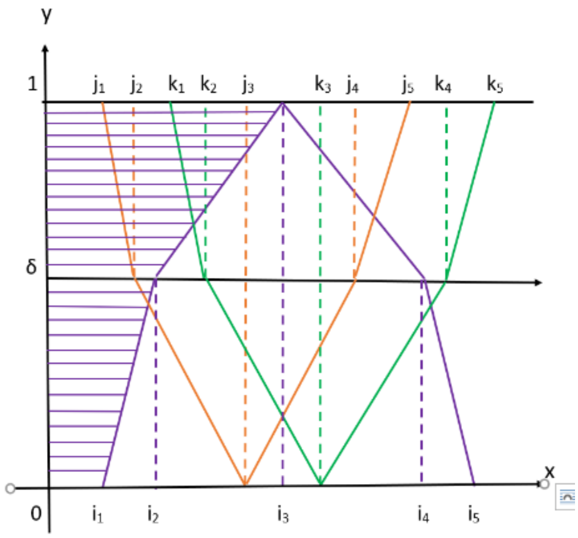

Figure 3.1(a)

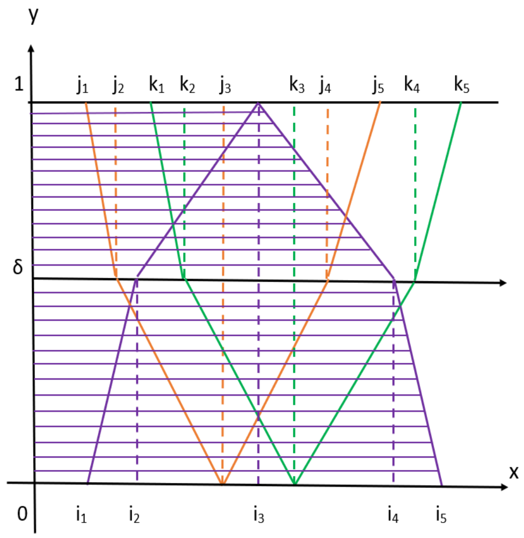

Figure 3.1(b)

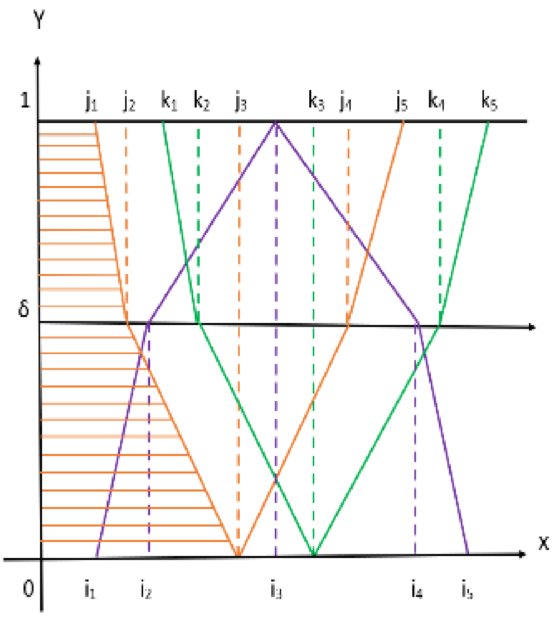

Figure 3.2(a)

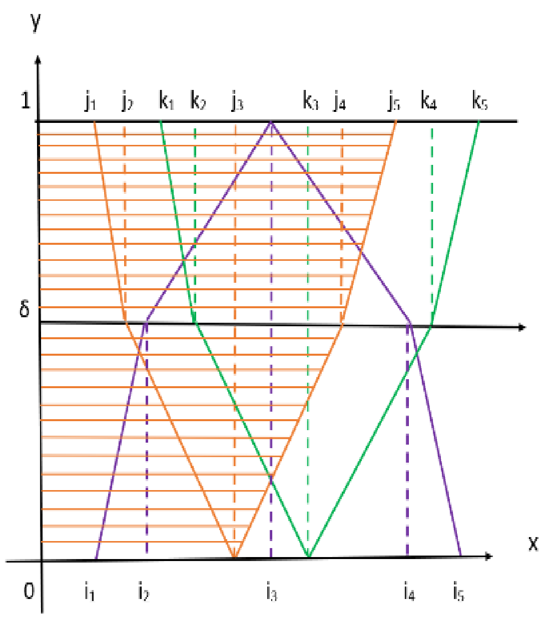

Figure 3.2(b)

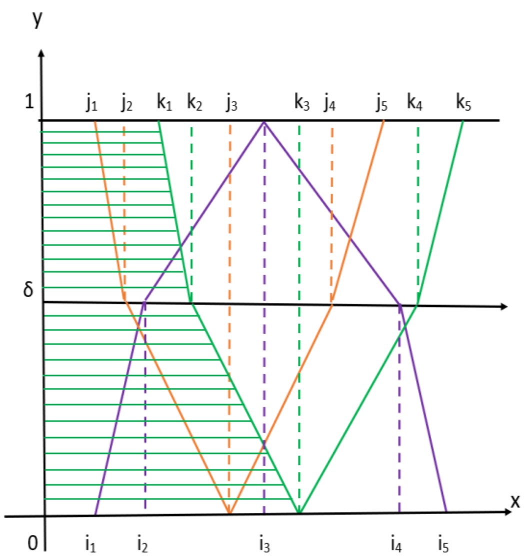

Figure 3.3(a)

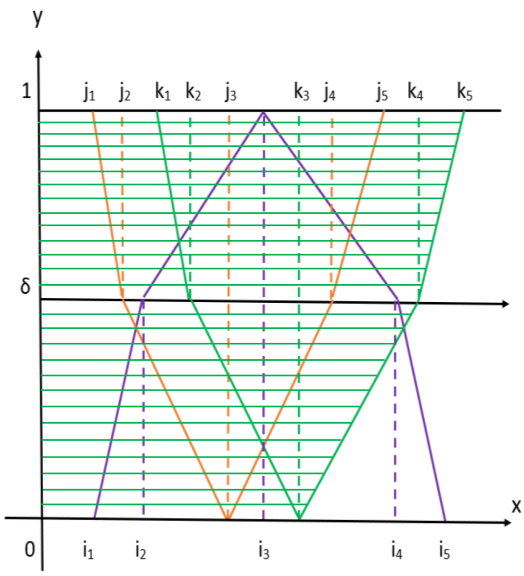

Figure 3.3(b)

Then,

$$A_{Neu\,l}(\breve{P},0) = \text{Area of Figure 3.1(a)} = \frac{(i_1+i_2)\delta}{2} + \frac{(i_2+i_3)(1-\delta)}{2}$$

$$A_{Neu\,r}(\breve{P},0) = \text{Area of Figure 3.1(b)} = \frac{(i_4+i_5)\delta}{2} + \frac{(i_3+i_4)(1-\delta)}{2}$$

$$A_{Neu\,l}(\breve{Q},0) = \text{Area of Figure 3.2(a)} = \frac{(j_1+j_2)(1-\delta)}{2} + \frac{(j_2+j_3)\delta}{2}$$

$$A_{Neu\,r}(\breve{Q},0) = \text{Area of Figure 3.2(b)} = \frac{(j_3+j_4)\delta}{2} + \frac{(j_4+j_5)(1-\delta)}{2}$$

$$A_{Neu\,l}(\breve{R},0) = \text{Area of Figure 3.3(a)} = \frac{(k_1+k_2)(1-\delta)}{2} + \frac{(k_2+k_3)\delta}{2}$$

$$A_{Neu\,r}(\breve{R},0) = \text{Area of Figure 3.3(a)} = \frac{(k_3+k_4)\delta}{2} + \frac{(k_4+k_5)(1-\delta)}{2}$$

$$\text{Hence, } A_{Neu}(\breve{P},0) = \frac{\frac{(i_1+i_2)\delta}{2}+\frac{(i_2+i_3)(1-\delta)}{2}+\frac{(i_4+i_5)\delta}{2}+\frac{(i_3+i_4)(1-\delta)}{2}}{2},$$

$$A_{Neu}(\breve{Q},0) = \frac{\frac{(j_1+j_2)(1-\delta)}{2}+\frac{(j_2+j_3)\delta}{2}+\frac{(j_3+j_4)\delta}{2}+\frac{(j_4+j_5)(1-\delta)}{2}}{2}, \quad A_{Neu}(\breve{R},0) = \frac{\frac{(k_1+k_2)(1-\delta)}{2}+\frac{(k_2+k_3)\delta}{2}+\frac{(k_3+k_4)\delta}{2}+\frac{(k_4+k_3)(1-\delta)}{2}}{2}$$

$$\text{So, } A_{Neu}(\widetilde{D_{Pen}},0) = \frac{(i_1+i_2+i_4+i_5+j_2+2j_3+j_4+k_2+2k_3+k_4)\delta+(i_2+2i_3+i_4+j_1+j_2+j_4+j_5+k_1+k_2+k_4+k_5)(1-\delta)}{12} \quad \text{..........(1)}$$

Table 3.1: Numerical computation of De-Neutrosophication value

Sl. No.	Pentagonal Neutrosophic Number	De-Neutrosophication value
1	(1,2,3,4,5;0.5,1.5,2.5,3.5,4.5;2.2,2.8,3.7,4.5,6)	3.091667
2	(0.5,1.5,2.5,3.5,4.5;0.3,1.3,2.3,3.3,4.3;1.8,2.8,3.8,4.8,5.8)	2.86667
3	(0.7,1.7,2.5,3.5,4.7;0.5,1.5,2.2,3.2,4;1.7,2.7,3.7,4.7,5.7)	2.86250
4	(1.2,2.2,3.2,4.2,5.2;1,2,3,4,5;2.5,3.5,4.5,5,6.5)	3.52500
5	(1,4,7,10,13;0.5,3.5,6.5,9.5,12.5;4.5,7.5,9,12,14.5)	7.66667

4. Minimal Spanning Tree in Pentagonal Neutrosophic Environment

Spanning Tree: Let, G is a graph and T is a subgraph of G. If T is a connected graph having no circuits and covers all vertices of G, then T is called a spanning tree.

Minimal Spanning Tree: A spanning tree which contains the least weight in G is defined as minimal spanning tree. Let us consider a graph in pentagonal neutrosophic domain. Here we developed an algorithm to search out the minimal spanning tree where the weights are pentagonal neutrosophic numbers. Thus this is a problem of neutrosophic graph.

Algorithm:

- Construct an adjacency matrix of the graph.
- Utilize de-Neutrosophication technique and construct crisp matrix.
- Select the least weight and if there is a tie in selection of least weight then take any one edge from the given graph.
- From the edges that are left behind select an edge containing the least edge that doesn't form a loop with the previous established figure.
- Continue this process until all vertices will be covered.
- Stop.

4.1 Illustrative Example:

To acquire a minimal spanning tree of the following graph

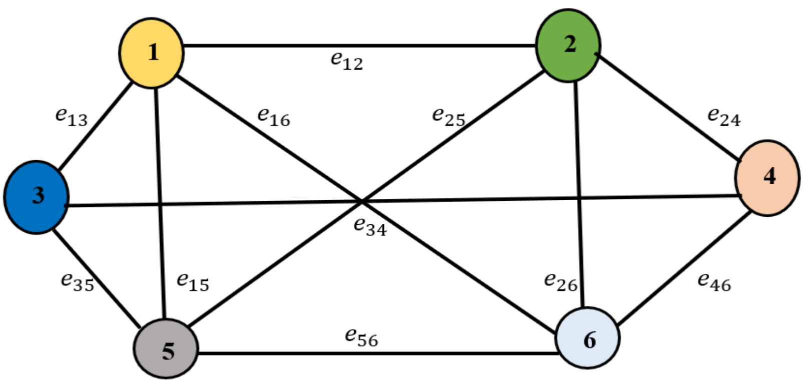

Figure 4.1.1: A Graph with pentagonal neutrosophic number Weight Edges

Table 1: The values of weights related with edges

Edges	Pentagonal Single-Valued Neutrosophic Weights
e_{12}	$< 0.5,1,1.5,2,2.5;\ 0.3,0.7,1.2,1.6,2.2;\ 0.8,1.3,1.8,2.3,3 >$
e_{13}	$< 0.7,1.2,1.8,2.4,3;\ 0.6,1,1.4,2,2.5;\ 1,1.5,2,2.5,3.5 >$
e_{15}	$< 0.3,0.8,1.4,2,2.6;\ 0.2,0.7,1.2,1.8,2.2;\ 0.5,1,1.5,2.4,3 >$
e_{16}	$< 1,1.5,2,2.5,3;\ 0.7,1.2,1.8,2.2,2.6;\ 1.2,1.6,2.2,2.8,3.5 >$
e_{25}	$< 0.8,1.4,2,2.6,3.2;\ 0.6,1.2,1.8,2.2,2.8;\ 1,1.8,2.4,2.8,3.5 >$
e_{26}	$< 0.6,1,1.5,2,2.5;\ 0.4,0.8,1.2,1.8,2.2;\ 0.8,1.4,2,2.4,3 >$
e_{24}	$< 0.9,1.4,2,2.5,3;\ 0.6,1.2,1.6,2.1,2.4;\ 1.2,1.5,2.3,2.8,3.5 >$
e_{34}	$< 1.1,1.5,1.9,2.3,2.7;\ 0.8,1.2,1.7,2.1,2.4;\ 1.4,1.8,2.2,2.6,3 >$
e_{46}	$< 0.7,1,1.3,1.6,2;\ 0.6,0.9,1.2,1.5,1.8;\ 1,1.4,1.8,2.2,2.5 >$
e_{35}	$< 0.8,1.2,1.5,1.8,2.4;\ 0.5,0.9,1.3,1.7,2.1;\ 1,1.4,1.8,2.2,2.6 >$
e_{56}	$< 1.2,1.5,1.8,2.4,2.6;\ 0.9,1.3,1.7,2,2.3;\ 1.4,1.8,2.2,2.5,2.8 >$

Step 1: The associated adjacency matrix of figure 1 is given as follows:

$$A = \begin{bmatrix}
\begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix} & \begin{matrix}< 0.5,1,1.5,2,2.5;\\ 0.3,0.7,1.2,1.6,2.2;\\ 0.8,1.3,1.8,2.3,3 >\end{matrix} & \begin{matrix}< 0.7,1.2,1.8,2.4,3;\\ 0.6,1,1.4,2,2.5;\\ 1,1.5,2,2.5,3.5 >\end{matrix} & \begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix} & \begin{matrix}< 0.3,0.8,1.4,2,2.6;\\ 0.2,0.7,1.2,1.8,2.2;\\ 0.5,1,1.5,2.4,3 >\end{matrix} & \begin{matrix}< 1,1.5,2,2.5,3;\\ 0.7,1.2,1.8,2.2,2.6;\\ 1.2,1.6,2.2,2.8,3.5 >\end{matrix} \\
\begin{matrix}< 0.5,1,1.5,2,2.5;\\ 0.3,0.7,1.2,1.6,2.2;\\ 0.8,1.3,1.8,2.3,3 >\end{matrix} & \begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix} & \begin{matrix}-\\ -\\ -\end{matrix} & \begin{matrix}< 0.9,1.4,2,2.5,3;\\ 0.6,1.2,1.6,2.1,2.4;\\ 1.2,1.5,2.3,2.8,3.5 >\end{matrix} & \begin{matrix}< 0.8,1.4,2,2.6,3.2;\\ 0.6,1.2,1.8,2.2,2.8;\\ 1,1.8,2.4,2.8,3.5 >\end{matrix} & \begin{matrix}< 0.6,1,1.5,2,2.5;\\ 0.4,0.8,1.2,1.8,2.2;\\ 0.8,1.4,2,2.4,3 >\end{matrix} \\
\begin{matrix}< 0.7,1.2,1.8,2.4,3;\\ 0.6,1,1.4,2,2.5;\\ 1,1.5,2,2.5,3.5 >\end{matrix} & \begin{matrix}-\\ -\\ -\end{matrix} & \begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix} & \begin{matrix}< 1.1,1.5,1.9,2.3,2.7;\\ 0.8,1.2,1.7,2.1,2.4;\\ 1.4,1.8,2.2,2.6,3 >\end{matrix} & \begin{matrix}< 0.8,1.2,1.5,1.8,2.4;\\ 0.5,0.9,1.3,1.7,2.1;\\ 1,1.4,1.8,2.2,2.6 >\end{matrix} & \begin{matrix}-\\ -\\ -\end{matrix} \\
\begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix} & \begin{matrix}< 0.9,1.4,2,2.5,3;\\ 0.6,1.2,1.6,2.1,2.4;\\ 1.2,1.5,2.3,2.8,3.5 >\end{matrix} & \begin{matrix}< 1.1,1.5,1.9,2.3,2.7;\\ 0.8,1.2,1.7,2.1,2.4;\\ 1.4,1.8,2.2,2.6,3 >\end{matrix} & \begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix} & \begin{matrix}-\\ -\\ -\end{matrix} & \begin{matrix}< 0.7,1,1.3,1.6,2;\\ 0.6,0.9,1.2,1.5,1.8;\\ 1,1.4,1.8,2.2,2.5 >\end{matrix} \\
\begin{matrix}< 0.3,0.8,1.4,2,2.6;\\ 0.2,0.7,1.2,1.8,2.2;\\ 0.5,1,1.5,2.4,3 >\end{matrix} & \begin{matrix}< 0.8,1.4,2,2.6,3.2;\\ 0.6,1.2,1.8,2.2,2.8;\\ 1,1.8,2.4,2.8,3.5 >\end{matrix} & \begin{matrix}< 0.8,1.2,1.5,1.8,2.4;\\ 0.5,0.9,1.3,1.7,2.1\\ 1,1.4,1.8,2.2,2.6 >\end{matrix} & \begin{matrix}-\\ -\\ -\end{matrix} & \begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix} & \begin{matrix}< 1.2,1.5,1.8,2.4,2.6;\\ 0.9,1.3,1.7,2,2.3;\\ 1.4,1.8,2.2,2.5,2.8 >\end{matrix} \\
\begin{matrix}< 1,1.5,2,2.5,3;\\ 0.7,1.2,1.8,2.2,2.6;\\ 1.2,1.6,2.2,2.8,3.5 >\end{matrix} & \begin{matrix}< 0.6,1,1.5,2,2.5;\\ 0.4,0.8,1.2,1.8,2.2;\\ 0.8,1.4,2,2.4,3 >\end{matrix} & \begin{matrix}-\\ -\\ -\end{matrix} & \begin{matrix}< 0.7,1,1.3,1.6,2;\\ 0.6,0.9,1.2,1.5,1.8;\\ 1,1.4,1.8,2.2,2.5 >\end{matrix} & \begin{matrix}< 1.2,1.5,1.8,2.4,2.6;\\ 0.9,1.3,1.7,2,2.3;\\ 1.4,1.8,2.2,2.5,2.8 >\end{matrix} & \begin{matrix}< 0,0,0,0,0;\\ 0,0,0,0,0;\\ 0,0,0,0,0 >\end{matrix}
\end{bmatrix}$$

Step 2: Using the De-neutrosophic value, the associated matrix becomes

$$A = \begin{bmatrix}
0 & 1.504 & 1.788 & - & 1.433 & 1.983 \\
1.504 & 0 & - & 1.933 & 2.012 & 1.570 \\
1.788 & - & 0 & 1.917 & 1.542 & - \\
- & 1.933 & 1.917 & 0 & - & 1.433 \\
1.433 & 2.012 & 1.542 & - & 0 & 1.900 \\
1.983 & 1.570 & - & 1.433 & 1.900 & 0
\end{bmatrix}$$

Step 3: After examining, the least value is 1.433. So, the edge is selected which is connected with the nodes (1, 5) and thus labelled it. This procedure is repeated till the final spanning tree is found.

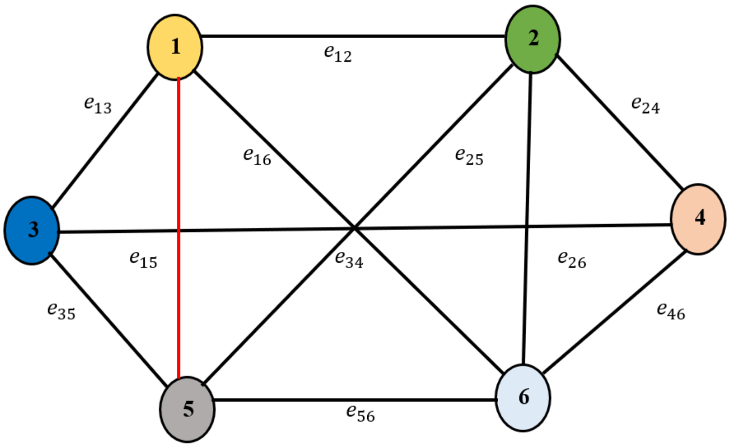

Figure 4.1.2 : Diagrammatic Presentation of Step 3

Step 4: After studying, the least value is 1.433 among the remaining weighted edges. Therefore the edge is selected connecting the nodes $(4,6)$ and label it.

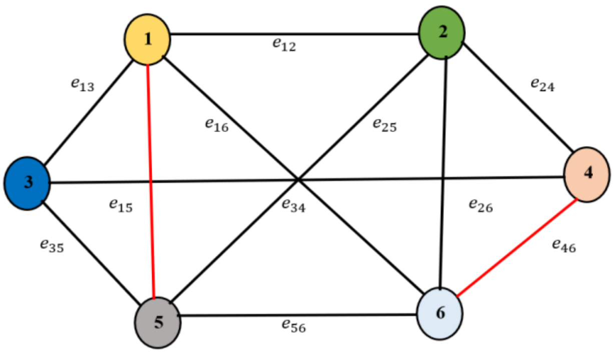

Figure 4.1.3: Diagrammatic Presentation of Step 4

Step 5: After examining, the least value is 1.504 amongst the remaining weighted edges. The edge is selected connecting nodes (1, 2) and thus marked it.

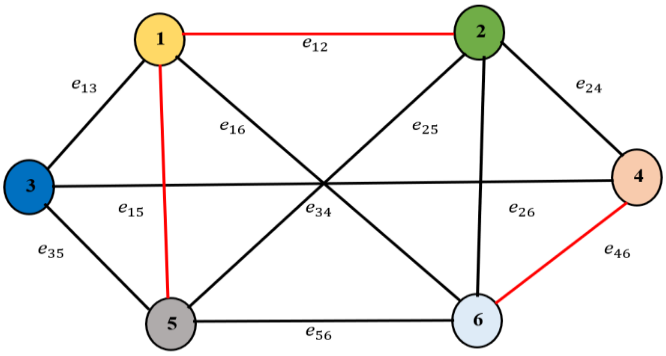

Figure 4.1.4: Diagrammatic Presentation of Step 5

Step 6: Examined that the least value is 1.542 among the remaining weighted edges. Hence, the edge is selected connecting with nodes (3, 5) and labelled it.

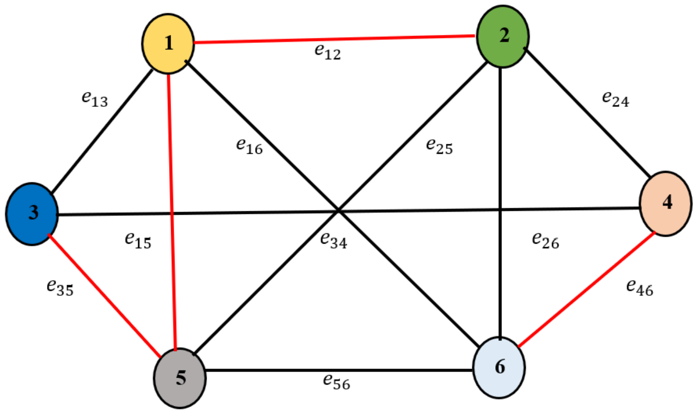

Figure 4.1.5: Diagrammatic Presentation of Step 6

Step 6: Examined that the least value is 1.570 out of the remaining weighted edges. Therefore, the edge is selected connecting the nodes $(2,6)$ and marked it.

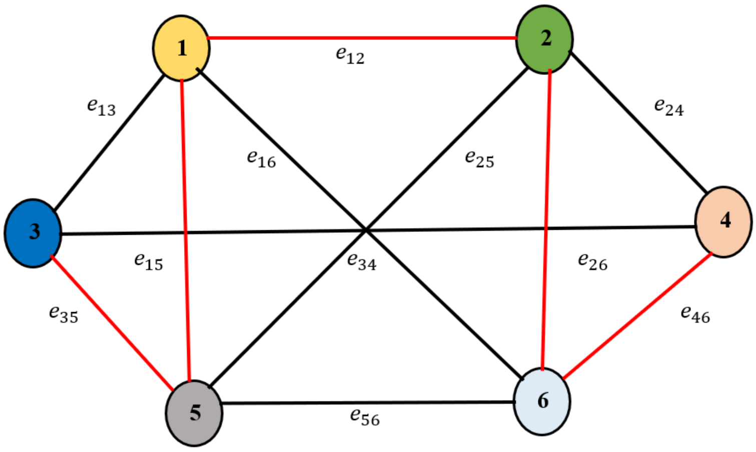

Figure 4.1.6: Diagrammatic Presentation of Step 7

Step 7: After examining all the nodes are joined and if more edges are to be joined it will form a circuit in the figure formed and as stated by the definition of a spanning tree it must not form any circuit but also all the nodes must be connected. Thus, the ultimate minimal spanning tree is followed:

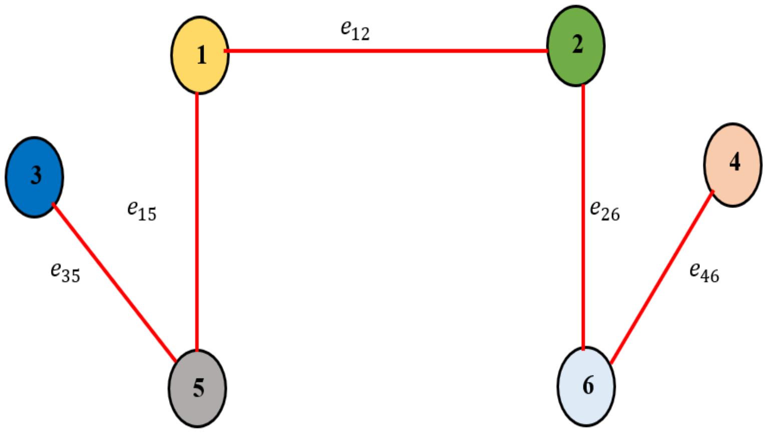

Figure 4.1.7: Diagrammatic Representation of ultimate minimal spanning tree

The least weight of the graph is – $(1.433 + 1.433 + 1.5041.542$+1.570$) = 7.482$ units.

5. Comparison:

Here, we compare our work with Mullai's [37] established algorithm. According to previous concept, the required minimal spanning tree can be obtained from the following steps.

Step 1: Let $S_1 = \{1\}\ then\ \bar{S_1} = \{2,3,4,5,6\}$

Step 2: Let $S_2 = \{1,5\}\ then\ \bar{S_2} = \{2,3,4,6\}$

Step 3: Let $S_3 = \{1,5,2\}\ then\ \bar{S_3} = \{3,4,6\}$

Step 4: Let $S_4 = \{1,5,2,3\}\ then\ \bar{S_4} = \{4,6\}$

Step 5: Let $S_5 = \{1,5,2,3,6\}\ then\ \bar{S_5} = \{4\}$

Step 6: Let $S_6 = \{1,5,2,3,6,4\}\ then\ \bar{S_6} = \{\phi\}$

The Required spanning tree is

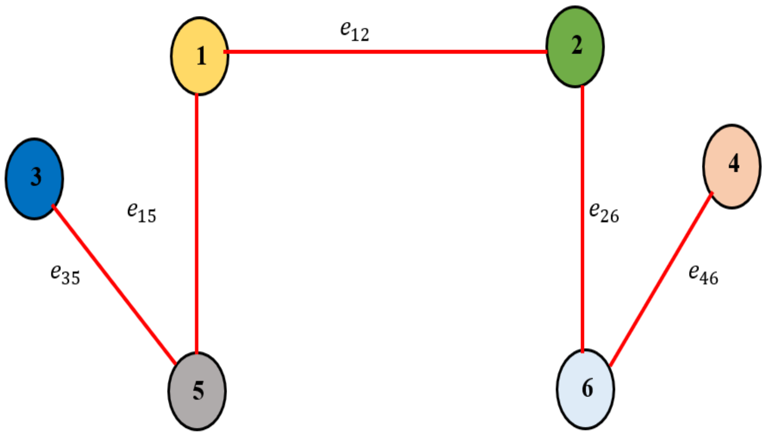

Figure 5.1: Diagrammatic Presentation of minimal spanning tree

Discussion: There is a contrast among the proposed approach and Mullai's technique is that Mullai's formulation is based on edges which are repeatedly evaluated at every steps of the algorithm which leads to the increase of time complexity. However, our technique relating with Matrix can be skillfully handled by utilizing Matlab software. In Mullai's method we need to consider each steps one by one manually but in our proposed method we can solve it using the help of computational software available in mathematics field with the help of computer as it is totally based on matrix concept. Thus we can claim that it will more useful and short time taking approach than any other established algorithm in this research domain.

6. Conclusion

In this research article, the concept of pentagonal neutrosophic number has been developed in a different aspect. Demonstration of De-Neutrosophication method utilizing the removal area technique has been introduced here for conversion of a pentagonal neutrosophic number into a real number. Further, this result is applied in the field of graph theory to evaluate the minimal spanning tree of a general graph. Comparison analysis is done with the established method which gave a crucial impact in this article for the evaluation of minimal spanning tree. Since, no work has been developed in this field so we can claim that this is the best method. Though the stated algorithm able to analyze the solutions of minimal spanning tree problem in pentagonal neutrosophic domain but more reliable, logical and short time taking algorithm maybe established in this field such that it can gives us much more fast, accurate and exact results after the total computation. Thus, these are the limitations of this stated algorithm in neutrosophic scenario.

In future, researchers can developed some interesting algorithms using pentagonal neutrosophic number in various fields like multi criteria decision making problem, image processing problem, pattern recognition problem, cloud computing problem and other mathematical modeling problems. Again, researcher may develop some new structural formulations of pentagonal neutrosophic number in different aspects. Also, researchers can compare this work with the new invented concept in pentagonal neutrosophic environment.

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Cyclic Associative Groupoids (CA-Groupoids) and Cyclic Associative Neutrosophic Extended Triplet Groupoids (CA-NET-Groupoids)

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Abstract: Group is the basic algebraic structure describing symmetry based on associative law. In order to express more general symmetry (or variation symmetry), the concept of group is generalized in various ways, for examples, regular semigroups, generalized groups, neutrosophic extended triplet groups and AG-groupoids. In this paper, based on the law of cyclic association and the background of non-associative ring, left weakly Novikov algebra and CA-AG-groupoid, a new concept of cyclic associative groupoid (CA-groupoid) is firstly proposed, and some examples and basic properties are presented. Moreover, as a combination of neutrosophic extended triplet group (NETG) and CA-groupoid, the notion of cyclic associative neutrosophic extended triplet groupoid (CA-NET-groupoid) is introduced, some important results are obtained, particularly, a decomposition theorem of CA-NET-groupoid is proved.

Keywords: Cyclic associative groupoid (CA-groupoid); CA-AG-groupoid; neutrosophic extended triplet group (NETG); CA-NET-groupoid; Decomposition theorem

1. Introduction

For algebraic operations, the associative law is very important, and it also characterizes the symmetry of operation: since from $(ab)c = a(bc)$, turn it upside down, we have $(cb)a = c(ba)$. This is also associative, that is, symmetry. Based on associative law, the concept of group is studied as basic algebraic structure describing symmetry. In order to express more general symmetry (or variation symmetry), group is generalized in various ways, for examples, regular semigroups, generalized groups, neutrosophic extended triplet groups and AG-groupoids (see [1, 16, 17, 22-24, 32]).

In many fields (such as non-associative rings and non-associative algebras [5, 18, 20, 21]), image processing [14] and networks [7]), non-associativity has important research significance. This paper focuses on non-associative algebraic structures satisfying the following operation law:

$$x(yz) = z(xy). \qquad \text{(Cyclic associative law)}$$

As early as 1995, M. Kleinfeld studied the rings with $x(yz) = z(xy)$ in [13], this research comes from the study of Novikov rings. After then, A. Behn, I. Correa, I. R. Hentzel and D. Samanta further investigated this kind of ring and algebra in [2, 3, 19]. Moreover, Zhan and Tan [34] introduced the notion of left weakly Novikov algebra: a non-associative algebra is called left weakly Novikov if it satisfies

$$(xy)z = (zx)y. \qquad \text{(Left weakly Novikov law)}$$

Obviously, the equation above is antithetical parallelism of the cyclic associative law (turn it upside down, $y(xz) = z(yx)$, that is cyclic associative).

Not only that, cyclic associativity is also applied to the research of AG-groupoids: in 2016, M. Iqbal, I. Ahmad, M. Shah and M.I. Ali [11] proposed the notion of cyclic associative AG-groupoid (CA-AG-groupoid), some new results are obtained in [9, 10].

Since cyclic associative law is widely used in algebraic systems, so we focus on basic algebra structure endow with a binary operation satisfying cyclic associative law in this paper, call it cyclic associative groupoid (CA-groupoid). We will also study the relationships between CA-groupoids and other related algebraic structures (see [4, 8, 12, 15, 26-31]).

The rest of this paper is organized as follows: in Section 2, we give some basic concepts and properties on semigroup, AG-groupoid and neutrosophic extended triplet groupoid (NETG); in Section 3, we give the definition of CA-groupoid and some interesting examples; in Section 4, we discuss the basic properties of CA-groupoids and analyze the relationships among some related algebraic systems; specially, we prove that every CA-groupoid with a left (or right) identity element is a commutative semigroup; in Section 5, we propose the new notion of cyclic associative neutrosophic extended triplet groupoid (CA-NET-groupoid), investigate basic properties of CA-NET-groupoids, and prove the composition theorem of CA-NET-groupoids.

2. Preliminaries

In this paper, a groupoid means that an algebraic structure consisting of a non-empty set with a single binary operation acting on it.

Let $(S, \cdot)$ be a groupoid. Some concepts are defined as follows (traditionally, the dot operator is omitted without confusion):

(1) S is called left nuclear square if for any $a, b, c\in S$, $a^2(bc) = (a^2b)c$; middle nuclear square if $a(b^2c) = (ab^2)c$; right nuclear square if $a(bc^2) = (ab)c^2$. S is called nuclear square if it is left, middle, and right nuclear square.

(2) S is called a Bol*-groupoid if $(\forall a, b, c, d\in S)$ $a((bc)d) = ((ab)c)d$.

(3) S is called left alternative if for all $a, b\in S$, $(aa)b = a(ab)$; and is called right alternative if $b(aa)= (ba)a$. S is called alternative, if it is both left alternative and right alternative.

(4) S is called right commutative if for all $a, b, c\in S$, $a(bc) = a(cb)$; and is called left commutative if $(ab)c = (ba)c$. S is called bi-commutative groupoid, if it is right and left commutative.

(5) An element $a\in S$ is called idempotent if $a^2=a$.

(7) S is called transitively commutative if $ab = ba$ and $bc = cb$ implies $ac = ca$ for all $a, b, c\in S$.

(8) S is called semigroup, if for any $a, b, c\in S$, $a(bc) = (ab)c$. A semigroup $(S, \cdot)$ is commutative, if for all $a, b\in S$, $ab = ba$. A semigroup $(S, \cdot)$ is called band, if for all $a\in S$, $a^2=a$.

Definition 1. ([24]) Assume that $(S, \cdot)$ is a groupoid. S is called an Abel-Grassmann's groupoid (or simply AG-groupoid), if S satisfying the left invertive law:

$$\forall a, b, c\in S,\ (ab)c = (cb)a.$$

For any AG-groupoid $(S, \cdot)$, the medial law holds, that is,

$$(ab)(cd) = (ac)(bd),\ \forall a, b, c\in S.$$

Definition 2. ([10, 11]) Let $(S, \cdot)$ be an AG-groupoid. (1) S is called an AG*-groupoid, if $(ab)c = b(ac)$ for all $a, b, c\in S$. (2) S is called an AG**-groupoid, if $(\forall a, b, c\in S)$ $a(bc) = b(ac)$. (3) S is called an T^1-AG-groupoid, if $(\forall a, b, c, d\in S)$ $ab = cd \Rightarrow ba = dc$.

Definition 3. ([22, 23]) Suppose that N is a non-empty set and $\cdot$ is a binary operation on N. If for any $a\in N$, there exist $neut(a)$, $anti(a)\in N$ such that

$$neut(a) \cdot a = a \cdot neut(a) = a;$$

$$anti(a) \cdot a = a \cdot anti(a) = neut(a).$$

Then $(N, \cdot)$ is called a neutrosophic extended triplet set, $neut(a)$ is called a neutral of "a", $anti(a)$ is called an opposite of "a", and $(a, neut(a), anti(a))$ is called a neutrosophic extended triplet.

Definition 4. ([22, 23]) Assume that $(N, \cdot)$ is a neutrosophic extended triplet set. If

(1) $(N, \cdot)$ is well-defined, that is, $(\forall a, b\in N)$ $a\cdot b\in N$.

(2) $(N, \cdot)$ is associative, that is, $(\forall a, b, c\in N)$ $(a\cdot b)\cdot c=a\cdot(b\cdot c)$.

Then, $(N, \cdot)$ is called a neutrosophic extended triplet group (NETG).

Theorem 1. ([30, 32]) Suppose that $(N, \cdot)$ is a neutrosophic extended triplet group (NETG). Then $(\forall a \in N)$ $neut(a)$ is unique.

3. Cyclic Associative Groupoids (CA-Groupoids)

Definition 5. Assume that $(S, \cdot)$ is a groupoid. If

$$a\cdot(b\cdot c) = c\cdot(a\cdot b),\ \forall\ a, b, c \in S,$$

then $(S, \cdot)$ is called a cyclic associative groupoid (shortly, CA-groupoid). By convention, operator $\cdot$ can be omitted without confusion.

Example 1. Considering the regular pentagon as shown in Figure 1, the center is at the origin of the x-y plane and the bottom side is parallel to the x-axis, the vertices are labeled a, b, c, d, e.

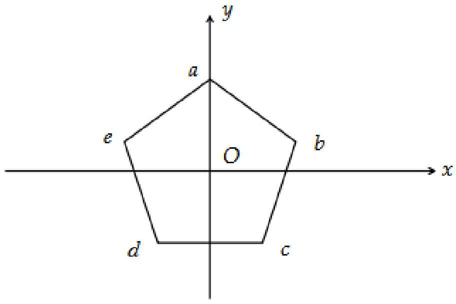

Figure 1. Regular pentagon

Denote $S=\{I, R, R^2, R^3, R^4\}$, representing some transformations of the regular pentagon, where I is 0 degrees clockwise around the center, R is 72 degrees clockwise around the center, R^2 is 144 degrees clockwise around the center, R^3 is 216 degrees clockwise around the center, R^4 is 288 degrees clockwise around the center. Define binary operation as a composition of functions in S, for arbitrary $U, V \in S$, $U \circ V$ is that the first transforming V and then transforming U. We can verity that $(S, \circ)$ is a CA-groupoid, the Cayley table can be presented as Table 1.

Table 1. The operation $\circ$ on $S = \{I, R, R^2, R^3, R^4\}$

$\circ$	$\boldsymbol{I}$	$\boldsymbol{R}$	$\boldsymbol{R^2}$	$\boldsymbol{R^3}$	$\boldsymbol{R^4}$
$\boldsymbol{I}$	I	R	R^2	R^3	R^4
$\boldsymbol{R}$	R	R^2	R^3	R^4	I
$\boldsymbol{R^2}$	R^2	R^3	R^4	I	R
$\boldsymbol{R^3}$	R^3	R^4	I	R	R^2
$\boldsymbol{R^4}$	R^4	I	R	R^2	R^3

Example 2. Suppose that Z is the set of all integer and $n \in Z$. Denote $W_n = \{a^2+nb^2 \mid a, b \in Z\}$, then $(W_n, \cdot)$ is a CA-groupoid, where $\cdot$ is the normal multiplication. In fact, for arbitrary element $w_1 = a_1^2+nb_1^2$, $w_2= a_2^2+nb_2^2$, $w_3 = a_3^2+nb_3^2 \in W_n$, we have

$$w_1\cdot(w_2\cdot w_3) = (a_1a_2a_3 - na_1b_2b_3 - nb_1a_2b_3 - nb_1b_2b_3)^2+n(a_1a_2b_3 + a_1b_2a_3 + b_1a_2a_3 - nb_1b_2b_3)^2 = w_3\cdot(w_1\cdot w_2).$$

Moreover, the result above can be extended and applied to solving binary indefinite equation, please see [6, 25]. We can obtain the following results (the proof is omitted).

Proposition 1. (1) Every commutative semigroup is a CA-groupoid. (2) Assume that $(S, \cdot)$ is a CA-groupoid. If S is commutative, then S is a commutative semigroup.

The following example shows that there exists CA-groupoid which is not a semigroup and not an AG-groupoid.

Example 3. Suppose $S = \{1, 2, 3, 4\}$, define a binary operation $\cdot$ on S in Table 2. Then, $(S, \cdot)$ is a CA-groupoid.

Table 2. The operation · on S

·	**1**	**2**	**3**	**4**
1	1	1	1	1
2	1	1	2	1
3	1	1	4	2
4	1	1	2	1

Moreover, S is not a AG-groupoid because $(4\cdot3)\cdot3 \neq (3\cdot3)\cdot4$. S isn't a semigroup because $(3\cdot4)\cdot 3 \neq 3\cdot(4\cdot3)$.

From the following example, we know that there exists CA-groupoid which is a semigroup and but it is not commutative.

Example 4. Assume $S = \{1, 2, 3, 4\}$, define a binary operation · on S by Table 3. Then, $(S, \cdot)$ is a CA-groupoid, and $(S, \cdot)$ is a semigroup, but · is not commutation because $2\cdot4 \neq 4\cdot2$.

Table 3. The operation · on S

·	**1**	**2**	**3**	**4**
1	1	1	1	1
2	1	3	1	3
3	1	1	1	1
4	1	1	1	3

Example 5. ([2]) Let A be an algebra (i.e. A be a linear space over a field F) with basis x_1, x_2, x_3, x_4, x_5, and the following nonzero products of basis elements

$$x_2\cdot x_1 = x_3,\ x_4\cdot x_2 = x_3,\ x_5\cdot x_1 = -x_3,\ x_3\cdot x_1=x_4,\ x_3\cdot x_2=x_5. \qquad \text{(NZP)}$$

For any $a, b \in A$, denote $a=\sum_{i=1}^{5} a_i x_i$, $b=\sum_{j=1}^{5} b_j x_j$, where $a_i, b_j \in F$ (i, j=1, 2, 3, 4, 5), then

$$a\cdot b = (\sum_{i=1}^{5} a_i x_i)\cdot(\sum_{j=1}^{5} b_j x_j) = a_2b_1x_3 + a_3b_1x_4 + a_3b_2x_5 + a_4b_2x_3 - a_5b_1x_3$$
$$= (a_2b_1 + a_4b_2 - a_5b_1)x_3 + a_3b_1x_4 + a_3b_2x_5$$

This means that $A^2=<x_3, x_4, x_5>$. Moreover, $AA^2=0$, since for any $c\in A$, $c=\sum_{k=1}^{5} c_k x_k$, where $c_k\in F$ (k=1, 2, 3, 4, 5),

$$c\cdot(a\cdot b) = (\sum_{k=1}^{5} c_k x_k)\cdot[(a_2b_1 + a_4b_2 - a_5b_1)x_3 + a_3b_1x_4 + a_3b_2x_5]$$

Note that, all nonzero products of basis elements are presented in (NZP), therefore, other products of basis elements are zero, that is, $x_1\cdot x_3 = x_1\cdot x_4 = x_1\cdot x_5 = \ldots = 0$. Hence, $(A, \cdot)$ is a CA-groupoid, since it satisfies the stronger identity $a\cdot(b\cdot c) = 0 = c\cdot(a\cdot b)$, $\forall a, b, c\in A$.

Example 6. ([2]) Let $N = \{x_1, x_2, x_3, \ldots\}$ a countably infinite set of indeterminates, for any element $x_i\in N$, call it is a letter. Denote P that is the set of the words in the letters x_i such that each letter occurs at most once in each word. For any word $u\in P$, if it is formed by k letters x_i, then say that u has length k, denote by length(u) = k. Obviously, length(u)≥1 for any $u\in P$. Suppose K is a field and A is the set of finite formal sums of words of P and with coefficient in K. For any $u, v\in P$, define multiplication · by:

(1) $u\cdot v = 0$, if length(v) >1, $u=v$ or v is a letter that is in the composition of u;

(2) $u\cdot v = uv$, if v is a letter that is not in the composition of u, where uv is the word obtained adding the letter v at the end of the word u.

For any $a, b\in A$, denote $a=\sum_{i=1}^{m} a_i p_i$, $b=\sum_{j=1}^{n} b_j q_j$, where $a_i, b_j\in K$, $p_i, q_j\in P$ (i=1, 2, …, m, j=1, 2, …, n), then

$$a\cdot b = (\sum_{i=1}^{m} a_i p_i)\cdot(\sum_{j=1}^{n} b_j q_j) = \sum_{t\leq nm} d_t u_t$$

Where, $d_t \in K$, $u_t \in P$. By the definition of the multiplication in A, $u_t = 0$ or length(u_t)>1. Therefore, $AA^2=0$, since for any $c \in A$, $c=\sum_{s=1}^{l} c_s v_s$, where $c_s \in K$, $v_s \in P$ (s=1, 2, …, l),

$$c \cdot (a \cdot b) = (\sum_{s=1}^{l} c_s v_s) \cdot (\sum_{t \le mn} d_t u_t) = 0$$

Hence, $(A, \cdot)$ is a CA-groupoid, since it satisfies the stronger identity $a\cdot(b\cdot c) = 0 = c\cdot(a\cdot b)$, $\forall a, b, c \in A$.

Example 7. Let S = [1, 2] (real number interval). For any $a, b \in S$, define the multiplication $\cdot$ by

$$a \cdot b = \begin{cases} a+b-1, & \text{if } \ a+b \le 3 \\ a+b-2, & \text{if } \ a+b > 3 \end{cases}$$

Then $(S, \cdot)$ is a CA-groupoid, since it satisfies $a\cdot(b\cdot c) = c\cdot(a\cdot b)$, $\forall a, b, c \in S$, the proof is as follows:

Case 1: $a+b+c-1 \le 3$. It follows that $b+c \le a+b+c-1 \le 3$ and $a+b \le a+b+c-1 \le 3$. Then $a\cdot(b\cdot c) = a\cdot(b+c-1) = a+b+c-2 = c\cdot(a+b-1) = c\cdot(a\cdot b)$.

Case 2: $a+b+c-1>3$, $b+c \le 3$ and $a+b \le 3$. Then $a\cdot(b\cdot c) = a\cdot(b+c-1) = a+b+c-3 = c\cdot(a+b-1) = c\cdot(a\cdot b)$.

Case 3: $a+b+c-1>3$, $b+c \le 3$ and $a+b>3$. It follows that $a+b+c-2 \le a+3-2 = a+1 \le 3$. Then $a\cdot(b\cdot c) = a\cdot(b+c-1) = a+b+c-3 = c\cdot(a+b-2) = c\cdot(a\cdot b)$.

Case 4: $a+b+c-1>3$, $b+c>3$ and $a+b \le 3$. It follows that $a+b+c-2 \le 3+c-2 = c+1 \le 3$. Then $a\cdot(b\cdot c) = a\cdot(b+c-2) = a+b+c-3 = c\cdot(a+b-1) = c\cdot(a\cdot b)$.

Case 5: $a+b+c-1>3$, $b+c>3$ and $a+b>3$. When $a+b+c-2 \le 3$, $a\cdot(b\cdot c) = a\cdot(b+c-2) = a+b+c-3 = c\cdot(a+b-2) = c\cdot(a\cdot b)$; When $a+b+c-2>3$, $a\cdot(b\cdot c) = a\cdot(b+c-2) = a+b+c-4 = c\cdot(a+b-2) = c\cdot(a\cdot b)$.

4. Some Properties of CA-Groupoids

Proposition 2. If $(S, \cdot)$ is a CA-groupoid, then, for any,

(1) $\forall a, b, c, d \in S$, $(ab)(cd) = (da)(cb)$;

(2) $\forall a, b, c, d, x, y \in S$, $(ab)((cd)(xy)) = (da)((cb)(xy))$.

Proof. Assume that $a, b, c, d, x, y \in S$, by Definition 5 we have

$$(ab)(cd) = d((ab)c) = c(d(ab)) = c(b(da)) = (da)(cb).$$

$$(ab)((cd)(xy)) = (xy)((ab)(cd)) = (xy)((da)(cb)) = (da)((cb)(xy)). \ \square$$

Theorem 2. Let $(S, \cdot)$ be a CA-groupoid.

(1) If S have a left identity element, that is, there exists $e \in S$ such that $e\cdot a=a$ for all $a \in S$, then S is a commutative semigroup.

(2) If $e \in S$ is a left identity element in S, then $e \in S$ is an identity element in S.

(3) If $e \in S$ is a right identity element in S, that is, $a\cdot e=a$ for all $a \in S$, then $e \in S$ is an identity element in S.

(4) If S have a right identity element, then S is a commutative semigroup.

Proof. (1) Suppose $a, b \in S$, $a\cdot b = a\cdot(e\cdot b) = b\cdot(a\cdot e) = e\cdot(b\cdot a) = b\cdot a$. It follows that $(S, \cdot)$ is a commutative CA-groupoid. By Proposition 1 (2) we know that $(S, \cdot)$ is a commutative semigroup.

(2) Assume that $e \in S$ is a left identity element in S, then for any $a \in S$, $a\cdot e = a\cdot(e\cdot e) = e\cdot(a\cdot e) = e\cdot(e\cdot a) = e\cdot a = a$. This means that $e \in S$ is an identity element in S.

(3) Assume that $e \in S$ is a right identity element in S, then for any $a \in S$, $e\cdot a = e\cdot(a\cdot e) = e\cdot(e\cdot a) = a\cdot(e\cdot e) = a\cdot e = a$. This means that $e \in S$ is an identity element in S.

(4) It follows from (1) and (3). $\square$

Theorem 3. Let $(S, \cdot)$ be a semigroup.

(1) When S is right commutative CA-groupoid, S is an AG-groupoid.

(2) When S is right commutative CA-groupoid, S is left commutative CA-groupoid.

(3) When S is left commutative CA-groupoid, S is right commutative CA-groupoid

(4) When S is left commutative CA-groupoid, S is an AG-groupoid.

(5) When S is left commutative AG-groupoid, S is an CA-groupoid.

(6) When S is left commutative AG-groupoid, S is right commutative AG-groupoid.

(7) When S is right commutative AG-groupoid, S is left commutative AG-groupoid.

(8) When $(S, *)$ is right commutative AG-groupoid, S is an CA-groupoid.

Proof. (1) If $(S, \cdot)$ is right commutative CA-groupoid, then, $\forall a, b, c \in S$, $(a \cdot b) \cdot c = a \cdot (b \cdot c) = c \cdot (a \cdot b) = c \cdot (b \cdot a)$. It follows that $(S, \cdot)$ is an AG-groupoid by Definition 1.

(2) If $(S, \cdot)$ is right commutative CA-groupoid, then, $\forall a, b, c \in S$, $(a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot (c \cdot b) = b \cdot (a \cdot c) = (b \cdot a) \cdot c$. That is, $(S, \cdot)$ is left commutative CA-groupoid.

(3) Assume that $(S, \cdot)$ is left commutative CA-groupoid. Then, for any $a, b, c \in S$, $a \cdot (b \cdot c) = c \cdot (a \cdot b) = (c \cdot a) \cdot b = (a \cdot c) \cdot b = a \cdot (c \cdot b)$. This means that $(S, \cdot)$ is right commutative CA-groupoid.

(4) It follows from (1) and (3).

(5) Suppose that $(S, \cdot)$ is left commutative AG-groupoid. Then, for any $a, b, c \in S$,

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c = (b \cdot a) \cdot c = c \cdot (a \cdot b).$$

Using Definition 5, $(S, \cdot)$ is a CA-groupoid.

(6) If $(S, \cdot)$ is left commutative AG-groupoid, then, $\forall a, b, c \in S$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c = (c \cdot b) \cdot a = (b \cdot c) \cdot a = (a \cdot c) \cdot b = a \cdot (c \cdot b)$. That is, $(S, \cdot)$ is right commutative AG-groupoid.

(7) If $(S, \cdot)$ is right commutative AG-groupoid, then, $\forall a, b, c \in S$,

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) = a \cdot (c \cdot b) = (a \cdot c) \cdot b = (b \cdot c) \cdot a = b \cdot (c \cdot a) = b \cdot (a \cdot c) = (b \cdot a) \cdot c.$$

This means that $(S, \cdot)$ is left commutative AG-groupoid.

(8) It follows from (5) and (7). □

Example 8. Let $S = \{a, b, c, d\}$. Define the operate $\cdot$ on S in Table 4. Then, $(S, \cdot)$ is a CA-groupoid, but isn't a CA-AG-groupoid because $(b \cdot d) \cdot d \neq (d \cdot d) \cdot b$.

Table 4. The operation on S

·	**a**	**b**	**c**	**d**
a	a	a	a	a
b	a	a	a	b
c	a	a	c	c
d	a	a	c	c

Example 9. Let $S = \{a, b, c, d, e\}$. Define the operate $\cdot$ on S in Table 5. Then, $(S, \cdot)$ is a CA-AG-groupoid, and $(S, \cdot)$ is not a semigroup, because $(a \cdot a) \cdot a \neq a \cdot (a \cdot a)$.

Table 5. The operation on S

·	**a**	**b**	**c**	**d**	**e**
a	b	c	c	c	e
b	d	c	c	c	e
c	c	c	c	c	e
d	c	c	c	c	e
e	e	e	e	e	c

From Proposition 1, Theorem 3, Example 4, Example 8 and Example 9, we know the relationships among some algebraic systems, we can present as Figure 2.

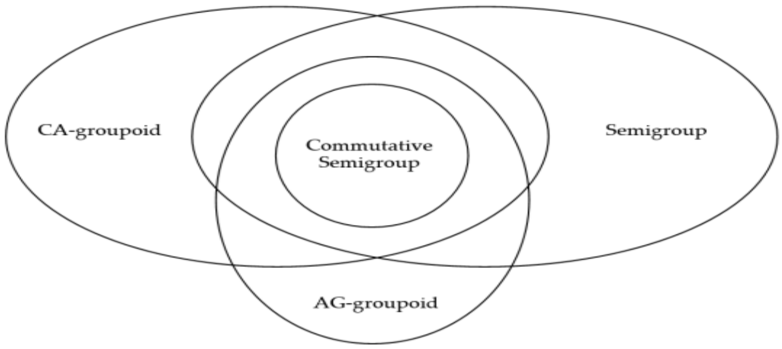

Figure 2. The relationships among some algebraic systems

Theorem 4. Let $(S, \cdot)$ be a CA-groupoid. If for all $a\in S$, $a^2=a$, then S is commutative.

Proof. Suppose that $(S, \cdot)$ is a CA-groupoid and $\forall a, b\in S$, we have

$$a\cdot b=(a\cdot a)\cdot(b\cdot b)=b\cdot((a\cdot a)\cdot b)=b\cdot(b\cdot(a\cdot a))=b\cdot(a\cdot(b\cdot a))=(b\cdot a)\cdot(b\cdot a)=b\cdot a;$$

hence S is commutative.

It follows that $(S, \cdot)$ is a commutative CA-groupoid, and it is a commutative semigroup.

Definition 6. Let $(S_1, \cdot_1)$ and $(S_2, \cdot_2)$ be two CA-groupoids, $S_1\times S_2 = \{(a, b)\mid a\in S_1, b\in S_2\}$. Define binary operation $\cdot$ on $S_1\times S_2$ as following: $(a_1, a_2)\cdot(b_1, b_2) = (a_1\cdot_1 b_1, a_2\cdot_2 b_2)$, $\forall(a_1, a_2), (b_1, b_2)\in S_1\times S_2$.

$(S_1\times S_2, \cdot)$ is called the direct product of $(S_1, \cdot_1)$ and $(S_2, \cdot_2)$, and S_1 and S_2 are called the direct factors of $S_1\times S_2$.

Theorem 5. Let $(S_1, \cdot_1)$ and $(S_2, \cdot_2)$ be two CA-groupoids. Then the direct product $(S_1\times S_2, \cdot)$ defined in Definition 7 is a CA-groupoid.

Proof. If $(a_1, a_2), (b_1, b_2), (c_1, c_2)\in S_1\times S_2$, then

$$(a_1, a_2)\cdot((b_1, b_2)\cdot(c_1, c_2)) = (a_1, a_2)\cdot(b_1\cdot_1 c_1, b_2\cdot_2 c_2) = (a_1\cdot_1(b_1\cdot_1 c_1), a_2\cdot_2(b_2\cdot_2 c_2))$$
$$= (c_1\cdot_1(a_1\cdot_1 b_1), c_2\cdot_2(a_2\cdot_2 b_2)) = (c_1, c_2)\cdot(a_1\cdot_1 b_1, a_2\cdot_2 b_2) = (c_1, c_2)\cdot((a_1, a_2)\cdot(b_1, b_2)).$$

Hence, $(S_1\times S_2, \cdot)$ is a CA-groupoid.

5. Cyclic Associative Neutrosophic Extended Triplet Groupoids (CA-NET-Groupoids)

In this section, we mainly study a class of important CA-groupoids, called CA-NET-groupoids. The research ideas are derived from regular semigroups in classical semigroup theory and the recent research results on neutrosophic extended triplet groupoids (NETGs, see [15, 22-23, 26, 30, 32-33]). After giving the basic definitions and properties, this section focuses on the structure of CA-NET-groupoids. The results show that every CA-NET-groupoid can be decomposed into disjoint union of some of its subgroups, which is actually an extension of the famous Clifford's theorem in semigroup theory.

Definition 7. Assume that $(N, \cdot)$ be a neutrosophic extended triplet set. If

(1) $(N, \cdot)$ is well-defined, that is, $(\forall a, b\in N)$ $a*b\in N$;

(2) $(N, \cdot)$ is cyclic associative, that is, $(\forall a, b, c\in N)$ $a\cdot(b\cdot c) = c\cdot(a\cdot b)$.

Then $(N, \cdot)$ is called a cyclic associative neutrosophic extended triplet groupoid (shortly, CA-NET-groupoid). A CA-NET-groupoid $(N, \cdot)$ is commutative, if $(\forall a, b\in N)$ $a\cdot b = b\cdot a$.

Theorem 6. If $(N, \cdot)$ is a CA-NET-groupoid and $a\in N$. Then the local unit element $neut(a)$ is unique in N.

Proof. Suppose that local unit element $neut(a)$ is not unique in S. Then, there exists $s, t\in\{neut(a)\}$ such that $(p, q\in N)$

$$as = sa = a \text{ and } ap=pa=s;\ at = ta = a \text{ and } aq = qa = t.$$

(1) $s = ts$. Since $s = pa = p(at) = t(pa) = ts$.

(2) $t = st$. Since $t = qa = q(as) = s(qa) = st$.

(3) $s = ss$ and $t = tt$. Since $s = pa = p(as) = s(pa) = ss$, and $t = qa = q(at) = t(qa) = tt$.

(4) $ts = st$. Since $ts = t(ts) = s(tt) = st$.

Hence s=t and $neut(a)$ is unique in N. □

From the following example, we know that $anti(a)$ may be not unique.

Example 10. Denote $N = \{1, 2, 3, 4\}$. Define the operate $\cdot$ on N in Table 6. Then, $(N, \cdot)$ is CA-NET-groupoid. Moreover, $neut(1) = 1$ and $\{anti(1)\} = \{1, 2, 3, 4\}$.

Table 6. The operation $\cdot$ on N

·	**1**	**2**	**3**	**4**
1	1	1	1	1
2	1	4	1	2
3	1	1	3	1
4	1	2	1	4

Theorem 7. If $(N, \cdot)$ be a CA-NET-groupoid, then

(1) $\forall a \in N$, $neut(a)neut(a) = neut(a)$;
(2) $\forall a \in N$, $neut(neut(a)) = neut(a)$;
(3) $\forall a \in N$, $\forall anti(neut(a)) \in \{anti(neut(a))\}$, $anti(neut(a))a = a$.

Proof. (1) By $a(anti(a)) = anti(a)a = neut(a)$, we get

$$neut(a)neut(a) = neut(a)(a(anti(a))) = anti(a)(neut(a)a) = anti(a)a = neut(a).$$

(2) For any $a \in N$, using the definition of $neut(neut(a))$ we have

$$neut(neut(a))neut(a) = neut(a)neut(neut(a)) = neut(a).$$

By the definition of $anti(neut(a))$ we have

$$anti(neut(a))neut(a) = neut(a)anti(neut(a)) = neut(neut(a)).$$

By (1) and Theorem 7, we get that $neut(neut(a)) = neut(a)$.

(3) Using Definition 5, Definition 8 and above (1), for all $a \in N$,

$$anti(neut(a))a = anti(neut(a))(neut(a)a) = a(anti(neut(a))neut(a)) = a(neut(neut(a))) = a(neut(a)) = a.$$

It follows that $anti(neut(a))a = a$. □

From the following example, $neut(anti(a))$ may be not equal to $neut(a)$.

Example 11. Denote $N = \{1, 2, 3, 4\}$. Define the operate $\cdot$ on N in Table 7. Then, $(N, \cdot)$ is CA-NET-groupoid. Moreover, $neut(1) = 1$, $neut(2) = 2$, $\{anti(1)\} = \{1, 2, 3, 4\}$. While $anti(1) = 2$, $neut(anti(a)) \neq neut(a)$, because $neut(anti(1)) = neut(2) = 2 \neq 1 = neut(1)$.

Table 7. The operation $\cdot$ on N

$\cdot$	**1**	**2**	**3**	**4**
1	1	1	1	1
2	1	2	1	4
3	1	1	3	1
4	1	4	1	2

Theorem 8. If $(N, \cdot)$ is a CA-NET-groupoid. Then
(1) $\forall a \in N$, $\forall p, q \in \{anti(a)\}$, $p(neut(a)) = q(neut(a))$;
(2) $\forall a \in N$, $\forall anti(a) \in \{anti(a)\}$, $anti(neut(a))anti(a) \in \{anti(a)\}$;
(3) $\forall a \in N$, $\forall q \in \{anti(a)\}$, $anti(q)neut(a) = a(neut(q))$;

Proof. (1) $\forall a \in N$, $\forall p, q \in \{anti(a)\}$, by the definition of neutral and opposite element, using Theorem 7, we get

$pa = ap = neut(a)$, $qa = aq = neut(a)$.

$p(neut(a)) = p(aq) = q(pa) = q(neut(a))$.

(2) $a \in N$, $\forall anti(a) \in \{anti(a)\}$, $\forall anti(neut(a)) \in \{anti(neut(a))\}$,

$a[anti(neut(a))anti(a)] = anti(a)\{a[anti(neut(a))]\} = anti(neut(a))[anti(a)a] = anti(neut(a))neut(a) = neut(neut(a)) = neut(a)$;

$[anti(neut(a))anti(a)]a = [anti(neut(a))anti(a)]\{a[neut(a)]\} = neut(a)\{[anti(neut(a))anti(a)]a\} = a\{neut(a)[anti(neut(a))anti(a)]\} = a\{anti(a)[neut(a)anti(neut(a))]\} = a[anti(a)neut(a)] = neut(a)[a(anti(a))] = neut(a)neut(a) = neut(a)$.

Thus, $anti(neut(a))anti(a) \in \{anti(a)\}$.

(3) $\forall a \in N$, $\forall q \in \{anti(a)\}$, by $aq = qa = neut(a)$ and $q(anti(q)) = anti(q)q = neut(q)$, we get

$a(neut(q)) = a[q(anti(q))] = anti(q)(aq) = anti(q)neut(a)$.

This shows that $anti(q)neut(a) = a(neut(q))$.

Proposition 3. If $(N, \cdot)$ is a CA-NET-groupoid. Then
(1) $\forall a, b, c \in N$, $ab = ac \Rightarrow b(neut(a)) = c(neut(a))$;
(2) $\forall a, b, c \in N$, $ba = ca$ *if and only if* $b(neut(a)) = c(neut(a))$.

Proof. (1) Assume $ab = ac$. For $a \in N$, by the definition of CA-NET-groupoid, $anti(a) \in N$. Multiply $anti(a)$ to the left side with $ab = ac$,

$anti(a)(ab) = anti(a)(ac)$, $b[anti(a)a] = c[anti(a)a]$, $b(neut(a)) = c(neut(a))$.

(2) Assume $ba = ca$. Then,
$anti(a)(ba) = anti(a)(ca)$, $a[anti(a)b] = a[anti(a)c]$, $b[a(anti(a))] = c[a(anti(a))]$, $b(neut(a)) = c(neut(a))$.
Conversely, suppose that $b(neut(a)) = c(neut(a))$. By Definition 5,
$a[b(neut(a))] = a[c(neut(a))]$, $neut(a)(ab) = neut(a)(ac)$, $b[neut(a)a] = c[neut(a)a]$, $ba = ca$.

Proposition 4. Suppose that $(N, \cdot)$ is a commutative CA-NET-groupoid. Then
$\forall\ a, b \in N$, $neut(a)neut(b) = neut(ab)$.

Proof. Because the local unit element of every element is unique in N, consider left hand side, $neut(a)neut(b)$. Now multiply to the left with ab,
$(ab)[neut(a)neut(b)] = neut(b)[(ab)neut(a)] = neut(a)[neut(b)(ab)] = neut(a)[b(neut(b)a)] =$
$= neut(a)\{a[b(neut(b))]\} = neut(a)(ab) = b[neut(a)a] = ba = ab$.
And multiply to the right with ab for $neut(a)neut(b)$, we can get
$[neut(a)neut(b)](ab) = b\{[neut(a)neut(b)]a\} = a\{b[neut(a)neut(b)]\} = a\{neut(b)[b(neut(a))]\} =$
$a\{neut(a)[neut(b)b]\} = a[neut(a)b] = b[a(neut(a))] = ba = ab$.
Therefore, $neut(a)neut(b) = neut(ab)$.

Definition 8. Let $(N, \cdot)$ be a CA-NET-groupoid. If $(\forall\ a, b \in N)$ $a(neut(b)) = neut(b)a$, then N is called a weak commutative CA-NET-groupoid (briefly, WC-CA-NET-groupoid).

Theorem 9. Assume that $(N, \cdot)$ is a CA-NET-groupoid. Then N is a commutative CA-NET-groupoid if and only if N is a weak commutative CA-NET-groupoid.

Proof. Suppose that N is a commutative CA-NET-groupoid. Obviously, N is a weak commutative CA-NET-groupoid. Conversely, if N is a weak commutative CA-NET-groupoid, then $(\forall\ a, b \in N)$

$$\begin{aligned} ab &= a[neut(b)b] = b[a(neut(b))] = neut(b)(ba) = neut(b)[b(neut(a)a)] = neut(b)\{a[b(neut(a))]\} \\ &= neut(b)[neut(a)(ab)] = (ab)[neut(b)neut(a)] = neut(a)[(ab)neut(b)] = neut(a)[neut(b)(ab)] \\ &= neut(a)[b(neut(b)a)] = neut(a)\{a(b[neut(b)]) = neut(a)(ab) = b[neut(a)a] = ba. \end{aligned}$$

Therefore, N is a commutative CA-NET-groupoid. □

Theorem 10. Suppose that $(N, \cdot)$ is a CA-NET-groupoid. Denote the set of all different neutral element in N by $E(N)$. For any $e \in E(N)$, denote $N(e) = \{a \in N \mid neut(a) = e\}$. Then

(1) $\forall\ e \in E(N)$, $N(e)$ is a subgroup of N.

(2) $\forall\ e_1, e_2 \in E(N), e_1 \neq e_2 \Rightarrow N(e_1) \cap N(e_2) = \phi$.

(3) $N = \bigcup_{e \in E(N)} N(e)$.

Proof. (1) $\forall\ x \in N(e)$, $neut(x) = e$. This means that e is an identity element in $N(e)$. Moreover, by Theorem 8 (1), $ee = e$.

If $x, y \in N(e)$, then $neut(x) = neut(y) = e$. We prove that $neut(xy) = e$. In fact, by Definition 5 and Proposition 2 (1) we have

$$(xy)e = (xy)(ee) = (ex)(ey) = xy;\ e(xy) = y(ex) = x(ye) = xy.$$

On the other hand, $\forall\ anti(x) \in \{anti(x)\}$, $\forall\ anti(y) \in \{anti(y)\}$, by Proposition 2 (1),

$$(xy)[anti(x)anti(y)] = (anti(y)x)(anti(x)y) = [y(anti(y))](anti(x)x) = neut(y)neut(x) = ee = e.$$

$$[anti(x)anti(y)](xy) = [y(anti(x))][x(anti(y))] = (anti(y)y)[x(anti(x))] = neut(y)neut(x) = ee = e.$$

Thus, by the definition of neutral element and Theorem 7, we know that $neut(xy) = e$. It follows that $xy \in N(e)$, that is, $N(e)$ is closed under operation $\cdot$.

Moreover, $\forall\ x \in N(e)$, there exists $q \in N$ and $q \in \{anti(x)\}$. Using Theorem 11 (1), $q(neut(x)) \in \{anti(x)\}$; and using Theorem 11 (5), $neut(q(neut(x))) = neut(x)$. Denote $t = q(neut(x))$, then

$$t = q(neut(x)) \in \{anti(x)\}, \text{ and } neut(t) = neut(q(neut(x))) = neut(x) = e.$$

This means that there exists $t \in \{anti(x)\}$, $neut(t) = e$, that is, $t \in N(e)$.

Combing above results, we know that $(N(e), \cdot)$ is a subgroup of N.

(2) Assume that $x \in N(e_1) \cap N(e_2)$ and $e_1, e_2 \in E(N)$. Then $neut(x) = e_1$, $neut(x) = e_2$. By Theorem 7 we get $e_1 = e_2$. Therefore, $e_1 \neq e_2 \Rightarrow N(e_1) \cap N(e_2) = \phi$.

(3) $\forall\ x \in N$, there exists $neut(x) \in N$. Denote $e = neut(x)$, then $e \in E(N)$ and $x \in N(e)$. This means that $N = \bigcup_{e \in E(N)} N(e)$. □

6. Conclusions

In this paper, the concept of cyclic associative groupoid (CA-groupoid) is introduced for the first time from various backgrounds, such as non-associative rings and non-associative algebras, weak Novikov algebras and CA-AG-groupoids. The research results of this paper show that CA-groupoid, as a non-associative algebraic structure, has typical representativeness and rich connotation, and is closely related to many kinds of algebraic structures. This paper obtains many interesting conclusions. Here are some important results：

(1) Every commutative semigroup is CA-groupoid, every commutative CA-groupoid is a semi-group. (see Example 1, 2 and Proposition 1)

(2) From some non-associative and non-commutative algebras (as vector spaces over fields), we can get some CA-groupoids. (see Example 5 and 6)

(3) Every CA-groupoid with left (or right) identity element is a commutative semigroup, every left cancellative element of a CA-groupoid is right cancellative. (see Theorem 2 and 4)

(4) CA-groupoids and AG-groupoids are closely related, but they do not contain each other. (see Theorem 3 and Figure 2)

(5) For cyclic associative neutrosophic extended triplet groupoids (CA-NET- groupoids), there are some interesting properties. (see Theorem 7, 8, 9 and 11)

(6) A CA-groupoid is weak commutative if and only if it is commutative CA-NET-groupoid. (see Definition 9 and Theorem 10)

(7) Every CA-NET- groupoid is a disjoint union of its subgroup. (Decomposition Theorem of the CA-NET-groupoids, see Theorem 12)

As a direction of future research, we'll investigate regularity, cancellability and the relationships among CA-groupoids, CA-NET-groupoids and related algebraic systems.

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An Integrated Neutrosophic and TOPSIS for Evaluating Airline Service Quality

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Abstract: This study applies the neutrosophic set theory to evaluate the service quality of airline. This research offers a novel approach for evaluating the service quality of airline under a group decision making (GDM) in a vague decision environment. The complexity of the selected decision criteria for the airline service quality is a significant feature of this analysis. To simulate these processes, a methodology that combines neutrosophic using bipolar numbers with Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) under GDM is suggested. Neutrosophic with TOPSIS approach is applied in the decision making process to deal with the vagueness, incomplete data and the uncertainty, considering the decisions criteria in the data collected by the decision makers (DMs). Service quality is a composite of various attributes, among them many intangible attributes are difficult to measure. This characteristic introduces the obstacles for respondent in replying to the survey. In order to overcome the issue, we invite neutrosophic set theory into the measurement of performance. We have introduced a real life example in the research of how to evaluate airline service according to opinion of experts. Through solution of a numerical example we present steps of how formulate problem in TOPSIS by neutrosophic. By applying TOPSIS in obtaining criteria weight and ranking, we found the most concerned aspects of service quality are tangible and the least is empathy. The most concerned attribute is courtesy, safety and comfort.

Keywords: Bipolar neutrosophic numbers; TOPSIS method; Service quality; Group decision making; Airline

1. Introduction

In Egypt, the air travel market, both domestic and international, have been experiencing great competition in recent years due to both the deregulation and the increasing of customers awareness of service quality. Under the situation, carriers endeavor to build up increasingly advantageous courses, yet in addition present progressively limited time motivations, including mileage rewards, long standing customer enrollment program, sweepstakes, etc. Carriers want to unite the piece of the pie and improve productivity. Nonetheless, the peripheral advantages of showcasing procedures step by step diminish on the grounds that the majority of the carriers demonstration also. Perceiving this confinement of the showcasing methodologies, some of air bearers currently will in general spotlight on the dedication of improving client administration quality. The air bearers give a scope of administrations to clients including ticket reservation, buy, airplane terminal ground administration, on-board administration and the administration at the goal.

Aircraft administration likewise comprises of the help related with interruptions, for example, lost-things taking care of and administration for deferred travelers. Administration quality can be viewed as a composite of different characteristics. It comprises of substantial traits, yet in addition elusive/emotional properties, for example, wellbeing, comfort, which are hard to quantify precisely. Diverse individual as a rule has wide scope of observations toward quality administration, contingent upon their inclination structures and jobs in procedure specialist organizations/recipients. To gauge administration quality, traditional estimation instruments are conceived on cardinal or ordinal scales. A large portion of the analysis about scale dependent on estimation is that scores don't really speak to client inclination. This is on the grounds that respondents need to inside proselyte inclination to scores and the transformation may present contortion of the inclination being caught.

Since administration industry contains elusiveness, perishability, connection and heterogeneity, it makes people groups progressively hard to gauge administration quality. To investigate the past related research record, a large portion of the strategies for assessing carrier administration quality utilizes measurements strategy. 5-point of Likert Scales is the significant method to assess administration quality previously.

These days, the neutrosophic set hypothesis has been connected to the field of the board science, similar to basic leadership nonetheless, it is hardly utilized in the field of administration quality. Lingual articulations, for instance, fulfilled, reasonable, disappointed, are viewed as the normal portrayal of the inclination or decisions. This study aims to suggest a set of valuation criteria for the service quality of airline in relationship to the selection of the best airlines. There are many resources that can be used for collecting the evaluation criteria, such as the judgments of academic experts, industrial and decision makers, the current scientific literature or available regulations and passengers. Decision making is mostly about choosing the preferable choice between a set of alternatives by considering the influence of many criteria altogether. In the last five decades, the multi criteria decision making (MCDM) methodology became one of the most important key in solving complicated and complex decision problems in the existence of multiple criteria and alternatives [1].

The MCDM methodology can be used to resolve multi valuation and ordering problems that combine a number of inconsistent criteria. After this progress, several types of MCDM methods are suggested to successfully solve various types of decision making problems. This powerful methodology often needs qualitative and quantitative data, which are used in the measurement of obtainable alternatives. In multi MCDM problems, interdependency, mutuality and interactivity features between decision criteria are of a vague nature, which obscures the task of a membership [2]. However, most methods proved inadequate and inappropriate in solving and explaining real life problems, mostly because they rely on crisp values. Many MCDM methods use the fuzzy or the intuitionistic fuzzy set theories to overcome this obstacle. Nevertheless, F and IF numbers are also not always appropriate. Classes of F and IF sets proved to be efficient in some implementations. Nevertheless, in our opinion that is a compromise, since the neutrosophic set offers major and better possibilities [3, 4-11].

The notion / concept of neutrosophic set provides a substitute approach where there is a lack of accuracy to the determinations imposed by the crisp sets or traditional fuzzy sets, and in situations where the presented information is not suitable to locate its inaccuracy. Neutrosophic sets are very powerful and successful in overcoming situations and cases in incomplete information environment, uncertainty, vagueness and imprecision, and it is described by a membership degree, an indeterminacy degree and a nonmembership degree [5]. Therefore, neutrosophic sets introduce a qualified tool for expressing DMs' preferences and priorities, completely determining the membership function in situations where DM opinions are subject to indeterminacy or lack of information. DMs use linguistic variables expressed in two parts, where the first part is employed to voice their preferences and the other part is used to convey the confirmation degree of linguistic variable according to each DM. Neutrosophic set is becoming a scientific key tool, receiving attention from many DMs and academic researchers for developing and improving the neutrosophic methodology.

The main accomplishments of this research are:

- The characterization and preparation of an effective evaluation framework to lead the marketing industry towards the suitable airline selection.
- It also contributes to the literature by providing a novel Neutrosophic with TOPSIS method under GDM setting, by considering the interactions among airlines selection criteria in a vague environment.

The research is organized as it is assumed up: Section 2 presents the TOPSIS method. Section 3 gives an insight into some basic definitions on neutrosophic sets. Section 4 explains the proposed methodology of neutrosophic TOPSIS group decision making model. Section 5 introduces numerical example. Finally, we close our research with some remarks.

2. TOPSIS

The TOPSIS was first proposed by Hwang and Yoon (1981). The hidden rationale of TOPSIS is to characterize the perfect arrangement and the negative perfect arrangement. The perfect arrangement is the arrangement that amplifies the advantage criteria and limits the cost criteria; while the negative perfect arrangement augments the cost criteria and limits the advantage criteria. The ideal option is the one, which is nearest to the perfect arrangement and most distant to the negative perfect arrangement. The positioning of choices in TOPSIS depends on 'the relative closeness to the perfect arrangement', which maintains a strategic distance from the circumstance of having same comparability to both perfect and negative perfect arrangements. To whole up, perfect arrangement is made out of every single best worth feasible of criteria, though negative perfect arrangement is comprised of every single most exceedingly awful worth achievable of criteria. During the procedures of elective determination, the best option would be the one that is closest to the perfect arrangement and most distant from the negative perfect arrangement.

3. Preliminaries

In this section, we give the fundamental meanings of neutrosophic set and bipolar neutrosophic numbers (BNNs).

Definition 1. A bipolar neutrosophic set A in X is defined as an object of the form A = {⟨x,T^+ (x), I^+ (x), F^+ (x), T^- (x), I^- (x), F^- (x) ⟩: x ∈ X}, where T^+, I^+, F^+ : X→ [1,0] and T^-, I^-, F^- : X→ [-1,0]. The positive membership degree T^+ (x), I^+ (x), F^+ (x) denotes the truth membership, the indeterminate membership and the false membership of an element x ∈ X corresponding to a bipolar neutrosophic set A, and the negative membership degree T^- (x), I^- (x), F^- (x) denotes the truth membership, the indeterminate membership and the false membership of an element x ∈ X to some implicit counter property corresponding to a bipolar neutrosophic set A.

Definition 2. Let A_1 = {⟨x, T_1^+(x),I_1^+ (x), F_1^+(x), T_1^-(x),I_1^- (x), F_1^-(x) ⟩ and A_2 = {⟨x, T_2^+(x),I_2^+ (x), F_2^+(x), T_2^-(x),I_2^- (x), F_2^-(x) ⟩ be two bipolar neutrosophic sets. Then, their union is defined as: $(A_1 \cup A_2)$(x)=(max(T_1^+(x) , T_2^+(x)),$\frac{I_1^+(x)+I_2^+(x)}{2}$, min(($F_1^+$(x) , F_2^+(x)), min(T_1^-(x), = T_2^+(x)) ,$\frac{I_1^-(x)+I_2^-(x)}{2}$, max(($F_1^-$(x) , F_2^-(x))), for all x ∈ X.

Definition 3. Let $\tilde{a}_1$= $(T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-)$ and $\tilde{a}_2$= $(T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^-)$ be two bipolar neutrosophic numbers. Then, the operations for NNs are defined as below:

$\lambda\tilde{a}_1 = (1-(1-T_1^+)^\lambda, (I_1^+)^\lambda, (F_1^+)^\lambda, -(-T_1^-)^\lambda, -(-I_1^-)^\lambda, -(1-(1-F_1^-))^\lambda))$

$\tilde{a}_1^\lambda = ((T_1^+)^\lambda, 1-(1-I_1^+)^\lambda, 1-(1-F_1^+)^\lambda, -(1-(1-T_1^-))^\lambda), -(I_1^-)^\lambda), -(-F_1^-)^\lambda)$

$\tilde{a}_1 + \tilde{a}_2 = (T_1^+ + T_2^+ - T_1^+ T_2^+, I_1^+ I_2^+, F_1^+ F_2^+, -T_1^- T_2^-, -(-I_1^- - I_2^- - I_1^- I_2^-), -(-F_1^- - F_2^- - F_1^- F_2^-))$

$\tilde{a}_1 . \tilde{a}_2 = (T_1^+ T_2^+, I_1^+ + I_2^+ - I_1^+ I_2^+ + F_1^+ + F_2^+ - F_1^+ F_2^+, -(-T_1^- - T_2^- - T_1^- T_2^-), -I_1^- I_2^-, -F_1^- F_2^-)$, where λ > 0.

Definition 4. Let $\tilde{a}_1$= $(T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-)$ be a bipolar neutrosophic number. Then, the score function s ($\tilde{a}_1$), accuracy function a ($\tilde{a}_1$) and certainty function c ($\tilde{a}_1$) of an NBN are defined as follows:

$$\tilde{S}(\tilde{a}_1) = (T_1^+ + 1 - I_1^+ + 1 - F_1^+ + 1 + T_1^- - I_1^- - F_1^-)/6 \quad (1)$$

$$\tilde{a}(\tilde{a}_1) = T_1^+ - F_1^+ + T_1^- - F_1^- \quad (2)$$

$$\tilde{c}(\tilde{a}_1) = T_1^+ - F_1^- \quad (3)$$

Definition 5. Let $\tilde{a}_1$= $(T_1^+, I_1^+, F_1^+, T_1^-, I_1^-, F_1^-)$ and $\tilde{a}_2$= $(T_2^+, I_2^+, F_2^+, T_2^-, I_2^-, F_2^-)$ be two bipolar neutrosophic numbers. The comparison method can be defined as follows:

- if $\tilde{s}(\tilde{a}_1) > \tilde{s}(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 > \tilde{a}_2$
- $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$ and $\tilde{a}(\tilde{a}_1) > \tilde{a}(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 < \tilde{a}_2$;
- if $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$, $\tilde{a}(\tilde{a}_1) = \tilde{a}(\tilde{a}_2)$) and $\tilde{c}(\tilde{a}_1) > \tilde{c}(\tilde{a}_2)$, then $\tilde{a}_1$ is greater than $\tilde{a}_2$, that is, $\tilde{a}_1$ is superior to $\tilde{a}_2$, denoted by $\tilde{a}_1 > \tilde{a}_2$;
- if $\tilde{s}(\tilde{a}_1) = \tilde{s}(\tilde{a}_2)$, $\tilde{a}(\tilde{a}_1) = \tilde{a}(\tilde{a}_2)$) and $\tilde{c}(\tilde{a}_1) = \tilde{c}(\tilde{a}_2)$, then $\tilde{a}_1$ is equal to $\tilde{a}_2$, that is, $\tilde{a}_1$ is indifferent to $\tilde{a}_2$, denoted by $\tilde{a}_1 = \tilde{a}_2$.

Definition 6. Let $\tilde{a}_j$= $(T_j^+, I_j^+, F_j^+, T_j^-, I_j^-, F_j^-)$ (*j* = 1, 2,…, *n*) be a family of bipolar neutrosophic numbers. A mappingA_ω: $Q_n \rightarrow Q$ is called bipolar neutrosophic weighted average operator if it satisfies the condition: $A_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} \omega_j \tilde{a}_j = \langle 1 - \prod_{j=1}^{n} (1 - T_j^+)^{\omega j}, \prod_{j=1}^{n} I_j^{+\omega j}, \prod_{j=1}^{n} F_j^{+\omega j}, -\prod_{j=1}^{n} (-T_j^-)^{\omega j}, -1(\prod_{j=1}^{n} (1-(-I_j^-))^{\omega j}), -(1-\prod_{j=1}^{n} (1-(-F_j^-))^{\omega j})$), where ω_j is the weight of $\tilde{a}_j$ (*j* = 1,2, … , *n*), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

4. Methodology

In this section, the steps of the suggested bipolar neutrosophic with TOPSIS framework are presented in details.

Step 1. Organize a committee of experts and determine the goal, the alternatives and the valuation criteria. Suppose that experts want to appreciate the collection of n criteria and m alternatives. Experts are symbolized by $Ex_E = \{Ex_1, Ex_2, Ex_3\}$, where E = 1, 2, ..., E, and alternatives by $A_i = \{A_1, A_2, \ldots, A_m\}$, where i = 1, 2, ..., m, assessed on n criteria $c_j = \{c_1, c_2, \ldots, c_n\}$, j = 1, 2, ..., n.

Step 2. Depict and design the linguistic scales to describe experts, and set the alternatives.

Step 3. Obtain experts' judgments on each element.

Based on previously knowledge and experience, experts are demanded to convey their judgments. Every expert gives his / her judgment on every of these elements.

Step 4. Obtain the conversion of (BNNs) bipolar neutrosophic numbers.

When all experts give their valuations on each element. Let $R^k{}_{ij}$ be a (BN) decision matrix of the K^{th} DMs for calculating weights of criteria by opinions of DMs, then:

$$R^k{}_{ij} = \begin{bmatrix} r^k{}_{11} & \cdots & r^k{}_{1n} \\ \vdots & \ddots & \vdots \\ r^k{}_{m1} & \cdots & r^k{}_{mn} \end{bmatrix}, k \in K \tag{4}$$

where $r^k{}_{ij} = [T^+(x), I^+(x), F^+(x)\ T^-(x), I^-(x), F^-(x)]$, k = 1, 2, …, K, i = 1, 2, …, m, j = 1,2, …, n.

Step 5. Calculating the weights of experts.

Experts' judgments are collected by using the following equation:

$$r^k{}_{ij} = \frac{[T^+(x)_{n1}, I^+(x)_{n1}, F^+(x)_{n1}, T^-(x)_{n1}, I^-(x)_{n1}, F^-(x)_{n1}]}{n} \tag{5}$$

Step 6. Construct the evaluation matrix.

Build the evaluation matrix $A_i \times C_j$ with the assistance of BNNS to evaluate the ratings of alternatives with respect to each criterion. Let $R^k{}_{ij}$ be a (BN) decision matrix of the K^{th}experts, then:

$$R^k{}_{ij} = \begin{bmatrix} r^k{}_{11} & \cdots & r^k{}_{1n} \\ \vdots & \ddots & \vdots \\ r^k{}_{m1} & \cdots & r^k{}_{mn} \end{bmatrix}, k \in K \tag{6}$$

where $r^k{}_{ij} = [T^+(x), I^+(x), F^+(x)\ T^-(x), I^-(x), F^-(x)]$, k = 1, 2, …, K, i = 1, 2, …, m, j = 1,2, …, n.

Step 7. Aggregate the final evaluation matrix.

Using Eq.7, aggregate the crisp values of evaluation matrices into a final matrix.

$$\tilde{a}_{ij} = \frac{\tilde{a}_{ij}{}^1 + \dots\dots\dots + \tilde{a}_{ij}{}^n}{n} \tag{7}$$

Then, normalize the obtained matrix by Eq. 8.

$$H_{rt} = \frac{x_{rt}}{\sqrt{\sum_{i=1}^{m} x_{rt}{}^2}};\ r = 1, 2\dots m;\ t = 1, 2\dots n. \tag{8}$$

After that, calculate the weight matrix by Eq. 9.

$$Q_{rt} = w_z \times H_{rt} \tag{9}$$

Step 8. Define Ideal Solution A^+, A^-.

Calculate the positive and negative ideal solution using Eqs. (10, 11).

$$A^+ = \{< \max(\delta_{ij} | i = 1, 2, \dots, m) | j \in J^+ >, < \min(\delta_{ij} | i = 1, 2, \dots, m) | j \in J^- >\} \tag{10}$$

$$A^- = \{< \min(\delta_{ij} | i = 1, 2, \dots, m) | j \in J^+ >, < \max(\delta_{ij} | i = 1, 2, \dots, m) | j \in J^- >\} \tag{11}$$

Step 9. Positive and Negative Ideal Solution $S^+{}_i$, $S^-{}_i$.

Calculate the Euclidean distance between positive solution ($S^+{}_i$) and negative ideal solution ($S^-{}_i$) using Eqs. (12, 13).

$$S^{+}_{i} = \sqrt{\sum_{j=1}^{n}(\delta_{ij} - \delta_{j}^{+})^2} \quad i = 1, 2, \dots, m, \tag{12}$$

$$S^{-}_{i} = \sqrt{\sum_{j=1}^{n}(\delta_{ij} - \delta_{j}^{-})^2} \quad i = 1, 2, \dots, m \tag{13}$$

Step 10. Rank the alternatives based on closeness coefficient.

$$P_i = \frac{S^{-}_{i}}{S^{+}_{i} + S^{-}_{i}} \quad i = 1, 2, \dots, m \tag{14}$$

5. Numerical Example

We presented in this area a numerical case, which requires techniques and information investigation to test the ability and effectiveness of proposed structure for determination of the best aircraft.

5.1. Case Study

In an exertion of leading the overview, 250 surveys are conveyed to authorize visit directs in 21 general travel offices. The reason of limiting the capability of respondents was that we wished respondents had the experience of going with all carriers to be assessed. The authorized visit aides were the most normal decisions because of their regular voyages. Among the 250 overviews, 211 were returned for an arrival pace of 47%. The other statistic measurements were: 21% were at their age of 21–41; 99.05% got in any event secondary school training; normal working knowledge in the travel industry was 5.9 years. The poll of administration quality assessment mostly was made out of two sections: inquiries for assessing the general significance of criteria and aircraft's presentation relating to every measure. TOPSIS technique was utilized in getting the overall load of criteria and positioning of options. Concerning the presentation comparing to criteria of each carrier, we utilized semantic articulation to quantify the communicated exhibition. So as to set up the enrollment capacity related with each semantic articulation term, we requested that respondents indicate the range from 0 to 1 comparing to etymological term 'disappointed', 'reasonable', 'fulfilled' and 'exceptionally fulfilled'. These score were later pooled to align the participation capacities. We picked three noteworthy air transporters as the objects of this experimental examination. Carrier A, the most established aircraft in Egypt, with over 30 year's history, gains the most noteworthy piece of the overall industry by about 30%. The piece of the pie of aircraft B, despite the fact that is just 20% as of now, is quickly developing a result of the positive picture and notoriety. Carrier C is a preferably youthful jetliner with less over 10 years of activity history. The piece of the pie of carrier C is the least out of the three aircrafts at about 13%. There are three experts: Ex_1, Ex_2, $Ex_3$and Ex_3, and three alternatives A, B and C .For evaluating the airlines alternatives, seven criteria are considered as selection factors: C_1 (Appearance of crew), C_2 (Food), C_3 (Professional skill of crew), C_4 (Customer complaints handling), C_5 (Responsiveness of crew), C_6 (Safety) and C_7 (Timeliness).

5.2. The Calculation Process

Step 1. Organize a committee of experts and determine the goal, alternatives and valuation criteria.

Step 2. Determine the appropriate linguistic variables for weights W_n of criteria C_n and alternatives A_n with regard to each criterion. Each linguistic variable is a bipolar neutrosophic number. For criteria weights and for compilation alternatives, the linguistic variables are as in Table 1.

Table 1. Linguistic terms for evaluation criteria and alternatives.

Linguistic terms	Bipolar neutrosophic number $[T^+(x), I^+(x), F^+(x)\ T^-(x), I^-(x), F^-(x)]$
Excessively Good (EG)	⟨0.9, 0.1, 0.0, 0.0 , - 0.8, - 0.9⟩
Very Good (VG)	⟨1.0, 0.0, 0.1, - 0.3 , - 0.8, - 0.9⟩
Midst Good (MG)	⟨0.8, 0.5, 0.6, - 0.1 , - 0.8, - 0.9⟩
Perfect (P)	⟨0.7, 0.6, 0.5, - 0.2 , - 0.5, - 0.6⟩
Approximately Similar (AS)	⟨0.5, 0.2, 0.3, - 0.3 , - 0.1, - 0.3⟩
Bad (B)	⟨0.4, 0.4, 0.3, - 0.5 , - 0.2, - 0.1⟩
Midst Bad (MB)	⟨0.3, 0.1, 0.9, - 0.4 , - 0.2, - 0.1⟩
Very Bad (VB)	⟨0.2, 0.3, 0.4, - 0.8 , - 0.6, - 0.4⟩
Excessively Bad (EB)	⟨0.1, 0.9, 0.8, - 0.9 , - 0.2, - 0.1⟩

Step 3. Calculating the weights of experts

Table 2 presents the criteria weights according to all experts, after deciding linguistic variables to each expert. Convert the linguistic variables into bipolar neutrosophic numbers. Use Eq. 5 to aggregate weights in BNNs. Then, employ Eq. 1 to calculate the crisp weight values. After that, make a normalization procedure on the previous values, as in Table 3.

Table 2. Criteria weights according to all experts.

Exs	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Ex_1	⟨EG⟩	⟨MG⟩	⟨AS⟩	⟨VG⟩	⟨MB⟩	⟨EG⟩	⟨EG⟩
Ex_2	⟨MB⟩	⟨B⟩	⟨VB⟩	⟨P⟩	⟨VB⟩	⟨MG⟩	⟨P⟩
Ex_3	⟨P⟩	⟨AS⟩	⟨MB⟩	⟨AS⟩	⟨MG⟩	⟨AS⟩	⟨EB⟩
Ex_4	⟨VG⟩	⟨EB⟩	⟨P⟩	⟨EG⟩	⟨VG⟩	⟨B⟩	⟨AS⟩

Table 3. The normalized criteria weights.

Weight $\tilde{w}_n$	Aggregation weights in BNNs	crisp	Normalized Weight
C_1	[0.725, 0.2, 0.375, - 0.225, - 0.575, - 0.625]	0.6875	0.17
C_2	[0.450 , 0.50, 0.500, - 0.45 , - 0.325, - 0.35]	0.4458	0.09
C_3	[0.425, 0.3, 0.525, - 0.425 , - 0.350, - 0.350]	0.4792	0.11
C_4	[0.775, 0.225, 0.225, - 0.20 , - 0.55, - 0.675]	0.7250	0.21
C_5	[0.575, 0.225, 0.500, - 0.4 , - 0.600, - 0.575]	0.6042	0.14
C_6	[0.650, 0.300, 0.3, - 0.225 , - 0.475, - 0.550]	0.6417	0.15
C_7	[0.550, 0.450, 0.40, - 0.35 , - 0.400, - 0.475]	0.5375	0.13

Step 4. Construct the evaluation matrix.

Obtain the final decision matrix by making the aggregation procedure of experts' priorities and preferences, as in Table 4. Calculate the crisp values of matrices and insert them into the aggregated matrix.

Table 4. The aggregated crisp values of decision matrix.

C_n/ A_n	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A	0.48	0.69	0.5	0.64	0.55	0.51	0.82
B	0.53	0.73	0.55	0.67	0.51	0.84	0.69
C	0.85	0.48	0.63	0.54	0.61	0.63	0.76

Apply the normalization process by using Eq. 8 to obtain the normalized evaluation matrix, as presented in Table 5.

Table 5. The normalized decision matrix.

C_n/ A_n	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A	0.43	0.62	0.51	0.60	0.57	0.44	0.62
B	0.48	0.66	0.56	0.62	0.53	0.72	0.53
C	0.77	0.43	0.65	0.50	0.63	0.54	0.58

Build the weighted matrix by multiplying the normalized evaluation matrix by the weights of criteria using Eq. 9, as in Table 6.

Table 6. The weighted matrix.

C_n/ A_n	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Weight	0.17	0.09	0.11	0.21	0.14	0.15	0.13
A	0.073	0.055	0.056	0.126	0.079	0.066	0.081
B	0.082	0.059	0.061	0.130	0.074	0.108	0.068
C	0.130	0.039	0.072	0.105	0.088	0.081	0.075

Step 5. Define Ideal Solution A^+, A^-.

Define the ideal solutions using Eqs. 10 and 11.

Step 6. Positive and Negative Ideal Solution $S^+{}_i$, $S^-{}_i$.

Calculate the Euclidean distance between positive solution ($S^+{}_i$) and negative ideal solution ($S^-{}_i$) using Eqs. 13 and 14.

Step 7. Rank the alternatives based on closeness coefficient.

Calculate the performance score using Eq. 14, and make the last ranking of alternatives as presented in Table 7 and in Figure.1.

Table 7. The TOPSIS result and ranking of alternatives.

C_n/ A_n	S^{+}_{1}	S^{-}_{1}	p_1	Rank
A	0.073	0.029	0.28	3
B	0.053	0.053	0.50	2
C	0.059	0.065	0.53	1

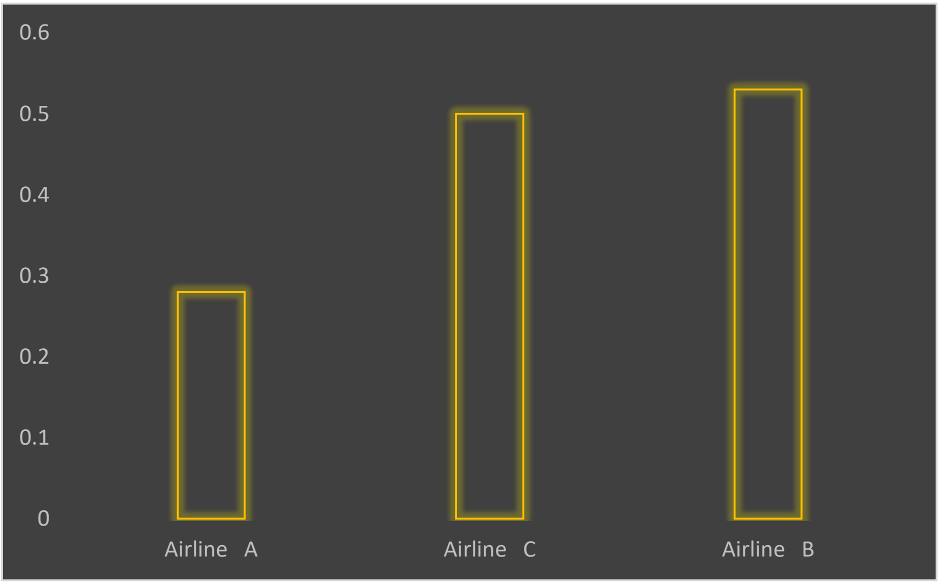

Figure 1. Ranking the alternatives using TOPSIS under Neutrosophic.

6. Conclusion

The idea of value administration goes past the specialized parts of giving the administration Fit incorporates clients' impression of what the administrations ought to be and how the administrations is to be passed on. In examining the two concerns, we build up the systems for recognizing the most significant characteristics of administration quality for clients and catch clients' evaluation of three aircrafts dependent on these traits.

The assessment methodology comprises of the accompanying advances: (1) distinguish the assessment criteria for carrier administration quality; (2) survey the normal significance of every model by TOPSIS over every one of the respondents. (3) Represent the presentation evaluation of air bearers for every paradigm by neutrosophic numbers, which expressly endeavors to precisely catch the genuine inclination of assessors. Singular appraisal at that point is amassed as a general evaluation for every carrier under every rule. (4) Use TOPSIS as the principle gadget in positioning the administration nature of the three air transporters.

The noteworthy discoveries of this investigation spread a few viewpoints. Clients are for the most part worried about the physical part of the administration and less worried about the sympathy perspective. The finding proposes that aircrafts ought to keep up their physical highlights about a specific level and keep redesign important.

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.NET Framework to deal with Neutrosophic $b^*g\alpha$-Closed Sets in Neutrosophic Topological Spaces

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Abstract: This article introduces a new computer based application for finding the values of the complement of neutrosophic sets, union of neutrosophic sets, intersection of neutrosophic sets and the inclusion of any two neutrosophic sets by using the software .NET Framework, Microsoft Visual Studio and C# Programming Language. In addition to this, the application has produces the values of neutrosophic topology [τ], neutrosophic α-closed set, neutrosophic $g\alpha$-closed set, neutrosophic $^*g\alpha$-closed set and neutrosophic $b^*g\alpha$-closed set values in neutrosophic topological spaces. Also it generates the values of its complement sets.

Keywords: .NET framework; Microsoft Visual Studio; C# Application; Neutrosophic Set Operations; Neutrosophic Topology; Neutrosophic α-Closed Set; Neutrosophic $g\alpha$-Closed Set; Neutrosophic $^*g\alpha$-Closed Set; Neutrosophic $b^*g\alpha$-Closed Set

1. Introduction

Nowadays the word `topology' is being commonly used and getting popularity day by day in the field of modern mathematics. It seems to be derived from Greek words: *topos* means *a surface* and *logos* means *a discourse*. The use of word `Topology' was first occurred in the title of the book `Vorstudien Zur Topologie' by Johann Benedict Listing in 1847. The general topology got its real start in 1906 due to Riesz, Frechet and Moore. By using the concept of neutrosophic set, which was introduced by Smarandache [24, 25]. Salama et al. [17] were introduced neutrosophic topological spaces by using the two most important concepts of Topology and neutrosophic sets in 2012.

In the last few decades many researchers has applied this effective concept in neutrosophic topology and they have introduced many neutrosophic sets, namely Arokiarani et al. [10] were introduced neutrosophic α-closed sets in neutrosophic topological spaces in 2017, which is the basic set for many researchers to produce various neutrosophic closed and neutrosophic open sets. In 2019, Saranya et al. [20] were introduced neutrosophic $g\alpha$-closed sets, neutrosophic $^*g\alpha$-closed sets and neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces in and developed a new C# application to deal with neutrosophic α-closed sets, neutrosophic $g\alpha$-closed sets; neutrosophic $^*g\alpha$-closed sets in neutrosophic topology. In 2014, Salama et al. [19] has developed some software programs for dealing with neutrosophic sets. Salama et al. [16] has designed and implemented a neutrosophic data operations by using object oriented programming in 2014. Neutrosophic theory was applied by various authors in different fields to produce some real world applications like time series, forecasting, decision making, etc [1-9, 11-15, 18, 21-23].

To reduce the manual calculations for finding the values of the complement, union, intersection and the inclusion of two neutrosophic sets in a neutrosophic field, we have developed a C# application by using .NET Framework, Microsoft Visual Studio and C# Programming Language. In this application the user can calculate the values of neutrosophic topology, neutrosophic α-closed set, neutrosophic $g\alpha$-closed set, neutrosophic $^*g\alpha$-closed set and neutrosophic $b^*g\alpha$-closed set values in each resultant screens. Also it generates the values of its complement sets.

The present study introduces the C# application for finding the neutrosophic closed sets and neutrosophic open sets in neutrosophic topological spaces via .NET Framework, Microsoft Visual Studio and C# Programming Language. The overall working process of this application have been shown as a flow chart in Figure:1. Individual Flow Chart of neutrosophic topology, neutrosophic α-closed sets, neutrosophic $g\alpha$-closed sets, neutrosophic $^*g\alpha$-closed sets and neutrosophic $b^*g\alpha$-closed sets are given in Figure:2, Figure:13, Figure:16, Figure:20 and in Figure:23. Figure:3 shows the initial resultant page[In this page, the user has to enter 0_N, 1_N and the neutrosophic sets of L and M values. Also, the results of neutrosophic topology(τ), neutrosophic α-closed set, neutrosophic $g\alpha$-closed set, neutrosophic $^*g\alpha$-closed set and neutrosophic $b^*g\alpha$-closed set via C# application are shown in Figure:12, Figure:15, Figure:19, Figure:22 and in Figure:25. It also produces the values of its complements of each closed sets.

2. Preliminaries

In this section, we recall some of the basic definitions which was already defined by various authors.

Definition: 2.1 [17]

Let X be a non empty fixed set. A neutrosophic set E is an object having the form

$$\mathrm{E} = \{< \mathrm{x}, \mathrm{mv}(\mathrm{E}(\mathrm{x})), \mathrm{iv}(\mathrm{E}(\mathrm{x})), \mathrm{nmv}(\mathrm{E}(\mathrm{x})) > \text{ for all } \mathrm{x} \in \mathrm{X}\},$$

where $\mathrm{mv}(\mathrm{E}(\mathrm{x}))$ represents the degree of membership, $\mathrm{iv}(\mathrm{E}(\mathrm{x}))$ represents the degree of indeterminacy and $\mathrm{nmv}(\mathrm{E}(\mathrm{x}))$ represents the degree of non-membership functions of each element $\mathrm{x} \in \mathrm{X}$ to the set E.

Definition: 2.2 [17]

Let E and F be two neutrosophic sets of the form,

$\mathrm{E} = \{< \mathrm{x}, \mathrm{mv}(\mathrm{E}(\mathrm{x})), \mathrm{iv}(\mathrm{E}(\mathrm{x})), \mathrm{nmv}(\mathrm{E}(\mathrm{x})) > \text{ for all } \mathrm{x} \in \mathrm{X}\}$ and

$F = \{< \mathrm{x}, \mathrm{mv}(\mathrm{F}(\mathrm{x})), \mathrm{iv}(\mathrm{F}(\mathrm{x})), \mathrm{nmv}(\mathrm{F}(\mathrm{x})) > \text{ for all } \mathrm{x} \in \mathrm{X}\}$.

Then,

1. $E \subseteq F$ if and only if $mv(E(x)) \leq mv(F(x))$, $iv(E(x)) \leq iv(F(x))$ and $nmv(E(x)) \geq nmv(F(x))$ for all $\mathrm{x} \in \mathrm{X}$,
2. $A^c = \{< x, nmv(E(x)), 1 - iv(E(x)), mv(E(x)) > \text{ for all } \mathrm{x} \in \mathrm{X}\}$,
3. $E \cup F = \{x, max[mv(E(x)), mv(F(x))],\ min[iv(E(x)), iv(F(x))],$ $min[nmv(E(x)), nmv(F(x))] \text{for all } \mathrm{x} \in \mathrm{X}\}$,
4. $E \cap F = \{x, min[mv(E(x)), mv(F(x))], max[iv(E(x)), iv(F(x))],$ $max[nmv(E(x)), nmv(F(x))] \text{ for all } \mathrm{x} \in \mathrm{X}\}$.

Definition: 2.3 [17]

A neutrosophic topology on a non-empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

i) $0_N,\ 1_N\ \in \tau$,

ii) $G_1 \cap G_2\ \in \tau$ for any $G_1, G_2\ \in \tau$,

iii) $\cup\, G_i\ \in \tau$ for all $\{G_i\ :\ i\ \in J\} \subseteq \tau$.

Then the pair (X, τ) or simply X is called a neutrosophic topological space.

3. Results

In this section we have shown the working process of C# application for finding the values of the complement, union, intersection and the inclusion of any two neutrosophic sets. Also it produces the values of neutrosophic topology (τ), neutrosophic α-closed set, neutrosophic $g\alpha$-closed set, neutrosophic $^*g\alpha$-closed set and neutrosophic $b^*g\alpha$-closed set values in neutrosophic topological spaces. The complements of neutrosophic α-closed set, neutrosophic $g\alpha$-closed set, neutrosophic $^*g\alpha$-closed set and neutrosophic $b^*g\alpha$-closed set values will be displayed at the end of the results of each sets.

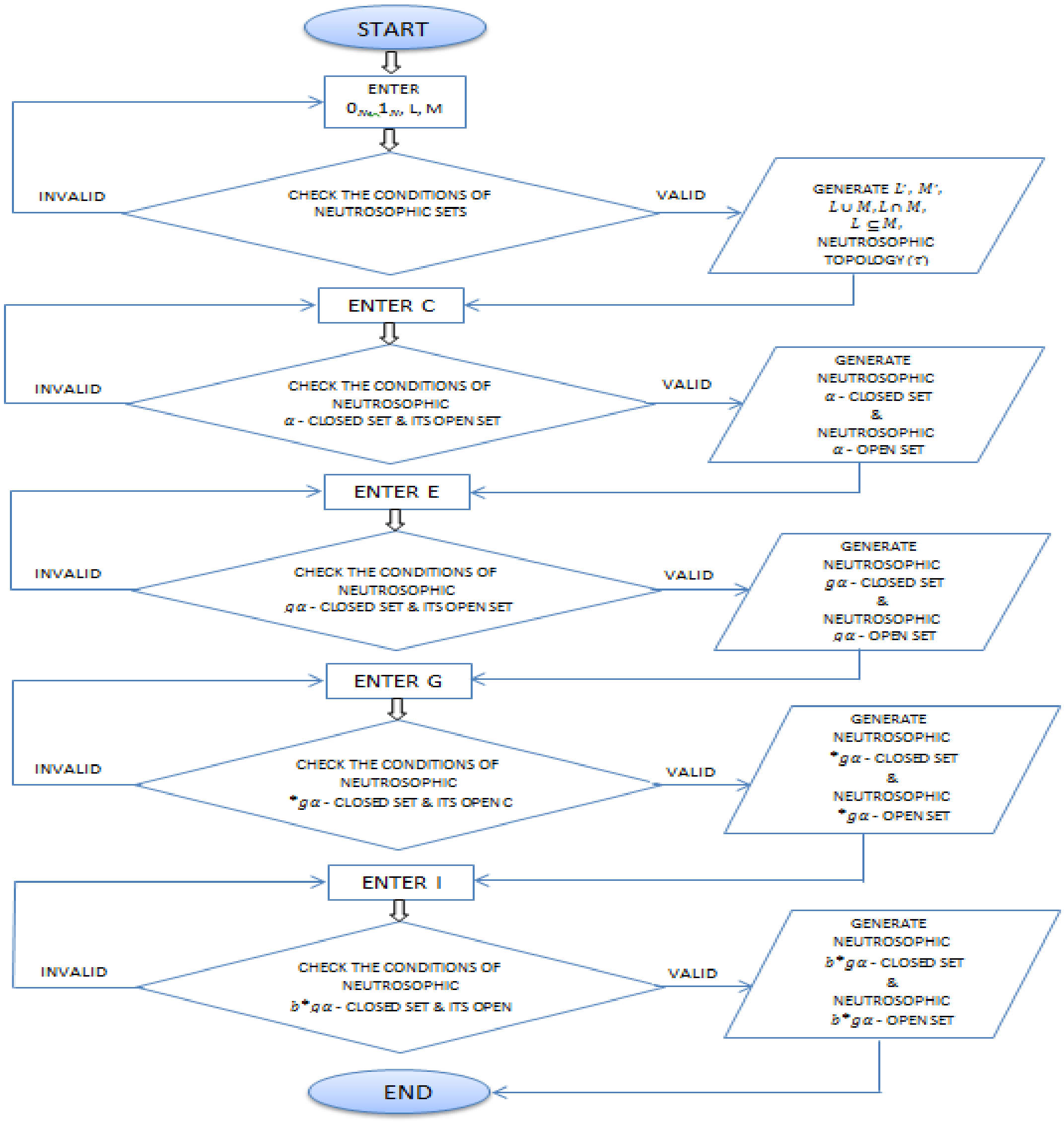

Figure.1: Flow Chart of the Existence of Neutrosophic Sets

3.1. Existence of Neutrosophic Topology via C# Application

3.1.1. Algorithm: Neutrosophic Topology

input	0_N, 1_N, L, M
output	complement of L and M, union of L and M intersection of L and M inclusion of L and M neutrosophic Topology

STEPS:

step-1: check 0_N and 1_N is valid
step-2: L and M should be a neutrosophic set
step-3: calculate the complement of L and M
step-4: calculate the union of L and M
step-5 calculate the intersection of L and M
step-6: check the inclusion of L and M
step-7: if the union and the intersection conditions satisfied then go to step-8 else repeat step-2
step-8: compute the neutrosophic topology for the assigned data.

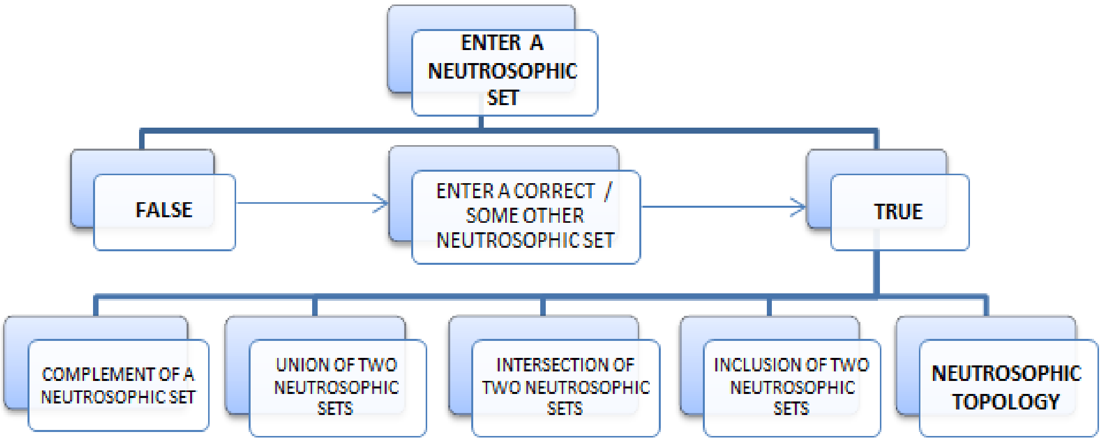

Figure.2: Flow Chart of Neutrosophic Topology [FC-NT]

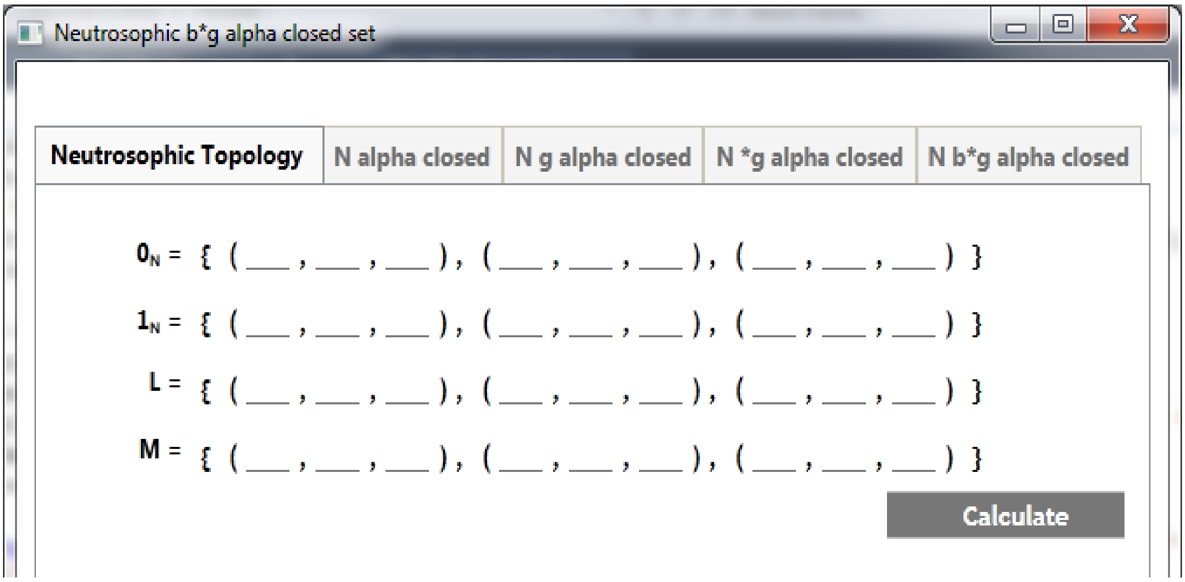

Figure.3: Screenshot of Initial Resultant Screen / User Screen

In the above resultant screen, the user has to enter all the values of 0_N, 1_N, L and M. Follow the below conditions to enter the values

- 0_N and 1_N values should be any three values of {(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1)} and {(1, 1, 1), (1, 1, 0), (1, 0, 1), (1, 0, 0)}.
- L and M values should be based on Definition 2.1 and Remark 2.2 of [20].

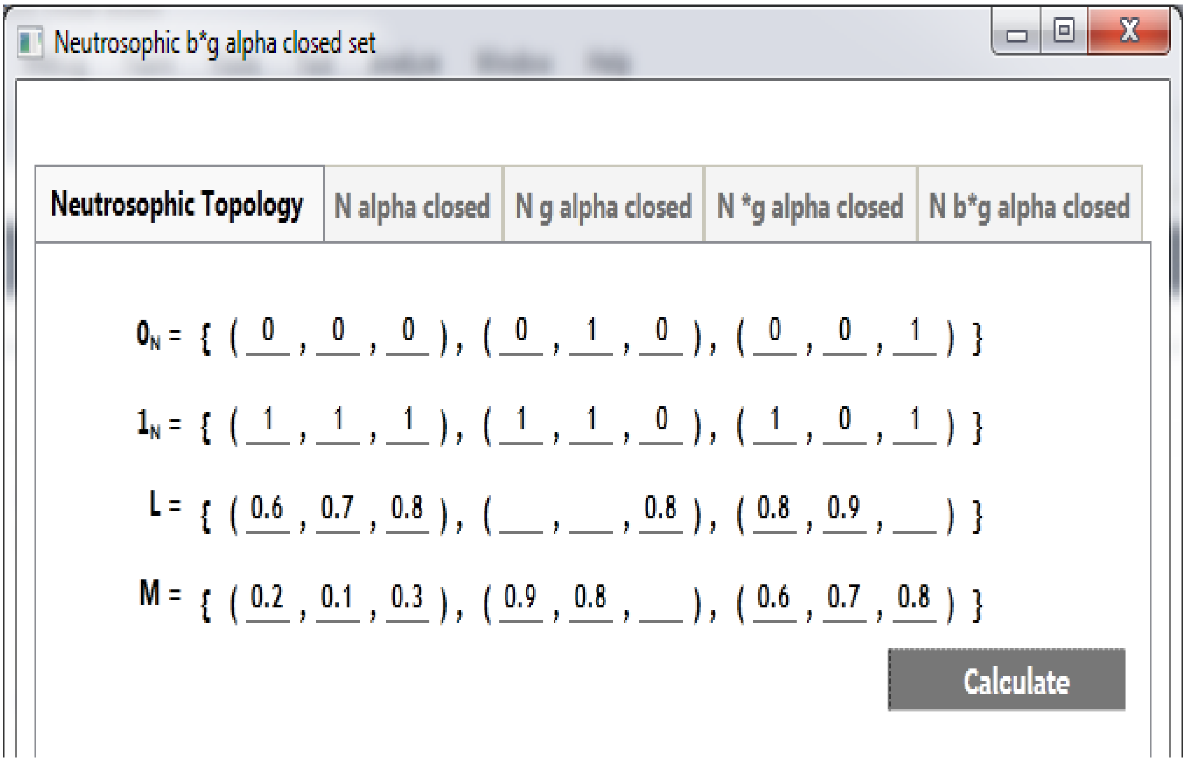

Figure.4: Screenshot of Incomplete Data in the Resultant Screen

The above figure shows the entered values of the initial resultant screen. In this, some of the values are not entered by the user. So the following command box intimates the user to enter all the values.

Figure.5: Screenshot of Dialog Box-1

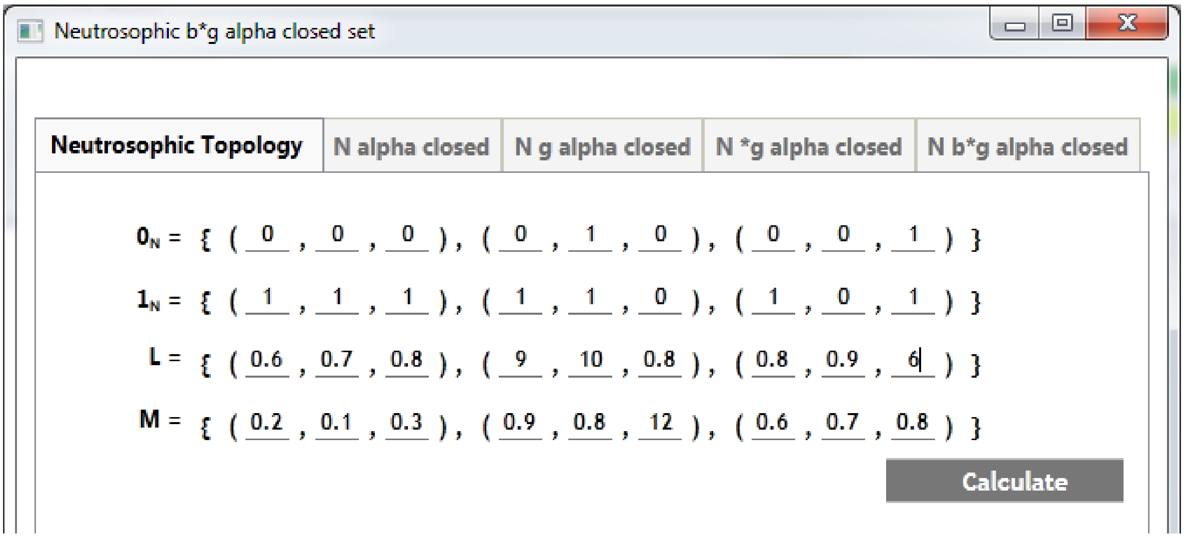

Figure.6: Screenshot of Invalid Data in the Resultant Screen

The above figure shows the entered values of the initial resultant screen. Here some of the values are not properly entered by the user. For this incorrect data the following command box intimate the user to enter the values in the non-standard unit interval 0 and 1 also the user did not follow the conditions to enter L and M. Both L and M should be a neutrosophic values.

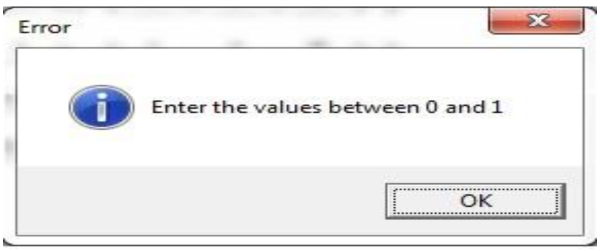

Figure.7: Screenshot of Dialog Box-2

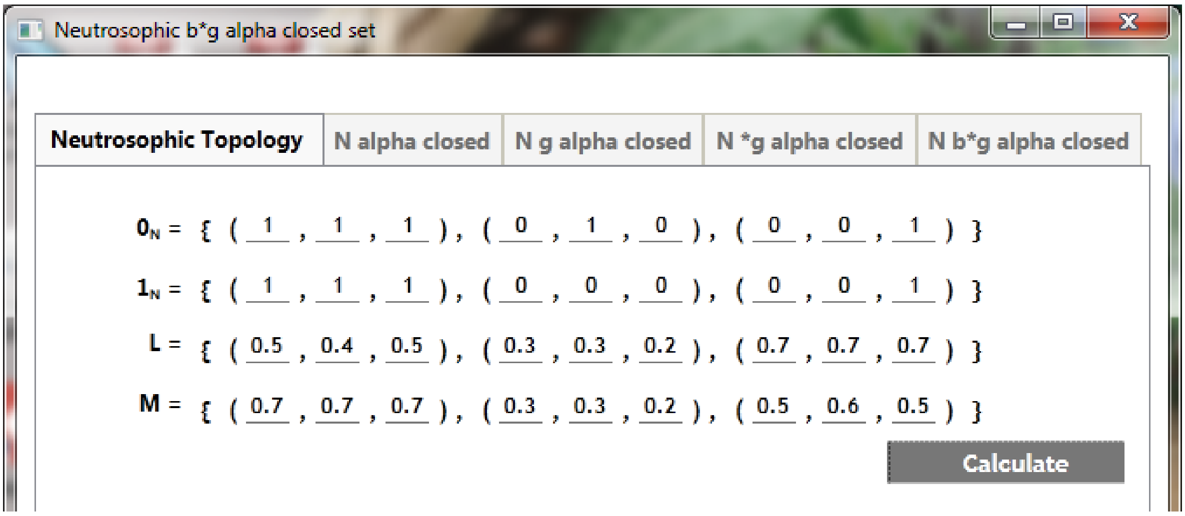

Figure.8: Screenshot of Invalid Data in the Resultant Screen

In the above figure, the entered values of 0_N are not followed by the conditions of 0_N. For this incorrect data the following command box intimate the user to enter the valid data in the 0_N th place.

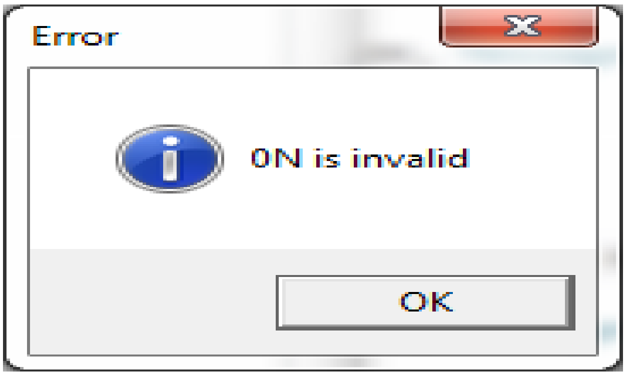

Figure.9: Screenshot of Dialog Box-3

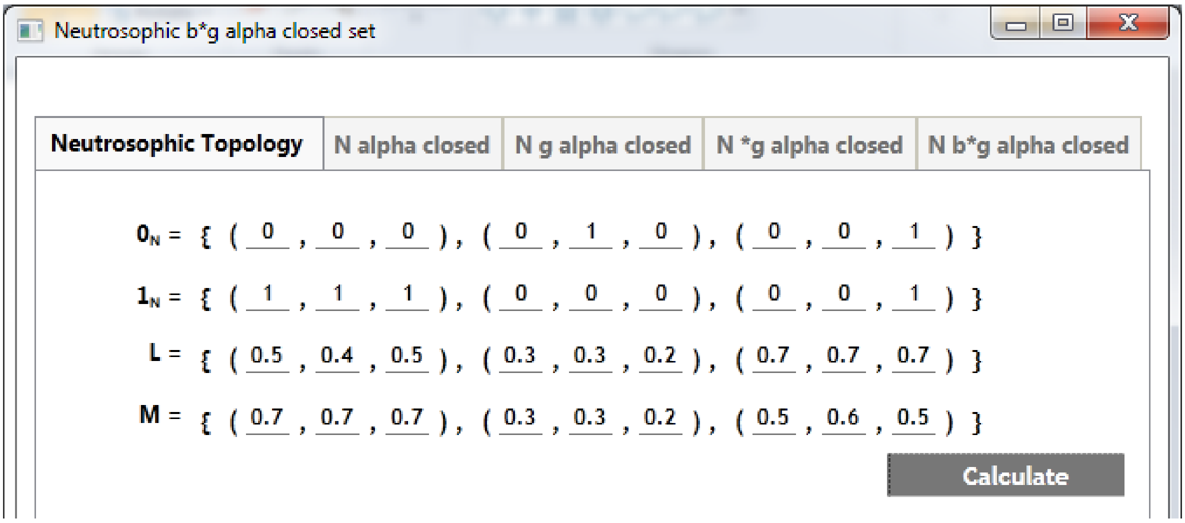

Figure.10: Screenshot of Invalid Data in the Resultant Screen

In the above figure, the entered values of 1_N are not followed by the conditions of 1_N. For this incorrect data the following command box intimate the user to enter the valid data in the 1_N^th^ place.

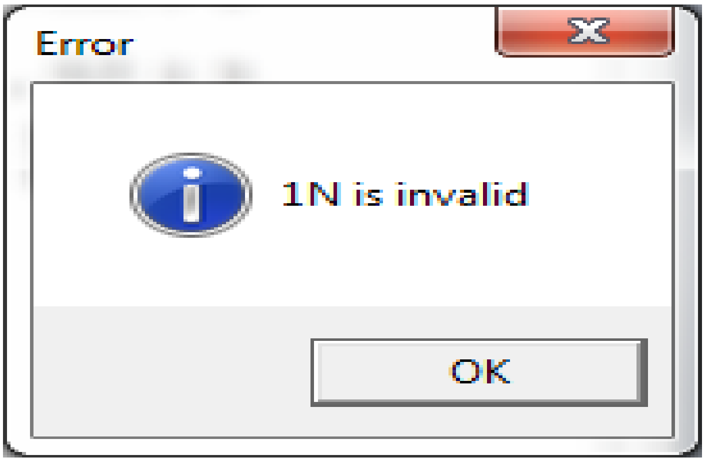

Figure.11: Screenshot of Dialog Box-4

The following figure shows the results of the complement of two neutrosophic sets L' and M, union of two neutrosophic sets L∪M, intersection of two neutrosophic sets L∩M and the inclusion of two neutrosophic sets L⊆M. Also it shows the result of neutrosophic topology.

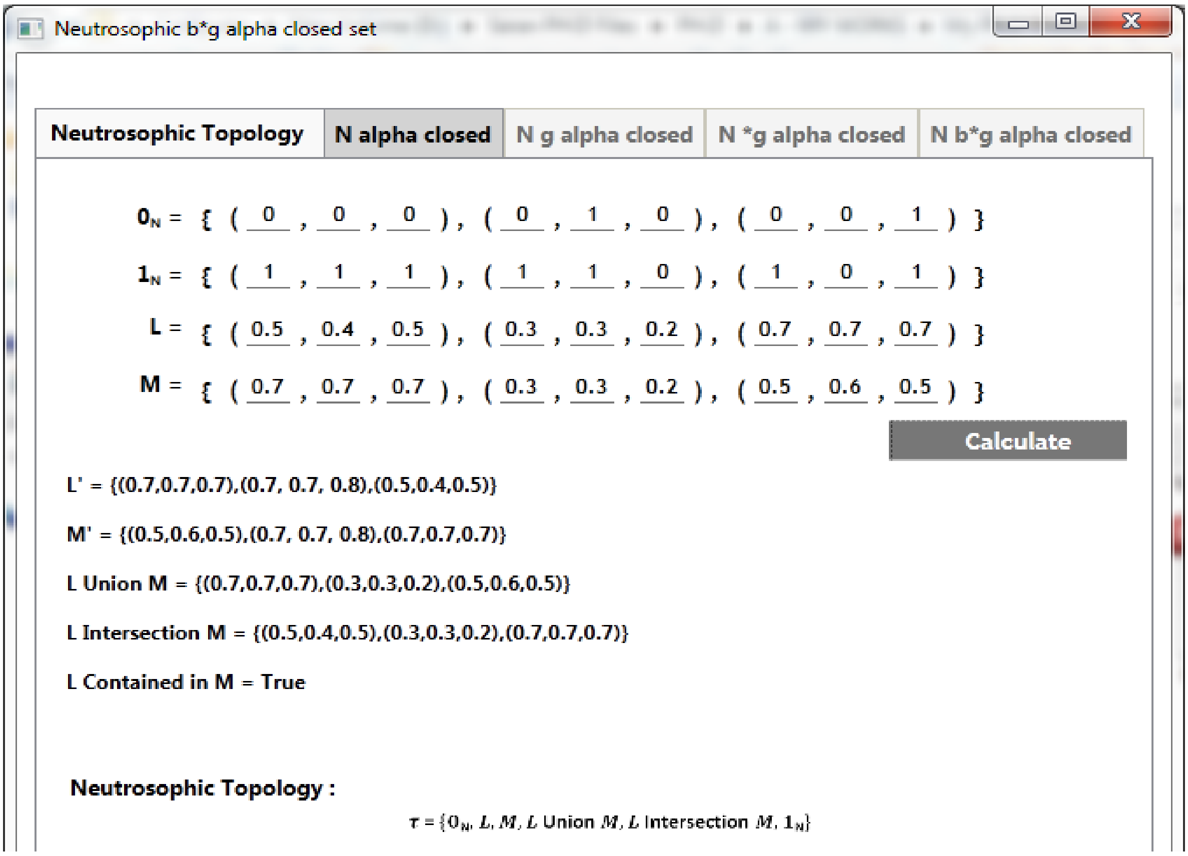

Figure.12: Screenshot of the Existence of Neutrosophic Topology via C# Application

3.2. Existence of Neutrosophic α-Closed Set via C# Application

3.2.1. Algorithm: Neutrosophic α-Closed Set

input	neutrosophic set $\mathbf{C}$
output	neutrosophic α-closed set; neutrosophic α-open set

STEPS:

step-1: check $\mathbf{C}$ is valid

step-2: find $\mathbf{Ncl}(\mathbf{C})$, if $\mathbf{Ncl}(\mathbf{C})$ satisfies the neutrosophic closure condition then go to step-3 else repeat step-1

step-3: find $\mathbf{Nint}[\mathbf{Ncl}[\mathbf{C}]]$, if $\mathbf{Nint}[\mathbf{Ncl}[\mathbf{C}]]$ satisfies the neutrosophic interior of neutrosophic closure condition then go to step-4 else repeat step-1

step-4: find $\mathbf{Ncl}[\mathbf{Nint}[\mathbf{Ncl}[\mathbf{C}]]]$, if $\mathbf{Ncl}[\mathbf{Nint}[\mathbf{Ncl}[\mathbf{C}]]]$ satisfies the neutrosophic closure of neutrosophic interior of neutrosophic closure condition then go to step-5 else repeat step-1

step-5: if $\boldsymbol{N\alpha cl}[\boldsymbol{C}] = \mathbf{C}$ then produce neutrosophic α-closed set else repeat step-1

step-6: compute the neutrosophic α-open set [D] for the assigned data.

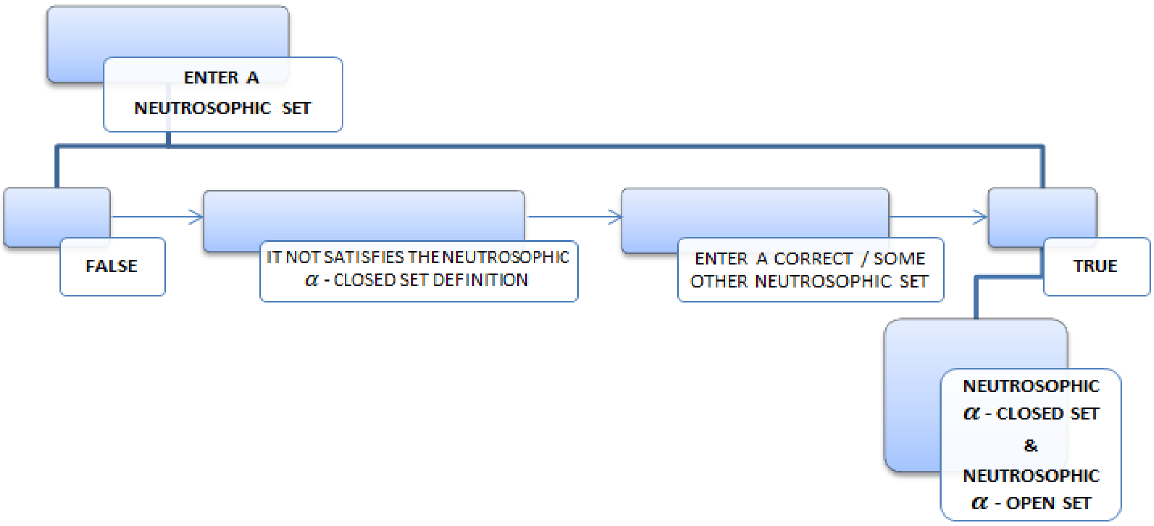

Figure.13: Flow Chart of Neutrosophic α-Closed Set [FC-**NαCS**]

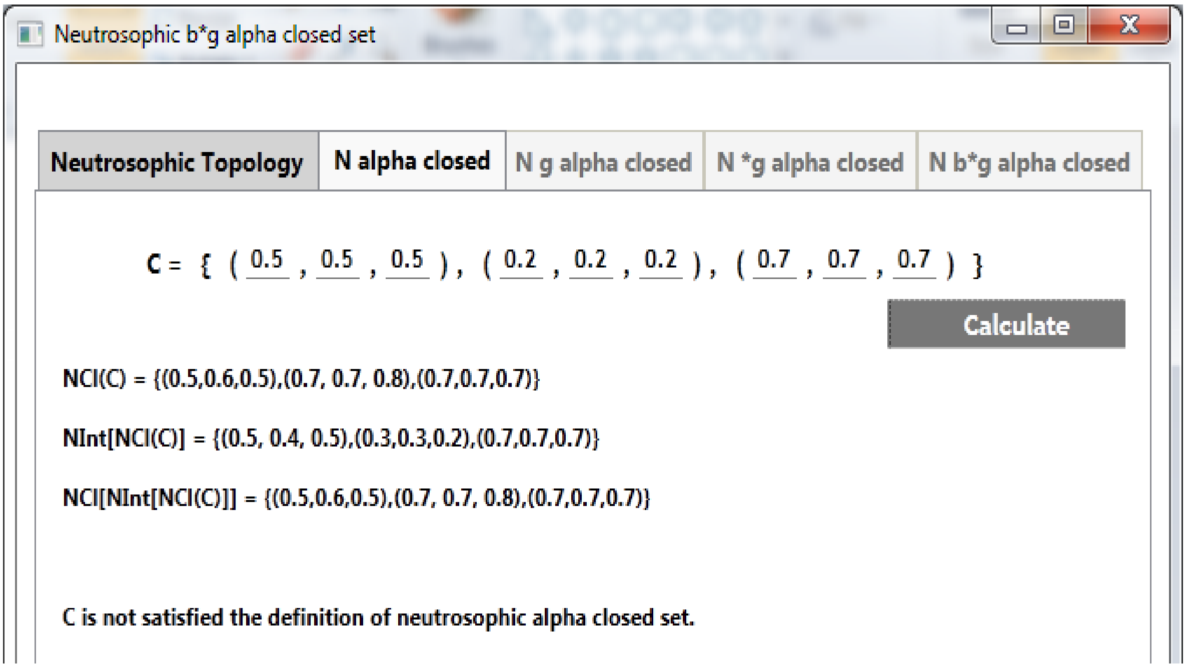

Figure.14: Screenshot of Dissatisfaction of the Definition of Neutrosophic α-Closed Set

The above figure shows that the entered neutrosophic set **C** and it is not satisfy the definition of neutrosophic α-closed set. To get a neutrosophic α-closed set and a neutrosophic α-open set, the user has to enter some other neutrosophic values. Repeat this process until to get the values of neutrosophic α-closed sets.

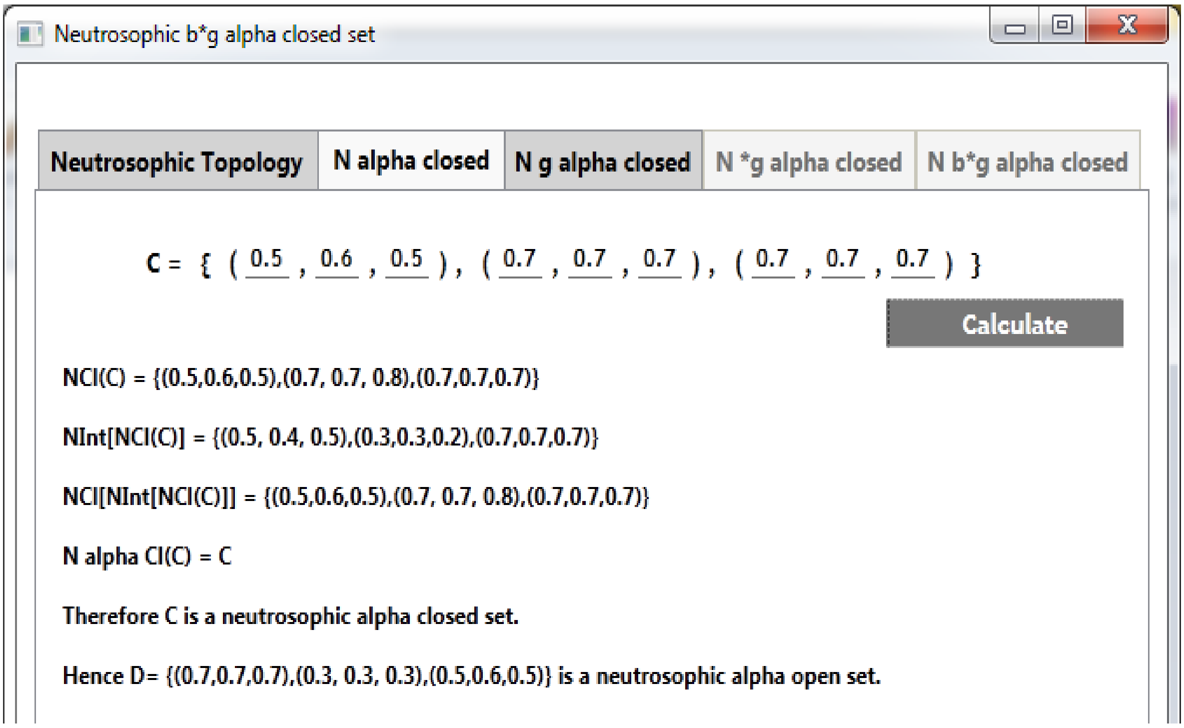

Figure.15: Screenshot of the Existence of Neutrosophic α-Closed Set [NαCS] via C# Application

3.3. Existence of Neutrosophic $g\alpha$-Closed Set via C# Application

3.3.1. Algorithm: Neutrosophic $g\alpha$-Closed Set

input	neutrosophic set **E**
output	neutrosophic $g\alpha$-closed set; neutrosophic $g\alpha$-open set

STEPS:

step-1: check **E** is valid
step-2: check E ⊆ D then go to step-3 otherwise repeat step-1
step-3: find $\mathbf{Ncl(E)}$, if $\mathbf{Ncl(E)}$ satisfies the neutrosophic closure condition then go to step-4 else repeat step-1
step-4: find $\mathbf{Nint[Ncl[E]]}$, if $\mathbf{Nint[Ncl[E]]}$ satisfies the neutrosophic interior of neutrosophic closure condition then go to step-5 else repeat step-1
step-5: find $\mathbf{Ncl[Nint[Ncl[E]]}$, if $\mathbf{Ncl[Nint[Ncl[E]]]}$ satisfies the neutrosophic closure of neutrosophic interior of neutrosophic closure condition then go to step-6 else repeat step-1
step-6: calculate $\mathbf{N\alpha cl[E]}$
step-7: if $\mathbf{N\alpha cl[E]}$ ⊆ **D**then produce neutrosophic $g\alpha$-closed set else repeat step-1
step-8: compute the neutrosophic $g\alpha$-open set [**F**] for the assigned data.

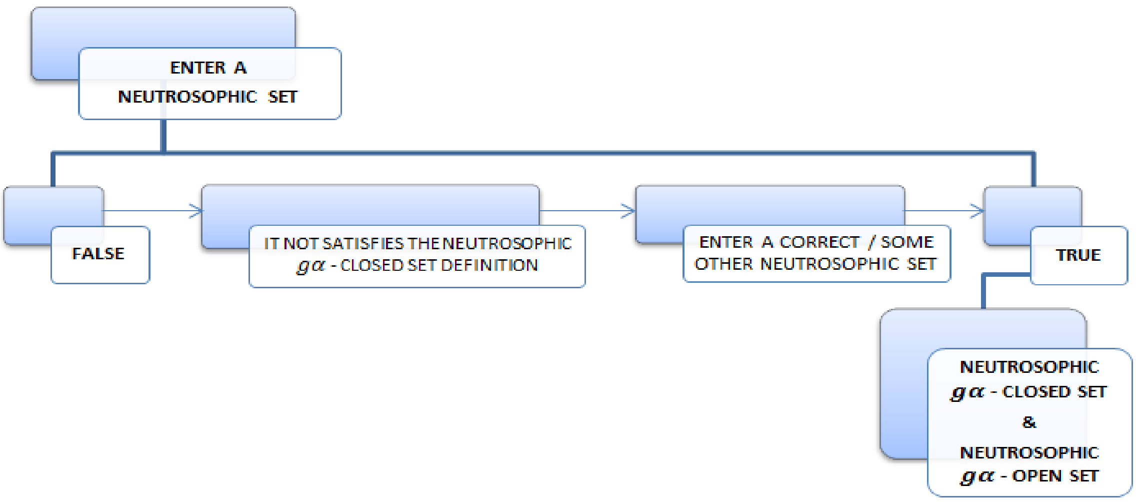

Figure.16: Flow Chart of Neutrosophic $g\alpha$-Closed Set [**NgαCS**]

The following two figures [Figure 17 & Figure 18] shows that the neutrosophic set E is not satisfy the definition of neutrosophic $g\alpha$-closed sets.

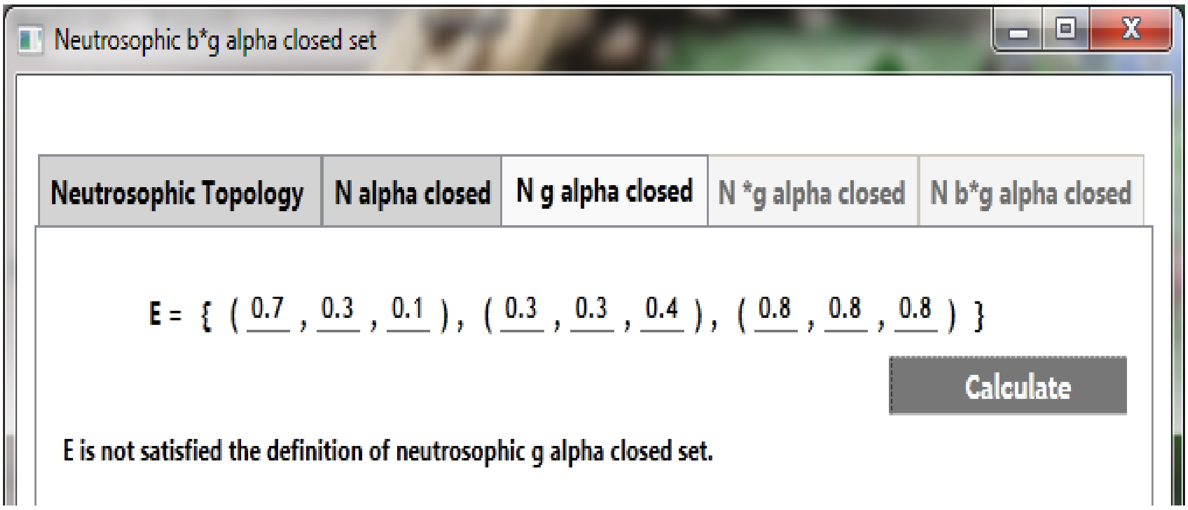

Figure.17: Screenshot of Dissatisfaction of the Definition of Neutrosophic $\boldsymbol{g\alpha}$-Closed Set

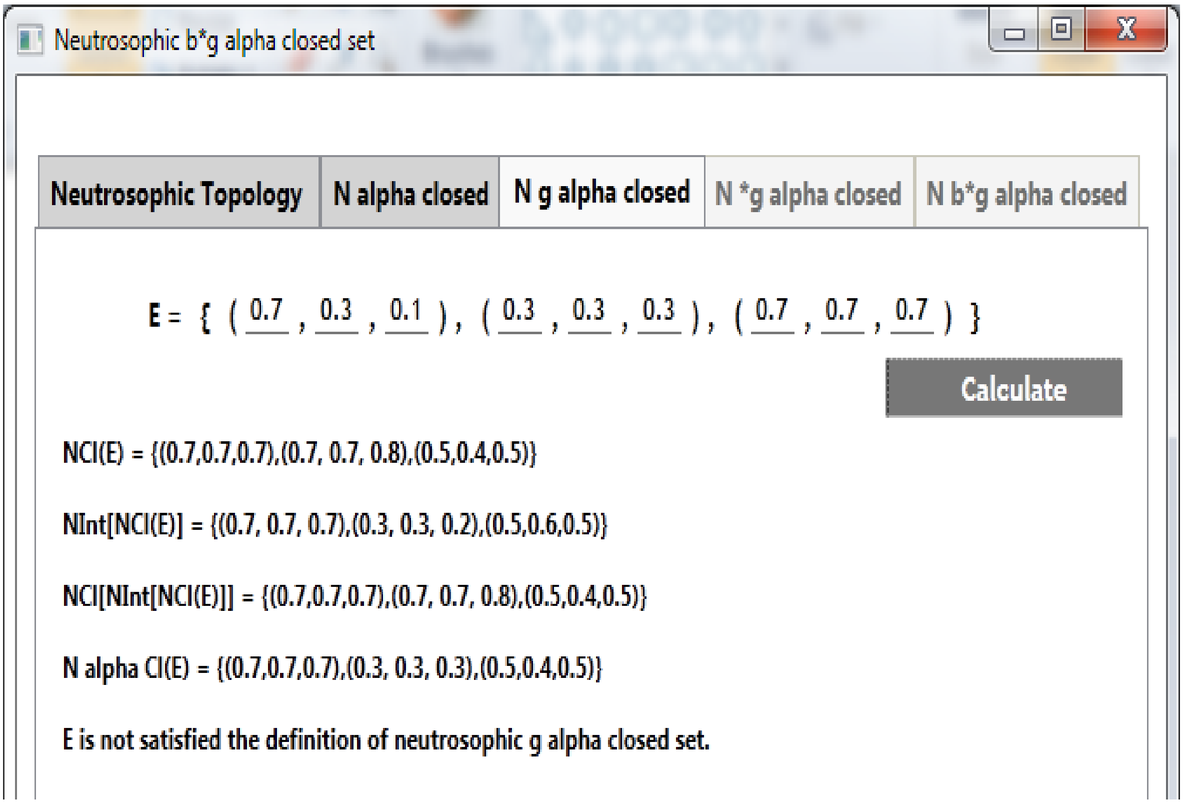

Figure.18: Screenshot of Dissatisfaction of the Definition of Neutrosophic $\boldsymbol{g\alpha}$-Closed Set

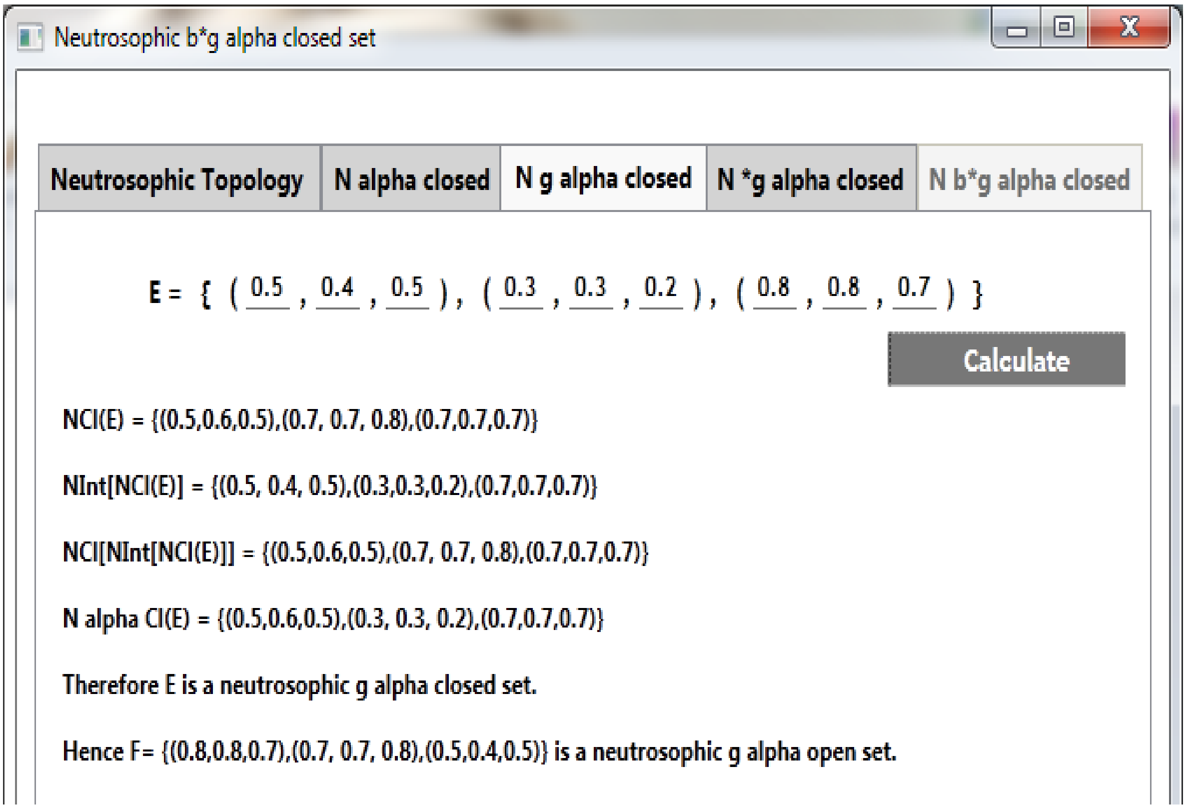

Figure.19: Screenshot of the Existence of Neutrosophic $\boldsymbol{g\alpha}$-Closed Set [**NgαCS**] via C#

3.4 Existence of Neutrosophic *$\boldsymbol{g\alpha}$-Closed Set via C# Application

*3.4.1. Algorithm: Neutrosophic * $\boldsymbol{g\alpha}$-Closed Set*

input	neutrosophic set $\mathbf{G}$
output	neutrosophic *$\boldsymbol{g\alpha}$-closed set; neutrosophic *$\boldsymbol{g\alpha}$-open set

STEPS:

step-1: check $\mathbf{G}$ is valid
step-2: check $\mathbf{G} \subseteq \mathbf{F}$ then go to step-3 otherwise repeat step-1
step-3: find $\mathbf{Ncl}(\mathbf{G})$, if $\mathbf{Ncl}(\mathbf{G})$ satisfies the neutrosophic closure condition then go to step-4 else repeat step-1
step-4: calculate $\mathbf{Ncl}[\mathbf{G}]$
step-5: if $\mathbf{Ncl}[\mathbf{G}] \subseteq \mathbf{F}$ then produce neutrosophic *$\boldsymbol{g\alpha}$-closed set else repeat step-1
step-6: compute the neutrosophic *$\boldsymbol{g\alpha}$-open set $[\mathbf{H}]$ for the assigned data.

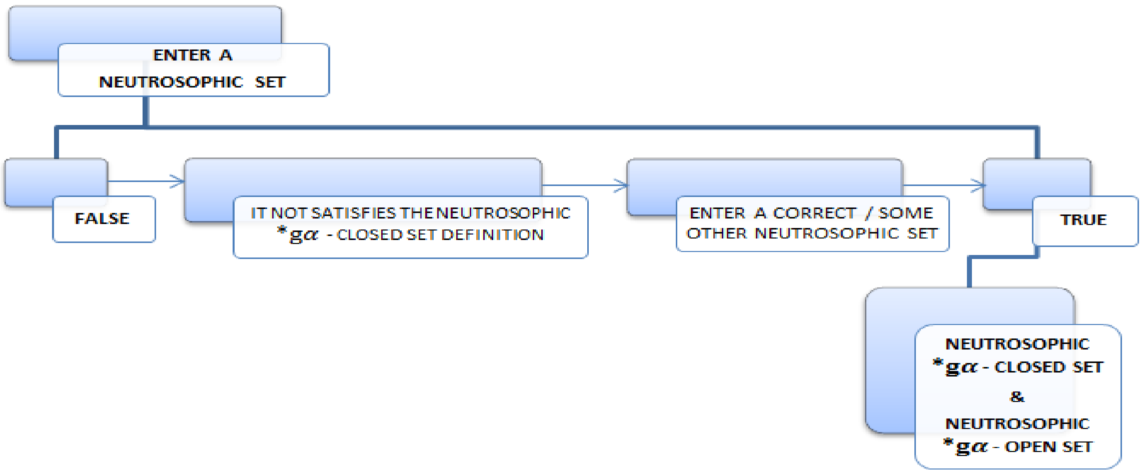

Figure.20: Flow Chart of Neutrosophic $*g\alpha$-Closed Set [**FC-N*** $g\alpha$**CS**]

The following figure shows that the neutrosophic set **G** does not satisfies the definition of neutrosophic $*g\alpha$-closed sets.

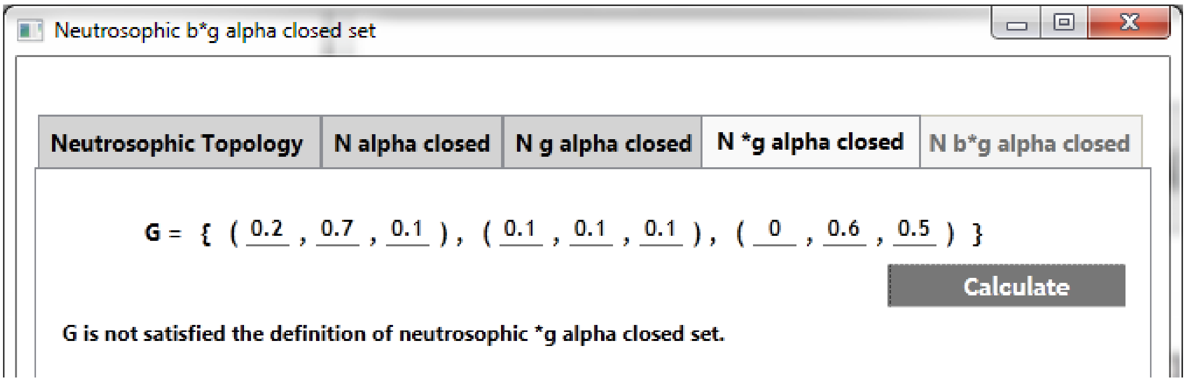

Figure.21: Screenshot of Dissatisfaction of the Definition of Neutrosophic $*g\alpha$-Closed Set

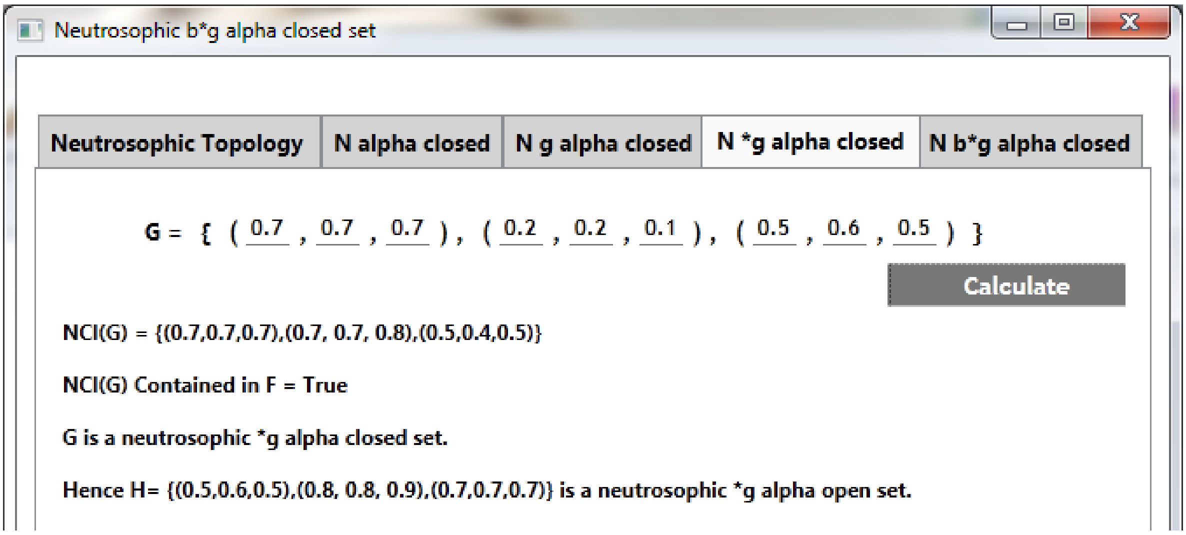

Figure.22: Screenshot of the Existence of Neutrosophic $*g\alpha$-Closed Set [**N*** $g\alpha$**CS**] via C# Application

3.5. Existence of Neutrosophic $b^*g\alpha$-Closed Set via C# Application

Algorithm: Neutrosophic $b^*g\alpha$-Closed Set

input	neutrosophic set $\mathbf{I}$
output	neutrosophic $\boldsymbol{b^*g\alpha}$-closed set; neutrosophic $\boldsymbol{b^*g\alpha}$-open set

<u>**STEPS**</u>:

step-1: check $\mathbf{I}$ is valid

step-2: check $\mathbf{I} \subseteq \mathbf{H}$ then goto step-3 otherwise repeat step-1

step-3: find $\mathbf{Ncl(I)}$, if $\mathbf{Ncl(I)}$ satisfies the neutrosophic closure condition then go to step-4 else repeat step-1

step-4: find $\mathbf{Nint[I]}$, if $\mathbf{Nint[I]}$ satisfies the neutrosophic interior condition then go to step-5 else repeat step-1

step-5: find $\mathbf{Nint[Ncl[I]]}$, if $\mathbf{Nint[Ncl[I]]}$ satisfies the neutrosophic interior of neutrosophic closure condition then go to step-6 else repeat step-1

step-6: find $\mathbf{Ncl[Nint[I]]}$, if $\mathbf{Ncl[Nint[I]]}$ satisfies the neutrosophic closure of neutrosophic interior condition then go to step-7 else repeat step-1

step-7: calculate $[\mathbf{Ncl[Nint[I]]}] \cup [\mathbf{Nint[Ncl[I]]}]$

step-8: if $[\mathbf{Ncl[Nint[I]]}]$ \cup $[\mathbf{Nint[Ncl[I]]}] \subseteq \mathbf{I}$ then goto step-9 else repeat step-1

step-9: calculate $\mathbf{Nbcl(I)}$

step-10: if $\mathbf{Nbcl[I]} \subseteq \mathbf{H}$ then produce neutrosophic $\boldsymbol{b^*g\alpha}$-closed set else repeat step-1

step-11: compute the neutrosophic $\boldsymbol{b^*g\alpha}$-open set $[\mathbf{F}]$ for the assigned data.

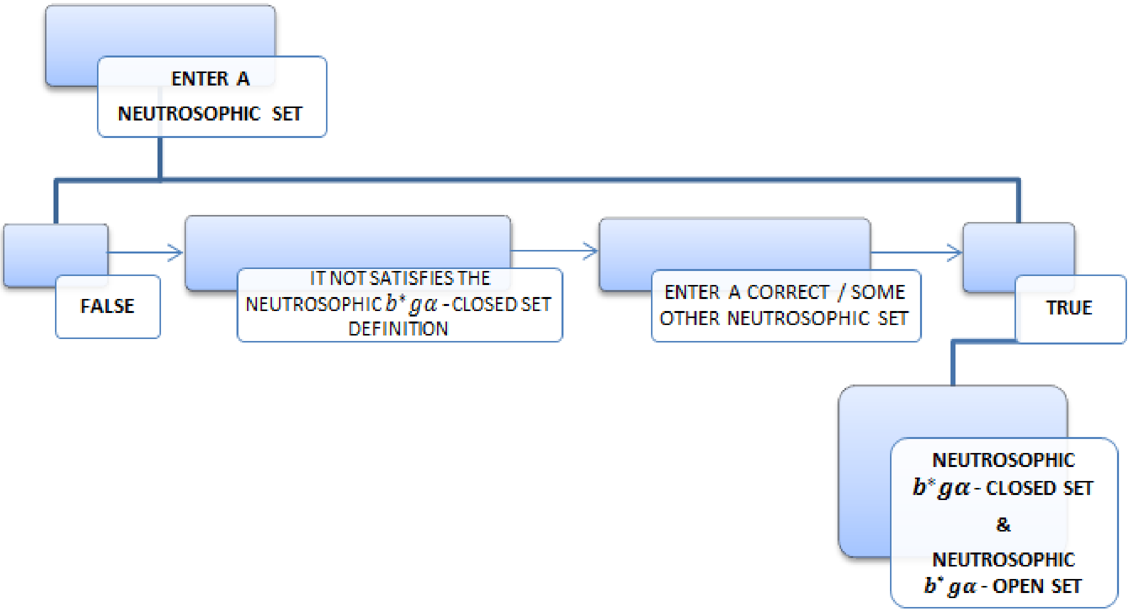

Figure.23: Flow Chart of Neutrosophic $\boldsymbol{b^*g\alpha}$-Closed Set $[\mathbf{FC\text{-}N}\boldsymbol{b^*g\alpha}\mathbf{CS}]$

The following figure shows that the neutrosophic set $\mathbf{I}$ is not satisfies the definition of neutrosophic $^{*}\boldsymbol{g\alpha}$-closed sets.

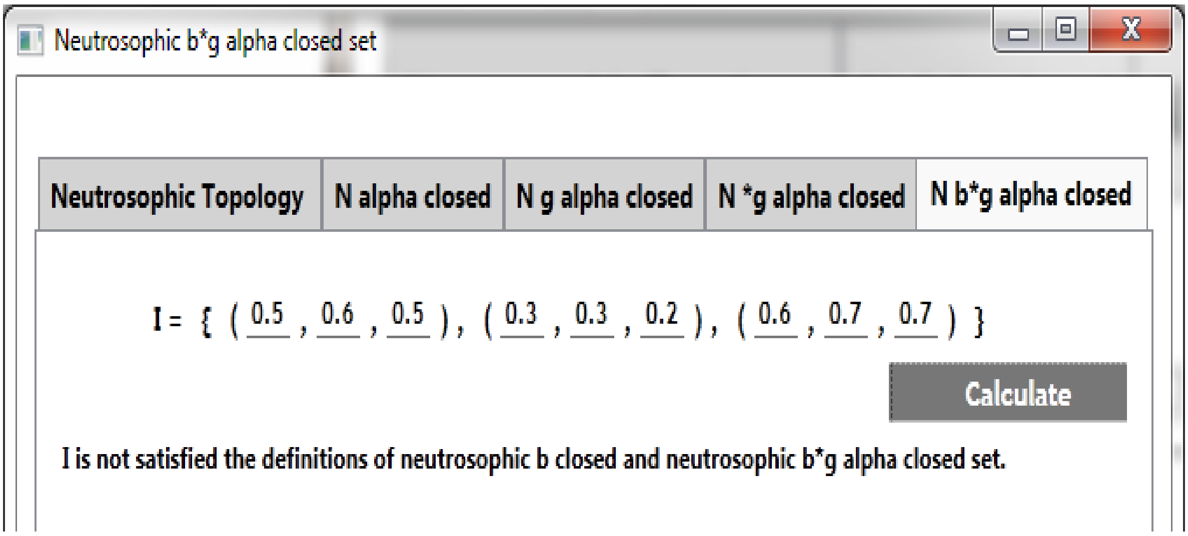

Figure.24: Screenshot of Dissatisfaction of the Definition of Neutrosophic $\boldsymbol{b^{*}g\alpha}$-Closed Set

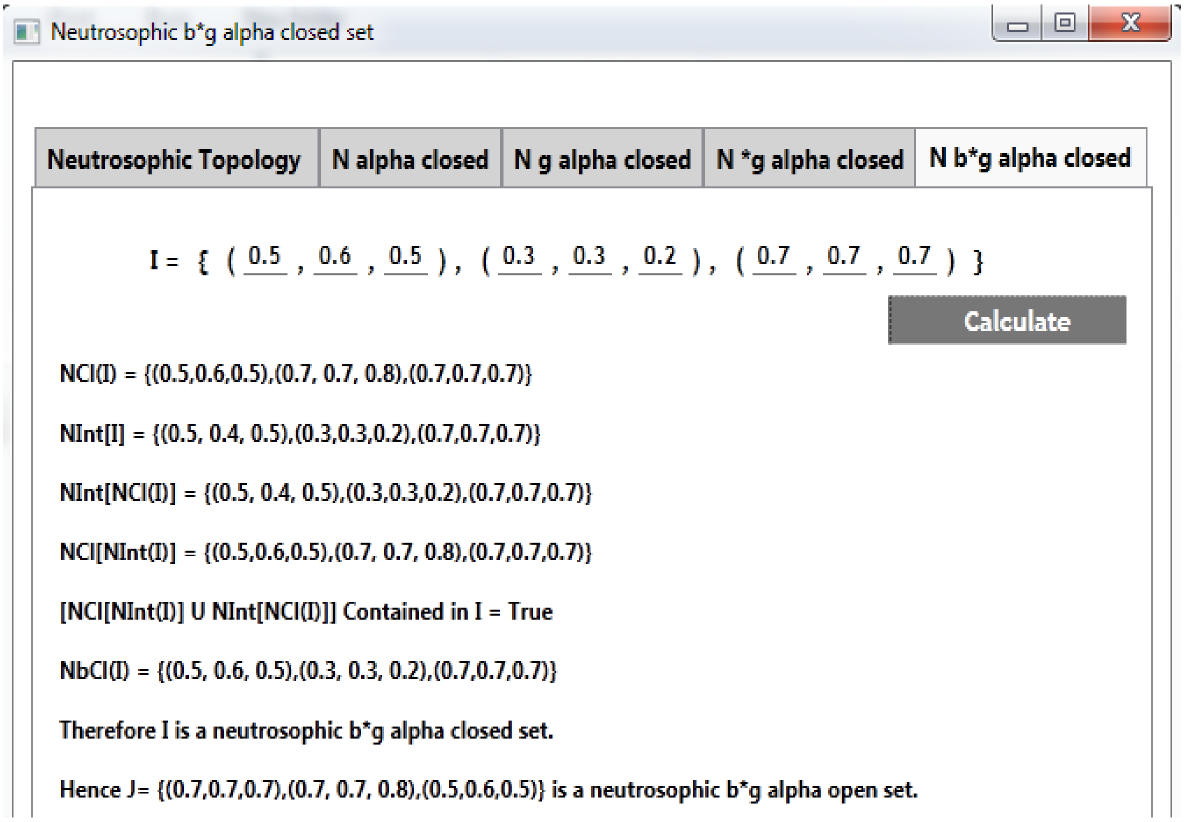

Figure.25: Screenshot of the Existence of Neutrosophic $\boldsymbol{b^{*}g\alpha}$-Closed Set $[\mathbf{N}\boldsymbol{b^{*}g\alpha}\mathbf{CS}]$ via C# application

We have assumed the values of neutrosophic sets 0_N, 1_N, $\mathbf{L}$, $\mathbf{M}$ as follows: If

$\mathbf{0}_{\boldsymbol{N}} = \{(\mathbf{0},\mathbf{0},\mathbf{0}),(\mathbf{0},\mathbf{0},\mathbf{1}),(\mathbf{0},\mathbf{1},\mathbf{0})\}$, $\mathbf{1}_{\boldsymbol{N}} = \{(\mathbf{1},\mathbf{1},\mathbf{1}),(\mathbf{1},\mathbf{1},\mathbf{0}),(\mathbf{1},\mathbf{0},\mathbf{1})\}$,
L = $\{(\mathbf{0.3},\mathbf{0.2},\mathbf{0.3}),(\mathbf{0.1},\mathbf{0.1},\mathbf{0}),(\mathbf{0.5},\mathbf{0.5},\mathbf{0.5})\}$ and
M = $\{(\mathbf{0.5},\mathbf{0.5},\mathbf{0.5}),(\mathbf{0.1},\mathbf{0.1},\mathbf{0}),(\mathbf{0.3},\mathbf{0.4},\mathbf{0.3})\}$.
After entered all the values of the above in the user screen, the current application has produced the complement set of $\mathbf{L}$ and $\mathbf{M}$, that is, $\mathbf{L}'$ and $\mathbf{M}'$. Also it has executed the union of $\mathbf{L}$ and $\mathbf{M}$, that is $[\mathbf{L} \cup \mathbf{M}]$ and the intersection of $\mathbf{L}$ and $\mathbf{M}$, that is $[\mathbf{L} \cap \mathbf{M}]$. Moreover, it has checked out the inclusion of $\mathbf{L}$ and $\mathbf{M}$, that is, whether $\mathbf{L}$ is contained in $\mathbf{M}$ or not. Finally it has produces the neutrosophic topology $[\boldsymbol{\tau}]$.
$\mathbf{L}' = \{(\mathbf{0.5},\mathbf{0.5},\mathbf{0.5}),(\mathbf{0.9},\mathbf{0.9},\mathbf{1}),(\mathbf{0.3},\mathbf{0.2},\mathbf{0.3})\}$,
$\mathbf{M}' = \{(\mathbf{0.3},\mathbf{0.4},\mathbf{0.3}),(\mathbf{0.9},\mathbf{0.9},\mathbf{1}),(\mathbf{0.5},\mathbf{0.5},\mathbf{0.5})\}$,
$\mathbf{L} \cup \mathbf{M} = \{(\mathbf{0.5},\mathbf{0.5},\mathbf{0.5}),(\mathbf{0.1},\mathbf{0.1},\mathbf{0}),(\mathbf{0.3},\mathbf{0.4},\mathbf{0.3})\}$,
$\mathbf{L} \cap \mathbf{M} = \{(\mathbf{0.3},\mathbf{0.2},\mathbf{0.3}),(\mathbf{0.1},\mathbf{0.1},\mathbf{0}),(\mathbf{0.5},\mathbf{0.5},\mathbf{0.5})\}$,
$\mathbf{L} \subseteq \mathbf{M}$ = True,
Then the Neutrosophic Topology $[\boldsymbol{\tau}] = \{\mathbf{0}_{\boldsymbol{N}}, \boldsymbol{L}, \boldsymbol{M}, \boldsymbol{L} \cup \boldsymbol{M}, \boldsymbol{L} \cap \boldsymbol{M}, \mathbf{1}_{\boldsymbol{N}}\}$.

By using this application we have checked out the following neutrosophic sets as neutrosophic $\boldsymbol{\alpha}$-closed set in neutrosophic topological spaces.

Table.1: Neutrosophic $\boldsymbol{\alpha}$-Closed Sets

$N_\alpha CS$	Membership	Indeterminacy	Non-Membership
C_1	$(0.3, 0.4, 0.3)$	$(0.5, 0.5, 0.6)$	$(0.5, 0.5, 0.5)$
C_2	$(0.3, 0.4, 0.3)$	$(0.5, 0.7, 0.7)$	$(0.5, 0.5, 0.5)$
C_3	$(0.3, 0.4, 0.3)$	$(0.9, 0.9, 0.9)$	$(0.5, 0.5, 0.5)$
C_4	$(0.3, 0.4, 0.3)$	$(0.1, 0.2, 0.9)$	$(0.5, 0.5, 0.5)$
C_5	$(0.3, 0.4, 0.3)$	$(0.4, 0.4, 0.4)$	$(0.5, 0.5, 0.5)$
C_6	$(0.3, 0.4, 0.3)$	$(0.5, 0.4, 0.4)$	$(0.5, 0.5, 0.5)$
C_7	$(0.3, 0.4, 0.3)$	$(0.5, 0.9, 0.4)$	$(0.5, 0.5, 0.5)$
C_8	$(0.3, 0.4, 0.3)$	$(0.2, 0.9, 0.4)$	$(0.5, 0.5, 0.5)$
C_9	$(0.3, 0.4, 0.3)$	$(0.35, 0.46, 0.39)$	$(0.5, 0.5, 0.5)$
C_{10}	$(0.3, 0.4, 0.3)$	$(0, 0.9, 0.4)$	$(0.5, 0.5, 0.5)$

By using this application we have checked out the following neutrosophic sets as neutrosophic $\boldsymbol{g\alpha}$-closed set in neutrosophic topological spaces.

Table.2: Neutrosophic $\boldsymbol{g\alpha}$-Closed Sets

$N_{g\alpha}CS$	Membership	Indeterminacy	Non-Membership
E_1	$(0.3, 0.213, 0.3)$	$(0.6594, 0.1, 0.517)$	$(0.671, 0.627, 0.5137)$
E_2	$(0.3, 0.4, 0.3)$	$(0, 0, 0)$	$(0.5, 0.5, 0.5)$
E_3	$(0.3, 0.2, 0.3)$	$(0.1, 0.1, 0)$	$(0.6, 0.6, 0.5)$
E_4	$(0.3, 0.2, 0.3)$	$(0.11, 0.1, 0)$	$(0.6, 0.6, 0.5)$
E_5	$(0.3, 0.2, 0.3)$	$(0.5, 0.1, 0.5)$	$(0.6, 0.6, 0.5)$
E_6	$(0.3, 0.2, 0.3)$	$(0.5, 0.1, 0.5)$	$(0.66, 0.6, 0.5)$
E_7	$(0.3, 0.2, 0.3)$	$(0.68, 0.1, 0.52)$	$(0.66, 0.63, 0.5)$
E_8	$(0.3, 0.2, 0.3)$	$(0.69, 0.1, 0.57)$	$(0.67, 0.67, 0.57)$
E_9	$(0.3, 0.2, 0.3)$	$(0.659, 0.1, 0.57)$	$(0.671, 0.627, 0.57)$
E_{10}	$(0.3, 0.213, 0.3)$	$(0.6594, 0.1, 0.57)$	$(0.671, 0.627, 0.57)$

We have assumed the values of neutrosophic sets $\mathbf{0}_N, \mathbf{1}_N, \mathbf{L}, \mathbf{M}$ as follows:
If $\mathbf{0}_N = \{(\mathbf{0},\mathbf{0},\mathbf{0}), (\mathbf{0},\mathbf{1},\mathbf{0}), (\mathbf{0},\mathbf{0},\mathbf{1})\}$, $\mathbf{1}_N = \{(\mathbf{1},\mathbf{1},\mathbf{1}), (\mathbf{1},\mathbf{1},\mathbf{0}), (\mathbf{1},\mathbf{0},\mathbf{1})\}$,
$\mathbf{L} = \{(\mathbf{0.5},\mathbf{0.4},\mathbf{0.5}), (\mathbf{0.3},\mathbf{0.3},\mathbf{0.2}), (\mathbf{0.7},\mathbf{0.7},\mathbf{0.7})\}$ and
$\mathbf{M} = \{(\mathbf{0.7},\mathbf{0.7},\mathbf{0.7}), (\mathbf{0.3},\mathbf{0.3},\mathbf{0.2}), (\mathbf{0.5},\mathbf{0.6},\mathbf{0.5})\}$.
After entered all the values of the above in the user screen, the current application has produced the complement set of $\mathbf{L}$ and $\mathbf{M}$, that is, $\mathbf{L}'$ and $\mathbf{M}'$. Also it has executed the union of $\mathbf{L}$ and $\mathbf{M}$, that is $[\mathbf{L} \cup \mathbf{M}]$ and the intersection of $\mathbf{L}$ and $\mathbf{M}$, that is $[\mathbf{L} \cap \mathbf{M}]$. Moreover, it has checked out the inclusion of $\mathbf{L}$ and $\mathbf{M}$, that is, whether $\mathbf{L}$ is contained in $\mathbf{M}$ or not. Finally it has produces the neutrosophic topology $[\boldsymbol{\tau}]$.
$\mathbf{L}' = \{(\mathbf{0.7},\mathbf{0.7},\mathbf{0.7}), (\mathbf{0.7},\mathbf{0.7},\mathbf{0.8}), (\mathbf{0.5},\mathbf{0.4},\mathbf{0.5})\}$,
$\mathbf{M}' = \{(\mathbf{0.5},\mathbf{0.6},\mathbf{0.5}), (\mathbf{0.7},\mathbf{0.7},\mathbf{0.8}), (\mathbf{0.7},\mathbf{0.7},\mathbf{0.7})\}$,
$\mathbf{L} \cup \mathbf{M} = \mathbf{M}$
$\mathbf{L} \cap \mathbf{M} = \mathbf{L}$
$\mathbf{L} \subseteq \mathbf{M}$ = True,
Then the Neutrosophic Topology $[\boldsymbol{\tau}] = \{\mathbf{0}_N, \boldsymbol{L}, \boldsymbol{M}, \mathbf{1}_N\}$.
$\mathbf{N\boldsymbol{\alpha}CS} = \{(\mathbf{0.5},\mathbf{0.6},\mathbf{0.5}), (\mathbf{0.7},\mathbf{0.7},\mathbf{0.7}), (\mathbf{0.7},\mathbf{0.7},\mathbf{0.7})\}$,
$\mathbf{N\boldsymbol{\alpha}OS} = \{(\mathbf{0.7},\mathbf{0.7},\mathbf{0.7}), (\mathbf{0.3},\mathbf{0.3},\mathbf{0.3}), (\mathbf{0.5},\mathbf{0.6},\mathbf{0.5})\}$,
$\mathrm{Ng}\boldsymbol{\alpha}\mathbf{CS} = \{(\mathbf{0.5},\mathbf{0.4},\mathbf{0.5}), (\mathbf{0.3},\mathbf{0.3},\mathbf{0.2}), (\mathbf{0.8},\mathbf{0.8},\mathbf{0.7})\}$ and
$\mathbf{Ng\boldsymbol{\alpha}OS} = \{(\mathbf{0.8},\mathbf{0.8},\mathbf{0.7}), (\mathbf{0.7},\mathbf{0.7},\mathbf{0.8}), (\mathbf{0.5},\mathbf{0.4},\mathbf{0.5})\}$.

By using this application we have checked out the following neutrosophic sets as neutrosophic $^*\boldsymbol{g\alpha}$-closed set in neutrosophic topological spaces.

Table.3: Neutrosophic $^{*}g\alpha$-Closed Sets

$N_{*g\alpha}CS$	Membership	Indeterminacy	Non-Membership
G_1	(0.1, 0.2, 0.3)	(0.23, 0.56, 0)	(0.7, 0.7, 0.7)
G_2	(0.13, 0.27, 0.3)	(0.23, 0.516, 0.4)	(0.7, 0.7, 0.7)
G_3	(0.13, 0.27, 0.35431)	(0.23, 0.516, 0.4)	(0.7, 0.7, 0.7)
G_4	(0.1113, 0.27, 0.35)	(0.23, 0.516, 0.4)	(0.72, 0.73, 0.71)
G_5	(0.1113, 0.27, 0.35)	(0.23, 0.516, 0.4)	(0.812, 0.83, 0.771)
G_6	(0.1113, 0.27, 0.35)	(0.23, 0.516, 0.456)	(0.812, 0.83, 0.771)
G_7	(0.1113, 0.25677, 0.35675)	(0.23, 0.516, 0.456)	(0.812, 0.83, 0.771)
G_8	(0.1113, 0.25677, 0.3567895)	(0.233, 0.516, 0.456)	(0.812, 0.85233, 0.771)
G_9	(0.2113, 0.256177, 0.3567895)	(0.233, 0.526, 0.55555)	(0.812, 0.85233, 0.771)
G_{10}	(0.2113, 0, 0.5)	(0.233, 0.526, 0.55555)	(0.812, 0.85233, 0.771)

By using this application we have checked out the following neutrosophic sets as neutrosophic $b^{*}g\alpha$-closed set in neutrosophic topological spaces.

Table.4: Neutrosophic $b^{*}g\alpha$-Closed Sets

$N_{b^*g\alpha}CS$	Membership	Indeterminacy	Non-Membership
I_1	(0.5, 0.6, 0.5)	(0.31, 0.32, 0.22)	(0.7, 0.7, 0.7)
I_2	(0.5, 0.5, 0.5)	(0.341, 0.362, 0.272)	(0.7, 0.7, 0.7)
I_3	(0.5, 0.432789, 0.5)	(0.341, 0.362, 0.272)	(0.7, 0.7, 0.7)
I_4	(0.5, 0.43279, 0.5)	(0.3441, 0.39, 0.272)	(0.7, 0.7, 0.7)
I_5	(0.5, 0.43279, 0.5)	(0.4, 0.39, 0.22)	(0.7, 0.7, 0.7)
I_6	(0.5, 0.43, 0.5)	(0.6, 0.3897, 0.3)	(0.7, 0.7, 0.7)
I_7	(0.5, 0.4509, 0.5)	(0.6, 0.37, 0.377777)	(0.7, 0.7, 0.7)
I_8	(0.5, 0.4509, 0.5)	(0.6123, 0.3123, 0.4123)	(0.7, 0.7, 0.7)
I_9	(0.5, 0.459, 0.5)	(0.612223, 0.314717, 0.418923)	(0.7, 0.7, 0.7)
I_{10}	(0.5, 0.459, 0.5)	(0.3, 0.387906, 0.418)	(0.7, 0.7, 0.7)

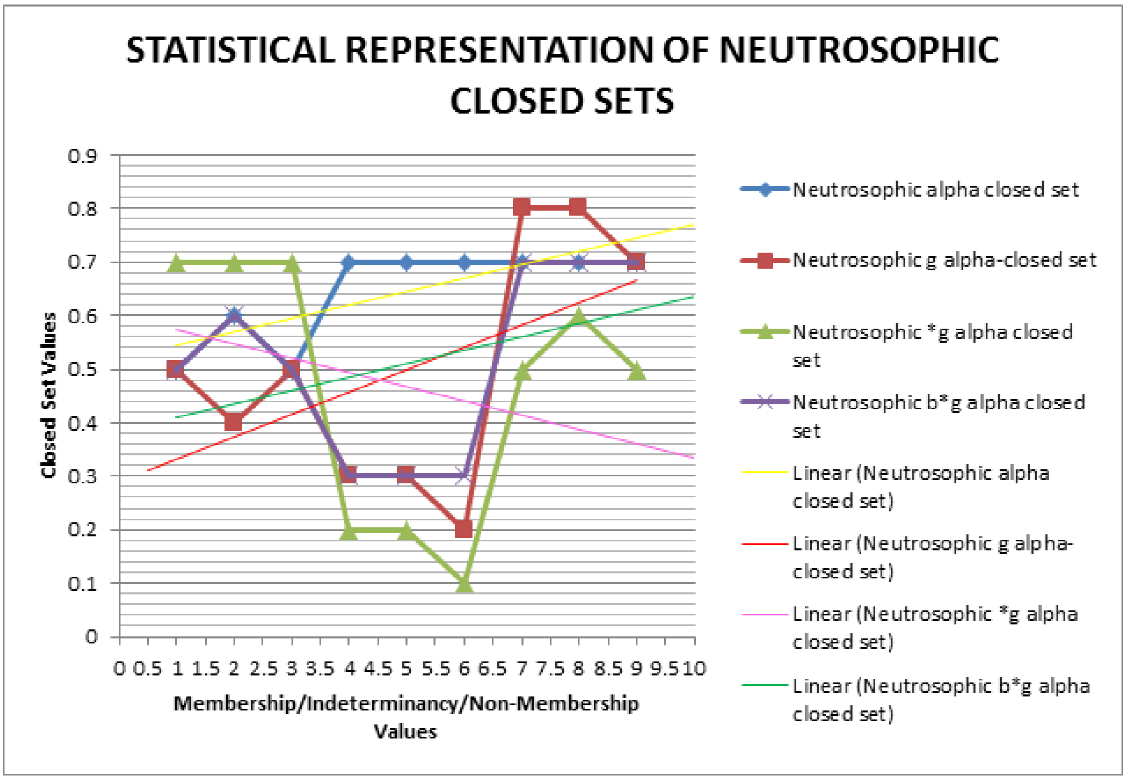

Figure.26: Statistical Representation of Neutrosophic Closed Sets

Linear Regression Line / Trend line Equations:

- Neutrosophic α-closed set: $y = 0.025x + 0.5194$, $R^2 = 0.6027$
- Neutrosophic gα-closed set: $y = 0.0417x + 0.2917$, $R^2 = 0.2604$
- Neutrosophic *gα-closed set: $y = -0.0267x + 0.6$, $R^2 = 0.0928$
- Neutrosophic b*gα-closed set: $y = 0.025x + 0.3861$, $R^2 = 0.1507$

Figure.27: Linear Regression Lines of Neutrosophic Closed Sets

In statistics, a linear regression line represents a straight line it describes how a response variable **y** changes as an explanatory variable **x** changes in the graph. Sometimes it is called as a trend line and its respective equations are denoted as a trend line equation. These type of trend lines are used in business to predict **y** value for the given value of **x**. Here we have used this regression line and its equations to predict the neutrosophic points in the non-standard interval to get the **n**-number of neutrosophic α closed sets, neutrosophic $g\alpha$ closed sets, neutrosophic $*g\alpha$-closed sets and neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces. Also we can check the stronger and weaker sets among the existing sets by using R^2 value.

4. Conclusion

This paper has introduced a new computer application for finding the neutrosophic closed sets and neutrosophic open sets in neutrosophic topological spaces via .NET Framework, Microsoft Visual Studio and C# Programming Language. Flow Chart's and the algorithm of neutrosophic topology, neutrosophic $\boldsymbol{\alpha}$-closed set, neutrosophic $\boldsymbol{g\alpha}$-closed set, neutrosophic * $\boldsymbol{g\alpha}$-closed set and neutrosophic $\boldsymbol{b^{*}g\alpha}$-closed set were presented. Also the existence of its results via C# application was shown in each figure. The complement sets were executed through this application. In future it will be extended to produce the values of the same in the neutrosophic supra topological spaces.

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An Introduction to Neutrosophic Bipolar Vague Topological Spaces

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Abstract: The main objective of this paper is to make known to a new concept of generalised neutrosophic bipolar vague sets and also defined neutrosophic bipolar vague topology in topological spaces. Also, we introduce generalized neutrosophic bipolar vague closed sets and conferred its properties.

Keywords: Bipolar set, Vague set, Neutrosophic set, Neutrosophic Bipolar Vague set, Neutrosophic Bipolar Vague Topological Spaces.

1. Introduction

Levine [24] studied the Generalized closed sets in general topology. Several investigations were conducted on the generalizations of the notion of the fuzzy set, after the introduction of the concept of fuzzy sets by Zadeh [34]. In the traditional fuzzy sets, the membership degree of component ranges over the interval [0, 1]. Few types of fuzzy set extensions in the fuzzy set theory are present, for example, intuitionistic fuzzy sets[12], interval-valued fuzzy sets[32], vague sets[30] etc. As a generalization of Zadeh's fuzzy set, the notion of vague set theory was first introduced by Gau W.L and Buehrer D.J [22]. In 1996, H.Bustince & P.Burillo indicated that vague sets are intuitionistic fuzzy sets [15].

Intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets can handle only unfinished information but not the indeterminate and unreliable information which happens normally in actual circumstances. Hence, the conception of a neutrosophic set is very common, and then it can overcome the aforesaid issues on the intuitionistic fuzzy set and the interval-valued intuitionistic fuzzy set. In 1995, the definition of Smarandache's neutrosophic set, neutrosophic sets and neutrosophic logic have been useful in many real applications to handle improbability. Neutrosophy is a branch of philosophy which studies the source, nature and scope of neutralities, as well as their interactions with different ideational scales [31]. The neutrosophic set uses one single value to indicate the truth-membership grade, indeterminacy-membership degree and falsity membership grade of an element in the universe X. The theory has been brought into extensive application in varieties of field [1-6, 8, 10, 11, 14, 17, 23, 27, 33, 35] for dealing with indeterminate and unreliable information in actual domain. The conception of Neutrosophic Topological space was introduced by A.A.Salama and S.A.Alblowi [29].

Bipolar-valued fuzzy sets, which was introduced by Lee [25, 26] is an extension of fuzzy sets whose membership degree range is extended from the interval [0, 1] to [-1, 1]. The membership degrees of the Bipolar valued fuzzy sets signify the degree of satisfaction to the property analogous to a fuzzy set and its counter-property in a bipolar valued fuzzy set, if the membership degree is 0 it means that the elements are unrelated to the corresponding property. Furthermore if the membership degree is on (0, 1] it indicates that the elements somewhat fulfil the property, and if the membership degree is on

[-1,0) it indicates that elements somewhat satisfy the entire counter property. After that, Deli et al. [21] announced the concept of bipolar neutrosophic sets, as an extension lead of neutrosophic sets. In the bipolar neutrosophic sets, the positive membership degree $T^+(x), I^+(x), F^+(x)$ signifies the truth membership, indeterminate membership and false membership of an element x ∈ X analogous to a bipolar neutrosophic set A and the negative membership degree $T^-(x),\ I^-(x), F^-(x)$ signifies the truth membership, indeterminate membership and false membership of an element x ∈ X to some implied counter-property analogous to a bipolar neutrosophic set A. There are quite a few extensions of Neutrosophic Bipolar sets such as Neutrosophic Bipolar Soft sets [7] and Rough Neutrosophic Bipolar sets [28].

Neutrosophic vague set is a combination of neutrosophic set and vague set which was well-defined by Shawkat Alkhazaleh [30]. Neutrosophic vague theory is a useful tool to practise incomplete, indeterminate and inconsistent information. In this paper, we introduced the perception of a neutrosophic bipolar vague set as a combination of neutrosophic set, Bipolar set and vague set and we also define the concept of generalised Neutrosophic Bipolar Vague set.

2. Preliminaries

Definition 2.1[16]: Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+: X→[0,1] and a negative membership function μ_A^-: X→[-1,0]. For sake of easiness, we shall practice the symbol A= {< x, μ_A^+(x), μ_A^-(x)>: x ∈ X}.

Definition 2.2[18]: Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are well-defined as follows:

(i)$\mu_{A\cup B}^+(x) = \max$ { μ_A^+(x), μ_B^+(x)}.

(ii) $\mu_{A\cup B}^-(x) = \min$ { μ_A^-(x), μ_B^-(x)}.

(iii) $\mu_{A\cap B}^+(x) = \min$ { μ_A^+(x), μ_B^+(x)}.

(iv) $\mu_{A\cap B}^-(x) = \max$ { μ_A^-(x), μ_B^-(x)}.

(v) $\mu_{\bar{A}}^+(x)$= 1-μ_A^+(x) and $\mu_{\bar{A}}^-(x)$= -1-μ_A^+(x) for all x∈ X.

Definition 2.3[15]: A vague set A in the universe of discourse U is a pair (t_A , f_A) where t_A : U→[0,1], f_A : U→[0,1] denote the mapping such that $t_A + f_A \le 1$ for all u∈ U .The function t_A and f_A are called true membership function and false membership function respectively. The interval [t_A ,1-f_A] is called the vague value of u in A, and denoted by ν_A(u), i.e ν_A(u)=[t_A ,1-f_A].

Definition 2.4[15]: Let A be a non-empty set and the vague set A and B in the form A= {<x, t_A ,1-f_A >:x∈ X}, B={<x, t_B ,1-f_B >:x∈ X}.

Then

(i)A⊆ B if and only if $t_A(x) \le t_A(x)$ and 1-f_B(x) ≤ 1-f_B(x).

(ii)A∪ B={<max(t_A(x),t_B(x)), max(1-f_A(x),1-f_B(x))>/x∈X}.

(iii) A∩ B={<min(t_A(x),t_B(x)), min(1-f_A(x),1-f_B(x))>/x∈X}.

(iv)$\bar{A}$={<x, f_A(x) ,1-t_A(x)>:x∈X}.

Definition 2.5[14]:Let X be a universe of discourse. Then a neutrosophic set is well-defined as: A = {⟨x, T_A(x), I_A(x), F_A(x)⟩:x ∈ X}, which is categorized by a truth-membership function T_A:X →]0−,1+[, an indeterminacy membership function I_A:X →]0−,1+[and a falsity-membership function F_A:X →]0−,1+[. There is no restriction to the sum of T_A(x), I_A(x) and F_A(x), so 0^-≤ supT_A(x) ≤ supI_A(x) ≤ supF_A(x) ≤ 3+.

Definition 2.6[30]: A neutrosophic vague set A_{NBV}(NVS in short) on the universe of discourse X written as,

$A_{NBV} = \{<\ \hat{T}_{NBV}(x),\ \hat{I}_{NBV}(x),\ \hat{F}_{NBV}(x) >: x \in X\}$ whose truth-membership, indeterminacy-membership and falsity-membership functions is defined as,

$\hat{T}_{NBV}(x) = [T^-, T^+]$, $\hat{I}_{NBV}(x) = [I^-, I^+]$, $\hat{F}_{NBV}(x) = [F^-, F^+]$ where

$T^+ = 1 - F^-$, $F^+ = 1$ −and T^-, $^-0 \le T^- + I^- + F^- \le 2^+$.

3. Bipolar Neutrosophic Vague Set:

Under this division, we present and well-defined the notion of neutrosophic bipolar vague set and its operations.

Definition 3.1: If A={< x, $[T_A^-,T_A^+]^+,[I_A^-,I_A^+]^+,[F_A^-,F_A^+]^+,[T_A^-,T_A^+]^-,[I_A^-,I_A^+]^-,[F_A^-,F_A^+]^-$ >} and B={< x, $[T_B^-,T_B^+]^+,[I_B^-,I_B^+]^+,[F_B^-,F_B^+]^+,[T_B^-,T_B^+]^-,[I_B^-,I_B^+]^-,[F_B^-,F_B^+]^-$>} where $(T^+)^+=1-(F^-)^+$, $(F^+)^+=1-(T^-)^+$ and $(T^+)^-=-1-(F^-)^-$, $(F^+)^-=-1-(T^-)^-$ T^+ , I^+ , F^+ : X→[0,1] and T^-,I^-,F^-:X→[-1,0] are two neutrosophic bipolar vague sets then their union, intersection and complement are well-defined as follows:

1.A∪B= { $\max[T_A^-,T_B^-]^+$, $\max[T_A^+,T_B^+]^+$,
$\min[I_A^-,I_B^-]^+$, $\min[I_A^+,I_B^+]^+$,
$\min[F_A^-,F_B^-]^+$, $\min[F_A^+,F_B^+]^+$,
$\min[T_A^-,T_B^-]^-$, $\min[T_A^+,T_B^+]^-$,
$\max[I_A^-,I_B^-]^-$, $\max[I_A^+,I_B^+]^-$,
$\max[F_A^-,F_B^-]^-$, $\max[F_A^+,F_B^+]^-$}.

2.A∩B={ $\min[T_A^-,T_B^-]^+$, $\min[T_A^+,T_B^+]^+$,
$\max[I_A^-,I_B^-]^+$,$\max[I_A^+,I_B^+]^+$,
$\max[F_A^-,F_B^-]^+$, $\max[F_A^+,F_B^+]^+$,
$\max[T_A^-,T_B^-]^-$, $\max[T_A^+,T_B^+]^-$,
$\min[I_A^-,I_B^-]^-$, $\min[I_A^+,I_B^+]^-$,
$\min[F_A^-,F_B^-]^-$, $\min[F_A^+,F_B^+]^-$}.

3. $\bar{A}$={<$[F_A^-,F_A^+]^+,[1-I_A^-,1-I_A^+]^+,[T_A^-,T_A^+]^+,[F_A^-,F_A^+]^-,[1-I_A^-,1-I_A^+]^-,[T_A^-,T_A^+]^-$>}.

Definition 3.2: Suppose A and B be two neutrosophic bipolar vague sets defined over a universe of disclosure X. We say that A ⊆ B if and only if $[T_A^-\le T_B^-]^+$, $[T_A^+\le T_B^+]^+$, $[I_A^-\ge I_B^-]^+$, $[I_A^+\ge I_B^+]^+$, $[F_A^-\ge F_B^-]^+$, $[F_A^+\ge F_B^+]^+$, $[T_A^-\ge T_B^-]^-$, $[T_A^+\ge T_B^+]^-$, $[I_A^-\le I_B^-]^-$, $[I_A^+\le I_B^+]^-$, $[F_A^-\le F_B^-]^-$, $[F_A^+\le F_B^+]^-$.

Definition 3.3: A bipolar vague topology NBVT on a nonempty set X is a family NBV_τ of Neutrosophic bipolar vague set in X sustaining the following axioms:

1. 0, 1 ∈ NBV_τ.
2. $G_1\cap G_2\in NBV_\tau$, for any $G_1,G_2\in NBV_\tau$.
3. $\cup G_i\in NBV_\tau$ for any arbitrary family { G_i: $G_i\in NBV_\tau$, i∈I}.

Under such case the pair (X, NBV_τ) is known as the neutrosophic bipolar vague topological space and any NBVS in NBV_τ is known as bipolar vague open set in X . The complement $\bar{A}$ of a neutrosophic bipolar vague open set (NBVOS) A in a neutrosophic bipolar vague topological space (X, NBV_τ) is referred as a neutrosophic bipolar vague closed (NBVCS) in X.

Example 3.4: Assume X={u,v},

A_{NBV}= $\{\frac{u}{[0.5,0.7][0.5,0.5][0.3,0.5][-0.4,-0.1][-0.5,-0.6][-0.9,-0.6]},\frac{v}{[0.3,0.6][0.4,0.4][0.4,0.7][-0.2,-0.2][-0.6,-0.8][-0.8,-0.8]}\}$,

B_{NBV}= $\{\frac{u}{[0.5,0.9][0.3,0.3][0.1,0.5][-0.4,-0.3][-0.4,-0.4][-0.7,-0.6]},\frac{v}{[0.4,0.6][0.2,0.2][0.4,0.6][-0.5,-0.3][-0.5,-0.5][-0.7,-0.5]}\}$.

Then the family NBV_τ={0,1,A,B} of neutrosophic bipolar vague sets in X is a NBVT on X.

Definition 3.5: Suppose (X, NBV_τ) is a neutrosophic bipolar vague topological space and A={< x, $[T_A^-,T_A^+]^+,[I_A^-,I_A^+]^+,[F_A^-,F_A^+]^+,[T_A^-,T_A^+]^-,[I_A^-,I_A^+]^-,[F_A^-,F_A^+]^-$ >} be a NBVS inX . Then the neutrosophic bipolar vague interior and neutrosophic bipolar vague closure of A are well-defined by,

NBVcl(A)=∩{K:K is a NBVCS in X and A⊆K},

NBVint(A)=∪{G:G is a NBVOS in X and G⊆A}.

Note that NBVcl(A) is a NBVCS and NBVint(A) is a NBVOS in X . Further,

1. A is a NBVCS in X iff NBVcl(A)=A
2. A is a NBVOS in X iff NBVint(A)=A.

Example 3.6: Assume that X={a, b},

A={x,$\frac{a}{[0.5,0.7][0.5,0.5][0.3,0.5][-0.4,-0.1][-0.5,-0.6][-0.9,-0.6]},\frac{b}{[0.3,0.6][0.4,0.4][0.4,0.7][-0.2,-0.2][-0.6,-0.8][-0.8,-0.8]}$}

B= $\{\frac{a}{[0.5,0.9][0.3,0.3][0.1,0.5][-0.4,-0.3][-0.4,-0.4][-0.7,-0.6]},\frac{b}{[0.4,0.6][0.2,0.2][0.4,0.6][-0.5,-0.3][-0.5,-0.5][-0.7,-0.5]}\}$.

Then the family NBV_τ={0,1,A,B} of a neutrosophic bipolar vague sets in X is NBVT on X. If,

F={<x,$\frac{a}{[0.5,0.4][0.5,0.5][0.6,0.5][-0.6,-0.4][-0.3,-0.3][-0.6,-0.4]},\frac{b}{[0.5,0.7][0.1,0.1][0.3,0.5][-0.3,-0.4][-0.2,-0.2][-0.6,-0.7]}$ >}

Then, NBVint(A)=∪{G:G is a NBVOS in X and G⊆F}=0 and NBVcl(A)=∩{K:K is a NBVCS in X and F⊆K}=1.

Proposition 3.7: For any NBVS A in (X, NBV_τ) we have,

1. NBVcl($\bar{A}$)=$\overline{\text{NBVint(A)}}$
2. NBVint($\bar{A}$)=$\overline{\text{NBVcl(A)}}$.

Proof: Let A={< x, $[T_A^-, T_A^+]^+, [I_A^-, I_A^+]^+, [F_A^-, F_A^+]^+, [T_A^-, T_A^+]^-, [I_A^-, I_A^+]^-, [F_A^-, F_A^+]^-$ >} and suppose that NBVOS's contained in A are indexed by the family

{<x, $[T_{G_i}^-, T_{G_i}^+]^+, [I_{G_i}^-, I_{G_i}^+]^+, [F_{G_i}^-, F_{G_i}^+]^+, [T_{G_i}^-, T_{G_i}^+]^-, [I_{G_i}^-, I_{G_i}^+]^-, [F_{G_i}^-, F_{G_i}^+]^-$ >: $i \in J$}. Then

NBVint(A)= $<x, \cup[T_{G_i}^-, T_{G_i}^+]^+, \cap[I_{G_i}^-, I_{G_i}^+]^+, \cap[F_{G_i}^-, F_{G_i}^+]^+, \cap[T_{G_i}^-, T_{G_i}^+]^-, \cup[I_{G_i}^-, I_{G_i}^+]^-, \cup[F_{G_i}^-, F_{G_i}^+]^- >$ and hence

$\overline{\text{NBVint(A}}$) $=< x, \cap[F_{G_i}^-, F_{G_i}^+]^+, \cup[1 - I_{G_i}^-, 1 - I_{G_i}^+]^+, \cup[T_{G_i}^-, T_{G_i}^+]^+, \cup[F_{G_i}^-, F_{G_i}^+]^-, \cap[1 - I_{G_i}^-, 1 - I_{G_i}^+]^-, \cap[T_{G_i}^-, T_{G_i}^+]^- >$ --------- (1)

Since,

$\bar{A}$={<$[F_A^-, F_A^+]^+, [1 - I_A^-, 1 - I_A^+]^+, [T_A^-, T_A^+]^+, [F_A^-, F_A^+]^-, [1 - I_A^-, 1 - I_A^+]^-, [T_A^-, T_A^+]^-$>}. Where

$[T_{G_i}^-, T_{G_i}^+]^+ \le [T_A^-, T_A^+]^+$, $[I_{G_i}^-, I_{G_i}^+]^+ \ge [I_A^-, I_A^+]^+$, $[F_{G_i}^-, F_{G_i}^+]^+ \ge [F_A^-, F_A^+]^+$, $[T_{G_i}^-, T_{G_i}^+]^- \ge [T_A^-, T_A^+]^-$,

$[I_{G_i}^-, I_{G_i}^+]^- \le [I_A^-, I_A^+]^-$, $[F_{G_i}^-, F_{G_i}^+]^- \le [F_A^-, F_A^+]^-$ for every i∈J we obtain that

$\{< x, [F_{G_i}^-, F_{G_i}^+]^+, [1 - I_{G_i}^-, 1 - I_{G_i}^+]^+, [T_{G_i}^-, T_{G_i}^+]^+, [F_{G_i}^-, F_{G_i}^+]^-, [1 - I_{G_i}^-, 1 - I_{G_i}^+]^-, [T_{G_i}^-, T_{G_i}^+]^- >$ i∈J}

Is the family of NBVS's containing$\bar{A}$, that is ,

NBVcl($\bar{A}$)= $< x, \cap[F_{G_i}^-, F_{G_i}^+]^+, \cup[1 - I_{G_i}^-, 1 - I_{G_i}^+]^+, \cup[T_{G_i}^-, T_{G_i}^+]^+, \cup[F_{G_i}^-, F_{G_i}^+]^-, \cap[1 - I_{G_i}^-, 1 - I_{G_i}^+]^-, \cap[T_{G_i}^-, T_{G_i}^+]^- >$ ------------ (2).

Hence from (1) and (2) we get NBVcl($\bar{A}$)=$\overline{\text{NBVint(A)}}$

(2) follows from (1).

Proposition 3.8: If (X, NBV_τ) is a NBVTS and A,B be are NBVS's in X. Then the following properties hold:

1. NBVint(A)⊆A
2. A⊆ NBvcl(A)
3. A⊆ B ⇒ NBVint(A) ⊆ NBVint(B)
4. A⊆ B ⇒ NBVcl(A) ⊆ NBVcl(B)
5. NBVint(NBVint(A))= NBVint(A)
6. NBVcl(NBVcl(A)= NBVcl(A)
7. NBVint($A \cap B$)= NBVint(A) ∩ NBVint(B)
8. NBVcl(A∪ B)= NBVcl(A)∪ NBVcl(B)
9. NBVint(1)=1
10. NBVcl(0)=0

Definition 3.9: Suppose (X, NBV_τ) and (Y, NBV_σ) be two neutrosophic bipolar vague topological spaces and $\psi: X \to Y$ be a function. Then ψ is referred to be a neutrosophic bipolar vague continuous iff the preimage of each neutrosophic bipolar vague open set in Y is a neutrosophic bipolar vague open set in X.

Proposition 3.10: Suppose A, {A_i: i∈J} be a neutrosophic bipolar vague set in X, and B, {B_j: j∈K} be a neutrosophic bipolar vague set in Y, and let $\psi: X \to Y$ be a function. Then,

(a) A_1⊆A_2⟺ $\psi(A_1) \subseteq \psi(A_2)$
(b) B_1⊆B_2⟺ $\psi^{-1}(B_1) \subseteq \psi^{-1}(B_2)$
(c) $\psi^{-1}(\cup B_i) = \cup\, \psi^{-1}(B_i)$ and $\psi^{-1}(\cap B_i) = \cap\, \psi^{-1}(B_i)$

Proof: Obvious.

Proposition 3.11: The subsequent are equivalent to each other.

1. $\psi: X \to Y$ is neutrosophic bipolar vague continuous.
2. ψ^{-1}(NBVint(B)) ⊆ NBVint(ψ^{-1}(B)) for each NBVOS B in Y.
3. NBVcl(ψ^{-1}(B)) ⊆ ψ^{-1}(NBVcl(B)) for each NBVOS B in Y.

Proof: (1)⟹(2) Given $\psi: X \to Y$ is neutrosophic bipolar vague continuous.

Then we have to show that ψ^{-1}(NBVint(B)) $\subseteq$ NBVint(ψ^{-1}(B)) for each NBVOS B in Y.
Let B={< y, $[T_B^-, T_B^+]^+, [I_B^-, I_B^+]^+, [F_B^-, F_B^+]^+, [T_B^-, T_B^+]^-, [I_B^-, I_B^+]^-, [F_B^-, F_B^+]^-$>} be NBVOS in Y.
NBVint(B)=
$\{ < y, \cup[T_{H_i}^-, T_{H_i}^+]^+, \cap[I_{H_i}^-, I_{H_i}^+]^+, \cap[F_{H_i}^-, F_{H_i}^+]^+, \cap[T_{H_i}^-, T_{H_i}^+]^-, \cup[I_{H_i}^-, I_{H_i}^+]^-, \cup[F_{H_i}^-, F_{H_i}^+]^- >$:i∈I}
Where,
$[T_{H_i}^-, T_{H_i}^+]^+ \leq [T_B^-, T_B^+]^+$, $[I_{H_i}^-, I_{H_i}^+]^+ \geq [I_B^-, I_B^+]^+$, $[F_{H_i}^-, F_{H_i}^+]^+ \geq [F_B^-, F_B^+]^+$, $[T_{H_i}^-, T_{H_i}^+]^- \geq [T_B^-, T_B^+]^-$, $[I_{H_i}^-, I_{H_i}^+]^- \leq [I_B^-, I_B^+]^-$, $[F_{H_i}^-, F_{H_i}^+]^- \leq [F_B^-, F_B^+]^-$ for every i ∈ I. By the definition of continuity ψ^{-1}(NBVint(B)) is a neutrosophic bipolar vague open set in NBV_τ. Now,
ψ^{-1}(NBVint(B)) ={
$\psi^{-1}(< y, \cup[T_{H_i}^-, T_{H_i}^+]^+, \cap[I_{H_i}^-, I_{H_i}^+]^+, \cap[F_{H_i}^-, F_{H_i}^+]^+, \cap[T_{H_i}^-, T_{H_i}^+]^-, \cup[I_{H_i}^-, I_{H_i}^+]^-, \cup[F_{H_i}^-, F_{H_i}^+]^- >)$}
= {(<x, $\psi^{-1}(\cup[T_{H_i}^-, T_{H_i}^+]^+)$, $\psi^{-1}(\cap[I_{H_i}^-, I_{H_i}^+]^+)$, $\psi^{-1}\left(\cap[F_{H_i}^-, F_{H_i}^+]^+\right)$, $\psi^{-1}(\cap[T_{H_i}^-, T_{H_i}^+]^-)$, $\psi^{-1}(\cup[I_{H_i}^-, I_{H_i}^+]^-)$, $\psi^{-1}(\cup[F_{H_i}^-, F_{H_i}^+]^-)$ >)}.
= {<x, $\cup[\psi^{-1}[[T_{H_i}^-, T_{H_i}^+]^+]$, $\cap[\psi^{-1}[I_{H_i}^-, I_{H_i}^+]^+]$, $\cap[\psi^{-1}[F_{H_i}^-, F_{H_i}^+]^+]$, $\cap[\psi^{-1}[T_{H_i}^-, T_{H_i}^+]^-]$, $\cup[\psi^{-1}[[I_{H_i}^-, I_{H_i}^+]^-]$, $\cup[\psi^{-1}[[F_{H_i}^-, F_{H_i}^+]^-]$>}.
$\subseteq$ NBVint(ψ^{-1}(B))
(2)⟹(1). Given ψ^{-1}(NBVint(B)) $\subseteq$ NBVint(ψ^{-1}(B)) for each NBVOS B in Y. Let
B={< y, $[T_B^-, T_B^+]^+, [I_B^-, I_B^+]^+, [F_B^-, F_B^+]^+, [T_B^-, T_B^+]^-, [I_B^-, I_B^+]^-, [F_B^-, F_B^+]^-$>} be NBVOS in Y. We know that B is a neutrosophic bipolar vague open in Y if and only if NBVint(B)=B and hence ψ^{-1}(NBVint(B))= ψ^{-1}(B). But according to our supposition ψ^{-1}(NBVint(B)) $\subseteq$ NBVint(ψ^{-1}(B)), therefore we get ψ^{-1}(B) $\subseteq$ NBVint(ψ^{-1}(B), i.e., ψ^{-1}(B) is a NBVS in X and thus ψ is a neutrosophic bipolar vague continuous.
(1)⟹ (3) Given $\psi: X \to Y$ is neutrosophic bipolar vague continuous.
Suppose B={< y, $[T_B^-, T_B^+]^+, [I_B^-, I_B^+]^+, [F_B^-, F_B^+]^+, [T_B^-, T_B^+]^-, [I_B^-, I_B^+]^-, [F_B^-, F_B^+]^-$>} be NBVOS in Y.
Also suppose NBVcl(B) =
{<y,$\cap[T_{K_i}^-, T_{K_i}^+]^+, \cup[I_{K_i}^-, I_{K_i}^+]^+, \cup[F_{K_i}^-, F_{K_i}^+]^+, \cup[T_{K_i}^-, T_{K_i}^+]^-, \cap[I_{K_i}^-, I_{K_i}^+]^-, \cap[F_{K_i}^-, F_{K_i}^+]^-$>:i∈ I},where
$[T_{K_i}^-, T_{K_i}^+]^+ \leq [T_B^-, T_B^+]^+$,$[I_{K_i}^-, I_{K_i}^+]^+ \geq [I_B^-, I_B^+]^+$,$[F_{K_i}^-, F_{K_i}^+]^+ \geq [F_B^-, F_B^+]^+$,
$[T_{K_i}^-, T_{K_i}^+]^- \geq [T_B^-, T_B^+]^-$,$[I_{K_i}^-, I_{K_i}^+]^- \leq [I_B^-, I_B^+]^-$, $[F_{K_i}^-, F_K^+]^- \leq [F_B^-, F_B^+]^-$ for every i ∈ I. Since ψ is a neutrosophic bipolar vague continuous iff the inverse image of each NBVCS in Y is a NBVCS in X, therefore ψ^{-1}(NBVcl(B)) is a NBVCS in X.
Now, ψ^{-1} (NBVcl(B))= {$\psi^{-1}(< y, \cap[T_{K_i}^-, T_{K_i}^+]^+, \cup[I_{K_i}^-, I_{K_i}^+]^+, \cup[F_{K_i}^-, F_{K_i}^+]^+, \cup[T_{K_i}^-, T_{K_i}^+]^-, \cap[I_{K_i}^-, I_{K_i}^+]^-, \cap[F_{K_i}^-, F_{K_i}^+]^- >)$}
={(<x, $\psi^{-1}(\cap[T_{K_i}^-, T_{K_i}^+]^+)$, $\psi^{-1}(\cup[I_{K_i}^-, I_{K_i}^+]^+)$, $\psi^{-1}\left(\cup[F_{K_i}^-, F_{K_i}^+]^+\right)$, $\psi^{-1}(\cup[T_{K_i}^-, T_{K_i}^+]^-)$, $\psi^{-1}(\cap[I_{K_i}^-, I_{K_i}^+]^-)$, $\psi^{-1}(\cap[F_{K_i}^-, F_{K_i}^+]^-)$ >)}.
= {<x, $\cap[\psi^{-1}[[T_{K_i}^-, T_{K_i}^+]^+]$, $\cup[\psi^{-1}[I_{K_i}^-, I_{K_i}^+]^+]$, $\cup[\psi^{-1}[F_{K_i}^-, F_{K_i}^+]^+]$, $\cup[\psi^{-1}[T_{K_i}^-, T_{K_i}^+]^-]$, $\cap[\psi^{-1}[[I_{K_i}^-, I_{K_i}^+]^-]$, $\cap[\psi^{-1}[[F_{K_i}^-, F_{K_i}^+]^-]$>}
$\supseteq$ NBVcl(ψ^{-1}(B))
(3)⟹(1)
Given NBVcl(ψ^{-1}(B))⊆ ψ^{-1}(NBVcl(B)), for each NBVOS B in Y. Let
B={< y, $[T_B^-, T_B^+]^+, [I_B^-, I_B^+]^+, [F_B^-, F_B^+]^+, [T_B^-, T_B^+]^-, [I_B^-, I_B^+]^-, [F_B^-, F_B^+]^-$>} be NBVCS in Y. Since NBVcl(B)=B. But it is given that NBVcl(ψ^{-1}(B))⊆ ψ^{-1}(NBVcl(B)), hence NBVcl(ψ^{-1}(B))⊆ ψ^{-1}(B). Hence ψ^{-1}(B)= NBVcl(ψ^{-1}(B)), i.e., ψ^{-1}(B) is a NBVCS in X and this proves that ψ is a neutrosophic bipolar vague continuous.

4. Generalized Neutrosophic Bipolar Vague Closed Sets:

Definition 4.1: Suppose if (X, NBV_τ) be a neutrosophic bipolar vague topological space. A neutrosophic bipolar vague set A in (X, NBV_τ) is referred to be a generalized neutrosophic bipolar vague closed set if NBVcl(A) $\subseteq G$ whenever A⊆G and G is a neutrosophic bipolar vague open. The complement of a generalized neutrosophic bipolar vague closed set is generalized neutrosophic bipolar vague open set.

Definition 4.2: Suppose let (X, NBV_τ) be a neutrosophic bipolar vague topological space and let A be a neutrosophic bipolar vague set in X. The generalized neutrosophic bipolar vague closure (GNBVcl for short) and the generalized neutrosophic bipolar vague interior (GNBVint for short) of A are well-defined by,

1) GNBVcl(A)=⋂{G:G is a generalized neutrosophic bipolar vague closed sets in X and A⊆ G},
2) GNBVint(A)=⋃{G:G is a generalized neutrosophic bipolar vague open sets in X and A⊇ G}.

Remark 4.3: Every NBVCS is generalized neutrosophic bipolar vague closed but not conversely.

Example 4.4: Assume that X= {u,v} and NBV_τ={0,1,F} is a NBVT on X where,

F=< $x, \frac{u}{[0.5,0.9][0.3,0.3][0.1,0.5][-0.4,-0.3][-0.4,-0.4][-0.7,-0.6]}, \frac{v}{[0.4,0.6][0.2,0.2][0.4,0.6][-0.5,-0.3][-0.5,-0.5][-0.7,-0.5]}$>

Then the neutrosophic bipolar vague set,

A=< $x, \frac{u}{[0.5,0.7][0.5,0.5][0.3,0.5][-0.4,-0.1][-0.5,-0.6][-0.9,-0.6]}, \frac{v}{[0.3,0.6][0.4,0.4][0.4,0.7][-0.2,-0.2][-0.6,-0.8][-0.8,-0.8]}$ > is a generalized neutrosophic bipolar vague closed but not NBVC in X.

Proposition 4.5: Suppose that (X, NBV_τ) be a neutrosophic bipolar vague topological space. If A is a generalized neutrosophic bipolar vague closed set and A⊆B⊆NBVcl(A), then B is a generalized neutrosophic bipolar vague closed set.

Proof: Suppose let G be a neutrosophic bipolar vague open set in (X, NBV_τ), such that B⊆G. Since A⊆B, A⊆ G. Now A is a generalized neutrosophic bipolar vague closed set and NBVcl(A) ⊆G. But NBVcl(B) ⊆ NBVcl(A). Since NBVcl(B) ⊆ NBVcl(A) ⊆G, NBVcl(B) ⊆G. Hence B is a generalized neutrosophic bipolar vague closed set.

Proposition 4.6: Suppose if A is a neutrosophic bipolar vague open set and generalized neutrosophic bipolar vague closed set in (X, NBV_τ), then A is said to be a neutrosophic bipolar vague closed set in X.

Proof: Assume that A is a neutrosophic bipolar vague open set in X. Since A⊆A, by hypothesis NBVcl(A) ⊆A. Then from definition A⊆ NBVcl(A). Therefore NBVcl(A)= A. Hence A is neutrosophic bipolar vague closed set in X.**Proposition 4.7**: Suppose that NBVint(A) ⊆B⊆A and assume A is a generalized neutrosophic bipolar vague open set then B is also a generalized neutrosophic bipolar vague open set.

Proof: Now, $\bar{A} \subseteq \bar{B} \subseteq \overline{\text{NBVint(A)}}$= NBVcl($\bar{A}$). As A is a generalized neutrosophic bipolar vague open, $\bar{A}$ is a generalized neutrosophic bipolar vague closed set. By proposition 4.5, $\bar{B}$ generalized neutrosophic bipolar vague closed set. That is, B is also a generalized neutrosophic bipolar vague open set.

Definition 4.8: Suppose (X, NBV_τ) and (Y, NBV_σ)be any two neutrosophic bipolar vague topological spaces.

1. A map ψ: (X, NBV_τ)→(Y, NBV_σ) is referred to be a generalized neutrosophic bipolar vague continuous if the inverse image of every neutrosophic bipolar vague open set in (Y, NBV_σ) is a generalized neutrosophic bipolar vague open set in (X, NBV_τ).
2. A map ψ: (X, NBV_τ)→(Y, NBV_σ) is called as a generalized neutrosophic bipolar vague irresolute if the inverse image of every generalized neutrosophic bipolar vague open set in (Y, NBV_σ) is a generalized neutrosophic bipolar vague open set in (X, NBV_τ).

Proposition 4.9: Suppose (X, NBV_τ) and (Y, NBV_σ)be any two neutrosophic bipolar vague topological spaces. A mapping ψ: (X, NBV_τ)→(Y, NBV_σ) is referred to be generalized neutrosophic bipolar vague continuous function mapping. Then for every neutrosophic bipolar vague set A in X, ψ(GNBVcl(A)) ⊆ NBVcl($\psi(A)$).

Proof: Assume A to be a neutrosophic bipolar vague set in (X, NBV_τ). Since NBVcl($\psi(A)$) is a neutrosophic bipolar vague closed set and since ψ is a generalized neutrosophic bipolar vague continuous mapping, the set ψ^{-1}(NBVcl($\psi(A)$)) is a generalized neutrosophic bipolar vague closed set and thus ψ^{-1}(NBVcl($\psi(A)$)) ⊇A.

Now, GNBVcl(A))⊆ $\psi^{-1}\left(\text{NBVcl}(\psi(A))\right)$. Therefore ψ(GNBVcl(A)) ⊆ NBVcl($\psi(A)$).

Proposition 4.10: If (X, NBV_τ) and (Y, NBV_σ) are two neutrosophic bipolar vague topological spaces. Let the mapping ψ : (X, NBV_τ) →(Y, NBV_σ) be a generalized neutrosophic bipolar vague continuous mapping. Then for every neutrosophic bipolar vague set A in Y, GNBVcl(ψ^{-1}(A)) ⊆ ψ^{-1}(NBVcl(A).

Proof: Assume A to be a neutrosophic bipolar vague set in (Y, NBV_σ). Let B= ψ^{-1}(A). Then, ψ(B)= ψ(ψ^{-1}(A)) ⊆A. By proposition 4.10, ψ(GNBVcl(ψ^{-1}(A))) ⊆NBV cl(ψ(ψ^{-1}(A))). Thus, GNBVcl(ψ^{-1}(A)) ⊆ ψ^{-1}(NBVcl(A).

Proposition 4.11: Suppose let (X, NBV_τ) and (Y, NBV_σ) be any two neutrosophic bipolar vague topological spaces. Let ψ : (X, NBV_τ) →(Y, NBV_σ) is referred to be a neutrosophic bipolar vague continuous mapping, then it is a generalized neutrosophic bipolar vague continuous mapping.

Proof: Suppose let A be a neutrosophic bipolar vague open set in (Y, NBV_σ). Since the mapping ψ is a neutrosophic bipolar vague continuous mapping, $\psi^{-1}(A)$ is a neutrosophic bipolar vague open set in (X, NBV_τ). Every neutrosophic bipolar vague open set is a generalized neutrosophic bipolar vague open set. Now, $\psi^{-1}(A)$ is a generalized neutrosophic bipolar vague open set in (X, NBV_τ). Hence ψ is thus a generalized neutrosophic bipolar vague continuous mapping.

The converse of the proposition need not be true as shown in example.

Example 4.12: Assume that X={a,b}, Y={u,v} and,

A=<x, $\frac{a}{[0.5,0.4][0.5,0.5][0.6,0.5][-0.6,-0.4][-0.3,-0.3][-0.6,-0.4]}, \frac{b}{[0.6,0.7][0.1,0.1][0.3,0.4][-0.3,-0.4][-0.2,-0.2][-0.6,-0.7]}$>,

B=<x, $\frac{a}{[0.5,0.3][0.5,0.5][0.7,0.5][-0.4,-0.2][-0.4,-0.4][-0.8,-0.6]}, \frac{b}{[0.5,0.4][0.2,0.2][0.6,0.5][-0.3,-0.4][-0.2,-0.2][-0.6,-0.7]}$>.

Then NBV_τ={0,1,A} and NBV_σ={0,1,B} are NBVT on X and Y respectively. Define a mapping ψ: (X, NBV_τ)→(Y, NBV_σ) by ψ(a)=u and ψ(b)=v. then ψ is a generalized neutrosophic bipolar vague continuous mapping but not bipolar vague continuous mapping.

Proposition 4.13: Suppose let (X, NBV_τ) and (Y, NBV_σ) be any two neutrosophic bipolar vague topological spaces. A mapping ψ: (X, NBV_τ)→(Y, NBV_σ) is said to be a generalized neutrosophic bipolar vague irresolute mapping, then it is a generalized neutrosophic bipolar vague continuous mapping.

Proof: Let A be a neutrosophic bipolar vague open set in (Y, NBV_σ). Since every neutrosophic bipolar vague open set is a generalized neutrosophic bipolar vague open set in (Y, NBV_σ), but ψ is a generalized neutrosophic bipolar vague irresolute mapping, $\psi^{-1}(A)$ is a generalized neutrosophic bipolar vague open set in (X, NBV_τ). Thus ψ is a generalized neutrosophic bipolar vague continuous mapping.

Proposition 4.14: Suppose let (X, NBV_τ), (Y, NBV_σ) and (Z, NBV_ρ) be any three bipolar vague topological spaces. Let ψ: (X, NBV_τ)→(Y, NBV_σ) be a generalized neutrosophic bipolar vague irresolute mapping and ψ_1: (Y, NBV_σ)→(Z, NBV_ρ) be a generalized neutrosophic bipolar vague continuous mapping. Then $\psi_1 \circ \psi$ is a generalized neutrosophic bipolar vague continuous mapping.

Proof: Let A be a neutrosophic bipolar vague open set in (Z, NBV_ρ). Since ψ_1 is a generalized neutrosophic bipolar vague continuous mapping, ψ_1^{-1} (A) is a generalized neutrosophic bipolar vague open set in (Y, NBV_σ) . Since ψ is a generalized neutrosophic bipolar vague irresolute mapping, $\psi^{-1}(\psi_1^{-1}$ (A)) is a generalized neutrosophic bipolar vague open set in (X, NBV_τ). Thus $\psi_1 \circ \psi$ is a generalized neutrosophic bipolar vague continuous mapping.

Conclusion:

This paper presented the new concept of Neutrosophic Bipolar Vague sets and studied some basic operational relation of Neutrosophic Bipolar Vague set. Then a generalization of NBVSs in closed set is done. As a future work, we shall continue to work in the application of NBVS to other domains, such as medical diagnosis, pattern recognition and decision making.

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Neutrosophic Almost Contra α-Continuous Functions

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Abstract: This study utilizes the notions of $\aleph$-α-open set to introduce and study new form of $\aleph$-ccontinuity termed as $\aleph$-almost contra α-continuous function. Besides, we also introduce $\aleph\alpha$-connected space, $\aleph$-weakly Hausdorff space, separation axioms, $\aleph\alpha$-normal and $\aleph$-strong normal spaces. Characterizations of $\aleph$-almost contra α-continuous functions is also discussed.

Keywords: $\aleph$-almost contra α-continuous function; $\aleph\alpha$-connected space; $\aleph$-weakly Hausdorff space; $\aleph$-locally αindiscrete space; $\aleph\alpha$-normal space; $\aleph$-strong normal space.

1. Introduction

Many real-world problems in Finance, Medical sciences, Engineering and Social sciences deals with uncertainties. There are difficulties in solving the uncertainties in these data by traditional mathematical models. There are approaches such as fuzzy sets [28], intuitionistic fuzzy sets [10], vague sets [13], and rough sets [18] which can be treated as mathematical tools to avert obstacles dealing with ambiguous data. But all these approaches have their implicit crisis in solving the problems involving indeterminate and inconsistent data due to inadequacy of parameterization tools. Smarandache [24] studied the idea of neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy and persistent data. Neutrosophic topological space was introduced by Salama et.al. [19] in 2012. Further Neutrosophic topological spaces are studied in [20]. Applications of neutrosophic topology depend upon the properties of neutrosophic open sets, neutrosophic closed sets, neutrosophic interior operator and neutrosophic closure operator. Topologists studied the sets that are near to neutrosophic open sets and neutrosophic closed sets. In this order, Arokiarani et.al.[9] defined neutrosophic semi-open (resp. pre-open and α-open) functions and investigated their relations. In [9], the characterizations of characterizations of neutrosophic pre continuous (resp. α-continuous) functions is also discussed.

The idea of almost continuous functions is done in 1968 [21] in topology. Similarly, the notion of fuzzy almost contra continuous and fuzzy almost contra α-continuous functions were discussed in [16]}. Recently, Al-Omeri and Smarandache [26, 27] introduced and studied a number of the definitions of neutrosophic closed sets, neutrosophic mapping, and obtained several preservation properties and some characterizations about neutrosophic of connectedness and neutrosophic connectedness continuity. More recently, in [1, 8] authors have given how new trend of Neutrosophic theory is applicable in the field of Medicine and multimedia with a novel and powerful model.

In this paper, we define Almost contra-continuity in the context of neutrosophic topology such as Neutrosophic Almost α-contra-continuous function. We also discuss some characterizations of this concept. Moreover $\aleph\alpha$-connected space, $\aleph$- weakly Hausdorff space, separation axioms and $\aleph\alpha$ -normal spaces are presented and investigated some properties.

2. Preliminaries

Definition: 2.1 [22, 23] Allow T,I,F as real standard or non standard members of $]0^-, 1^+[$, with
$sup_T = t_{sup}, inf_T = t_{inf}$,
$sup_I = i_{sup}, inf_I = i_{inf}$,
$sup_F = f_{sup}, inf_F = f_{inf}$
$n - sup = t_{sup} + i_{sup} + f_{sup}$
$n - inf = t_{inf} + i_{inf} + f_{inf}$. T,I,F are neutrosophic components.

Definition: 2.2 [22, 23] Let S_1 be a non-empty fixed set. A definition set (in short N-set) Λ is an object such that $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \gamma_{\Lambda}(x)\rangle : x \in S_1\}$ wherein $\mu_{\Lambda}(x), \sigma_{\Lambda}(x)$ and $\gamma_{\Lambda}(x)$ which represents the degree of membership function (viz $\mu_{\Lambda}(x)$), the degree of indeterminacy (viz $\sigma_{\Lambda}(x)$) as well as the degree of non-membership (viz $\gamma_{\Lambda}(x)$) respectively of each element $x \in S_1$ to the set Λ.

Remark: 2.3[22, 23]

I. *An N-set* $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$ *can be identified to an ordered triple* $\langle \mu_{\Lambda}, \sigma_{\Lambda}, \Gamma_{\Lambda}\rangle$ *in* $]0^-, 1^+[$ *on* S_1.

II. *In this paper, we use the symbol* $\Lambda = \langle \mu_{\Lambda}, \sigma_{\Lambda}, \Gamma_{\Lambda}\rangle$ *for the* N*-set* $\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$.

Definition: 2.4*[12]* Let $S_1 \neq \emptyset$ and the N-sets Λ and Γ be defined as

$\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$, $\Gamma = \{\langle x, \mu_{\Gamma}(x), \sigma_{\Gamma}(x), \Gamma_{\Gamma}(x)\rangle : x \in S_1\}$. Then

I. $\Lambda \subseteq \Gamma$ *iff* $\mu_{\Lambda}(x) \leq \mu_{\Gamma}(x)$, $\sigma_{\Lambda}(x) \leq \sigma_{\Gamma}(x)$ *and* $\Gamma_{\Lambda}(x) \geq \Gamma_{\Gamma}(x)$ *for all* $x \in S_1$;

II. $\Lambda = \Gamma$ *iff* $\Lambda \subseteq \Gamma$ *and* $\Gamma \subseteq \Lambda$;

III. $\bar{\Lambda} = \{\langle x, \Gamma_{\Lambda}(x), \sigma_{\Lambda}(x), \mu_{\Lambda}(x)\rangle : x \in S_1\}$; *[Complement of* Λ*]*

IV. $\Lambda \cap \Gamma = \{\langle x, \mu_{\Lambda}(x) \wedge \mu_{\Gamma}(x), \sigma_{\Lambda}(x) \wedge \sigma_{\Gamma}(x), \Gamma_{\Lambda}(x) \vee \Gamma_{\Gamma}(x)\rangle : x \in S_1\}$;

V. $\Lambda \cup \Gamma = \{\langle x, \mu_{\Lambda}(x) \vee \mu_{\Gamma}(x), \sigma_{\Lambda}(x) \vee \sigma_{\Gamma}(x), \Gamma_{\Lambda}(x) \wedge \gamma_{\Gamma}(x)\rangle : x \in S_1\}$;

VI. $[\,]\Lambda = \{\langle x, \mu_{\Lambda}(x), \sigma_{\Lambda}(x), 1 - \mu_{\Lambda}(x)\rangle : x \in S_1\}$;

VII. $\langle\rangle\, \Lambda = \{\langle x, 1 - \Gamma_{\Lambda}(x), \sigma_{\Lambda}(x), \Gamma_{\Lambda}(x)\rangle : x \in S_1\}$.

Definition: 2.5[12] Let $\{\Lambda_i : i \in J\}$ be an arbitrary family of N-sets in S_1. Thereupon

I. $\cap \Lambda_i = \{\langle p, \wedge \mu_{\Lambda_i}(p), \wedge \sigma_{\Lambda_i}(p), \vee \Gamma_{\Lambda_i}(p)\rangle : p \in S_1\}$;

II. $\cup \Lambda_i = \{\langle p, \vee \mu_{\Lambda_i}(p), \vee \sigma_{\Lambda_i}(p), \wedge \Gamma_{\Lambda_i}(p)\rangle : p \in S_1\}$.

The main theme is to construct the tools for developing NTS, so we establish the neutrosophic sets $0_{\aleph}$ along with $1_{\aleph}$ in X as follows:

Definition: 2.6[12] $0_{\aleph} = \{\langle q, 0,0,1\rangle : q \in X\}$ and $1_{\aleph} = \{\langle q, 1,1,0\rangle : q \in X\}$.

Definition: 2.7[12] A definition topology (in short,ℵ-topology) on $S_1 \neq \emptyset$ is a family ξ_1 of N-sets in S_1 satisfying the laws given below:

I. $0_N, 1_N \in \xi_1$,

II. $W_1 \cap W_2 \in T$ *being* $W_1, W_2 \in \xi_1$,

III. $\cup W_i \in \xi_1$ *for arbitrary family* $\{W_i | i \in \Lambda\} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is termed as NTS and each N-set in ξ_1 is named as neutrosophic open set (in short,ℵ-open set) . The complement $\overline{\Lambda}$ of an ℵ-open set Λ in S_1 is known as neutrosophic closed set (briefly,ℵ-closed set) in S_1.

Definition: 2.8[12] Let Λ be an $\aleph$-set in an $NTSS_1$. Thereupon

$\aleph int(\Lambda) = \cup \{G|G$ is an $\aleph$-open set in S_1 and $G \subseteq \Lambda\}$ is termed as neutrosophic interior (in brief $\aleph$-interior) of Λ;

$\aleph cl(\Lambda) = \cap \{G|G$ is an $\aleph$-closed set in S_1 and $G \supseteq \Lambda\}$ is termed as neutrosophic closure (shortly $\aleph cl$) of Λ.

Definition: 2.9[12] Let X be a nonempty set. Whenever r, t, s be real standard or non standard subsets of $]0^{-}, 1^{+}[$ then the neutrosophic set $x_{r,t,s}$ is termed as neutrosophic point(in short NP)in X given by $x_{r,t,s}(x_p) = \begin{cases} (r,t,s), & if \quad x = x_p \\ (0,0,1), & if \quad x \neq x_p \end{cases}$ for $x_p \in X$ is termed as the support of $x_{r,t,s}$, wherein r indicates the degree of membership value, t indicates the degree of indeterminacy along with s as the degree of non-membership value of $x_{r,t,s}$.

Definition: 2.10[12] Allow (S_1, ξ_1) be a NTS. A neutrosophic set Λ in (S_1, ξ_1) is termed as $\mathfrak{g}\aleph$ closed set if $Ncl(\Lambda) \subseteq \Gamma$ whenever $\Lambda \subseteq \Gamma$ and Γ is a $\aleph$-open set. The complement of a $\mathfrak{g}\aleph$-closed set is named as $\mathfrak{g}\aleph$-open set.

Definition: 2.11[12] Let (X, T) be a NTS and Λ be a neutrosophic set in X. Subsequently, the neutrosophic generalized closure and neutrosophic generalized interior of Λ are defined by,

$$(i) NGcl(\Lambda) = \cap \{G: G \text{ is a generalized neutrosophic closed set in } S_1 \text{ and } \Lambda \subseteq G\}.$$
$$(ii) NGint(\Lambda) = \cup \{G: G \text{ is a generalized neutrosophic open set in } S_1 \text{ and } \Lambda \supseteq G\}.$$

3. Neutrosophic Almost Contra α-Continuous Functions.

A new form of $\aleph\alpha$-continuity termed as $\aleph$-almost contra α-continuity is discussed along with some of their properties.

Definition 3.1 *Let (S_1, ξ_1) and (S_2, ξ_2) be any two NTS . A function $g: (S_1, \xi_1) \to (S_2, \xi_2)$ is named as $\aleph$-almost contra α-continuous if inverse image of each $\aleph$-regular open set in S_2 is $\aleph\alpha$-closed set in S_1.*

Recall that, for a function $f: S_1 \to S_2$, the subset $G_f = \{x, f(x): x \in S_1\} \subset S_1 \times S_2$ is said to be graph of f.

Theorem 3.2 *Let $f: S_1 \to S_2$ be a function along with $g: S_1 \to S_1 \times S_2$ be the graph function defined by $g(x) = (x, f(x))$ being each $x \in S_1$. Whenever g is $\aleph$-almost contra α-continuous function, thereupon f is $\aleph$-almost contra α-continuous function.*

Proof. Let M be a $\aleph$-regular closed set in S_2 accordingly $S_1 \times M$ is a $\aleph$-regular closed set in $S_1 \times S_2$. In view of g is $\aleph$-almost contra α-continuous, so that $f^{-1}(M) = g^{-1}(S_1 \times M)$ is a $\aleph\alpha$-open in S_1. Thus f is $\aleph$-almost contra α-continuous.

Definition: 3.3

1. *A nonempty family $\mathbb{F}$ of $\aleph$ –open sets on (S_1, ξ_1) is known as $\aleph$ –filter if*
   - I. $0_\aleph \notin \mathbb{F}$
   - II. *If $A, B \in \mathbb{F}$ then $A \cap B \in \mathbb{F}$*
   - III. *If $A \in \mathbb{F}$ and $A \subset B$ then $B \in \mathbb{F}$*
2. *A nonempty family $\mathbb{B}$ of $\aleph$ – open sets on $\mathbb{F}$ is named as $\aleph$ – filter base if*
   - I. $0_\aleph \notin \mathbb{B}$

II. *If* $B_1, B_2 \in \mathbb{B}$ *then* $B_3 \subset B_1 \cap B_2$ *for some* $B_3 \in \mathbb{B}$

3. *A* $\aleph$ *−filter* $\mathbb{F}$ *is known as* $\aleph$ *−convergent to a* $\aleph$ *−point* $x_{r,s,t}$ *of a* $NTS(S_1, \xi_1)$ *if for each* $\aleph$ *−open set A of* (S_1, ξ_1)*containing*$x_{r,s,t}$*, there exists a* $\aleph$ *−set* $B \in \mathbb{F}$ *so as* $B \subseteq A$.
4. *A* $\aleph$ *−filter* $\mathbb{F}$ *is said to be* $\aleph\alpha$*-convergent to a* $\aleph$ *−point* $x_{r,s,t}$ *of a* $NTS(S_1, \xi_1)$ *if for each* $\aleph\alpha$ *−open set A of* (S_1, ξ_1)*containing*$x_{r,s,t}$*,there exists a* $\aleph - setB \in \mathbb{F}$ *thereby* $B \subseteq A$.
5. *A* $\aleph$ *−filter* $\mathbb{F}$ *is said to be* $\aleph$ *rc-convergent to a* $\aleph$ *−point* $x_{r,s,t}$ *of a* $NTS(S_1, \xi_1)$ *if for each* $\aleph$ *regular closed set A of* (S_1, ξ_1) *containing* $x_{r,s,t}$*,there exists a* $\aleph$ *−set* $B \in \mathbb{F}$ *so as* $B \subseteq A$.

Proposition 3.4 *If a function* $\mu: S_1 \to S_2$ *is* $\aleph$*-almost contra* α*-continuous function and each* $\aleph$ *−filter base* $\mathbb{F}$ *in* S_1 *is* $\aleph\alpha$*-converging to* $x_{r,s,t}$*, the* $\aleph$ *−filter base* $\mu(\mathbb{F})$ *is* $\aleph$*rc-convergent to* $\mu(x_{r,s,t})$.

Proof. Let $x_{r,s,t} \in S_1$ and $\mathbb{F}$ be any $\aleph$-filter base in S_1 is $\aleph\alpha$-converging to $x_{r,s,t}$. As μ is $\aleph$-almost contra α-continuous, subsequently for any $\aleph$ regular closed R in S_2 including $\mu(x_{r,s,t})$, there exists $\aleph\alpha$-open W in S_1 involving $x_{r,s,t}$ so as $\mu(W) \subset R$. As $\mathbb{F}$ is $\aleph\alpha$-convergent to $x_{r,s,t}$, there occurs $A \in \mathbb{F}$ thereby $A \subset W$. This means that $\mu(A) \subset R$ and consequently the $\aleph$-filter base $\mu(\mathbb{F})$ is $\aleph$rc-convergent to $\mu(x_{r,s,t})$.

Definition: 3.5

1. *A space* S_1 *is termed as* $\aleph\alpha$*-connected if* S_1 *can't be expressed as union of two disjoint non-empty* $\aleph\alpha$*-open sets.*
2. *A space* S_1 *is named as* $\aleph$*-connected if* S_1 *cannot be written as union of two disjoint non-empty* $\aleph$*-open sets.*

Theorem 3.6 *If* $f: S_1 \to S_2$ *is a* $\aleph$*-almost contra* α*-continuous surjection along with* S_1 *is* $\aleph\alpha$*-connected space, then*S_2 *is* $\aleph$*-connected.*

Proof. Let $f: S_1 \to S_2$ be a $\aleph$-almost contra α-continuous surjection with S_1 is $\aleph\alpha$-connected space. Assuming S_2 is a not $\aleph$-connected space. Accordingly, there existdisjoint$\aleph$-open sets W and Rsuch that $S_2 = W \cup R$. Then, W and R are $\aleph$-clopen in S_2. As f is $\aleph$-almost contra α-continuous, $f^{-1}(W)$and$f^{-1}(R)$ are $\aleph\alpha$-open sets in S_1. In addition $f^{-1}(W)$and$f^{-1}(R)$ are disjoint non-empty and$S_1 = f^{-1}(W) \cup f^{-1}(R)$. It is contradiction to the fact thatS_1 is $\aleph\alpha$-connected space. Hence, S_2 is $\aleph$-connected.

Definition 3.6 *A space* S_1 *is named as* $\aleph$*-locally* α*-indiscrete if every* $\aleph\alpha$*-open set is* $\aleph$*-closed in* S_1.

Definition 3.7 *A function* $g: S_1 \to S_2$ *is termed as* $\aleph$*-almost continuous if* $g^{-1}(V)$ *is* $\aleph$*-open in* S_1 *for each* $\aleph$*-regular open set* V *in* S_2.

Definition 3.8 *A function* $f: S_1 \to S_2$ *is known as* $\aleph$*-almost* α*-continuous if* $f^{-1}(V)$ *is* $\aleph\alpha$*-open in* S_1 *for each* $\aleph$*-regular open set* V *in* S_2.

Theorem 3.9 *If a function*$\eta: S_1 \to S_2$ *is* $\aleph$*-almost contra* α*-continuous function and* S_1 *is* $\aleph$*-locally* α*-indiscrete space, then* f *is* $\aleph$*-almost continuous function.*

Proof. Let$W$ be a $\aleph$-regular closed set in S_2. Since η is $\aleph$-almost contra α-continuous function, $\eta^{-1}(W)$ is $\aleph\alpha$-open set in S_1 and S_1 is $\aleph$-locally α-indiscrete space, which implies $\eta^{-1}(W)$ is a $\aleph$-closed set in S_1. Hence,η is $\aleph$-almost continuous function.

Definition 3.10 *A space S_1 named as ℵ-weakly Hausdorff if each element of S_1 is an intersection of ℵ-regular closed sets.*

Definition 3.11 *A space S_1 is named as*

1. *$\aleph\alpha$-T_0 if for each pair of distinct ℵ-points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exists $\aleph\alpha$-open set U such that $x_{r,s,t} \in U$, $y_{r,s,t} \notin U$ or $x_{r,s,t} \notin U$, $y_{r,s,t} \in U$.*

2. *$\aleph\alpha$-T_1 if for each pair of distinct ℵ-points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exist $\aleph\alpha$-open sets U and V containing $x_{r,s,t}$ and $y_{r,s,t}$ respectively, so as $y_{r,s,t} \notin U$ and $x_{r,s,t} \notin V$.*

3. *$\aleph\alpha$-T_2 if for each pair of distinct ℵ-points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exists $\aleph\alpha$-open set U containing $x_{r,s,t}$ and $\aleph\alpha$-open set V containing $y_{r,s,t}$ so as $U \cap V = 0_N$.*

4. *A space S_1 is termed as $\aleph\alpha$-normal if each pair of non-empty disjoint ℵ-closed sets can be separated by disjoint $\aleph\alpha$-open sets.*

5. *A space S_1 is termed as ℵ-strongly-normal ifr each pair of disjoint non-empty ℵ-closed sets U and V there exists disjoint ℵ-open sets W and R such that $U \subset W$, $V \subset R$ and $\aleph cl(W) \cup \aleph cl(R) = 0_\aleph$.*

6. *A space S_1 is called a ℵ-ultra normal if each pair of non-empty disjoint ℵ-closed sets can be separated by disjoint ℵ-clopen sets.*

Theorem 3.12 *If $f: S_1 \to S_2$ is an ℵ-almost contra α-continuous injection and S_2 is ℵ-weakly Hausdorff space, then S_1 is $\aleph\alpha$-T_1.*

Proof. Let S_2 be a ℵ-weakly Hausdorff space. For any distinct ℵ points $x_{r,s,t}$ and $y_{r,s,t}$ in S_1, there exist V and W, ℵ-regular closed sets in S_2 such that $f(x_{r,s,t}) \in V$, $f(y_{r,s,t}) \notin V$, $f(y_{r,s,t}) \in W$ and $f(x_{r,s,t}) \notin W$. As f is ℵ-almost contra α-continuous, $f^{-1}(V)$and$f^{-1}(W)$ are $\aleph\alpha$-open subsets of S_1 such that $x_{r,s,t} \in f^{-1}(V)$, $y_{r,s,t} \notin f^{-1}(V)$, $y_{r,s,t} \in f^{-1}(W)$ and $x_{r,s,t} \notin f^{-1}(W)$. Hence, S_1 is $\aleph\alpha$-T_1.

Theorem 3.13 *If $h: S_1 \to S_2$ is a ℵ-almost contra α-continuous injective mapping from space S_1 into a ℵ-Ultra Hausdroff space S_2, thernS_1 is $\aleph\alpha$-T_2.*

Proof. Let$x_{r,s,t}$and$y_{r,s,t}$ be any two distinct ℵ elements in S_1. As f is an injective $h(x_{r,s,t}) \neq h(y_{r,s,t})$ and S_2 is ℵ-Ultra Hausdroff space, there exist disjoint ℵ-clopen sets U and V of S_2 containing $h(x_{r,s,t})$ and $h(y_{r,s,t})$ respectively. Subsequently, $x_{r,s,t} \in h^{-1}(U)$ and $y_{r,s,t} \in h^{-1}(V)$, wherein $h^{-1}(U)$ and $h^{-1}(V)$ are disjoint $\aleph\alpha$-open sets in S_1. Then,S_1 is $\aleph\alpha$-T_2.

Proposition 3.14 *If S_2 is ℵ strongly-normal and$\mu: S_1 \to S_2$ is a ℵ almost contra-α-continuous closed injection, thenS_1 is $\aleph\alpha$-normal.*

Proof. Suppose J and $K$are disjoint ℵ-closed members of S_1. Let μ is ℵ-closed and injective $f(J)$ and $f(K)$ are disjoint ℵ-closed sets in S_2. As S_2 is ℵ strongly-normal, there exist ℵ-open sets W and R in Y so that $\mu(J) \subset W$ and $\mu(K) \subset R$ and $\aleph cl(W) \cap \aleph cl(R) = 0_\aleph$. Then, since $\aleph cl(W)$ and $\aleph cl(V)$ are ℵ regular closed, and μ is an ℵ almost contra α-continuous, $\mu^{-1}(\aleph cl(W))$ and $\mu^{-1}(\aleph cl(R))$ are $\aleph\alpha$-open sets in S_1. This implies $J \subseteq \mu^{-1}(\aleph cl(W))$, $K \subseteq \mu^{-1}(\aleph cl(R))$ and $\mu^{-1}(\aleph cl(W))$ and $\mu^{-1}(\aleph cl(R))$ are disjoint,so S_1 is $\aleph\alpha$-normal.

Theorem 3.15 *If $f: S_1 \to S_2$ is a $\aleph$-almost contra α-continuous, $\aleph$-closed injection along with S_2 is $\aleph$-ultra normal, then S_1 is $\aleph\alpha$-normal.*

Proof. Let P and Q be disjoint $\aleph$-closed sets of S_1. As f is $\aleph$-closed as well as injective, $f(P)$ along with $f(Q)$ are disjoint $\aleph$-closed sets in S_2. Since S_2 is $\aleph$-ultra normal, there exist disjoint $\aleph$-clopen sets U and V in S_2such that$f(P) \subset U$ and $f(Q) \subset V$. This implies $P \subset f^{-1}(U)$ with $Q \subset f^{-1}(V)$. As f is a $\aleph$-almost contra α-continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $\aleph\alpha$-open sets in S_1. Therefore, S_1 is $\aleph\alpha$-normal.

Definition 3.16 *A function $f: S_1 \to S_2$ is called $\aleph$-weakly almost contra α continuous if for each $\aleph$-point $x_{r,s,t}$ in S_1 and each $\aleph$ regular closed set V of S_2containing$f(x_{r,s,t})$, there exists a $\aleph\alpha$-open set U in S_1, such that $\aleph cl(f(U)) \subseteq V$.*

Definition 3.17 *A function $f: S_1 \to S_2$ is termed as $\aleph(\alpha, S)$-open if the image of each $\aleph$-open set is $\aleph$-semi open.*

Theorem 3.18 *If $f: S_1 \to S_2$ is a $\aleph$-weakly almost contra α-continuous and $\aleph(\alpha, S)$-open then, f is $\aleph$-almost contra α continuous.*

Proof. Let $x_{r,s,t}$ be a $\aleph$ point in S_1 and V be a $\aleph$-regular closed set containing $f(x_{r,s,t})$. Since f is $\aleph$-weakly almost contra α continuous, there exist a $\aleph\alpha$-open set U in S_1 containing $x_{r,s,t}$ so as $\aleph cl(f(U)) \subseteq V$. Since f is a $\aleph(\alpha, S)$-open, $f(U)$ is a $\aleph$-semi open set in S_2 and $f(U) \subseteq \aleph cl(\aleph int(f(U))) \subseteq V$.This shows f is $\aleph$ almost contra α continuous.

4. Conclusions

In this paper, we have introduced and studied the concepts like, Neutrosophic Almost α-contra-continuous function, $\aleph\alpha$-connected space, $\aleph$-weakly Hausdorff space, separation axioms and $\aleph\alpha$-normal spaces and investigated some properties. Some preservation theorems are also discussed. It will be necessary to carry out more theoretical research to establish a general framework for decision-making and to define patterns for complex network conceiving and practical application.

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Neutrosophic Cognitive Maps for Situation Analysis

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Abstract. There are various factors which lead to the criminal behaviour in humans. Prominent researchers monitoring the situation of crime in Nigeria cite poverty, unemployment, family-breakdown, bribing & corruption, lack of co-operation from public and negative perception of police to be the major causes behind criminal behaviour. The factors like underemployment, inadequate equipment, NGOs are not taken into account by the researchers because these are considered to be indeterminate. To show how these indeterminate factors are actually related to crime in Nigeria we model the situation mathematically using FCMs and NCMs. The work also shows how efficient is the technique of Neutrosophic Cognitive Maps (NCM) against Fuzzy Cognitive Maps (FCM) to deal with the uncertainties and indeterminacy in Situation Analysis. The obtained results are interpreted which demonstrate the importance of indeterminate factors in analysing the situation of crime in Nigeria. This shows how indeterminate factors when taken into consideration could enhance the accuracy and efficiency of mathematical models using the concept of Neutrosophic Cognitive Maps.

Keywords: Fuzzy logic, Fuzzy Cognitive Maps, Neutrosophy, Neutrosophic Cognitive Maps, Situation Analysis, Crime in Nigeria.

1. Introduction

The term situation from situation (Medieval Latin) is defined as placed in certain location. Situation also represents dispositions of a person, set of circumstances and surrounding environment. According to Pew (2000), a situation is "a set of environmental conditions and system states with which the participant is interacting that can be characterized uniquely by a set of information, knowledge, and response options". For Roy (2001) "Situation Analysis is a process, the examination of a situation, its elements, and their relations, to provide and maintain a product, i.e. a state of Situation Awareness (SAW) for the decision maker". Situation analysis plays a vital role in deciding our actions which are needed to progress further based on our current situation. It is important since it forecast results based on current decisions being taken by the agent. Situation analysis though appears to be simple in predicting the results based on current scenario, but on the other side there exist challenges that are being faced by the agent who is analyzing the situation.

An agent who analyses an event for Situation Analysis apprehends data from various sources like reports, databases, various devices, surroundings and people etc. Based on the data collected together with expert's opinion, conclusions have been drawn by the agent. These conclusions are of great importance in Situation Analysis. The problem arises where raw, conflictual and paradoxical datum is being transformed into statements which are understood by man and machine. Hence measuring the world i.e. quantitative measurement of factors that affect any situation and reasoning about the world i.e. qualitative inferences being drawn from information, co-exists in Situation Analysis. It poses a great challenge to combine these two important aspects in logical and mathematical frame-works. Hence a framework general enough is needed to take into account various uncertainties and indeterminacies arising during information processing, being done in Situation Analysis.

Netrosophic theory is not limited to the field of situation analysis but it is spreading its wings in various other fields. The researchers around the globe have employed the neutrosophic techniques to solve a number of problems prevailing in current scenario i.e. in [23] [29] [30] it is being used to solve the problem in multi-criteria decision making. In this authors have proposed a hybrid technique to detect disease based on certain criteria. In [28] authors have used Bipolar neutrosophic sets in solving the multi-attribute decision making problem. The applications of neutrosophy is not confined as the authors in [24] [25] have used this to obtain solutions to a given mathematical problem. In [24] it is used to find an optimal solution to a given linear programming problem and in [25] it is used in solving the differential equation in neutrosophic environment. In [26] authors have used neutrosophic time series in forecasting the different phenomenon happening all around us. Authors in [27] have used neutrosophic sets in understanding and enhancing the supply chain sustainability in current scenario. The proposed approach claims to be efficient in solving decision making problems while meeting the supply chain sustainability requirement. Authors in [31] have used IoT and Fog computing to propose a health care system for the prediction and diagnosis of diseases. For this purpose they have introduced a neutrosophic multi-criteria decision making technique. The above work by prominent researchers proves that the application of neutrosophic theory in various fields of research is the need of the hour. Some of the problems are discussed below:

1.2 Obstacles in situation analysis

A lot of hurdles exist in prediction and estimation of Situation Analysis described by Anne-Laure Jousselme and Patrick Maupin (2004). These hurdles comprises of ontological limits i.e. due nature of objects, epistemic limits that originate because of cognitive limitation of agents, anarchy when situation is not governed by law, ignorance, vagueness of concepts, Chance and Chaos as per exact estimation is sought, data ignorance and of course uncertainty which is an unavoidable obstacle. Indeterminacy arises from paradoxical conclusions to a given inference from impossible physical measurements. Uncertainty is regarded as discoloration of information, as misconception in measurement and does not rely on state of mind. G´erald Bronner a sociologist (1997) regards uncertainty as a mind's state that depends on our potential to bypass it. He proposes two types of uncertainties: uncertainty in finality (or uncertainty in material) and uncertainty of sense. The first one is defined as "state of mind of a person, who wants to achieve a desire, and is in opposition with the open possibilities" (e.g. Will my rail ticket get confirm?) or it is our understanding of the world, whereas the other one is "state of a person where a part or whole of its system of representation is deteriorated or may be" or it refers to the representation of the world. Agents in situation analysis tackle with uncertainty of sense (i.e. data driven) and uncertainty in finality (i.e. goal driven) from the bottom-up and the top-down perspective respectively.

The rest of this paper is organized as follows: Section 2 presents related work. Section 3 gives a brief description of proposed solution. In section 4 we illustrate proposed work. Section 5 interprets the results obtained. In section 6 we have compared previous solution to proposed work and section 7 concludes the work.

2 Related works

A lot of research work is carried out by the researchers where they needed modelling of real life situations and representing them mathematically for interpretation and drawing conclusions. We present the work done by well-known researchers in this field. Igor Bagány and Márta Takács [12] explored the correlations among various factors being involved in education system so that its functionality can be modelled. It is being done to effectively examine various education systems. Here authors have employed fuzzy cognitive map (FCM) technology, since it aids in determining qualitative illustration of the relationships and parameters. C. Enrique Peliez and John B. Bowles [13] seek to determine the behavior of a system in case of device failure. It requires the combination of various tasks by the expert to choose components for the purpose of analysis, find out failure modes, predict effects and put forward the corrective actions etc. Fuzzy Cognitive Maps and Fuzzy Set Theory provide foundation for automating the reasoning that is required to do a Failure Modes Effects Analysis on a system. The information processing model described by G. Jiang et al. [14] is cantered on the cognitive behavior of human brains. They have recommended two ways of modelling situation cognitively, which are representation and reasoning about Situation Analysis with ontology and using fuzzy cognitive maps (FCM) to develop a Situation Analysis model. Mentioned work done by prominent authors revolves around the factors which govern a particular situation, they accordingly have simulated behavior of the system. This shows that factors or sources play an important role in describing the situation and accordingly system is modelled and various inferences are drawn. If all the factors are not taken into consideration the results can be fatal. Almost all work by researchers in analyzing a situation employs Fuzzy Cognitive Maps (FCMs) introduced by B. Kosko [11]. These fuzzy structures resemble neural networks and mathematically model complex systems where situation analysis is needed. We briefly describe the FCM in the next section.

Though all the above mentioned approaches have significantly achieved wonderful results but these all lack somewhere in considering the indeterminate factors while modelling the situation. These indeterminate factors are of same importance as the determinate factors. When all these are taken into consideration it would aid in achieving the desirable goals. Later in the paper it is being proved mathematically.

2.1 Fuzzy Cognitive Maps

Fuzzy Cognitive Map (FCM) is a directed graph introduced by Bart Kosko [11]. Nodes are represented as concepts and relationship among them as edges. It portrays relationship among concepts. FCMs with weights assigned to the edges are in the set {-1, 0, 1} are known as simple FCMs. Let us assume that $C_1,\ldots, C_n$ are the nodes of FCM. Using edges $e_{ij} \in \{0, 1, -1\}$, a graph that is directed is drawn. The matrix E where $E = (e_{ij})$ is called the adjacency matrix (connection matrix) of the Fuzzy Cognitive Map. Fuzzy cognitive maps (FCMs) are employed in case of unsupervised data. FCMs perform on expert's opinion. FCMs are used to model the world as the set of different classes together with the relationship among these classes. An edge that is directed from concept C_i to C_j ascertains the extent of C_i causing C_j. FCMs aid in modeling various problems varying from socio-economic to popular political developments etc. The edges e_{ij} are in the set [-1, 0, 1], $e_{ij} = 0$ shows that casualty is absent, $e_{ij} > 0$ shows that C_j increments as C_i gets incremented (or C_j decrements as C_i decrements), $e_{ij} < 0$ shows negative causality i.e. C_j gets decremented as C_i decrements (or C_j increments as C_i gets decremented). Now let us consider a real life situation to further understand the application of FCM in Situation Analysis.

2.2 Application of FCM in Situation Analysis

To analyze the situation we have taken into consideration the factors nourishing crime in Nigeria, put forward by various researchers. Anthony Abayomi Adebayo [17] has examined the increasing wave of crime in Nigeria. Study reveals that factors such as inadequately equipped police, unemployment, and breakdown of family values, poverty, Bribery and corruption have made it difficult to prevent and control crime in Nigeria. Ime Okon Utuk [19] has studied the effect of NGO on economic development which in turn has effect on crime. Recent facts from the 'Nigeria Economic Report' of World Bank [20] reveal that the challenge to country's employment is more in line with underemployment than unemployment. Taking into account all factors which nourish crime in Nigeria, a representational model has been shown in following figure 1.

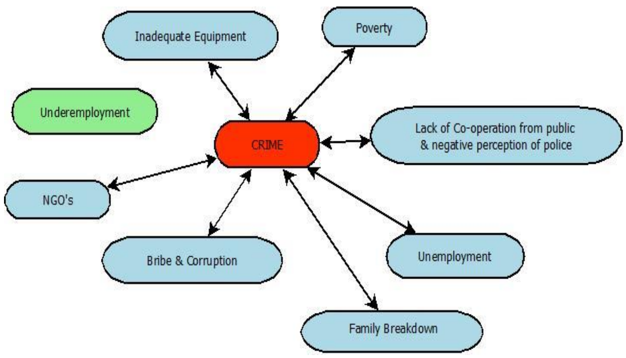

Figure 1: Factors effecting crime

Let us consider the following nodes:

$$A = Inadequate\_equipment$$

$$B = Lack\_of\_co-operation\_from\_public\_\&$$
$$\_negative\_perception\_of\_police$$

$$C = Poverty$$

$$D = Unemployment$$

$$E = Family\_breakdown$$

$$F = Bribery\_\&\_corruption$$

$$G = Underemployment$$

$$H = NGOs$$

$$I = Crime$$

These factors govern a situation that is being analyzed by the agent. In Situation Analysis using FCMs, experts present their views about the existence of relationship or non-existence of relationship. Based on the expert's opinion together with his own knowledge, agent draws the inferences. Now we model the problem of crime prevailing in Nigeria by using the technique of FCM in the following figure 2.

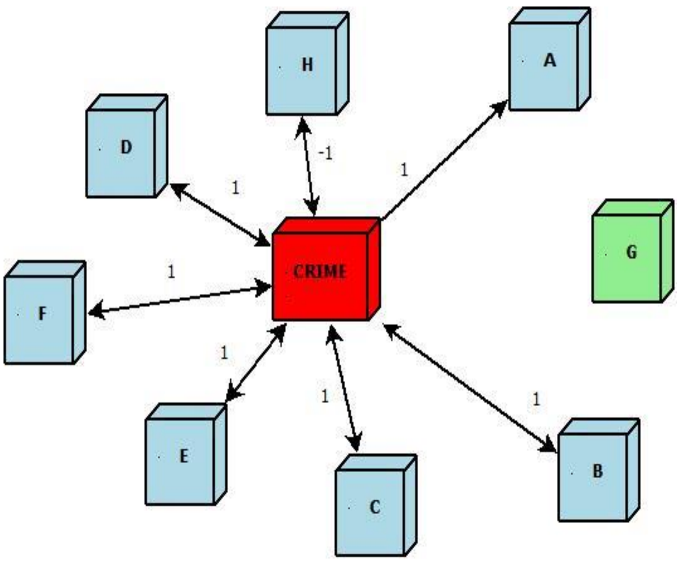

Figure: 2 An instance of FCM model

Here casual increase (or decrease) of A increases (or decreases) I and is marked with "1" as allowed in FCMs. Similarly casual increase (or decrease) of H decreases (or increases) I and is marked with "-1". As indicated in above figure neither anything about the effect of G on I, D on G, nor G on E is mentioned. The Fuzzy Adjacency matrix (E) that is the representation of above Situation is presented in Figure 3.

$$E = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure: 3 Related connection matrix of the graph in Figure 1

Suppose we have taken the state vector X_1i.e. $X_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$. Now we will see its effects on E. The following resultant vector is obtained after thresholding and updating. The symbol '→' symbolizes the updating and thresholding of the resultant vector.

$X_1E = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & -1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} = X_2$

$X_2E = \begin{pmatrix} 6 & 1 & 1 & 1 & 1 & 1 & 0 & -1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} = X_2$

Thus crime has affect or is affected by lack of co-operation from public & negative perception of police, poverty, unemployment, family breakdown, bribing & corruption but underemployment, Inadequate equipment, NGOs are absent in this plot. This means crime flourishes with lack of co-operation from public & negative perception of police, poverty, unemployment, family breakdown and bribing & corruption. The state vector gives fixed point.

2.3 Role of Indeterminacy in Situation Analysis

Practically speaking, when Situation Analysis is being done in real life, the unpredictability and indeterminacy of things happening in life, affects every sphere almost as determined factors. It is a restriction of mathematical modeling that it assigns weightage to only known concepts; and is unconcern about indeterminate relationships between concepts; thereby our views are sometimes biased and skewed. Keeping in mind all factors we present an indeterminate model. Authors Anne-Laure Jousselme and Patrick Maupin [3] have studied situation analysis, various obstacles, governing principles and methods. Authors have described Kripke model [16] that assumes ϕ to be a propositional atom. This *model* is represented by triple structure<*S, Π, R*>where

- *S* is collection of worlds which is non-empty;
- $\Pi : S \to (\phi \to \{0,1\})$ represents truth assigned to atoms of world;
- $R \subseteq S \times S$ is the accessibility relation.

Here'*0*','*1*' represents 'True' , 'False'.

Authors have introduced Neutrosophy in Kripke model [16] and presented a new model that has taken into account the indeterminacy. Earlier in Kripke model ' ϕ ' can only have TRUE or FALSE as values. In Neutrosophic logic ' ϕ 'can be True (*T*%), False (*F*%) and Indeterminate (*I*%). Therefore ' ϕ ' is having triplet of truth values referred to as *neutrosophical values*.

Indeterminacy plays a crucial role in real life as stated by W. B. Vasantha Kandasamy [5][3], therefore when Situation Analysis is being done using FCMs, it does not reflect the true picture since fuzzy theory evaluates the existence or non-existence of associateship but it has failed to attribute the indeterminate relations among concepts. Therefore in Situation Analysis, when data under scrutiny contains concepts which are indeterminate, we are not able to formulate mathematical expression using FCMs.

3 Proposed Solution

The proposed solution to indeterminacy uses the concept of Neutrosophic Cognitive Map (NCMs). It is a technique in Neutrosophy introduced by W. B. Vasantha Kandasamy [5]. The concept of Neutrosophic logic introduced by Florentine Smarandache [6 - 8], which is a merger of the fuzzy logic together with the inclusion of indeterminacy. When data under scrutiny contains concepts which are indeterminate, we are not able to formulate mathematical expression. Presentation of Neutrosophic logic by Florentine Smarandache [6][7][8] has put forward a panacea to this problem. It is the reason Neutrosophy has been introduced as an additional notion in Situation Analysis. Fuzzy theory evaluates the existence or non-existence of associateship but it has failed to attribute the indeterminate relations among concepts. Therefore one can say that the indeterminate situation together with fuzzy will result in Neutrosophic logic. Further we have employed Neutrosophic Cognitive Maps (NCMs) in place of Fuzzy Cognitive Maps (FCMs) to represent the real life situation in Situation Analysis. Earlier researches in Situation Analysis have not included the indeterminacy which is a part and parcel of real life. Hence when working on Situation Analysis, indeterminacy need to be considered. Contemplating the importance of indeterminacy we propose to use NCM in Situation Analysis.

4 Proposed Work

This research work assesses the power of Neutrosophic logic proposed by Florentin Smarandache to tackle hindrances encountered while performing Situation Analysis. An agent observing a scene for situation analysis gathers information from various sources. Here agent tries to reach at the level where he can make decisions about the situation under consideration. While dealing with unsupervised data there always comes a point where no relation can be determined among the concepts. Here person faces Neutrosophic questions like "can you find any relation among concepts" or "are you not in a position to determine any relationship among concepts" and so on. In this way we try to introduce an idea of indeterminacy to them. We have underlined one basic principle that guides the modernization in Situation Analysis by introducing the concept of uncertainty by A.L. Jousselme et al. [15].

4.1 Stating uncertainty

a. Uncertainty as a mind state refers to an agent not having enough information to make a decision i.e. "Agent is not sure about the object".
b. Uncertainty as a tangible feature of information representing the loopholes of perception system i.e. "The dimension of this object is uncertain".

4.2 Methodology used in proposed work

Now indeterminacy has been introduced in Fuzzy Cognitive Maps (FCMs) and the generalized structure so obtained is referred as Neutrosophic Cognitive Maps (NCMs) by W. B. Vasantha Kandasamy [5]. NCM is a neutrosophic directed graph (a directed graph with dotted edge representing indeterminacy) with concepts represented as nodes of the directed graph and relationship or indeterminacy as edge of the graph. Let us suppose C_1, C_2,……,C_n are n nodes from Neutrosophic vector space V. The nodes of graph are represented by $(x_1,x_2,\ldots,x_n)$ where x_i's can be'0' or '1' or 'I' (I shows indeterminacy) where $x_i= 1$ indicates the ON state of the node whereas $x_i=0$ indicates the OFF state and $x_i= I$ indicates the indeterminate state of node in that situation. Suppose C_i and C_j are two nodes in this model (NCM), a directed edge from C_i to C_j represents the relationship of C_i and C_j. The edges of directed graph in NCM are weighted having value in set {-1, 0, 1, I}. When e_{ij} is the weight assigned to the directed edge from C_i to C_j then if the value of e_{ij} is '0' it shows C_i does not affect C_j, it is '1' representing increase (or decrease) of C_i leads to increase (or decrease) of C_j, when it is '-1' representing increase (or decrease) of C_i leads decrease (or increase) of C_j and when the value is 'I' it shows effect of C_i on C_j is indeterminate. These NCMs are called simple NCMs. Let N (E) be a matrix defined as N (E) = (e_{ij}) then N (E) is called as Neutrosophic adjacency matrix.

4.3 Reformulating Problems encountered in Situation Analysis using NCM

Now we present a graphical model of situation by considering the factors which nourish crime in Nigeria. This was earlier represented by FCM. The recent facts from the 'Nigeria Economic Report' of World Bank [20] reveal that employment challenge faced by the country is more in line with underemployment than presumed unemployment. Furthermore Adeleke Adegbami [18] has concluded that effect of underemployment causes same level of anxiety as unemployment itself. The workers who are underemployed are not provided with the opportunities to utilize their educational qualification, experience and skills that they possess. They assume that their ability and capability are not up to the mark with the work they are assigned to. Therefore these workers experience lower job satisfaction and get frustrated. This can be referred to as disguised unemployment. Further Kimberly Amadeo a U.S. Economy expert [21] has studied underemployment and its effects on poverty and found that underemployment leads to higher levels of poverty. Hence underemployment has indeterminate relationship with crime which is being shown in NCM but not in FCM. Now we include indeterminacy in Figure 1. Dotted lines represent indeterminate relation between the nodes.

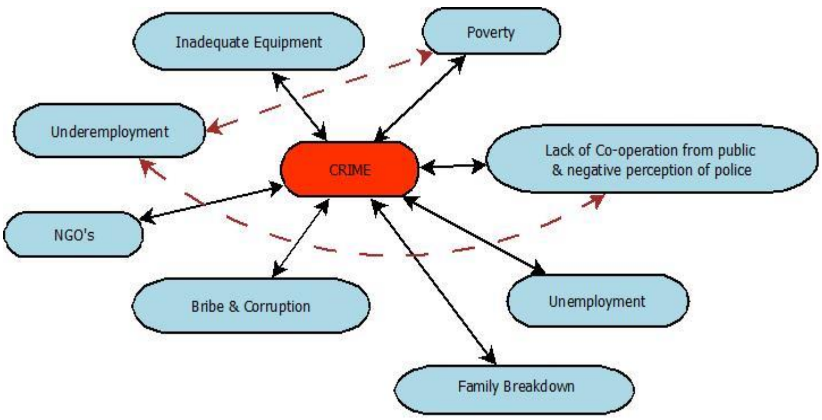

Figure: 4 Factors effecting crime and indeterminate relations

Now we reformulate previous logic of FCM used in analyzing the situation into NCM in Figure 5.

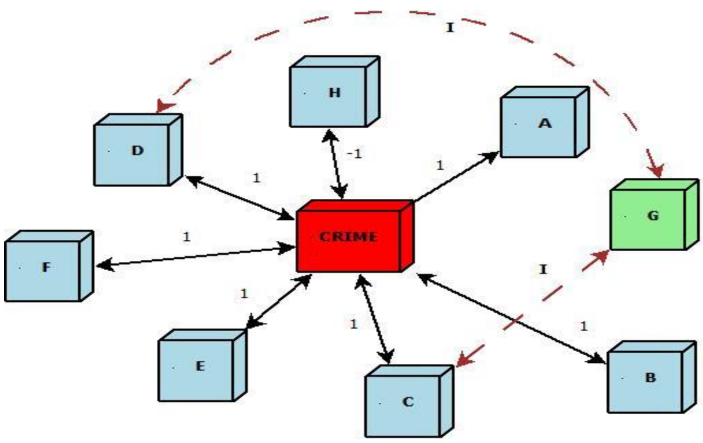

Figure: 5 An instance of NCM model

Neutrosophic Cognitive Maps not only represent the existence or non-existence of relationship among concepts but also represent indeterminate relations among the concepts as shown above. Further we represent Neutrosophic Augmented Matrix N (E) in Figure 6.

$$N(E) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & I & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure: 6 Related connection matrix to the graph in Figure 5.

Earlier we have studied effect of X_1 on E. Now we will try to find what effect does $X_1 = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)$ has on N (E). After resultant vector is updated and thresholded we have the following.

$X_1 N(E) = (0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ -1 \ 1) \rightarrow (1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1) = X_2$

$X_2 N(E) = (6 \ 1 \ 1 \ 1 \ 1 \ 1 \ I \ -1 \ 1) \rightarrow (1 \ 1 \ 1 \ 1 \ 1 \ 1 \ I \ 0 \ 1) = X_3$

$X_3 N(E) = (6 \ 1 \ 1+I \ 1+I \ 1 \ I \ 1 \ -1 \ 1) \rightarrow (1 \ 1 \ 1 \ 1 \ 1 \ 1 \ I \ 0 \ 1) = X_3$

The symbol '→' represents the thresholded and updated resultant vector. This shows that crime has affect or is affected by lack of co-operation from public & negative perception of police, poverty, unemployment, family breakdown, bribing & corruption and the factor underemployment is indeterminate to crime. However results obtained using FCM show as if there is no effect of underemployment on crime. Hence NCMs are better than FCMs in analyzing situation in Situation Analysis.

5 Interpretations of the Results Obtained Using FCM and NCM

Work done in Situation Analysis earlier was based on FCMs. FCMs do not consider indeterminate relations. Since in situation analysis there is uncertainty of sense i.e. data driven (bottom-up perspective) together with uncertainty in finality i.e. goal driven (top-down perspective) which comes as a challenge to the agent. It is a limitation in FCMs modeling that only assigns weightage to known concepts and unconcern about indeterminate relationships between concepts; thereby our views are sometimes biased and skewed. Further with NCMs we include indeterminacy in FCMs. Now experts face Neutrosophic questions like "Is there any relationship among concepts?" or "Are you not in a state to determine any relation among concepts?" and so on. In this way they get familiar with the idea of indeterminacy. The problem formulated by FCM is considered and we reformulate questionnaire in different format so that the experts are allowed to answer like "the relationship among certain concepts is indeterminable or not known". On the grounds of expert's opinion together with the notion of indeterminacy a model is obtained which is referred to as Neutrosophic model. The result obtained is mentioned in the table below:

Table 1: Results obtained from FCM and NCM

Effect of X_1 on E using FCM	Effect of X_1 on N(E) using NCM
$(1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1)$	$(1 \ 1 \ 1 \ 1 \ 1 \ 1 \ I \ 0 \ 1)$

Earlier when problem was formulated using FCM we got resultant vector as $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1\end{pmatrix}$ where A was ON state which shows that crime in ON state affects or is affected by lack of co-operation from public & negative perception of police, poverty, unemployment, family breakdown, bribing and corruption, but underemployment, Inadequate equipment, Non-Governmental Organization (NGOs) are absent in this plot. The sate Vector leads to a fixed point. But in real life underemployment has effect on crime. We have employed Neutrosophic Cognitive Maps (NCMs) in place of Fuzzy Cognitive Maps (FCMs) to represent the real life situation in Situation Analysis. When indeterminacy is included and Neutrosophic Adjacency Matrix is formulated, we again studied the effect of factors on crime. This time the resultant vector is $\begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & I & 0 & 1\end{pmatrix}$. This clearly shows that crime is affected by lack of co-operation from public & negative perception of police, poverty, unemployment, family breakdown, bribing and corruption, but underemployment is indeterminate to crime. In FCMs, the values assigned to edges of graph are the results of knowledge and experience possessed by the expert. These values are functions of engineering judgments and common sense. Moreover in FCM structure the parameters are tunable. Now as FCMs are replaced by NCMs, we allow the experts to make statement of indeterminacy among concepts. If FCM is employed, these edges do not get any value except a '0' but in case of NCM, certainly they do have a weight 'I'; an element of indeterminacy.

6 Proposed Solution versus Previous Solution

The work done earlier in the field of Situation Analysis has not included the indeterminacy which could occur in modeling the situation. In parameter analysis of educational model only factors which have effect or no effect are considered. The experts are put forward with questions like "this factor affects another or not?" the expert responds with positive, negative or absence of impacts, but indeterminacy of impacts is not taken into consideration. In Failure Mode Effect Analysis nothing about the uncertainty of system design is mentioned. In contrast uncertainty in system design is of much importance since changes In Design of the system under consideration will have corresponding changes in the modes of failure of the system. In Information Processing Model Fuzzy Cognitive Maps (FCMs) are used for acquisition of causal knowledge and guide the reasoning process. Indeterminate relations are not considered. Taking indeterminacy into account; improves the evaluation and hence valid inferences are drawn. Now further modeling the situation using Neutrosophic Cognitive Maps (NCMs) allow us to model indeterminacy. In this model experts face Neutrosophic questions like "is there any relation among concepts" or "are you not in a state to determine any relation among concepts and so on". These questions led to the introduction of indeterminacy to the experts. The problem formulated by FCM is considered and we reformulate questionnaire in different format so that the experts are allowed to answers like "the relationship among certain concepts is indeterminable or not known". On the grounds of opinion of the expert together with the notion of indeterminacy, we have obtained the Neutrosophic model.

7 Conclusion

One of the great scientists Albert Einstein [22] quoted, "**So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality**". Earlier used FCM technique does not take into account indeterminacy. When unsupervised data is analyzed we are not in a position to say anything for certain. At some point of time we come across the indeterminacy of facts when analyzing the unsupervised data. The only powerful tool that aids in understanding and applying the concept of indeterminacy is the notion of Neutrosophy. This paper discusses NCM technique and a comparison with FCM is presented. The presented Neutrosophic Cognitive Map approach in analyzing the situation has led to the inclusion of indeterminacy in Situation Analysis and gives a better understanding of how indeterminacy plays a vital role in this field. By exploring various concepts and relationships among them, NCM is designed and corresponding Neutrosophic

Adjacency Matrix is formulated. Through examining the Adjacency matrix a valid inference can be drawn. Future work in this regard might be exploring the structure of NCM and corresponding adjacency matrix, applying learning algorithms to refine structure and carrying out simulation where Situation Analysis is needed to validate the output.

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Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity

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Abstract: Smarandache introduced and developed the new concept of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama introduced Neutrosophic topological spaces by using the Neutrosophic crisp sets. Aim of this paper is we introduce and study the concepts Neutrosophic generalized b closed sets and Neutrosophic generalized b continuity in Neutrosophic topological spaces and its Properties are discussed details.

Keywords: Neutrosophic gb closed sets, Neutrosophic gb continuity, Neutrosophic continuity mapping, Neutrosophic gb continuity mapping.

1. Introduction

Smarandache's neutrosophic system have wide range of real time applications for the fields of Computer Science ,Information Systems, Applied Mathematics , Artificial Intelligence, Mechanics, decision making. Medicine, Electrical & Electronic, and Management Science etc. [20-25]. Topology is a classical subject, as a generalization topological spaces many type of topological spaces introduced over the year. Smarandache [9] defined the Neutrosophic set on three component Neutrosophic sets (T Truth, F -Falsehood, I- Indeterminacy). Neutrosophic topological spaces (N-T-S) introduced by Salama [17] et al., R.Dhavaseelan [6], Saied Jafari are introduced Neutrosophic generalized closed sets. Neutrosophic b closed sets are introduced C. Maheswari[14] et al.Aim of this paper is we introduce and study about Neutrosophic generalized b closed sets and Neutrosophic generalized b continuity in Neutrosophic topological spaces and its properties and Characterization are discussed with details.

2. Preliminaries

In this section, we recall needed basic definition and operation of Neutrosophic sets and its fundamental Results

Definition 2.1 *[9]* Let X be a non-empty fixed set. A Neutrosophic set P is an object having the form

$P = \{< x, \mu_P(x), \sigma_P(x), \gamma_P(x) >: x \in X\}$,

$\mu_P(x)$-represents the degree of membership function

$\sigma_P(x)$-represents degree indeterminacy and then

$\gamma_P(x)$-represents the degree of non-membership function

Definition 2.2 *[9].* Neutrosophic set $P = \{< x, \mu_P(x), \sigma_P(x), \gamma_P(x) >: x \in X\}$, on X and $\forall x \in X$ then complement of P is $P^C = \{< x, \gamma_P(x), 1 - \sigma_P(x), \mu_P(x) >: x \in X\}$

Definition 2.3 *[9].* Let P and Q are two Neutrosophic sets, $\forall x \in X$

$P = \{< x, \mu_P(x), \sigma_P(x), \gamma_P(x) >: x \in X\}$

$Q = \{< x, \mu_Q(x), \sigma_Q(x), \gamma_Q(x) >: x \in X\}$

Then $P \subseteq Q \Leftrightarrow \mu_P(x) \le \mu_Q(x), \sigma_P(x) \le \sigma_Q(x) \& \gamma_P(x) \ge \gamma_Q(x)\}$

Definition 2.4 *[9].* Let X be a non-empty set, and Let P and Q be two Neutrosophic sets are

$P = \{< x, \mu_P(x), \sigma_P(x), \gamma_P(x) >: x \in X\}$, $Q = \{< x, \mu_Q(x), \sigma_Q(x), \gamma_Q(x) >: x \in X\}$Then

1. $P \cap Q = \{< x, \mu_P(x) \cap \mu_Q(x), \sigma_P(x) \cap \sigma_Q(x), \gamma_P(x) \cup \gamma_Q(x) >: x \in X\}$
2. $P \cup Q = \{< x, \mu_P(x) \cup \mu_Q(x), \sigma_P(x) \cup \sigma_Q(x), \gamma_P(x) \cap \gamma_Q(x) >: x \in X\}$

Definition 2.5 *[17].* Let X be non-empty set and τ_N be the collection of Neutrosophic subsets of X satisfying the following properties:

1.$0_N, 1_N \in \tau_N$

2. $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$

3. $\cup T_i \in \tau_N$ for every $\{T_i: i \in j\} \subseteq \tau_N$

Then the space (X, τ_N) is called a Neutrosophic topological space(N-T-S).

The element of τ_N are called Neu-OS (Neutrosophic open set)

and its complement is Neu-CS(Neutrosophic closed set)

Example 2.6. Let X ={x} and $\forall x \in X$

$A_1 = \langle x, \frac{6}{10}, \frac{6}{10}, \frac{5}{10}\rangle$, $A_2 = \langle x, \frac{5}{10}, \frac{7}{10}, \frac{9}{10}\rangle$

$A_3 = \langle x, \frac{6}{10}, \frac{7}{10}, \frac{5}{10}\rangle$,$A_4 = \langle x, \frac{5}{10}, \frac{6}{10}, \frac{9}{10}\rangle$

Then the collection $\tau_N = \{0_N, A_1, A_2, A_3, A_{4,} 1_N\}$ is called a N-T-S on X.

Definition 2.7. Let (X, τ_N)be a N-T-S and $P = \{< x, \mu_P(x), \sigma_P(x), \gamma_P(x) >: x \in X\}$ be a Neutrosophic set in X. Then P is said to be

1. Neutrosophic b closed set [14] (Neu-bCS in short) if Neu-cl(Neu-int(P))∩Neu-int(Neu-cl(P))⊆P,
2. Neutrosophic α-closed set [2] (Neu- αCS in short) if Neu-cl(Neu-int(Neu-cl(P)))⊆P,
3. Neutrosophic pre-closed set [20] (Neu-Pre-CS in short) if Neu-cl(Neu-int(P))⊆P,
4. Neutrosophic regular closed set [9] (Neu-RCS in short) if Neu-cl(Neu-int(P)) = P,
5. Neutrosophic semi closed set [11] (Neu-SCS in short) if Neu-int(Neu-cl(P))⊆P,
6. Neutrosophic generalized closed set [6] (Neu-GCS in short) if Neu-cl(P⊆H whenever P⊆H and H is an Neu-OS,
7. Neutrosophic generalized pre closed set [13] (Neu-GPCS in short) if Neu-Pcl(P) ⊆ H whenever P ⊆ H and H is an Neu-OS,
8. Neutrosophic α generalized closed set [12] (Neu- αGCS in short) if Neu α-cl(P)⊆H whenever P ⊆ H and H is an Neu-OS,
9. Neutrosophic generalized semi closed set [19](Neu-GSCS in short) if Neu-Scl(P)⊆H whenever P⊆H and H is an Neu-OS.

Definition 2.8 [*9]* (X, τ_N)be a N-T-S and $P = \{< x, \mu_P(x), \sigma_P(x), \gamma_P(x) >: x \in X\}$ be a Neutrosophic set in X.Then

Neutrosophic closure of P is Neu-Cl(P)=∩{H:H is a Neu-CS in X and P⊆H}

Neutrosophic interior of P is Neu-Int(P)=∪{M:M is a Neu-OS in X and M⊆P}.

Definition 2.9 *[14]*Let (X,τ_N) be a N-T-S and $P=\{<x,\mu_P(x),\sigma_P(x),\gamma_P(x)>: x\in X\}$ be a Neutrosophic set in X. Then the Neutrosophic b closure of P (Neu-bcl(P)in short) and Neutrosophic b interior of P (Neu-bint(P) in short) are defined as Neu-bint(P)= ∪{ G/G is a Neu-bOS in X and G⊆P}, Neu-bcl(P)= ∩{ K/K is a Neu-bCS in X and P⊆K }.

Proposition 2.10 Let (X,τ_N) be any N-T-S. Let P and Q be any two Neutrosophic sets in (X,τ_N).Then the Neutrosophic generalized b closure operator satisfies the following properties.

1. Neu-bcl(0_N)=0_N and Neu-bcl(1_N) = 1_N,
2. P⊆Neu-bcl(P),
3. Neu-bint(P)⊆P,
4. If P is a Neu-bCS then P=Neu-bcl(Neu-bcl(P)),
5. P⊆Q⇒Neu-bcl(P) ⊆Neu-bcl(Q),
6. P⊆Q⇒Neu-bint(P)⊆Neu-bint(Q).

3. Neutrosophic Generalized b Closed Sets

Definition 3.1. A Neutrosophic set P in a N-T-S (X,τ_N) is said to be a Neutrosophic generalized b closed set(Neu-GbCS in short) if Neu-bcl(P) ⊆H whenever P⊆H and H is a Neu-OS in (X,τ_N).The family of all Neu-GbCSs of a N-T-S (X,τ_N) is denoted by Neu-gbC(X).

Example 3.2. Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ is be a N.T.on X where $E_1=\langle x,\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right)\rangle$. Then the Neutrosophic set $P=\langle x,\left(\frac{7}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)\rangle$is a Neu-GbCS in X.

Example 3.3. Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ isbe a N.T.on X. where $E_1=\langle x,\left(\frac{6}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$. Then the Neutrosophic set $P=\langle x,\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right)\rangle$is not a Neu-GbCS in X.

Theorem 3.4. Every Neu-CS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X,τ_N). Since P is a Neu-CS and Neu-bcl(P) ⊆Neu-cl(P), Neu-bcl(P) ⊆Neu-cl(P)=P⊆H. Therefore P is a Neu-GbCS in X.

Example 3.5. Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ is be a N.T. on X .where $E_1=\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{7}{10}\right)\rangle$. Then the Neutrosophic set $P=\langle x,\left(\frac{5}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{3}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-CS in X, since Neu-cl(P)=E_1≠P

Theorem 3.6. Every Neu-αCS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X,τ_N). Since P is a Neu-αCS, Neu-αcl(P)= P. Therefore Neu-bcl(P) ⊆Neu-αcl(P)=P⊆H. Hence P is a Neu-GbCS in X.

Example 3.7. Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ is be a N.T. on X .where $E_1=\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$. Then the Neutrosophic set $P=\langle x,\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{3}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-αCS in X, since Neu-cl(Neu-int(Neu-cl(P)))= ${E_1}^C \not\subseteq$ P.

Theorem 3.8. Every Neu-Pre-CS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X, τ_N).Since P is a Neu-Pre-CS,Neu-cl(Neu-int(P)) ⊆P. Therefore Neucl(Neu-int(P))∩Neu-int(Neu-cl(P) ⊆Neu-cl(P)∩Neu-cl(Neu-int(P) ⊆P. This implies Neu-bcl(P)
⊆H. Hence P is a Neu-GbCS in X.

Example 3.9.Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X.where $E_1 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$. Then the Neutrosophic set $P = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-pre closed set in X, since Neu-cl(Neu-int(P))= ${E_1}^C \nsubseteq$ P.

Theorem 3.10. Every Neu-bCS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X, τ_N). Since P is a Neu-bCS,Neu-bcl(P)=P. Therefore Neu-bcl(P)=P ⊆ H. Hence P is a Neu-GbCS in X.

Example 3.11 Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X .where $E_1 = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-bCS in X, since Neu-cl (Neu-int(P))∩ Neu-int(Neu-cl(P))=$1_N \nsubseteq$ P.

Theorem 3.12. Every Neu-RCS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X, τ_N). Since P is a Neu-RCS , Neu-cl(Neu-int(P))=P. This implies Neu-cl(P)=Neu-cl(Neu-int(P)). Therefore Neu-cl(P)=P. Hence P is a Neu-CS in X. By theorem 3.4, P is a Neu-GbCS in X.

Example 3.13.Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X where $E_1 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-RCS in X, since Neu-cl(Neu-int(P))=$E_1{}^C \neq$ P.

Theorem 3.14. Every Neu-GCS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X, τ_N). Since P is a Neu-GCS, Neu-cl(P) ⊆H. Therefore Neu-bcl(P) ⊆Neu-cl(P), Neu-bcl(P)⊆H. Hence P is a Neu-GbCS in X.

Example 3.15 Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X. where $E_1 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{1}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-GCS in X, since Neu-cl(P)= $E_1^C \nsubseteq E_1$.

Theorem 3.16. Every Neu-αGCS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X, τ_N).Since P is a Neu-αGCS, Neu-αcl(P)⊆H.Therefore Neubcl(P) ⊆ Neu-αcl(P), Neu-bcl(P)⊆H. Hence P is a Neu-GbCS in X.

*Example 3.17.*Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X. where $E_1 = \langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-αGCS in X, since Neu-cl(Neu-int(Neu-cl(A)))= $1_N \nsubseteq E_1$

Theorem 3.18. Every Neu-GPCS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X, τ_N). Since P is a Neu-GPCS, Neu-Pcl(P)⊆H. Therefore Neubcl(P) ⊆ Neu-Pcl(P), Neu-bcl(P) ⊆ H. Hence P is a Neu-GbCS in X.

Example 3.19. Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, E_2, 1_N\}$ is be a N.T.on X.where $E_1 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$, $E_2 = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ Then the Neutrosophic set $P = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-Gp closed set in X, since Neu-Pcl(P)= $E_2^C \nsubseteq E_2$.

Theorem 3.20. Every Neu-SCS is a Neu-GbCS but not conversely.

Proof. Let P⊆H and H is a Neu-OS in (X, τ_N).Since P is a Neu-SCS, Neu-bcl(P) ⊆ Neu-Scl(P) ⊆ H. Therefore P is a Neu-GbCS in X.

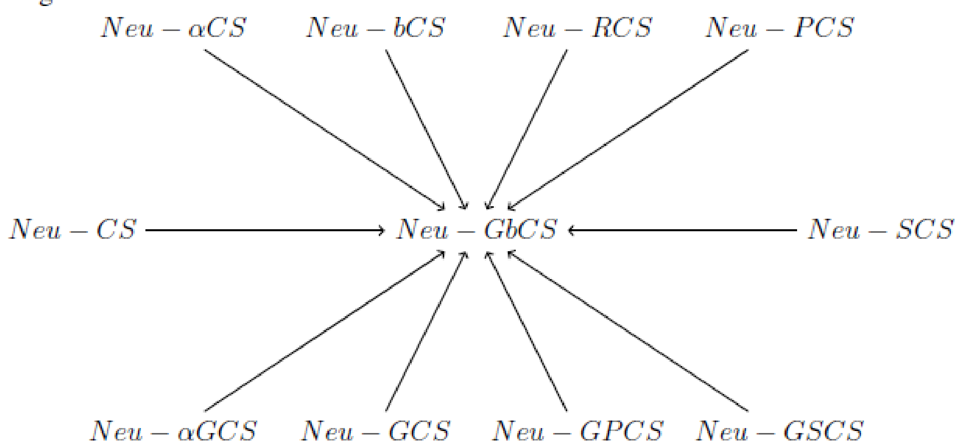
Example 3.21. Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X where $E_1 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-SCS in X, since Neu-int(Neu-cl(P))=$1_N \nsubseteq P$

Theorem 3.22. Every Neu-GSCS is a Neu-GbCS but not conversely.

Proof. Obivious

Example 3.23.Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X where $E_1 = \langle x, \left(\frac{8}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{0}{10}, \frac{5}{10}, \frac{1}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right)\rangle$ is a Neu-GbCS but not a Neu-GSCS in X, since Neu-int(Neu-cl(P))=$1_N \nsubseteq P$ The following implications are true:

Diagram:I



Theorem 3.24. The union of any two Neu-GbCSs need not be a Neu-GbCS in general as seen from the following example.

Example 3.25. Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X where $E_1 = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{1}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$, $Q = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right)\rangle$ is a are Neu-GbCSs but P∩Q is not a Neu-GbCS in X, since Neu-bcl(P∩Q) =$1_N \nsubseteq E_1$

Theorem 3.26. `If P is a Neu-GbCS in (X,τ_N).such that P⊆Q⊆Neu-bcl(P) then Q is a Neu-GbCS in (X,τ_N).

Proof. Let Q be a Neutrosophic set in a N-T-S (X,τ_N).such that Q⊆H and H is a Neu-OS in X. This implies P ⊆ H. Since P is a Neu-GbCS, Neu-bcl(P)⊆H. By hypothesis, we have Neu-bcl(Q)⊆Neu-bcl(Neu-bcl(P))= Neu-bcl(P)⊆H. Hence Q is a Neu-GbCS in X.

Theorem 3.27. If P is Neutrosophic b open and Neutrosophic generalized b closed in a N-T-S (X,τ_N).then P is Neutrosophic b closed in (X,τ_N).

Proof. Since P is Neutrosophic b open and Neutrosophic generalized b closed in (X,τ_N)., Neu-bcl(P)⊆P. but P ⊆ Neu-bcl(P). Thus Neu-bcl(P)=P and hence P is Neutrosophic b closed in (X,τ_N).

4. Neutrosophic generalized b open sets

In this section, we introduce Neutrosophic generalized b open sets in Neutrosophic topological space and study some of their properties.

Definition 4.1. A Neutrosophic set P is said to be a Neutrosophic generalized b open set (Neu-GbOS in short)in (X,τ_N).if the complement P^C is a Neu-GbCS in X. The family of all Neu-GbOSs of a N-T-S (X,τ_N) is denoted by Neu-GbO (X).

Example 4.2.Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ is be a N.T.on X ,where $E_1=\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right)\rangle$.Then the Neutrosophic set $P=\langle x,\left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$ is a Neu-GbOS in X.

Theorem 4.3. For any N-T-S (X,τ_N)., we have the following:

1. Every Neu-OS is a Neu-GbOS.
2. Every Neu-bOS is a Neu-GbOS.
3. Every Neu-αOS is a Neu-GbOS.
4. Every Neu-GOS is a Neu-GbOS.
5. Every Neu-GPOS is a Neu-GbOS.

Proof. Straight forward.

The converse part of the above results need not be correct in common as seen from using following **examples.**

Example 4.4. Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ is be a N.T.on X where $E_1=\langle x,\left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{4}{10}\right)\rangle$.Then the Neutrosophic set $P=\langle x,\left(\frac{4}{10},\frac{6}{10},\frac{6}{10}\right),\left(\frac{4}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$ is a Neu-GbOS but not a Neu-OS in X.

Example 4.5. Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ is be a N.T.on X where $E_1=\langle x,\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$.Then the Neutrosophic set $P=\langle x,\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right),\left(\frac{1}{10},\frac{5}{10},\frac{9}{10}\right)\rangle$ is a Neu-GbOS but not a Neu-bOS in X.

Example 4.6. Let $X=\{p_1,p_2\}$ $\tau_N=\{0_N,E_1,1_N\}$ is be a N.T.on X

where $E_1=\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$.Then the Neutrosophic set $P=\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right)\rangle$ is a Neu-GbOS but not a Neu-bOS in X.

Example 4.7. Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X

where $E_1 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right)\rangle$.Then the Neutrosophic set $P = \langle x, \left(\frac{8}{10}, \frac{5}{10}, \frac{0}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right)\rangle$ is a Neu-GbOS but not a Neu-GOS in X.

Example 4.8. Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, E_2, 1_N\}$ is be a N.T.on X where

$E_1 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$, $E_2 = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ .Then the Neutrosophic set $P = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right)\rangle$ is a Neu-GbOS but not a Neu-GPOS in X.

The intersection of any two Neu-GbOSs need not be a Neu-GbOS in general

Example 4.9. Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X

where $E_1 = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$, .Then the Neutrosophic sets $P = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{9}{10}, \frac{5}{10}, \frac{1}{10}\right)\rangle$

and $Q = \langle x, \left(\frac{7}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{2}{10}\right)\rangle$are Neu-GbOSs but P∩Q is not a Neu-GbOS in X.

Theorem 4.10. A Neutrosophic set P of a N-T-S (X, τ_N)., is a Neu-GbOS if and only if H⊆Neu-bint(P) whenever H is a Neu-CS and H⊆P.

Proof. Necessity: Suppose P is a Neu-GbOS in X. Let G be a Neu-CS and H⊆P. Then F^C is a Neu-OS in X such that $P^C \subseteq H^C$. Since P^C is a Neu-GbCS, Neu-bcl(P^C) ⊆H^C.Hence (Neu-bint(P))^C ⊆H^C . This implies H⊆Neu-bint(P).

Sufficiency: Let P be any Neutrosophic set of X and let H⊆Neu-bint(P) whenever H is a Neu-CS and H⊆P.Then P⊆H^C and H^C is a Neu-OS. By hypothesis, (Neu-bint(P))^C⊆H^C . Hence Neu-bcl(P^C) ⊆H^C . Hence P is a Neu-GbOS in X.

Theorem 4.11. If P is a Neu-GbOS in (X, τ_N)., such that Neu-bint(P) ⊆Q⊆P then Q is a Neu-GbOS in (X, τ_N)

Proof. By hypothesis, we have Neu-bint(P)⊆Q⊆P. This implies $P^C \subseteq Q^C \subseteq$(Neu-bint(P)) ^{C}. That is, $P^C \subseteq Q^C \subseteq$Neubcl($P^C$). Since P^C is a Neu-GbCS, by theorem 3.26, Q^C is a Neu-GbCS. Hence Q is a Neu-GbOS in X.

5. Applications of Neutrosophic Generalized b Closed Sets

In this section, we introduce Neutrosophic $bU_{1/2}$ spaces, Neutrosophic $gbU_{1/2}$ spaces and Neutrosophic gbU_b spaces in Neutrosophic topological space and study some of their properties.

Definition *5.1.* A N-T-S (X, τ_N)., is called a Neutrosophic $bU_{1/2}$ space (Neu- $bU_{1/2}$space in short) if every Neu-bCS in X is a Neu-CS in X.

Definition *5.2.* A N-T-S (X, τ_N)., is called a Neutrosophic $gbU_{1/2}$ space (Neu-$gbU_{1/2}$ space in short) if every Neu-GbCS in X is a Neu-CS in X.

Definition 5.3. A N-T-S (X, τ_N)., is called a Neutrosophic gbU_b space (Neu- gbU_b space in short) if every Neu-GbCS in X is a Neu-bCS in X.

Theorem 5.4. Every Neu-$gbU_{1/2}$ space is a Neu- gbU_b space.

Proof. Let (X, τ_N)be a Neu-$gbU_{1/2}$ space and let P be a Neu-GbCS in X. By hypothesis, P is a Neu-CS in X.Since every Neu-CS is a Neu-bCS, P is a Neu-bCS in X. Hence (X, τ_N)., is a Neu- gbU_b space.

The converse of the above theorem need not be true in general as seen from the following example.

Example 5.5. Let $X = \{p_1, p_2\}$ $\tau_N = \{0_N, E_1, 1_N\}$ is be a N.T.on X where $E_1 = \langle x, \left(\frac{9}{10}, \frac{5}{10}, \frac{9}{10}\right), \left(\frac{1}{10}, \frac{5}{10}, \frac{1}{10}\right)\rangle$,.Then the Neutrosophic set

$P = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{8}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$ is a Neu- gbU_bspace but not a Neu-$gbU_{1/2}$space,

Theorem 5.6. Let (X, τ_N) ., be a N-T-S and (X, τ_N) .,a Neu- $gbU_{1/2}$ space. Then the following statements hold.

1. Any union of Neu-GbCS is a Neu-GbCS.
2. Any intersection of Neu-GbOS is a Neu-GbOS.

Proof. 1. Let $\{A_i\}_{i\in j}$ be a collection of Neu-GbCS in a Neu-$gbU_{1/2}$space (X, τ_N)., Therefore every Neu-GbCS is a Neu-CS. but the union of Neu-CS is a Neu-CS. Hence the union of Neu-GbCS is a Neu-GbCS in X.

2. It can be proved by taking complement in (1).

Theorem 5.7. A N-T-S (X, τ_N)., is a Neu- gbU_b space if and only if Neu-Gb(X)=Neu-bO(X).

Proof. Necessity: Let P be a Neu-GbOS in X. Then P^C is a Neu-GbCS in X. By hypothesis, P^C is a Neu-bCS in X. Therefore P is a Neu-bOS in X. Hence Neu-GbO (X)=Neu-bO(X).

Sufficiency: Let P be a Neu-GbCS in X. Then P^C is a Neu-GbOS in X. By hypothesis, P^C is a Neu-bOS in X. Therefore P is a Neu-bCS in X. Hence(X, τ_N)., is a Neu- gbU_b space.

Theorem 5.8. A N-T-S (X, τ_N) is a Neu-$gbU_{1/2}$ space if and only if Neu-GbO(X) = Neu-O(X).

Proof. Necessity: Let P be a Neu-GbOS in X. Then P^C is a Neu-GbCS in X. By hypothesis, P^C is a Neu-CS in X. Therefore P is a Neu-OS in X. Hence Neu-GbO(X)=Neu-O(X).

Sufficiency: Let P be a Neu-GbCS in X.Then P^C is a Neu-GbOS in X. By hypothesis, P^C is a Neu-OS in X. Therefore P is a Neu-CS in X. Hence (X, τ_N) is a Neu-$gbU_{1/2}$space.

6. Neutrosophic generalized b continuity mapping

In this section we have introduced Neutrosophic generalized b continuity mapping and studied some of its properties.

Definition 6.1. A mapping $f: (X, \tau_N) \to (Y, \sigma_N)$is called a Neutrosophic generalized b continuity (Neu-Gbcontinuity in short) if $f^{-1}(Q)$ is a Neu-Gb CS in $(X, \tau_N)$for every Neu-CS Q of (Y, σ_N).

Example 6.2. Let $X = \{p_1, p_2\}, Y = \{q_1, q_2\}, E_1 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right)\rangle$ $E_2 = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right)\rangle$, $\tau_N = \{0_N, E_1, 1_N\}$ and $\sigma_N = \{0_N, E_2, 1_N\}$ are N-T-S on X and Y respectively. Define a mapping $f: (X, \tau_N) \to (Y, \sigma_N)$ by $f(p_1)=q_1$ and $f(p_2)=q_2$ .Then f is a Neu-Gb continuity mapping.

Theorem 6.3. Every Neutrosophic continuity mapping is a Neu-Gb continuity mapping but not conversely.

Proof. Let $f: (X, \tau_N) \to (Y, \sigma_N)$ be a Neutrosophic continuity mapping. Let P be a Neu-CS in Y. Since f is Neutrosophic continuity mapping, $f^{-1}(P)$ is a Neu-CS in X. Since every Neu-CS is a Neu-GbCS, $f^{-1}(P)$ is a Neu-Gb CS in X. Hence f is a Neu-Gb continuity mapping

Example 6.4. Let $X = \{p_1, p_2\}, Y = \{q_1, q_2\}, E_1 = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{7}{10}\right) \rangle$ $E_2 = \langle x, \left(\frac{4}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right) \rangle$, $\tau_N = \{0_N, E_1, 1_N\}$ and $\sigma_N = \{0_N, E_2, 1_N\}$ are N-T-S on X and Y respectively. Define a mapping $f: (X, \tau_N) \rightarrow (Y, \sigma_N)$ by $f(p_1)=q_1$ and $f(p_2)=q_2$ .The Neutrosophic set $P = \langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{4}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right) \rangle$ is Neu-CS in Y. Then $f^{-1}(P)$ is Neu-GbCS in X but not Neu-CS in X. Therefore, f is a Neu-Gb continuity mapping but not a Neutrosophic continuity mapping.

Theorem 6.5. Every Neu-α continuity mapping is a Neu-Gb continuity mapping but not conversely.

Proof. Let $f: (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a Neu- α continuity mapping. Let P be a Neu-CS in Y. Then $f^{-1}(P)$ is a Neu-αCS in X. Since every Neu-αCS is a Neu-GbCS, $f^{-1}(P)$ is a Neu-GbCS in X. Hence, f is a Neu-Gb continuity mapping.

Example 6.6. Let $X = \{p_1, p_2\}, Y = \{q_1, q_2\}, E_1 = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right) \rangle$ $E_2 = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{5}{10}\right), \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right) \rangle$, $\tau_N = \{0_N, E_1, 1_N\}$ and $\sigma_N = \{0_N, E_2, 1_N\}$ are N-T-S on X and Y respectively. Define a mapping

$f: (X, \tau_N) \rightarrow (Y, \sigma_N)$ by $f(p_1)=q_1$ and $f(p_2)=q_2$ .The Neutrosophic set $P = \langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right) \rangle$ is Neu-CS in Y. Then $f^{-1}(P)$ is Neu-Gb CS in X but not Neu- αCS in X. Then f is Neu-Gb continuity mapping but not a Neu- α continuity mapping.

Theorem 6.7. Every Neu-R continuity mapping is a Neu-Gb continuity mapping but not conversely.

Proof. Let $f: (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a Neu-R continuity mapping. Let P be a Neu-CS in Y. Then by hypothesis $f^{-1}(P)$ is a Neu-RCS in X. Since every Neu-RCS is an Neu-GbCS, $f^{-1}(P)$ is a Neu-Gb CS in X. Hence, f is a Neu-Gb continuity mapping.

Example 6.8. Let $X = \{p_1, p_2\}, Y = \{q_1, q_2\}, E_1 = \langle x, \left(\frac{2}{10}, \frac{5}{10}, \frac{8}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right) \rangle$ $E_2 = \langle x, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$, $\tau_N = \{0_N, E_1, 1_N\}$ and $\sigma_N = \{0_N, E_2, 1_N\}$ are N-T-S on X and Y respectively. Define a mapping $f: (X, \tau_N) \rightarrow (Y, \sigma_N)$ by $f(p_1)=q_1$ and $f(p_2)=q_2$ .The Neutrosophic set $P = \langle x, \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}\right) \rangle$ is Neu-CS in Y. Then $f^{-1}(P)$ is Neu-Gb CS in X but not Neu-RCS in X. Then f is Neu-Gb continuity mapping but not a Neu-R continuity mapping

Theorem 6.9. Every Neu-GS continuity mapping is a Neu-Gb continuity mapping but not conversely.

Proof. Let $f: (X, \tau_N) \rightarrow (Y, \sigma_N)$ be a Neu-GS continuity mapping. Let P be a Neu-CS in Y. Then by hypothesis $f^{-1}(P)$ is a Neu-GCS in X. Since every Neu-GSCS is a Neu-Gb CS, $f^{-1}(P)$ is a Neu-GbCS in X. Hence f is a Neu-Gb continuity mapping

Example 6.10. Let $X = \{p_1, p_2\}, Y = \{q_1, q_2\}, E_1 = \langle x, \left(\frac{5}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right) \rangle$ $E_2 = \langle x, \left(\frac{6}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{6}{10}, \frac{5}{10}, \frac{2}{10}\right) \rangle$, $\tau_N = \{0_N, E_1, 1_N\}$ and $\sigma_N = \{0_N, E_2, 1_N\}$ are N-T-S on X and Y respectively. Define a mapping $f:(X,\tau_N)\to(Y,\sigma_N)$ by $f(p_1)=q_1$ and $f(p_2)=q_2$ .The Neutrosophic set $P=\langle x,\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right),\left(\frac{2}{10},\frac{5}{10},\frac{6}{10}\right)\rangle$ is Neu-CS in Y. Then $f^{-1}(P)$ is Neu-Gb CS in X but not Neu-GSCS in X. Then f is Neu-Gb continuity mapping but not a Neu-GS continuity mapping.

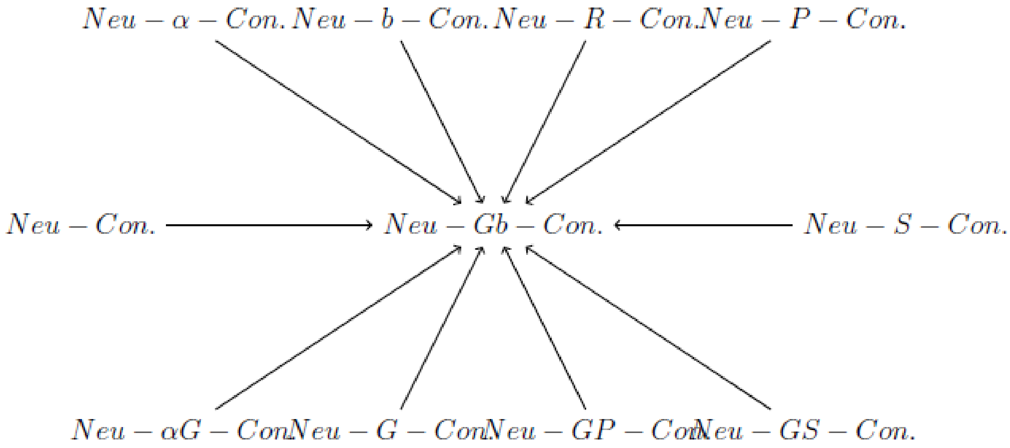
Theorem 6.11. Every Neu-αG continuity mapping is a Neu-Gb continuity mapping but not conversely.

Proof. Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be anNeu-αG continuity mapping. Let P be a Neu-CS in Y. Then, by hypothesis $f^{-1}(P)$ is a Neu- αgcs in X. Since, every Neu- αGCS is a Neu-GSCS and every Neu-GSCS is a Neu-GbCS, $f^{-1}(P)$ is a Neu-Gb CS in X. Hence f is a Neu-Gb continuity mapping.

Example 6.12. Let $X=\{p_1,p_2\}, Y=\{q_1,q_2\}, E_1=\langle x,\left(\frac{5}{10},\frac{5}{10},\frac{4}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$ $E_2=\langle x,\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{7}{10},\frac{5}{10},\frac{3}{10}\right)\rangle$, $\tau_N=\{0_N,E_1,1_N\}$ and $\sigma_N=\{0_N,E_2,1_N\}$ are N-T-S on X and Y respectively. Define a mapping $f:(X,\tau_N)\to(Y,\sigma_N)$ by $f(p_1)=q_1$ and $f(p_2)=q_2$ .The Neutrosophic set $P=\langle x,\left(\frac{5}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{3}{10},\frac{5}{10},\frac{7}{10}\right)\rangle$ is Neu-CSin Y. Then $f^{-1}(P)$ is Neu-Gb CS in X but not Neu-αGCS in X. Then f is Neu-Gb continuity mapping but not an Neu-αG continuity mapping.

The following implications are true:

Diagram-II



Theorem 6.13. A mapping $f: X \to Y$ is Neu-Gb continuity then the inverse image of each Neu-OS in Y is a Neu-αGOS in X.

Proof. Let P be a Neu-OS in Y. This implies P^C is Neu-CS in Y. Since f is Neu-Gb continuity, $f^{-1}(P^C)$ is Neu-Gb CS in X. Since $f^{-1}(P^C)=(f^{-1}(P))^C$, $f^{-1}(P)$ is a Neu-Gb OS in X.

Theorem 6.14. Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be a Neu-Gb continuity mapping, then f is a Neutrosophic continuity mapping if X is a Neu-$bU_{1/2}$ space.

Proof. Let P be a Neu-CS in Y. Then $f^{-1}(P)$ is a Neu-Gb CS in X, since f is a Neu-Gb Continuity. Since X is a Neu-$bU_{1/2}$ space, $f^{-1}(P)$ is a Neu-CS in X. Hence f is a Neutrosophic continuity mapping.

Theorem 6.15. Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be a Neu-Gb continuity function, then f is a Neu-G continuity mapping if X is a Neu-gb$U_{1/2}$space

Proof. Let P be a Neu-CS in Y. Then $f^{-1}(P)$ is a Neu-GbCS in X, by hypothesis. Since X is a Neu-gb$U_{1/2}$ space, $f^{-1}(P)$ is a Neu-GCS in X. Hence f is a Neu-G continuity mapping.

Theorem 6.16. Let $f:(X,\tau_N)\to(Y,\sigma_N)$ be a Neu-Gb continuity mapping and $g:(X,\tau_N)\to(Z,\rho_N)$ is Neutrosophic continuity, then gof: $(X,\tau_N)\to(Z,\rho_N)$is a Neu-Gb continuity.

Proof. Let P be a Neu-CS in Z. Then, $g^{-1}(P)$ is a Neu-CS in Y, by hypothesis. Since, f is a Neu-Gb continuity mapping, $f^{-1}(g^{-1}(P))$ is a Neu-Gb CS in X. Hence, g of is a Neu-Gb continuity mapping.

7 .Conclusion

Many different forms of closed sets have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, in this paper we have introduced Neutrosophic generalized b closed sets in Neutrosophic Topological Spaces and then we presented Neutrosophic generalized b continuity mapping and studied some of its properties. Also we investigate the relationships between the other existing Neutrosophic continuity functions. This shall be extended in the future Research with some applications

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On Parametric Divergence Measure of Neutrosophic Sets with its Application in Decision-making Models

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Abstract: In various decision-making models the divergence measure is found to be a useful information measure in handling impreciseness and uncertainty among the qualitative and quantitative factors of the decision-making process. In the proposed work, a novel parametric divergence measure for neutrosophic sets has been proposed along with its various properties. On the basis of the proposed parametric divergence measure, we have outlined some methodologies along with its implementing procedural steps for classification problem (pattern recognition problem, medical diagnosis problem) and multi criteria decision making problem. Also, numerical examples for the application problems have been provided for illustration of the proposed methodologies. Comparative remarks along with necessary observations and advantages have also been presented in view of the existing appraoches.

Keywords: Neutrosophic set; Divergence measure; Decision-making; Medical diagnosis; Pattern recognition.

1. Introduction

In the applications of expert system, fusion of information and belief system, the notion of truth-membership of fuzzy set (FS) [1] is not the only parameter to be supported by the evident but there is need of falsity-membership against by the evident. The intuitionistic fuzzy sets (IFSs) [2] consider both types of memberships and can manage the incomplete and imprecise information except the indeterminate/inconsistent information which may exist in case of a belief system. The concept of FSs and IFSs have been widely applied to model such uncertainties and hesitancy inherent in many practical circumstances having a comprehensive application in the area of decision processes, classification problems, econometrics, selection processes etc.

The notion of a neutrosophic set (NS) introduced by Smarandache [3] is a more generalized platform for handling and presenting the uncertainty, impreciseness, incompleteness and inconsistency inherited in a real world problem. As per the statement of Smarandache - "*Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra*"[3]. From a philosophical point of view, the neutrosophic set can be understood as a formal generalized framework of the crisp set, fuzzy set, intuitionistic fuzzy set (IFS) etc. A special case of neutrosophic set is single valued neutrosophic set (SVNS) which has been given by Wang et al. [4]. In literature, various extensions of SVNSs have been available with a hybrid approach such as soft set analogous to NS, rough NS, neutrosophic hesitant fuzzy set, etc.

Various researchers have extensively studied different information measures (similarity measures, entropy, distance measures, divergence measures etc.) for different types of fuzzy sets/intuitionistic fuzzy sets because of their wider applicability in the different fields of science and engineering. In 1993, Bhandari and Pal [5] first studied the directed divergence based on the mutual information measure given by Kullback and Leibler [6]. Fan and Xie [7] provided a divergence measure based on exponential operation and established relation with the fuzzy exponential entropy. Further, Montes et al. [8] studied the special classes of measures of divergence in connecting with fuzzy and probabilistic uncertainty. Next, Ghosh et al. [9] have successfully implemented the fuzzy divergence measure in automated leukocyte recognition. Besides this, four fuzzy directed divergence measures were proposed by Bhatia and Singh [10] with important properties and particular cases.

Vlachos and Sergiadis [11] successfully presented an intuitionistic fuzzy directed divergence measure analogous to Shang and Jiang [12]. Further, a set of axioms for the distance measure of IFSs is provided by Wang and Xin [13] and then Hung and Yang [14] proposed a set of axioms for intuitionistic divergence measure by applying Hausdorff metric. Li [15] provided the intuitionistic fuzzy divergence measure and Hung and Yang [16] proposed intuitionistic *J*-divergence measure with its application in pattern recognition. In intuitionistic fuzzy setup, Montes et. al. [17] established some important relationships among divergence measures, dissimilarity measures and distance measures. In literature, the fuzzy divergence measures and intuitionistic fuzzy divergence measures have been widely applied in various applications – decision-making problems [18, 19], medical diagnosis [20], logical reasoning [21] and pattern recognition [22, 23] etc. Kaya and Kahraman [24] have provided comparison of fuzzy multi-criteria decision-making methods for intelligent building assessment along with detailed ranking results.

It may be noted that the degree of indeterminacy/hesitancy in case of IFSs is dependent on the other two uncertainty parameters of membership degree and non-membership degree. This gives a sense of limitation and boundedness for the decision makers to quantify the impreciseness factors. To overcome such limitations, the NS theory found to be more advantageous and effective tool in the field of information science and applications. Broumi and Smarandache [25] studied various types of similarity measures for neutrosophic sets. On the basis of the distance measure between two single valued neutrosophic sets, Majumdar and Samanta [26] proposed some similarity measures and studied their characteristics. Ye [27] studied various similarity measures for interval neutrosophic sets (INSs) on the basis of distance measures and used them in group decision-making [28]. Further, by using distance based similarity measures for single valued neutrosophic multisets, Ye et al. [29] solved the medical diagnosis problem. Also, Ye [30] studied various measures of similarity measures on the basis of cotangent function for SVNSs & utilized to solve MCDM problem and fault detection. Dhivya and Sridevi [31] studied a new single valued neutrosophic exponential similarity measure and its weighted form to overcome some drawbacks of existing measures and applied in decision making and medical diagnosis problem. Wu et al. [32] established a kind of relationship among entropy, similarity measure and directed divergence based on the three axiomatic definitions of information measure by involving a cosine function. Also, a new multi-attribute decision making method has been developed based on the proposed information measures with a numerical example of city pollution evaluation. Thao and Smarandache [33] proposed new divergence measure for neutrosophic set with some properties and utilized to solve the medical diagnosis problem and the classification problem.

Recently, the notion of NSs theory and its various generalizations have been explored in various field of research by different researchers. Abdel-Basset et al. [34] developed a new model to handle the hospital medical care evaluation system based on plithogenic sets and also studied intelligent medical decision support model [35] based on soft computing and internet of things. In addition to this, a hybrid plithogenic approach [36] by utilizing the quality function in the supply chain management has also been developed. Further, a new systematic framework for providing aid and support to the cancer patients by using neutrosophic sets has been successfully suggested by Abdel-Basset et al. [37]. Based on neutrosophic sets, some new decision-making models have also been successfully presented for project selection [38] and heart disease diagnosis [39] with advantages and defined limitations. In subsequent research, Abdel-Basset et al. [40] have proposed a modified forecasting model based on neutrosophic time series analysis and a new model for linear fractional programming based on triangular neutrosophic numbers [41]. Also, Yang et al. [42] have studied some new similarity and entropy measures of the interval neutrosophic sets on the basis of new axiomatic definition along with its application in MCDM problem.

In view of the above discussions on the recent trends in the field of neutrosophic set theory, it may be observed that the neutrosophic information measures such as distance measures, similarity measures, entropy, divergence measures, have been successfully utilized and implemented to handle the issues related to uncertainty and vagueness. For the sake of wider applicability and the desired flexibility, we need to develop some parametric information measures for SVNSs. These parametric measures will give rise to a family of information measures and we can have selections based on the desired requirements. Subsequently, they can be utilized in various soft computing applications. This approach is novel in its kind where we propose a parametric divergence measure for the neutrosophic sets with various properties so that these can be well utilized in different classification problem and decision-making problems.

The rest of the paper is structured as - In Section 2, some fundamental preliminaries of the neutrosophic sets, information measures are presented with its properties. In Section 3, a new parametric divergence measure for neutrosophic sets has been introduced with its proof. In Section 4, various properties of the proposed divergence measure have also been discussed along with their proofs. Further, in Section 5, application examples of classification problems and decision-making problem have been solved by providing the necessary steps of the proposed methodologies based on the proposed parametric divergence measure. In view of the results obtained in contrast with the existing methodologies related to these fields, some comparative remarks have also been stated for the problems under consideration. The presented work and its results have been summarized in Section 6 with scope for the future work.

2. Preliminaries

Here, some basic definitions and fundamental notions in reference with neutrosophic set, information measures and its properties are presented. Smarandache [3] introduced the notion of neutrosophic set as follows:

Definition 1. [3] Let X be a fixed class of points (objects) with a generic element x in X. A neutrosophic set M in X is specified by a truth-membership function $T_M(x)$, an indeterminacy-membership function $I_M(x)$ and a falsity-membership function $F_M(x)$, where $T_M(x), I_M(x)$ and $F_M(x)$ are real standard or nonstandard subsets of the interval $\left({}^{-}0,1^{+}\right)$ such that $T_M(x): X \to \left({}^{-}0,1^{+}\right), I_M(x): X \to \left({}^{-}0,1^{+}\right), F_M(x): X \to \left({}^{-}0,1^{+}\right)$ and the sum of these functions viz. $T_M(x)+I_M(x)+F_M(x)$ satisfies the requirement ${}^{-}0 \le \sup T_M(x)+\sup I_M(x)+\sup F_M(x) \le 3^{+}.$ We denote the neutrosophic set $M=\left\{(x,T_M(x),I_M(x),F_M(x) \mid x \in X\right\}$.

In case of neutrosophic set, indeterminacy gets quantified in an explicit way, while truth-membership, indeterminacy-membership and falsity-membership are independent terms. Such framework is found to be very useful in the applications of information fusion where the data are logged from different sources. For scientific and engineering applications, Wang et al. [4] defined a single valued neutrosophic set (SVNS) as an instance of a neutrosophic set as follows:

Definition 2 *[4] Let X be a fixed class of points (objects) with a generic element x in X. A single valued neutrosophic set M in X is characterized by a truth-membership function $T_M(x)$, an indeterminacy membership function $I_M(x)$ and a falsity-membership function $F_M(x)$. For each point $x \in X$, $T_M(x)$, $I_M(x)$, $F_M(x) \in [0,1]$. A single valued neutrosophic set M can be denoted by*

$M = \{ < T_M(x), I_M(x), F_M(x) \mid x \in X \}.$

It may be noted that $T_M(x) + I_M(x) + F_M(x) \in [0,3]$.

We denote $SVNS(X)$ as the set of all the SVNSs on X. For any two SVNSs $M, N \in SVNS(X)$, some of the basic and important operations and relations may be defined as follows (Refer [4]):

- **Union of** M **and** N: $M \cup N = \{ (x, T_{M \cup N}(x), I_{M \cup N}(x), F_{M \cup N}(x) \mid x \in X \};$

  where $T_{M \cup N}(x) = \max\{T_M(x), T_N(x)\}$, $I_{M \cup N}(x) = \min\{I_M(x), I_N(x)\}$ and

  $F_{M \cup N}(x) = \min\{F_M(x), F_N(x)\};$ *for all* $x \in X$.

- **Intersection of** M **and** N: $M \cap N = \{ (x, T_{M \cap N}(x), I_{M \cap N}(x), F_{M \cap N}(x) \mid x \in X \};$

  where $T_{M \cap N}(x) = \min\{T_M(x), T_N(x)\}$, $I_{M \cap N}(x) = \max\{I_M(x), I_N(x)\}$ and

  $F_{M \cap N}(x) = \max\{F_M(x), F_N(x)\};$ *for all* $x \in X$.

- **Containment:** $M \subseteq N$ if and only if

  $T_M(x) \le T_N(x), I_M(x) \ge I_N(x), F_M(x) \ge F_N(x),$ *for all* $x \in X$.

- **Complement:** The complement of a neutrosophic set M, denoted by $\overline{M}$, defined by

  $T_{\overline{M}}(x) = 1 - T_M(x),\ I_{\overline{M}}(x) = 1 - I_M(x),\ T_{\overline{M}}(x) = 1 - T_M(x);$ *for all* $x \in X$.

Definition 3. *[32] Consider* M *and* N *be two single-valued neutrosophic sets, then the cross entropy between* M *and* N *must satisfy the following two axioms:*

- $C(M, N) \ge 0$;
- $C(M, N) = 0$ if $M = N$.

Based on the above stated axioms, Wu et al. [32] proposed the divergence measure for two SVNS M and N, given by

$$C_1(M, N) = 1 - \frac{1}{3(\sqrt{2}-1)} \sum_{t=1}^{3} (\sqrt{2} \cos\left(\frac{M_t - N_t}{4}\right)\pi - 1).$$

Also, Thao and Smarandache [33] have put forward various properties and axiomatic definition for divergence measure of single valued neutrosophic sets M and N with four axioms as follows:

- DivAxiom 1: $D(M, N) = D(N, M)$;
- DivAxiom 2: $D(M, N) \ge 0$; and $D(M, N) = 0$ if $M = N$.
- DivAxiom 3: $D(M \cap P, N \cap P) \le D(M, N) \, \forall P \in SVNS(X)$.
- DivAxiom 4: $D(M \cup P, N \cup P) \le D(M, N) \, \forall P \in SVNS(X)$.

3. Parametric Divergence Measure of Neutrosophic Sets

In this section, we present a new parametric divergence measure for two arbitrary SVNSs and discuss its properties. Recently, Ohlan et al. [43] proposed the generalized Hellinger's divergence measure for fuzzy sets A and B as follows:

$$h_\alpha(A, B) = \sum_{i=1}^{n} \left(\frac{\left(\sqrt{\mu_A(x_i)} - \sqrt{\mu_B(x_i)}\right)^{2(\alpha+1)}}{\sqrt{\mu_A(x_i)\mu_B(x_i)}} + \frac{\left(\sqrt{\mu_{\overline{A}}(x_i)} - \sqrt{\mu_{\overline{B}}(x_i)}\right)^{2(\alpha+1)}}{\sqrt{\mu_{\overline{A}}(x_i)\mu_{\overline{B}}(x_i)}} \right), \alpha \in \mathbb{N}. \quad (1)$$

Analogous to the above proposed divergence measure for fuzzy sets given by Equation (1), we propose the following parametric divergence measure for single valued neutrosophic set:

$$Div_\alpha(M,N)=\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_N(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_N(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_N(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_N(x_i)\right)^{\alpha}}\right],\alpha\in\mathbb{N}. \quad (2)$$

Next, we need to prove that the proposed parametric divergence measure for single valued neutrosophic sets is a valid information measure.

Theorem 1. *The divergence measure* $Div_\alpha(M,N)$ *given by Equation (2) is a valid divergence measure for*
two SVNSs.

Proof: In order to prove the theorem, we need to show that the divergence measure given by Equation (2) satisfies the four axioms (Divaxiom (1) - (4) [33]) stated in Section 2.

• **Divaxiom 1:** Since Equation (2) is symmetric with respect to M and N, therefore it is quite obvious that $Div_\alpha(M,N)=Div_\alpha(N,M)$.

• **Divaxiom 2:** In view of Equation (2), we observe that $Div_\alpha(M,N)=0 \Leftrightarrow$ $T_M(x)=T_N(x),\ I_M(x)=I_N(x),\ F_M(x)=F_N(x)\ for\ all\ x\in X.$ It remains to show that $Div_\alpha(M,N)\geq 0$. For this, we first show the convexity of Div_α. Since Div_α is of the Csiszar's f -divergence type with generating mapping $f_\alpha:(0,\infty)\to\mathbb{R}^+$, defined by,

$$f_\alpha(t)=\frac{2^{\alpha}\left(\sqrt{t}-1\right)^{2(\alpha+1)}}{(t+1)^{\alpha}}\ with\ f_\alpha(1)=0. \quad (3)$$

Differentiating Equation (3) two times with respect to t and on simplification, we get

$$f_\alpha''(t)=\left(\frac{2^{\alpha}}{2}\right)\frac{\left(2t+2\alpha\sqrt{t}+2\alpha t^{3/2}+4\alpha t+t^2+1\right)(\alpha+1)\left(\sqrt{t}-1\right)^{2\alpha}}{(t+1)^{\alpha+2}\,t^{3/2}}.$$

Since $\alpha\in\mathbb{N}$ and $t\in(0,\infty)$, therefore, $f_\alpha''(t)\geq 0$ which proves the convexity of $f_\alpha(t)$. Thus, $Div_\alpha(M,N)\geq 0.$

• **Divaxiom 3:** For this purpose we decompose the collection X into two disjoint subsets X_1 and X_2 such that,

$$X_1=\{x_i\in X\,|\,T_M(x_i)\geq T_N(x_i)\geq T_P(x_i),I_M(x_i)\leq I_N(x_i)\leq I_P(x_i),F_M(x_i)\leq F_N(x_i)\leq F_P(x_i)\}; \quad (4)$$

and

$$X_2=\{x_i\in X\,|\,T_M(x_i)\leq T_N(x_i)\leq T_P(x_i),I_M(x_i)\geq I_N(x_i)\geq I_P(x_i),F_M(x_i)\geq F_N(x_i)\geq F_P(x_i)\}. \quad (5)$$

Using the definition of intersection of neutrosophic sets and Equation (2) in connection of Equations (4) and (5), the component terms with respect to X_1 will vanish while the component terms with respect to X_2 only will remain in left hand side. Therefore, the left hand side term will have only one term while the right hand side will have two regular terms. The detailed calculation may be shown easily. In view of this, Divaxiom 3 is satisfied.

• **Divaxiom 4:** This axiom can similarly be proved by using the definition of union on the basis of proof of Divaxiom 3. This implies that $Div_\alpha(M,N)$ is a valid divergence measure between the single valued neutrosophic sets M and N.

4. Properties of New Parameterized Neutrosophic Divergence Measure

In this section some important properties of the proposed parametric measures of neutrosophic fuzzy divergence are given and proved.

Theorem 2. *For any* $M, N \text{ and } P \in SVNS(X)$, *the proposed divergence measure (2) satisfies the following properties:*

1. $Div_\alpha(M \cup N, M \cap N) = Div_\alpha(M,N)$
2. $Div_\alpha(M \cup N, M) + Div_\alpha(M \cap N, M) = Div_\alpha(M,N)$
3. $Div_\alpha(M \cup N, P) + Div_\alpha(M \cap N, P) = Div_\alpha(M,P) + Div_\alpha(N,P)$
4. $Div_\alpha(M, M \cup N) = Div_\alpha(N, M \cap N)$
5. $Div_\alpha(M, M \cap N) = Div_\alpha(N, M \cup N)$.

Proof : For this purpose we decompose the collection X into two disjoint subsets X_1 & X_2 s.t.,

$$X_1 = \left\{x_i \in X \mid T_M(x_i) \le T_N(x_i), I_M(x_i) \ge I_N(x_i), F_M(x_i) \ge F_N(x_i)\right\}; \tag{6}$$

$$X_2 = \left\{x_i \in X \mid T_M(x_i) \ge T_N(x_i), I_M(x_i) \le I_N(x_i), F_M(x_i) \le F_N(x_i)\right\}. \tag{7}$$

1. $Div_\alpha(M \cup N, M \cap N)$

$$= \sum_{i=1}^{n} 2^\alpha \left[\frac{\left(\sqrt{T_{M\cup N}(x_i)} - \sqrt{T_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(T_{M\cup N}(x_i) + T_{M\cap N}(x_i)\right)^\alpha} + \frac{\left(\sqrt{1-T_{M\cup N}(x_i)} - \sqrt{1-T_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_{M\cup N}(x_i) - T_{M\cap N}(x_i)\right)^\alpha} \right]$$
$$+ \sum_{i=1}^{n} 2^\alpha \left[\frac{\left(\sqrt{I_{M\cup N}(x_i)} - \sqrt{I_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(I_{M\cup N}(x_i) + I_{M\cap N}(x_i)\right)^\alpha} + \frac{\left(\sqrt{1-I_{M\cup N}(x_i)} - \sqrt{1-I_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_{M\cup N}(x_i) - I_{M\cap N}(x_i)\right)^\alpha} \right]$$
$$+ \sum_{i=1}^{n} 2^\alpha \left[\frac{\left(\sqrt{F_{M\cup N}(x_i)} - \sqrt{F_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(F_{M\cup N}(x_i) + F_{M\cap N}(x_i)\right)^\alpha} + \frac{\left(\sqrt{1-F_{M\cup N}(x_i)} - \sqrt{1-F_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_{M\cup N}(x_i) - F_{M\cap N}(x_i)\right)^\alpha} \right], \alpha \in \mathbb{N}.$$

In view of the Equation (6) and Equation (7), we have

$$\Rightarrow Div_\alpha(M \cup N, M \cap N) = \sum_{x_i \in X_1}^{n} 2^\alpha \left[\frac{\left(\sqrt{T_N(x_i)} - \sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_N(x_i) + T_M(x_i)\right)^\alpha} + \frac{\left(\sqrt{1-T_N(x_i)} - \sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_N(x_i) - T_M(x_i)\right)^\alpha} \right]$$

$$+ \sum_{x_i \in X_1}^{n} 2^\alpha \left[\frac{\left(\sqrt{I_N(x_i)} - \sqrt{I_M(x_i)}\right)^{2(\alpha+1)}}{\left(I_N(x_i) + I_M(x_i)\right)^\alpha} + \frac{\left(\sqrt{1-I_N(x_i)} - \sqrt{1-I_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i) - I_M(x_i)\right)^\alpha} \right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_N(x_i)}-\sqrt{F_M(x_i)}\right)^{2(\alpha+1)}}{\left(F_N(x_i)+F_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_N(x_i)}-\sqrt{1-F_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_N(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_N(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_N(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_N(x_i)\right)^{\alpha}}\right]$$

$$=Div_{\alpha}\left(M,N\right).$$

2. $Div_{\alpha}(M\cup N,M)+Div_{\alpha}(M\cap N,M)$

$$=\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_{M\cup N}(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_{M\cup N}(x_i)+T_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_{M\cup N}(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_{M\cup N}(x_i)-T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_{M\cap N}(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_{M\cap N}(x_i)+T_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_{M\cap N}(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_{M\cap N}(x_i)-T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_{M\cup N}(x_i)}-\sqrt{I_M(x_i)}\right)^{2(\alpha+1)}}{\left(I_{M\cup N}(x_i)+I_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_{M\cup N}(x_i)}-\sqrt{1-I_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_{M\cup N}(x_i)-I_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_{M\cap N}(x_i)}-\sqrt{I_M(x_i)}\right)^{2(\alpha+1)}}{\left(I_{M\cap N}(x_i)+I_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_{M\cap N}(x_i)}-\sqrt{1-I_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_{M\cap N}(x_i)-I_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_{M\cup N}(x_i)}-\sqrt{F_M(x_i)}\right)^{2(\alpha+1)}}{\left(F_{M\cup N}(x_i)+F_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_{M\cup N}(x_i)}-\sqrt{1-F_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_{M\cup N}(x_i)-F_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_{M\cap N}(x_i)}-\sqrt{F_M(x_i)}\right)^{2(\alpha+1)}}{\left(F_{M\cap N}(x_i)+F_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_{M\cap N}(x_i)}-\sqrt{1-F_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_{M\cap N}(x_i)-F_M(x_i)\right)^{\alpha}}\right].$$

$$\Rightarrow Div_{\alpha}(M\cup N,M)+Div_{\alpha}(M\cap N,M)$$

$$=\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_N(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_N(x_i)+T_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_N(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_N(x_i)-T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_N(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_N(x_i)+T_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_N(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_N(x_i)-T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_N(x_i)}-\sqrt{I_M(x_i)}\right)^{2(\alpha+1)}}{\left(I_N(x_i)+I_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_N(x_i)}-\sqrt{1-I_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i)-I_M(x_i)\right)^{\alpha}}\right]+0+0$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_N(x_i)}-\sqrt{I_M(x_i)}\right)^{2(\alpha+1)}}{\left(I_N(x_i)+I_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_N(x_i)}-\sqrt{1-I_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i)-I_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_N(x_i)}-\sqrt{F_M(x_i)}\right)^{2(\alpha+1)}}{\left(F_N(x_i)+F_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_N(x_i)}-\sqrt{1-F_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_M(x_i)\right)^{\alpha}}\right]+0+0$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_N(x_i)}-\sqrt{F_M(x_i)}\right)^{2(\alpha+1)}}{\left(F_N(x_i)+F_M(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_N(x_i)}-\sqrt{1-F_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_M(x_i)\right)^{\alpha}}\right]$$

$$=Div_{\alpha}(M,N).$$

3. $Div_{\alpha}(M\cup N,P)+Div_{\alpha}(M\cap N,P)$

$$= \sum_{i=1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{T_{M\cup N}(x_i)} - \sqrt{T_P(x_i)}\right)^{2(\alpha+1)}}{\left(T_{M\cup N}(x_i) + T_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-T_{M\cup N}(x_i)} - \sqrt{1-T_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_{M\cup N}(x_i) - T_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{i=1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{T_{M\cap N}(x_i)} - \sqrt{T_P(x_i)}\right)^{2(\alpha+1)}}{\left(T_{M\cap N}(x_i) + T_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-T_{M\cap N}(x_i)} - \sqrt{1-T_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_{M\cap N}(x_i) - T_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{i=1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{I_{M\cup N}(x_i)} - \sqrt{I_P(x_i)}\right)^{2(\alpha+1)}}{\left(I_{M\cup N}(x_i) + I_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-I_{M\cup N}(x_i)} - \sqrt{1-I_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_{M\cup N}(x_i) - I_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{i=1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{I_{M\cap N}(x_i)} - \sqrt{I_P(x_i)}\right)^{2(\alpha+1)}}{\left(I_{M\cap N}(x_i) + I_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-I_{M\cap N}(x_i)} - \sqrt{1-I_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_{M\cap N}(x_i) - I_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{i=1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{F_{M\cup N}(x_i)} - \sqrt{F_P(x_i)}\right)^{2(\alpha+1)}}{\left(F_{M\cup N}(x_i) + F_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-F_{M\cup N}(x_i)} - \sqrt{1-F_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_{M\cup N}(x_i) - F_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{i=1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{F_{M\cap N}(x_i)} - \sqrt{F_P(x_i)}\right)^{2(\alpha+1)}}{\left(F_{M\cap N}(x_i) + F_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-F_{M\cap N}(x_i)} - \sqrt{1-F_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_{M\cap N}(x_i) - F_P(x_i)\right)^{\alpha}} \right]$$

$$= \sum_{x_i \in X_1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{T_N(x_i)} - \sqrt{T_P(x_i)}\right)^{2(\alpha+1)}}{\left(T_N(x_i) + T_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-T_N(x_i)} - \sqrt{1-T_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_N(x_i) - T_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{x_i \in X_2}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{T_M(x_i)} - \sqrt{T_P(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i) + T_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-T_M(x_i)} - \sqrt{1-T_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i) - T_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{x_i \in X_1}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{T_M(x_i)} - \sqrt{T_P(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i) + T_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-T_M(x_i)} - \sqrt{1-T_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i) - T_P(x_i)\right)^{\alpha}} \right]$$

$$+ \sum_{x_i \in X_2}^{n} 2^{\alpha} \left[\frac{\left(\sqrt{T_N(x_i)} - \sqrt{T_P(x_i)}\right)^{2(\alpha+1)}}{\left(T_N(x_i) + T_P(x_i)\right)^{\alpha}} + \frac{\left(\sqrt{1-T_N(x_i)} - \sqrt{1-T_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_N(x_i) - T_P(x_i)\right)^{\alpha}} \right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{I_N(x_i)}-\sqrt{I_P(x_i)}\right)^{2(\alpha+1)}}{\left(I_N(x_i)+I_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_N(x_i)}-\sqrt{1-I_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i)-I_P(x_i)\right)^{\alpha}}\right]+0+0$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_P(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_P(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_P(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_P(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{I_N(x_i)}-\sqrt{I_P(x_i)}\right)^{2(\alpha+1)}}{\left(I_N(x_i)+I_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_N(x_i)}-\sqrt{1-I_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i)-I_P(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{F_N(x_i)}-\sqrt{F_P(x_i)}\right)^{2(\alpha+1)}}{\left(F_N(x_i)+F_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_N(x_i)}-\sqrt{1-F_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_P(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_P(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_P(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_P(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_P(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{F_N(x_i)}-\sqrt{F_P(x_i)}\right)^{2(\alpha+1)}}{\left(F_N(x_i)+F_P(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_N(x_i)}-\sqrt{1-F_P(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_P(x_i)\right)^{\alpha}}\right]$$

$$= Div_{\alpha}(M,P)+Div_{\alpha}(N,P).$$

4. $Div_{\alpha}(M, M\cup N)$

$$=\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_{M\cup N}(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_{M\cup N}(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_{M\cup N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_{M\cup N}(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_{M\cup N}(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_{M\cup N}(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_{M\cup N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_{M\cup N}(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_{M\cup N}(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_{M\cup N}(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_{M\cup N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_{M\cup N}(x_i)\right)^{\alpha}}\right]$$

$$=\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_N(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_N(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_N(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_N(x_i)\right)^{\alpha}}\right]=Div_{\alpha}\left(N,M\cap N\right).$$

5. $Div_{\alpha}(M,M\cap N)$

$$=\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_{M\cap N}(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_{M\cap N}(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_{M\cap N}(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_{M\cap N}(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{i=1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_{M\cap N}(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_{M\cap N}(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_{M\cap N}(x_i)\right)^{\alpha}}\right]$$

$$=\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_N(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_N(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_N(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_N(x_i)\right)^{\alpha}}+\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_N(x_i)\right)^{\alpha}}\right]=Div_{\alpha}\left(N,M\cup N\right).$$

Theorem 3. *For any* $M,N\in SVNS(X)$, *the proposed divergence measure (2) satisfies the following properties:*

1. $Div_{\alpha}\left(\overline{M},\overline{N}\right)=Div_{\alpha}\left(M,N\right)$
2. $Div_{\alpha}\left(\overline{M\cup N},\overline{M\cap N}\right)=Div_{\alpha}\left(\overline{M}\cap\overline{N},\overline{M}\cup\overline{N}\right)=Div_{\alpha}\left(M,N\right)$
3. $Div_{\alpha}\left(M,\overline{N}\right)=Div_{\alpha}\left(\overline{M},N\right)$
4. $Div_{\alpha}\left(M,\overline{N}\right)+Div_{\alpha}\left(\overline{M},\overline{N}\right)=Div_{\alpha}\left(M,N\right)+Div_{\alpha}\left(\overline{M},N\right)$

Proof:

1. As per the definition of the *complement* given in Section 2, this result holds.
2. In view of the Equation (6) and Equation (7), we get $Div_{\alpha}(\overline{M\cup N},\overline{M\cap N})$

$$=\sum_{x_i\in X_1}^{n}2^{\alpha}\frac{\left(\sqrt{1-T_N(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_N(x_i)-T_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_N(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_N(x_i)+T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_N(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_N(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\frac{\left(\sqrt{1-I_N(x_i)}-\sqrt{1-I_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i)-I_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_N(x_i)}-\sqrt{I_M(x_i)}\right)^{2(\alpha+1)}}{\left(I_N(x_i)+I_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n}2^{\alpha}\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_N(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n}2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_N(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n}2^{\alpha}\frac{\left(\sqrt{1-F_N(x_i)}-\sqrt{1-F_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_1}^{n}2^{\alpha}\left[\frac{\left(\sqrt{F_N(x_i)}-\sqrt{F_M(x_i)}\right)^{2(\alpha+1)}}{\left(F_N(x_i)+F_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_N(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_N(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_N(x_i)\right)^{\alpha}}\right]=Div_{\alpha}(M,N).$$

On the other hand, $Div_{\alpha}(\overline{M}\cap\overline{N},\overline{M}\cup\overline{N})$

$$=\sum_{x_i\in X_1}^{n} 2^{\alpha}\frac{\left(\sqrt{1-T_N(x_i)}-\sqrt{1-T_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_N(x_i)\right)^{\alpha}}+\sum_{x_i\in X_1}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{T_N(x_i)}-\sqrt{T_M(x_i)}\right)^{2(\alpha+1)}}{\left(T_N(x_i)+T_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{1-T_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_M(x_i)-T_N(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{T_M(x_i)}-\sqrt{T_N(x_i)}\right)^{2(\alpha+1)}}{\left(T_M(x_i)+T_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\frac{\left(\sqrt{1-I_N(x_i)}-\sqrt{1-I_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i)-I_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_1}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{I_N(x_i)}-\sqrt{I_M(x_i)}\right)^{2(\alpha+1)}}{\left(I_N(x_i)+I_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{1-I_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_M(x_i)-I_N(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{I_M(x_i)}-\sqrt{I_N(x_i)}\right)^{2(\alpha+1)}}{\left(I_M(x_i)+I_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\frac{\left(\sqrt{1-F_N(x_i)}-\sqrt{1-F_M(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_1}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{F_N(x_i)}-\sqrt{F_M(x_i)}\right)^{2(\alpha+1)}}{\left(F_N(x_i)+F_M(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\alpha}\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{1-F_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_M(x_i)-F_N(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{F_M(x_i)}-\sqrt{F_N(x_i)}\right)^{2(\alpha+1)}}{\left(F_M(x_i)+F_N(x_i)\right)^{\alpha}}\right]=Div_{\alpha}(M,N).$$

Therefore, $Div_{\alpha}\left(\overline{M\cup N},\overline{M\cap N}\right)=Div_{\alpha}\left(\overline{M}\cap\overline{N},\overline{M}\cup\overline{N}\right)=Div_{\alpha}(M,N)$.

3. $$Div_{\alpha}(M,\overline{N})=\sum_{x_i\in X_1}^{n} 2^{\alpha}\frac{\left(\sqrt{T_M(x_i)}-\sqrt{1-T_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-T_N(x_i)-T_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{1-T_M(x_i)}-\sqrt{T_N(x_i)}\right)^{2(\alpha+1)}}{\left(1-T_M(x_i)+T_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\frac{\left(\sqrt{I_M(x_i)}-\sqrt{1-I_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-I_N(x_i)-I_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{1-I_M(x_i)}-\sqrt{I_N(x_i)}\right)^{2(\alpha+1)}}{\left(1-I_M(x_i)+I_N(x_i)\right)^{\alpha}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\alpha}\frac{\left(\sqrt{F_M(x_i)}-\sqrt{1-F_N(x_i)}\right)^{2(\alpha+1)}}{\left(2-F_N(x_i)-F_M(x_i)\right)^{\alpha}}+\sum_{x_i\in X_2}^{n} 2^{\alpha}\left[\frac{\left(\sqrt{1-F_M(x_i)}-\sqrt{F_N(x_i)}\right)^{2(\alpha+1)}}{\left(1-F_M(x_i)+F_N(x_i)\right)^{\alpha}}\right]=Div_{\alpha}\left(M,\overline{N}\right).$$

4. Using (a) and (c), $Div_{\alpha}\left(M,\overline{N}\right)+Div_{\alpha}\left(\overline{M},\overline{N}\right)=Div_{\alpha}\left(M,N\right)+Div_{\alpha}\left(\overline{M},N\right)$ holds.

5. Application of Parametric Divergence Measure in Decision Making Problems

We study some important applications of the proposed divergence measure for neutrosophic sets in the area of classication problems and decision-making.

5.1 Pattern Recognition

In order to illustrate an application of the proposed divergence measure in the field of pattern recognition, we refer to a well posed problem which has been discussed in literature [33]. Consider three existing patterns A_1, A_2 and A_3 representing the classes C_1, C_2 and C_3 respectively and being described by the following SVNSs in $X=\{x_1,x_2,x_3\}$:

$$A_1=\{(x_1,0.7,0.7,0.2),(x_2,0.7,0.8,0.4),(x_3,0.6,0.8,0.2)\};$$

$$A_2=\{(x_1,0.5,0.7,0.3),(x_2,0.7,0.7,0.5),(x_3,0.8,0.6,0.1)\};$$

$$A_3=\{(x_1,0.9,0.5,0.1),(x_2,0.7,0.6,0.4),(x_3,0.8,0.5,0.2)\}.$$

Consider an unknown sample pattern B which is given by

$$B=\{(x_1,0.7,0.8,0.4),(x_2,0.8,0.5,0.3),(x_3,0.5,0.8,0.5)\}.$$

Now, the main objective of the problem is to find out the class to which *B* belongs. As per the principle of minimum divergence measure [44], the procedure for allocation of *B* to C_{β^*} is determined by

$$\beta^*=arg\min_{\beta}(Div_{\alpha}(A_k,B)). \tag{8}$$

Table 1: Values of $Div_{\alpha}\left(A_k,B\right)$ with $\beta\in\{1,2,3\}$

	α	A_1	A_2	A_3
B	1	**0.035200913**	0.109091158	0.116197599
B	4	**0.0001939**	0.010382714	0.003212291

Clearly, from the Table 1, it may be observed that *B* has to get into the class C_1. The obtained result is based on the proposed parametric divergence measure and is perfectly consistent with the results achieved by [33].

5.2 Medical Diagnosis

In a classical problem of medical diagnosis, assume that if a doctor needs to diagnose some of patients' $"P=\{Al, Bob, Joe, Ted\}"$ under some defined diagnoses

$"D=\{Viral\ fever,\ Malaria,\ Typhoid,\ Stomach\ problem,\ Cough,\ Chest\ problem\}"$,

& a set of symptoms $"S=\{Temperature,\ Headache,\ Stomach\ pain,\ Cough,\ Chest\ pain\}"$.

The following tables (Refer Table 2 & Table 3) serve the purpose of the proposed computational application:

Table 2: Symptoms characteristic for the diagnoses considered [33]

	"Viral Fever"	*"Malaria"*	*"Typhoid"*	*"Stomach Problem"*	*"Chest Problem"*
"Temperature"	(0.7,0.5,0.6)	(0.7,0.9,0.1)	(0.3,0.7,0.2)	(0.1,0.6,0.7)	(0.1,0.9,0.8)
"Headache"	(0.8,0.2,0.9)	(0.4,0.5,0.5)	(0.6,0.9,0.2)	(0.7,0.4,0.3)	(0.1,0.6,0.7)
"Stomach Pain"	(0.8,0.1,0.1)	(0.5,0.9,0.2)	(0.2,0.5,0.5)	(0.7,0.7,0.8)	(0.5, 0. 7, 0.6)
"Cough"	(0.45,0.8,0.7)	(0.7,0.8,0.6)	(0.2,0.5,0.5)	(0.2,0.8,0.65)	(0.2,0.8,0.6)
"Chest Pain"	(0.2,0.6,0.5)	(0.1,0.6,0.8)	(0.1,0.8,0.8)	(0.5,0.8,0.6)	(0.8,0.8,0.2)

Table 3: Symptoms for the diagnose under consideration

	"Temperature"	*"Headache"*	*"Stomach pain"*	*"Cough"*	*"Chest pain"*
"Al"	(0.7,0.6,0.5)	(0.6,0.3,0.5)	(0.5,0.5,0.75)	(0.8,0.75,0.5)	(0.7,0.2,0.6)
"Bob"	(0.7,0.3,0.5)	(0.5,0.5,0.8)	(0.6,0.5,0.5)	(0.65,0.4,0.75)	(0.2,0.85,0.65)
"Joe"	(0.75,0.5,0.5)	(0.2,0.85,0.7)	(0.7,0.6,0.4)	(0.7,0.55,0.5)	(0.5,0.9,0.64)
"Ted"	(0.4,0.7,0.6)	(0.7,0.5,0.7)	(0.6,0.7,0.5)	(0.5,0.9,0.65)	(0.6,0.5,0.85)

In order to have a proper diagnose, we evaluate the value of the proposed divergence measure $Div_\alpha(P,d_\beta)$ between the patient's symptoms & the defined symptoms for each diagnose $d_\beta \in D$, with $\beta=\{1,\ 2,...,\ 5\}$. Similar to the Equation (8), the proper diagnose d_β for the patient P may be based on the following equation:

$$\beta^* = \arg\min_\beta \left(Div_\alpha\left(P,d_\beta\right)\right). \tag{9}$$

Table 4: Values of $Div_\alpha(A_k,B)$, with $\beta \in \{1,2,3\}$

	"Viral Fever"	*"Malaria"*	*"Typhoid"*	*"Stomach Problem"*	*"Chest Problem"*
"Al"	0.29738	0.27867	0.41362	**0.26433**	0.37028

"Bob"	**0.09767**	0.23291	0.13969	0.18798	0.41526
"Joe"	0.29928	0.16506	**0.13089**	0.26385	0.26343
"Ted"	0.20289	0.16876	0.20367	**0.03168**	0.27893

In view of Table 4, it is being concluded that the patient *Al* and *Ted* are suffering from the *stomach problem*, *Bob* is suffering from *viral fever* and *Joe* is suffering from *Typhoid*.

It may be observed that the result obtained through the proposed method is perfectly consistent with the results achieved by [33].

Comparative Remarks: It may be observed that the proposed method is found to be perfectly competent to provide the desired result with an added advantage of the parameters involvement in the proposed divergence measure. The parameters may provide a better variability in the selection of a divergence measure for achieving a better specificity and accuracy.

5.2 Multi-criteria Decision-Making Problem

The main purpose of MCDM problem is to identify the alternative from the available alternatives under consideration. Here, on the basis of the proposed parametric divergence measure for neutrosophic sets, an algorithm for solving MCDM problem is being outlined. Consider the available m-alternatives, i.e., $Z = \{Z_1, Z_2, \ldots, Z_m\}$ and n-criterion, i.e., $O = \{o_1, o_2, \ldots, o_n\}$. The target of a decision maker is to pick the optimal alternative out of the available *m*-alternatives fulfilling the *n*-criterion. The perspectives/opinions of decision makers have been taken in the form of a matrix $A = [a_{ij}]_{m\times n}$ called neutrosophic decision matrix where $a_{ij} = (T_{ij}, I_{ij}, F_{ij})$.

Procedural Steps of Algorithm for MCDM Problem:

Step 1: Construct the neutrosophic decision matrix based on the available data.

Step 2: Sometimes heterogeneity in the type of criterions in a MCDM problem is observed. In order to resolve this issue, it is required to make them homogeneous before applying any methodology. Mainly, the criteria may be categorized into two types: benefit criteria and cost criteria. We need to transform the decision matrix, for this we transform the cost criteria into the benefit criteria. Thus the decision matrix $A = [a_{ij}]_{m\times n}$ is converted into a new decision matrix, say, $B = [b_{ij}]_{m\times n}$ where b_{ij} is given by

$$b_{ij} = (T_{ij}, I_{ij}, F_{ij}) = \begin{cases} a_{ij} & \textit{for benefits criteria}; \\ a_{ij}^{c} & \textit{for cost criteria}; \end{cases} \tag{10}$$

where $B = [b_{ij}]_{m\times n}$ representing the alternatives in the form of

$$Z_i = \{(o_j, 1-T_{ij}, 1-I_{ij}, 1-F_{ij}) \mid o_j \in O\};\ i = 1, 2, \ldots, m \textit{ and } j = 1, 2, \ldots, n. \tag{11}$$

Step 3: Evaluate the best preferred solution as

$$Z^{+} = \{\sup(T_{ij}(Z_i)), \inf(I_{ij}(Z_i)), \inf(F_{ij}(Z_i))\}\ i = 1, 2, \ldots, m \textit{ and } j = 1, 2, \ldots, n. \tag{12}$$

Step 4: Determine the value of divergence measure of alternatives $Z_i^{'}s$ from Z^{+} using Equation (2).

Step 5. Now, sorting the computed values of the divergence measure, we can find the preference order of the alternatives $Z_i^{'}s$. The best alternative is the one which corresponds to the least value of the divergence measure.

For the sake of illustration of the proposed methodology, a multi-criteria decision-making problem [45] related to a manufacturing company which needs to hire the best supplier. Assume that there are four available suppliers $Z = \{Z_1, Z_2, Z_3, Z_4\}$ whose capabilities and competencies have been evaluated with the help of four laid down criteria $O = \{o_1, o_2, o_3, o_4\}$. Based on the information available about the suppliers w.r.t. the individual criteria, we determine a neutrosophic decision matrix as given below:

1. In the given MCDM problem, all criterions are of same kind. Therefore, we need not to transform the cost criteria into the benefit criteria or vice versa by using Equation (10). The constructed neutrosophic decision matrix based on the available information is in the following Table 5.

Table 5: Neutrosophic Decision Matrix

	o_1	o_2	o_3	o_4
Z_1	(0.5, 0.1, 0.3)	(0.5, 0.1, 0.4)	(0.7, 0.1, 0.2)	(0.3, 0.2, 0.1)
Z_2	(0.4, 0.2, 0.3)	(0.3, 0.2, 0.4)	(0.9,0.0,0.1)	(0.5, 0.3, 0.2)
Z_3	(0.4, 0.3, 0.1)	(0.5, 0.1, 0.3)	(0.5, 0.0, 0.4)	(0.6, 0.2, 0.2)
Z_4	(0.6,0.1,0.2)	(0.2, 0.2, 0.5)	(0.4, 0.3, 0.2)	(0.7, 0.2, 0.1)

2. The best preferred solution obtained by using equation (12) is given by

$$Z^+ = \{(0.6, 0.1, 0.1), (0.5, 0.1, 0.3), (0.9, 0.0, 0.1), (0.7, 0.2, 0.1)\}.$$

3. Compute the values of divergence measure between $Z_i^{'}s$ $(i = 1, 2, 3, 4)$ and Z^+ using Equation (2) and tabulate them in the following Table 6.

Table 6: Values of Proposed Divergence Measure between $Z_i^{'}s$ and Z^+

4. Now, the ranking of the alternatives can be performed. The best alterative is one which has the lowest value of the divergence measure from the best preferred solution. The sequence of the alternatives has been particularly obtained as: $Z_2 > Z_1 > Z_3 > Z_4$.

Divergence Measure	(Z_1, Z^+)	(Z_2, Z^+)	(Z_3, Z^+)	(Z_4, Z^+)
Proposed Divergence Measure	0.42409	0.27570	0.4791	0.80810
Ye's Divergence Measure [33]	1.1101	1.1801	0.9962	1.2406

Hence, among all the four suppliers, Z_2 is supposed to be the best one.

6. Conclusions and Scope for Future Work

The parametric divergence measure for SVNSs has been successfully proposed along with discussions on its various properties. In literature, this parametric measure for the neutrosophic set is for the first time where the applications of the proposed divergence measure have been successfully utilized in the computational fields of pattern analysis, medical diagnosis & MCDM problem. The procedural steps of the proposed methodologies for solving these application problems have been well illustrated with numerical examples for each. The results hence obtained are found to be equally and firmly consistent in comparison with the existing methodologies.

In order to have a direction for the scope of future work, it has been observed that there is a notion of another set called rough set, which do not conflict the concept of neutrosophic set, can be mutually incorporated. Sweety and Arockiarani [46] combined the mathematical tools of fuzzy sets, rough sets and neutrosophic sets and introduced a new notion termed as fuzzy neutrosophic rough sets. In future, the following important research contributions can be systematically carried out

- The study on the various information measures - entropy, similarity measures and divergence measures for fuzzy neutrosophic rough sets can be done with their various possible applications.
- In recent years, various researchers have duly utilized the notion of neutrosophic sets to relations, theory of groups and rings, theory of soft sets and so on. On the basis of this, the theoretical contribution related to fuzzy neutrosophic rough sets in the field of algebra may be proposed.

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Single-Valued Neutrosophic Hyperrings and Single-Valued Neutrosophic Hyperideals

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Abstract. In this paper, we introduced the concepts of Single-valued neutrosophic hyperring and Single-valued neutrosophic hyperideal. The algebraic properties and structural characteristics of the single-valued neutrosophic hyperrings and hyperideals are investigated and verified.

Keywords: Hyperring, Hyperideal, Single-valued neutrosophic set, Single-valued neutrosophichyperring and Single-valued neutrosophichyperideal.

1 Introduction

Hyperstructure theory was introduced by Marty in 1934 [16]. The concept of hyperring and the general form of hyperring for introducing the notion of hyperring homomorphism was developed by Corsini [11]. Vougiouklis [31] coined different type of hyperrings called H_v-ring, H_v-subring, and left and right H_v-ideal of a H_v-ring, all of which are generalizations of the corresponding concepts related to hyperrings introduced by Corsini [11].

In general fuzzy sets [34] the grade of membership is represented as a single real number in the interval [0,1]. The uncertainty in the grade of membership of the fuzzy set model was overcome using the interval-valued fuzzy set modelintroduced by Turksen [29]. In 1986, Atanassov [8] introduced intuitionistic fuzzy sets which is a generalization of fuzzy sets. This model was equivalent to interval valued fuzzy sets in [32]. Intuitionistic fuzzy sets can only handle incomplete information, and not indeterminate information which commonly exists in real-life [32]. To overcome these problems, Smarandache introduced the neutrosophic model. Some new trends of neutrosophic theory were introduced in [1,2,3,4,5,6,7] .Wang et al. [32] introduced the concept of single-valued neutrosophic sets (SVNSs), whereas Smarandache introduced plithogenic set as generalization of neutrosophic set model in [13].

The theory of hyperstructures are widely used in various mathematical theories. The study on fuzzy algebra began by Rosenfeld [17], and this was subsequently expanded to other fuzzy based models such as intuitionistic fuzzy sets, fuzzy soft sets and vague soft sets. Some of the recent works related to fuzzy soft rings and ideal, vague soft groups, vague soft rings and vague soft ideals can be found in [21; 22; 23; 26, 27]. Research on fuzzy algebra led to the development of fuzzy hyperalgebraic theory. The concept of fuzzy ideals of a ring introduced by Liu [15]. The generalization of the fuzzy hyperideal introduced by Davvaz[12]. The concepts of fuzzy γ-ideal was then introduced by Bharathi

and Vimala [10], and the fuzzy γ-ideal was subsequently expanded in [33]. The hypergroup and hyperring theory for vague soft sets were developed by Selvachandran et al. in [18,19,20,24,25]
In this paper we develop the theory of single-valued neutrosophic hyperrings and single-valued neutrosophic hyperideals to furter contribute to the development of the body of knowledge in neutrosophic hyperalgebraic theory.

2 Preliminaries

Let X be a space of points (objects) with a generic element in X denoted by x.

Definition 2.1. [32] A SVNS $A$is a neutrosophic set that is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$. This set A can thus be written as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}. \quad (1)$$

The sum of $T_A(x), I_A(x)$ and $F_A(x)$ must fulfill the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. For a SVNS A in U, the triplet $\big(T_A(x), I_A(x), F_A(x)\big)$ is called a single-valued neutrosophic number (SVNN). Let $x = (T_x, I_x, F_x)$ to represent a SVNN .

Definition 2.2. [32] Let A and B be two SVNSs over a universe U.

(i) A is contained in B, if $T_A(x) \le T_B(x), I_A(x) \le I_B(x)$, and $F_A(x) \ge F_B(x)$, for all $x \in U$. This relationship is denoted as $A \subseteq B$.

(ii) A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$.

(iii) $A^c = \langle x, \big(F_A(x), 1 - I_A(x), T_A(x)\big)\rangle$, for all $x \in U$.

(iv) $A \cup B = \big(x, (\max(T_A, T_B), \max(I_A, I_B), \min(F_A, F_B))\big)$, for all $x \in U$.

(v) $A \cap B = \big(x, (\min(T_A, T_B), \min(I_A, I_B), \max(F_A, F_B))\big)$, for all $x \in U$.

Definition 2.3. [16] A *hypergroup* $\langle H, \circ\rangle$ is a set H with an associative hyperoperation $(\circ) : H \times H \to P(H)$ which satisfies $x \circ H = H \circ x = H$ for all x in H (reproduction axiom) .

Definition 2.4.[12] A hyperstructure $\langle H, \circ\rangle$ is called an H_v*-group* if the following axioms hold:

(i) $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \emptyset$ for all $x, y, z \in H$, (H_v-semigroup)

(ii) $x \circ H = H \circ x = H$ for all x in H.

Definition 2.5.[16] A subset K of H is called a *subhypergroup* if $\langle K, \circ\rangle$ is a hypergroup.

Definition 2.6.[11]A H_v*-ring* is a multi-valued system $(R, +, \circ)$ which satisfies the following axioms:

(i) $(R, +)$ is a H_v-group,

(ii) $(R, \circ)$ is a H_v-semigroup,

(iii) The hyperoperation "$\circ$" is weak distributive over the hyperoperation "+", that is for each $x, y, z \in R$ the conditions $x \circ (y + z) \cap \big((x \circ y) + (x \circ z)\big) \neq \phi$ and $(x + y) \circ z \cap \big((x \circ z) + (y \circ z)\big) \neq \phi$ holds true.

Definition 2.7. [11]A nonempty subset R' of R is a *subhyperring* of $(R, +, \circ)$ if $(R', +)$ is a subhypergroup of $(R, +)$ and for all $x, y, z \in R'$, $x \circ y \in P^*(R')$, where $P^*(R')$ is the set of all non-empty subsets of R'.

Definition 2.8. [11] Let R be a H_v-ring. A nonempty subset I of R is called a *left* (respectively *right*) H_v-*ideal* if the following axioms hold:

(i) $(I, +)$ is a H_v-subgroup of $(R, +)$,

(ii) $R \circ I \subseteq I$(resp. $I \circ R \subseteq I$).

If I is both a left and right H_v-ideal of R, then I is said to be a H_v*-ideal* of R.

3 Single-Valued NeutrosophicHyperrings

Throughout this section, we denote the hyperring $(R, +, \circ)$by R.

Definition 3.1. Let A be a SVNS over R. A is called a single-valued neutrosophic hyperringover R, if ,

(i) $\forall \quad a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a + b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a + b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a + b\}$

(ii) $\forall \quad x, a \in R$, there exists $b \in R$ such that $a \in x + b$ and $\min\{T_A(x), T_A(a)\} \leq T_A(b)$, $\max\{I_A(x), I_A(a)\} \geq I_A(b)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(b)$

(iii) $\forall \quad x, a \in R$, there exists $c \in R$ such that $a \in c + x$ and $\min\{T_A(x), T_A(a)\} \leq T_A(c)$, $\max\{I_A(x), I_A(a)\} \geq I_A(c)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(c)$

(iv) $\forall \ a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a \circ b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a \circ b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a \circ b\}$

Example 3.2. The family of t-level sets of SVNSs over R is a subhyperring of R is given below:

$$A_t = \{a \in R: T_A(a) \geq t, I_A(a) \geq t, F_A(a) \leq t\}, \text{for all } t \in [0, 1].$$

Then A is a single-valued neutrosophic hyperring over R.

Theorem 3.3. A is a SVNS over R. Then A is a single-valued neutrosophichyperring over R iff A is single-valued neutrosophic semi hyper group over $(R, \circ)$ and also a single-valued neutrosophic hypergroup over $(R, +)$.

Proof. This is obvious by Definition 3.1. ■

Theorem 3.4. Let A and B be single-valued neutrosophic hyperrings over R. Then$A \cap B$ is a single-valued neutrosophichyperring over R if it is non-null.

Proof. Let A and B are single-valued neutrosophic hyperrings over R.By Definition 3.1, $A \cap B = \{\langle a, T_{A\cap B}(a), I_{A\cap B}(a), F_{A\cap B}(a)\rangle: a \in R\}$, where $T_{A\cap B}(a) = \min(T_A(a), T_B(a))$, $I_{A\cap B}(a) = \max(I_A(a), I_B(a))$, $F_{A\cap B}(a) = \max(F_A(a), F_B(a))$. Then for all $a, b \in R$, we have the following. We only prove all the four conditions for the truth membership terms T_A, T_B. The proof for the I_A, I_B and F_A, F_B membership functions obtained in a similar manner.

(i)
$$\begin{aligned}\min\{T_{A\cap B}(a), T_{A\cap B}(b)\} &= \min\{\min(T_A(a), T_B(a)), \min(T_A(b), T_B(b))\} \\ &\leq \min\{\min(T_A(a), T_A(b)), \min(T_B(a), T_B(b))\} \\ &\leq \min\{\inf\{T_A(c): c \in a + b\}, \inf\{T_B(c): c \in a + b\}\} \\ &\leq \inf\{\min(T_A(c), T_B(c)): c \in a + b\} \\ &= \inf\{T_{A\cap B}(c): c \in a + b\}\end{aligned}$$

Similarly, $\max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \geq \sup\{I_{A\cap B}(c): c \in a + b\}$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \geq \sup\{F_{A\cap B}(c): c \in a + b\}$.

(ii) $\forall x, a \in R$, there exists $b \in R$ such that $a \in x + b$. Then it follows that:

$$\begin{aligned}\min\{T_{A\cap B}(a), T_{A\cap B}(b)\} &= \min\{\min(T_A(a), T_B(a)), \min(T_A(b), T_B(b))\} \\ &\leq \min\{\min(T_A(a), T_A(b)), \min(T_B(a), T_B(b))\} \\ &\leq \min(T_A(c), T_B(c)) \\ &= T_{A\cap B}(c)\end{aligned}$$

Similarly, $\max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \geq I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \geq F_{A\cap B}(c)$.

(iii) It can be easily verified that $\forall \ x, a \in R$, there exists $c \in R$ such that $a \in c + x$ & $\min\{T_{A\cap B}(x), T_{A\cap B}(a)\} \leq T_{A\cap B}(c), \max\{I_{A\cap B}(x), I_{A\cap B}(a)\} \geq I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(x), F_{A\cap B}(a)\} \geq$

$F_{A\cap B}(c)$.

(iv) $\forall\ a \in R, \min\{T_{A\cap B}(a), T_{A\cap B}(b)\} \leq \inf\{T_{A\cap B}(c): c \in a \circ b\}, \max\{I_{A\cap B}(a), I_{A\cap B}(b)\} \geq \sup\{I_{A\cap B}(c): c \in a \circ b\}$ and $\max\{F_{A\cap B}(a), F_{A\cap B}(b)\} \geq \sup\{F_{A\cap B}(c): c \in a \circ b\}$.

Hence, $A \cap B$ is single-valued neutrosophichyperring over R. ■

Theorem 3.5. Let A be a single-valued neutrosophic hyperring over R. Then for every $t \in [0,1], A_t \neq \emptyset$ is a subhyperring over R.

Proof. Let A be a single-valued neutrosophichyperring over R. $\forall\ t \in [0,1]$, let $a, b \in A_t$. Then $T_A(a), T_A(b) \geq t, I_A(a), I_A(b) \leq t$and $F_A(a), F_A(b) \leq t$. Since A is a single-valued neutrosophic sub hyper group of $(R, +)$, we have the following:

$$\inf\{T_A(c): c \in a + b\} \geq \min\{T_A(a), T_A(b)\} \geq \min\{t, t\} = t,$$

$$\sup\{I_A(c): c \in a + b\} \leq t,$$

and

$$\sup\{F_A(c): c \in a + b\} \leq t.$$

This implies that $c \in A_t$ and then for every $c \in a + b$, we obtain $a + b \subseteq A_t$. As such, for every $c \in A_t$, we obtain $c + A_t \subseteq A_t$. Now let $a, c \in A_t$. Then $T_A(a), T_A(c) \geq t, I_A(a), I_A(c) \leq t$and $F_A(a), F_A(c) \leq t$.

A is a single-valued neutrosophic subhypergroup of $(R, +)$, there exists $b \in R$ such that $a \in c + b$ and $T_A(b) \geq \min\big(T_A(a), T_A(c)\big) \geq t$, $I_A(b) \leq \max\big(I_A(a), I_A(c)\big) \leq t$, $F_A(b) \leq \max\big(F_A(a), F_A(c)\big) \leq t$, and this implies that $b \in A_t$. Therefore, we obtain $A_t \subseteq c + A_t$. As such, we obtain $c + A_t = A_t$. As a result, A_t is a subhypergroup of $(R, +)$.

Let $a, b \in A_t$,then $T_A(a), T_A(b) \geq t, I_A(a), I_A(b) \leq t$ and $F_A(a), F_A(b) \leq t$.Since A is a single-valued neutrosophic subsemihypergroup of $(R, \circ)$, then for all $a, b \in R$, we have the following:

$$\inf\{T_A(c): c \in a \circ b\} \geq \min\{T_A(a), T_A(b)\} = t,$$

$$\sup\{I_A(c): c \in a \circ b\} \leq \max\big(I_A(a), I_A(b)\big) = t,$$

and

$$\sup\{F_A(c): c \in a \circ b\} \leq \max\big(F_A(a), F_A(b)\big) = t.$$

This implies that $c \in A_t$ and consequently $a \circ b \in A_t$. Therefore, for every $a, b \in A_t$ we obtain $a \circ b \in P^*(R)$. Hence A_t is a subhyperring over R.

Theorem 3.6. Let A be a single-valued neutrosophic set over R.Then the following statements are equivalent:

(i) $A$is a single-valued neutrosophic hyperring over R.

(ii) $\forall t \in [0,1]$, a non-empty A_t is a sub hyperring over R.

Proof.

$(i) \Longrightarrow (ii)$ $\forall\ t \in [0,1]$, by Theorem 3.5, A_t is sub hyperring over R.

$(ii) \Longrightarrow (i)$ Assume that A_t is a subhyperring over R. Let $a, b \in A_t$ and therefore $a + b \subseteq A_{t_0}$. Then for every $c \in a + b$ we have $T_A(c) \geq t_0, I_A(c) \leq t_0$and $F_A(c) \leq t_0$, which implies that:

$$\min\big(T_A(a), T_A(b)\big) \leq \inf\{T_A(c): c \in a + b\},$$

$$\max\big(I_A(a), I_A(b)\big) \geq \sup\{I_A(c): c \in a + b\},$$

and

$$\max\big(F_A(a), F_A(b)\big) \geq \sup\{F_A(c): c \in a+b\}$$

Therefore, condition (i) of Definition 3.1 has been verified.

Next, let $x, a \in A_{t_1}$ for every $t_1 \in [0,1]$ which means that there exists $b \in A_{t_1}$ such that $a \in x \circ b$. Since$b \in A_{t_1}$, we have $T_A(b) \geq t_1, I_A(b) \leq t_1$ and $F_A(b) \leq t_1$, and thus we have

$$T_A(b) \geq t_1 = \min\big(T_A(a), T_A(c)\big),$$

$$I_A(b) \leq t_1 = \max\big(I_A(a), I_A(c)\big),$$

and

$$F_A(b) \leq t_1 = \max\big(F_A(a), F_A(c)\big).$$

Therefore, condition (ii) of Definition 3.1 has been verified. Compliance to condition (iii) of Definition 3.1 can be proven in a similar manner. Thus, A is a single-valued neutrosophic subhypergroup of $(R,+)$. Now since A_t is a subsemihypergroup of the semihypergroup $(R, \circ)$, we have the following. Let $a, b \in A_{t_2}$ and therefore we have $a \circ b \in A_{t_2}$. Thus for every $c \in a \circ b$, we obtain $T_A(c) \geq t_2, I_A(c) \leq t_2$ and $F_A(c) \leq t_2$, and therefore it follows that:

$$\min\big(T_A(a), T_A(b)\big) \leq \inf\{T_A(c): c \in a \circ b\},$$

$$\max\big(I_A(a), I_A(b)\big) \geq \sup\{I_A(c): c \in a \circ b\},$$

and

$$\max\big(F_A(a), F_A(b)\big) \geq \sup\{F_A(c): c \in a \circ b\},$$

which proves that condition (iv) of Definition 3.1 has been verified. Hence A is a single-valued neutrosophic hyperring over R.

4 Single-Valued Neutrosophic Hyperideals

Definition 4.1. Let A be a SVNS over R. Then A is single-valued neutrosophic left (resp. right) hyperideal over R, if ,

(i) $\forall \; a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a+b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a+b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a+b\}$

(ii) $\forall \; x, a \in R$, there exists $b \in R$ such that $a \in x+b$ and $\min\{T_A(x), T_A(a)\} \leq T_A(b), \max\{I_A(x), I_A(a)\} \geq I_A(b)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(b)$

(iii) $\forall \; x, a \in R$, there exists $c \in R$ such that $a \in c+x$ and $\min\{T_A(x), T_A(a)\} \leq T_A(c), \max\{I_A(x), I_A(a)\} \geq I_A(c)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(c)$

(iv) $\forall \; a, b \in R, T_A(b) \leq \inf\{T_A(c): c \in a \circ b\}$ (resp. $T_A(a) \leq \inf\{T_A(c): c \in a \circ b\}$), $I_A(b) \geq \sup\{I_A(c): c \in a \circ b\}$ (resp. $I_A(a) \geq \sup\{I_A(c): c \in a \circ b\}$) and $F_A(b) \geq \sup\{F_A(c): c \in a \circ b\}$ (resp. $F_A(a) \geq \sup\{F_A(c): c \in a \circ b\}$)

A is a single-valued neutrosophic left (resp. right) hyperidealof R. From conditions (i), (ii) and (iii) A is a single-valued neutrosophic subhypergroup of $(R,+)$.

Definition 4.2. Let A be a SVNS over R. Then A is a single-valued neutrosophic hyper ideal over R, if the following conditions are satisfied:

(i) $\forall \; a, b \in R, \min\{T_A(a), T_A(b)\} \leq \inf\{T_A(c): c \in a+b\}, \max\{I_A(a), I_A(b)\} \geq \sup\{I_A(c): c \in a+b\}$ and $\max\{F_A(a), F_A(b)\} \geq \sup\{F_A(c): c \in a+b\}$

(ii) $\forall \; x, a \in R$, there exists $b \in R$ such that $a \in x+b$ and $\min\{T_A(x), T_A(a)\} \leq T_A(b), \max\{I_A(x), I_A(a)\} \geq I_A(b)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(b)$

(iii) $\forall \; x, a \in R$, there exists $c \in R$ such that $a \in c+x$ and $\min\{T_A(x), T_A(a)\} \leq T_A(c), \max\{I_A(x), I_A(a)\} \geq I_A(c)$ and $\max\{F_A(x), F_A(a)\} \geq F_A(c)$

(iv) $\forall \; a, b \in R, \max\big(T_A(a), T_A(b)\big) \leq \inf\{T_A(c): c \in a \circ b\}, \max\big(I_A(a), I_A(b)\big) \geq \sup\{I_A(c): c \in a \circ b\}$ and $\max\big(F_A(a), F_A(b)\big) \geq \sup\{F_A(c): c \in a \circ b\}$

From conditions (i), (ii) and (iii) A is a single-valued neutrosophic sub hyper group of $(R,+)$. Condition (iv) indicate both single-valued neutrosophic left hyperideal and single-valued neutrosophic right hyperideal. Hence A is a single-valued neutrosophic hyper ideal of R.

Theorem 4.3. Let A be a non-null SVNS over R. A is a single-valued neutrosophic hyperideal over R iff A is a single-valued neutrosophic hyper group over $(R,+)$ and also $A$is both a single-valued neutrosophic left hyper ideal and a single-valued neutrosophic right hyper ideal of R.

Proof. This is straight forward by Definitions 4.1 and 4.2.

Theorem 4.4. Let $A$and B be two single-valued neutrosophic hyper ideals over R. Then $A\cap B$ is a single-valued neutrosophichyperideal over R if it is non-null.

Proof. Let A and B are single-valued neutrosophic hyper ideals over R.By Definition 4.2, $A\cap B=\{\langle a,T_{A\cap B}(a),I_{A\cap B}(a),F_{A\cap B}(a)\rangle: a\in R\}$, where $T_{A\cap B}(a)=\min\big(T_A(a),T_B(a)\big)$, $I_{A\cap B}(a)=\max\big(I_A(a),I_B(a)\big)$ and$F_{A\cap B}(a)=\max\big(F_A(a),F_B(a)\big)$. Then $\forall\ a,b\in R$, we have the following. We only prove all the four conditions for the truth membership terms T_A,T_B. The proof for the I_A,I_B and F_A,F_B membership functions obtained in a similar manner.

(i)
$$\begin{aligned}\min\{T_{A\cap B}(a),T_{A\cap B}(b)\}&=\min\big\{\min\big(T_A(a),T_B(a)\big),\min\big(T_A(b),T_B(b)\big)\big\}\\&\le\min\big\{\min\big(T_A(a),T_A(b)\big),\min\big(T_B(a),T_B(b)\big)\big\}\\&\le\min\{\inf\{T_A(c): c\in a+b\},\inf\{T_B(c): c\in a+b\}\}\\&\le\inf\big\{\min\big(T_A(c),T_B(c)\big): c\in a+b\big\}\\&=\inf\{T_{A\cap B}(c): c\in a+b\}\end{aligned}$$

Similarly, it can be proven that $\max\{I_{A\cap B}(a),I_{A\cap B}(b)\}\ge\sup\{I_{A\cap B}(c): c\in a+b\}$ and $\max\{F_{A\cap B}(a),F_{A\cap B}(b)\}\ge\sup\{F_{A\cap B}(c): c\in a+b\}$.

(ii) $\forall x,a\in R$, there exists $b\in R$ such that $a\in x+b$. Then:

$$\begin{aligned}\min\{T_{A\cap B}(a),T_{A\cap B}(b)\}&=\min\big\{\min\big(T_A(a),T_B(a)\big),\min\big(T_A(b),T_B(b)\big)\big\}\\&\le\min\big\{\min\big(T_A(a),T_A(b)\big),\min\big(T_B(a),T_B(b)\big)\big\}\\&\le\min\big(T_A(c),T_B(c)\big)\\&=T_{A\cap B}(c)\end{aligned}$$

Similarly, $\max\{I_{A\cap B}(a),I_{A\cap B}(b)\}\ge I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(a),F_{A\cap B}(b)\}\ge F_{A\cap B}(c)$.

(iii) $\forall\ x,a\in R$, there exists $c\in R$ such that $a\in c+x$ and $\min\{T_{A\cap B}(x),T_{A\cap B}(a)\}\le T_{A\cap B}(c)$, $\max\{I_{A\cap B}(x),I_{A\cap B}(a)\}\ge I_{A\cap B}(c)$ and $\max\{F_{A\cap B}(x),F_{A\cap B}(a)\}\ge F_{A\cap B}(c)$.

(iv) $\forall\ a\in R$, $\max\{T_{A\cap B}(a),T_{A\cap B}(b)\}\le\inf\{T_{A\cap B}(c): c\in a\circ b\}$, $\min\{I_{A\cap B}(a),I_{A\cap B}(b)\}\ge\sup\{I_{A\cap B}(c): c\in a\circ b\}$ and $\min\{F_{A\cap B}(a),F_{A\cap B}(b)\}\ge\sup\{F_{A\cap B}(c): c\in a\circ b\}$.

Hence, it is verified that $A\cap B$ is a single-valued neutrosophichyperideal over R.

5. Conclusion

We developed hyperstructure for the SVNS model through several hyperalgebraic structures such as hyperrings and hyperideals. The properties of these structures were studied and verified. The future work is on the development of hyperalgebraic theory for Plithogenic sets which is the generalization of neutrosophic set and also planned to develop some real life applications.

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Technique for Reducing Dimensionality of Data in Decision-Making Utilizing Neutrosophic Soft Matrices

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Abstract: The decision-making problems in which there are large numbers of qualitative and quantitative factors involved, the technique of dimensionality reduction plays an important role for simplicity and wider applicability. The impreciseness in the information about these factors are being considered in the neutrosophic perception with the parameters - degree of truth-membership, degree of indeterminacy (neutral) and degree of falsity for a better span of the information. In the present communication, we first propose a technique for finding the threshold value for the information provided in the form of neutrosophic soft matrix. Further, utilizing the proposed definitions of the object-oriented neutrosophic soft matrix and the parameter-oriented neutrosophic soft matrix, we present a new algorithm for the dimensionality reduction process. The proposed algorithm has also been applied in an illustrative example of decision-making problem. Further, a comparative analysis in contrast with the existing methodologies has been successfully presented with comparative remarks and additional advantages.

Keywords: Neutrosophic soft matrix, Dimensionality reduction, multiple criteria decision-making, Object-oriented matrix, Parameter-oriented matrix.

1. Introduction

The methodology of dimensionality reduction is to set out an arrangement of set of high dimensional vectors to a lower dimensionality space while holding systematic measures among them. Due to the inherited disadvantage of dimensionality, there are limitations over using the techniques of machine learning as well as the techniques of data mining for high dimensional data. However, there are two noteworthy dimensionality reduction – procedure where the process of *feature selection* and *feature extraction/feature reduction* is involved. In the procedural steps of feature selection, we select a subset of optimal/most useful features as per the need of the objective function. The prime necessity of the feature selection is to enhance the process of data mining and to increase the speed of learning by reducing the dimensionality and obliterate the noise. Feature extraction or

Feature reduction is the task of mapping the large dimensional data to a smaller dimensional data. The major goals of the dimensionality reduction techniques are to enhance the ability to handle both irrelevant and redundant features, to enhance the cost efficiency in contrast with the existing subset evaluation methods etc. It may be noted that the higher the number of factors, the harder it will be to visualize and work on it.

In case of extreme data modality, dimensionality reduction becomes the center of curiosity to a significant point of study in various fields of application. In the field of soft sets, Chen at al. [1] presented a novel concept of parameterization reduction and compared with the reduction of attributes in rough set theory. There are sequential and simultaneous perspectives to consolidate the selection of samples and for reduction of dimensionality of data whose application structure has been given by Xu et al. [2]. This almost gives the best results while processing of the large-scale training data in comparison to the original data models. In addition to this, they also reached to the conclusion that the selection of samples and the reduction of the data dimensionality are mandatory and helpful for handling the modern large-scale databases. Su et al. [3] introduced a new approach called linear sequence discriminant analysis (LSDA) for reducing the dimensionality of the sequences and devised two new algorithms which differs in the extraction of the statistics. Perfilieva [3] introduced the technique of fuzzy transforms which are in agreement with the technique of dimensionality reduction, based on Laplacian eigenmaps along with an application of fuzzy transform to the mathematical finance.

Konate et al. [5] utilized the principal component analysis (PCA) and linear discriminant analysis (LDA) for the reduction of the dimensionality of the original log set of Chinese Continental Scientific Drilling Main Hole to a convenient size, and then feed these reduced-log set into the three classifiers, i.e., support vector machines, feed forward back propagation and radial basis function neural networks. Further, they also demonstrate and discussed the utilization of the combination of dimensionality reduction methods & classifiers and come up with the result that the reduced log set found from dimensionality reduction separate the metamorphic rocks types better or almost as well as the original log set. Sabitha et al. [6] utilized the three different dimensionality reduction techniques, i.e., principal component analysis, singular value decomposition & learning vector quantization. They applied these three techniques to solar irradiance data set which consists of temperature, solar irradiance, and humidity data and evaluated the efficiency and attain the best technique to be applicable for the data set. Chaterjee et al. [7] proposed a novel hybrid method surround factor relationship and multi-attributive border approximation area comparison (MABAC) methods for selection and evaluation of non-traditional machining processes. The technique condenses the problem of pair wise comparisons for estimating criteria weights in multi-criteria decision-making problem significantly.

Mukhametzyanov and Pamucar [8] presented a model to check the result consistency of MCDM methods and in the process of choosing the best one. Further, issue of sensitivity in the process of decision-making using the different ranking algorithms, e.g., "*SAW, MOORA, VIKOR, COPRAS, CODAS, TOPSIS, D'IDEAL, MABAC, PROMETHEE-I,II, ORESTE-II*" have been analyzed by making necessary perturbations in the entries of the decision matrix within a permissible imprecision value.

In order to deal with the vagueness and impreciseness in various engineering applications, socio-economic problems and other decision-making problems, there are many theories available in literature which have their own limitations due to the involvement of the parameterization tools. Molodtsov [9] proposed a new kind of set, termed as soft set, which has the capability to overcome such limitations and put forward important deliberations based on this. Next, Maji et al. [10-12] extended the notion of soft set to fuzzy soft set & intuitionistic fuzzy soft set and proposed various standard binary operations over it with applications in decision-making. Kahraman et al. [13] studied the fuzzy multi-criteria decision-making literature in detail and presented a literature review on the MCDM techniques. Liu et al. [14] proposed a model for evaluation and selection of a transport service provider based on a single valued neutrosophic number (an extension of interval valued intuitionistic fuzzy number). It was a modified version of the DEMATEL method (Decision-making Trial and Evaluation Laboratory Method) for ranking alternative solutions. Kumar and Bajaj [15] introduced the concept of complex intuitionistic fuzzy soft set and proposed some important distance measures with applications.

Hooda and Hooda [16] used the entropy optimization principles for establishing some criteria for dimension reduction over multivariate data with no external variables. A new criterion for maximum entropy and its relation with other criteria have been established for the selection of principal variables. Maji et al. [17] first introduced the notion of neutrosophic soft set, operations for handling the imprecise & inconsistent information which was further redfined by Deli and Broumi [18] for a better understanding of the belief systems. Further, Peng et al. [19] extended the concept to the Pythagorean fuzzy soft set (PyFSS) with different binary operations and utilized them to solve decision-making problems. Cuong [20] extended the notion of intuitionistic fuzzy soft sets to picture fuzzy soft set. Recently, Guleria and Bajaj [21] successfully proposed the notion of *T*-spherical fuzzy soft set and studied some new aggregation operators along with some applications in the field of decision-making.

The concept of soft matrices was first introduced by Naim and Serdar [22] for representing the notion of soft set with its successful application in the decision-making problems. This matrix representation of soft set was further extended by Yong et al. [23] and Chetia et al. [24] by incorporating the fuzzy and intuitionistic fuzzy setup to deal the decision-making problems respectively. Also, Deli and Broumi [18] have proposed neutrosophic soft matrices and operators which are more functional to make theoretical studies and application in the neutrosophic soft set theory. Such matrices are helpful in representing a neutrosophic soft set in the memory of computers for a wider applicability. Hooda and Kumari [25] proposed a dimensionality reduction model for finding coherent and logical solution to various real-life problems containing uncertainty, impreciseness and vagueness by utilizing the fuzzy soft set. Recently, Guleria and Bajaj [26] studied the Pythagorean fuzzy soft matrices and its various types along with a new decision-making algorithm to deal the medical diagnosis problem and decision-making problem. In the field of neutrosophic set theory, the new trends have brought important field of research. Abdel-Basset et al. [27] developed a multi-criteria group decision-making method under neutrosophic environment based on analytic network process and VIKOR method to solve a supplier selection problem. Many researchers have worked on neutrosophic set theory and applied these notions in solving various multi-criteria decision-making problems, viz., selection processes [28-30], green supply chain management [31], IoT based problems [32,33].

In the literature available, the problem of dimensionality reduction has not been addressed using the notion of neutrosophic soft matrices yet. In the proposed research work, in order to handle the parameterization tool in a more elaborative way, we have proposed a new methodology to handle the dimensionality reduction of the data in a decision-making problem using the notion of neutrosophic sets in a well structure way and compared it with the existing methodologies & example.

The paper has been organized as follows. The basic notions related to the definitions and operations of neutrosophic soft sets and soft matrices have been presented in Section 2. The definitions of the object-oriented neutrosophic soft matrix, the parameter-oriented neutrosophic soft matrix and its threshold value have been proposed along with an algorithm for the dimensionality reduction in Section 3. In Section 4, an application by taking a decision-making problem into account has been dealt with the help of a numerical example using the proposed methodology. Some comparative remarks depicting the advantages and limitations have also been listed. Finally, the paper is concluded in Section 5 by stating the scope for the future work.

2. Basic Notions & Preliminaries

Some of the basic definitions and fundamental notions related to the neutrosophic soft set and matrix are briefly presented in this section which is easily available in literature. The geometrical extensions and generalizations of fuzzy set are being presented by Figure 1 below:

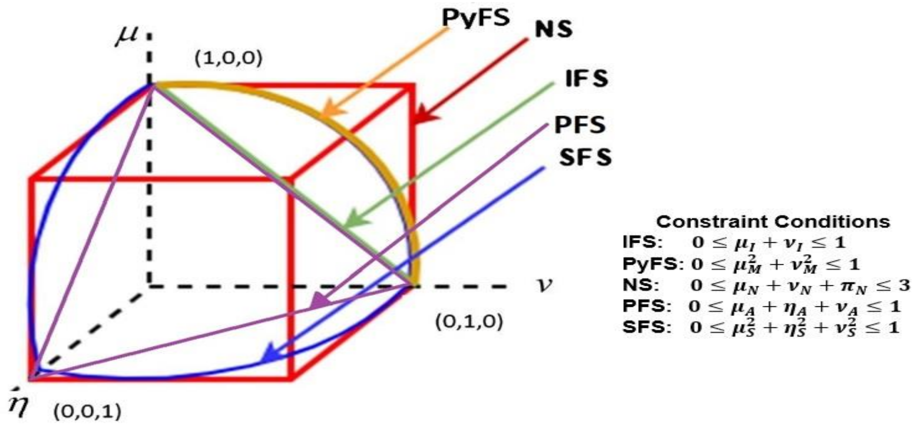

Figure 1: Geometrical Representation of Extensions and Generalizations of Fuzzy Set

In the above Figure 1, the different constraint conditions for the various generalized types of fuzzy sets for intuitionistic fuzzy set (IFS), Pythagorean fuzzy set (PyFS), neutrosophic set (NS), picture fuzzy set (PFS) and spherical fuzzy set (SFS) in terms of membership degree (μ), non-membership degree (ν), indeterminacy or hesitation (η) have been presented. The constraints have been figured out geometrically as per the conditions.

There are different basic notions of matrices, e.g., fuzzy matrices, intuitionistic fuzzy matrices and neutrosophic matrices whose formal definitions are as follows:

Definition 1

Let $U=\{u_1,u_2,\ldots,u_m\}$ be the set of alternatives and $V=\{v_1,v_2,\ldots,v_n\}$ be the set of attributes of every element of U.

- A ***fuzzy matrix*** [34] is defined by $M=\{(u_i,v_j),\mu_M(u_i,v_j)\}$ for all $i=1,2,\ldots,m$ & $j=1,2,\ldots,n$ where, $\mu_M:U\times V\to[0,1]$.
- A ***intuitionistic fuzzy matrix*** [35] is defined by $M=\{(u_i,v_j),\mu_M(u_i,v_j),\nu_M(u_i,v_j)\}$ for all $i=1,2,\ldots,m$ & $j=1,2,\ldots,n$ where, $\mu_M:U\times V\to[0,1]$ and $\nu_M:U\times V\to[0,1]$ satisfying the condition $0\le\mu_M(u_i,v_j)+\nu_M(u_i,v_j)\le 1$.
- A ***neutrosophic fuzzy matrix*** [36] is defined by $M=\{[(a_{ij})]_{m\times n}\,|\,a_{ij}\in K(I)\}$ for all $i=1,2,\ldots,m$ & $j=1,2,\ldots,n$ where, $K(I)$ is the neutrosophic field.

For detailed description, the cited references may be referred.

Definition 2 [37] *A single valued neutrosophic set M in* U *(universal set) is defined by* $M=\{<u,\,T_M(u),\,I_M(u),F_M(u)>|\,u\in U\}$; *with* $T_M:U\to[0,1]$, $I_M:U\to[0,1]$ *and* $F_M:U\to[0,1]$ *being the degree of truth membership, degree of indeterminacy and degree of falsity membership respectively and satisfy the condition*

$$0\le T_M(u)+I_M(u)+F_M(u)\le 3;\ \forall\ u\in U.$$

The sequential development of the notion of soft sets and soft matrices to the concept of Neutrosophic soft sets/matrices can be easily found with necessary illustrative examples in literature [9, 17, 18, 22, 23].

Suppose $U=\{u_1,u_2,u_3,\ldots,u_n\}$ is the universe of discourse and let the collection of parameters $P=\{p_1,p_2,p_3,\ldots,p_n\}$ be under consideration.

- The pair (F,P) is defined to be a ***soft set*** over $U \Leftrightarrow F:P\to\wp(U)$, where $\wp(U)$ is the power set of U.
- Let $FS(U)$ represents the collection of all fuzzy sets of U. A pair $(F,\,P)$ is defined as a fuzzy ***soft set*** over $FS(U)$, where F is a function $F:P\to\wp(FS(U))$.
- The pair $(F,\,P)$ is termed as the ***neutrosophic soft set*** over U if $F:P\to NS(U)$ and can be defined by $(F,\,P)=\{(p,\,F(p)):p\in P,\,F(p)\in NS(U)\}$, where $NS(U)$ is the collection of all neutrosophic sets of U.

- Suppose (F, P) be a soft set on U . Then the set $U \times P$ is represented by $R = \{(u, p), p \in P, u \in F(p)\}$ .The characterizing function of R is $\chi_R : U \times P \to [0, 1]$ defined as

$$\chi_R(u, p) = \begin{cases} 1 & if \ (u, p) \in R; \\ 0 & if \ (u, p) \notin R. \end{cases}$$

If $a_{ij} = \chi_R(u_i, p_j)$, then a matrix $[a_{ij}] = [\chi_R(u_i, p_j)]$ is defined as ***soft matrix*** of the soft set (F, P) on U of size $m \times n$.

- If (F, P) be a neutrosophic soft set on U, then the set $U \times P$ is represented by

$$R = \{(u, p), p \in P, u \in F(p)\}.$$

The set R may defined by its characterizing functions- truth function, indeterminacy and falsity function given by $T_R : U \times P \to [0, 1]$, $I_R : U \times P \to [0, 1]$ and $F_R : U \times P \to [0, 1]$ respectively.

If $(T_{ij}, I_{ij}, F_{ij}) = (T_R(u_i, p_j), I_R(u_i, p_j), F_R(u_i, p_j))$, where $T_R(u_i, p_j)$ represents the belongingness of u_i , $I_R(u_i, p_j)$ represents the indeterminacy of u_i and $F_R(u_i, p_j)$ represents the non-belongingness of u_i in the neutrosophic set $F(p_j)$ respectively, then the **neutrosophic soft matrix** of order $m \times n$ over U, is given by

$$[m_{ij}]_{m\times n} = \left[(T_{ij}^M, I_{ij}^M, F_{ij}^M)\right]_{m\times n} = \begin{bmatrix} (T_{11}, I_{11}, F_{11}) & (T_{12}, I_{12}, F_{12}) & \cdots & (T_{1n}, I_{1n}, F_{1n}) \\ (T_{21}, I_{21}, F_{21}) & (T_{22}, I_{22}, F_{22}) & \cdots & (T_{2n}, I_{2n}, F_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (T_{m1}, I_{m1}, F_{m1}) & (T_{m2}, I_{m2}, F_{m2}) & \cdots & (T_{mn}, I_{mn}, F_{mn}) \end{bmatrix}.$$

In order to have a better understanding for constructing a neutrosophic soft matrix, let us consider $U = \{u_1, u_2, u_3\}$ as a universal set and $P = \{p_1, p_2, p_3\}$ as a set of parameters and

$$F(p_1) = \{(u_1, 0.4, 0.5, 0.4), (u_2, 0.5, 0.5, 0.3), (u_3, 0.9, 0.6, 0.2)\},$$
$$F(p_2) = \{(u_1, 0.2, 0.6, 0.5), (u_2, 0.5, 0.6, 0.3), (u_3, 0.5, 0.4, 0.2)\},$$
$$F(p_3) = \{(u_1, 0.9, 0.6, 0.2), (u_2, 0.5, 0.4, 0.2), (u_3, 0.5, 0.4, 0.3)\},$$

then (F, P) represents the family of $F(p_1), F(p_2), F(p_3)$ on U after parameterization.
Hence, the neutrosophic soft matrix $[M(F, P)]$ may be given by

$$[m_{ij}]_{m\times n} = \left[(T_{ij}^M, I_{ij}^M, F_{ij}^M)\right]_{3\times 3} = \begin{bmatrix} (0.4, 0.5, 0.4) & (0.2, 0.6, 0.5) & (0.5, 0.3, 0.2) \\ (0.5, 0.5, 0.3) & (0.5, 0.6, 0.3) & (0.5, 0.6, 0.6) \\ (0.9, 0.6, 0.2) & (0.5, 0.4, 0.2) & (0.5, 0.4, 0.3) \end{bmatrix}.$$

Throughout this paper, we take $NSM_{m\times n}$ to represent the collection of all the neutrosophic soft matrices of order $m \times n$.

Operations over Neutrosophic Soft Matrices:

Different types of binary operations for two Neutrosophic soft matrices $M=\left[\left(T_{ij}^{M},\ I_{ij}^{M},F_{ij}^{M}\right)\right]$ and $N=\left[\left(T_{ij}^{N},\ I_{ij}^{N},F_{ij}^{N}\right)\right]\in NSM_{m\times n}$ are as follows [18]:

- $M^{c}=\left[\left(F_{ij}^{M},1-I_{ij}^{M},T_{ij}^{M}\right)\right]\quad \forall i\ \&\ j.$
- $M\cup N=\left[\max\left(T_{ij}^{M},\ T_{ij}^{N}\right),\ \min\left(I_{ij}^{M},\ I_{ij}^{N}\right),\min\left(F_{ij}^{M},\ F_{ij}^{N}\right)\right]\quad \forall i\ \&\ j.$
- $M\cap N=\left[\min\left(T_{ij}^{M},\ T_{ij}^{N}\right),\ \max\left(I_{ij}^{M},\ I_{ij}^{N}\right),\ \max\left(F_{ij}^{M},\ F_{ij}^{N}\right)\right]\quad \forall\ i\ \&\ j.$

3. Algorithm for Dimensionality Reduction

In this section, we first propose two types of matrices - *object-oriented neutrosophic soft matrix* and *parameter-oriented neutrosophic soft matrix*, and then by proposing a new definition for the threshold value we provide a new algorithm for the dimensionality reduction. In general, let $U=\{u_1,u_2,\dots,u_m\}$ be the universe of discourse and $P=\{p_1,p_2,p_3,\dots,p_n\}$ be the set of parameters. Consider M to be the neutrosophic soft matrix of the neutrosophic soft set $(F,\ P)$.

Definition 3 *The object-oriented neutrosophic soft matrix with respect to the parameter is defined as:*

$$O_i=\left[\sum_j\frac{T_{ij}}{|P|},\ \sum_j\frac{I_{ij}}{|P|},\sum_j\frac{F_{ij}}{|P|}\right];\ i=1,\ 2,\dots,m\ \&\ j=1,\ 2,...,n; \qquad (3.1)$$

where $|\cdot|$ *denotes the cardinality of the set.*

Definition 4 *The parameter-oriented neutrosophic soft matrix with respect to the object is defined as:*

$$P_j=\left[\sum_i\frac{T_{ij}}{|U|},\ \sum_i\frac{I_{ij}}{|U|},\sum_i\frac{F_{ij}}{|U|}\right];\ i=1,\ 2,\dots,m\ \&\ j=1,\ 2,...,n; \qquad (3.2)$$

where $|\cdot|$ *denotes the cardinality of the set.*

Definition 5 *If* $M=[(T_{ij}^{M},\ I_{ij}^{M},\ F_{ij}^{M})]\in NSM_{m\times n}$, *then the respective score matrix of neutrosophic soft matrix M is*

$$S(M)=\left[s_{ij}\right]=\left[\left(T_{ij}-I_{ij}F_{ij}\right)\right];\ \forall\ i\ \&\ j. \qquad (3.3)$$

Definition 6 *The threshold value of neutrosophic soft matrix is defined as*

$$S(T)=T_{ij}^{M}-I_{ij}^{M}F_{ij}^{M},\ \textit{where}$$

$$T=\left(T_T,\ I_T,F_T\right)=\left[\sum_{i,j}\frac{T_{ij}}{|U\times P|},\ \sum_{i,j}\frac{I_{ij}}{|U\times P|},\sum_{i,j}\frac{F_{ij}}{|U\times P|}\right];\ i=1,2,\dots,m\ \&\ j=1,2,\dots,n. \quad (3.4)$$

where $|\cdot|$ *denotes the cardinality of the set.*

Procedural steps of the proposed algorithm:

The methodology of the proposed algorithm for dimensionality reduction is given by:

- **Step 1.** We first construct the neutrosophic soft matrix as outlined in the beginning of the section.
- **Step 2.** Find the object-oriented matrix for the object O_i and the parameter-oriented matrix for the parameters P_j. Next, compute their score matrix using equation (3.1).
- **Step 3.** Find the threshold element and threshold value of the neutrosophic soft matrix as proposed in equation (3.2).
- **Step 4.** Remove those objects for which $S(O_i) < S(T)$ and those parameters for which $S(P_j) > S(T)$.
- **Step 5.** The new neutrosophic soft matrix is the desired dimensionality reduced matrix.

Based on the neutrosophic soft matrix, the *object-oriented neutrosophic soft matrix*, the *parameter-oriented neutrosophic soft matrix* and the score matrix, the proposed algorithm for dimensionality reduction may be represented with the help of the following flow chart (Figure 2):

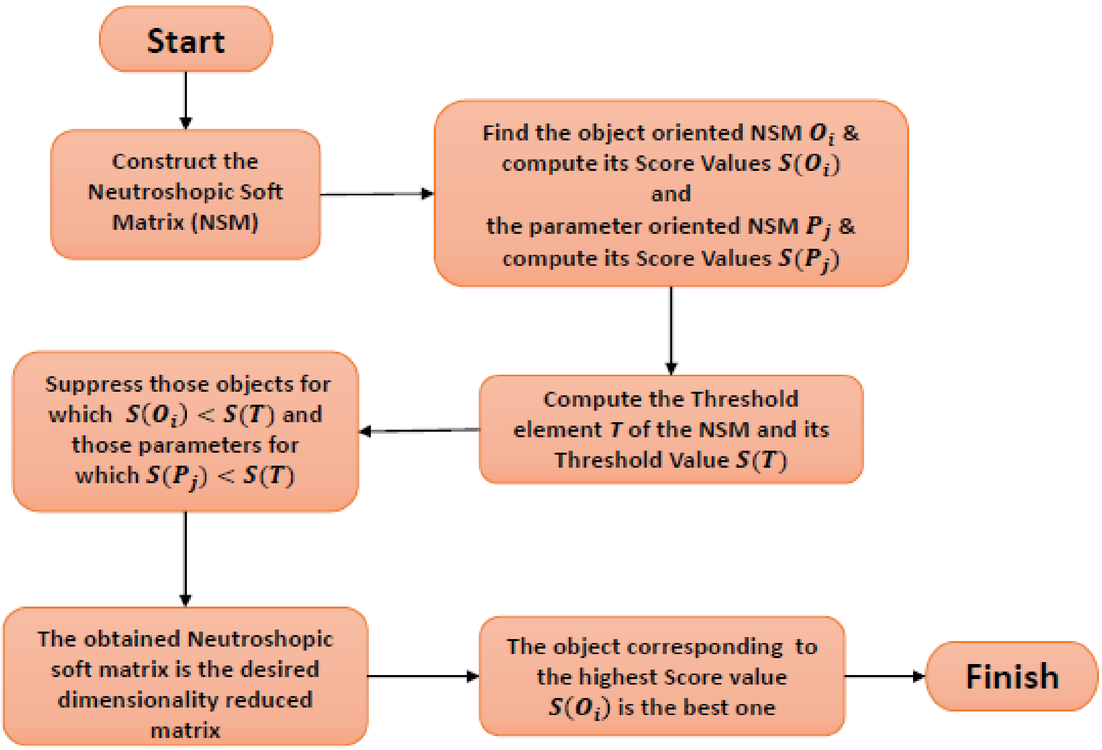

Figure 2: Algorithm for Dimensionality Reduction Using Neutrosophic Soft Matrix

4. Application of Dimensionality Reduction in Decision-Making

We consider an illustrative numerical example in this section for showing the step by step implementation of the proposed algorithm.

Example: Consider there are 5 suppliers (say) $U = \{u_1, u_2, u_3, u_4, u_5\}$ whose proficiencies are being evaluated on the criteria given by $P = \{p_1, p_2, p_3\}$, where" p_1: level of technology innovation",

" p_2 : ability of management", " p_3 : level of services". The available data in the form of a neutrosophic soft set is shown below:

$$(F,P)=\{\{F(p_1)=(u_1,0.5,0.6,0.4),(u_2,0.9,0.4,0.1),(u_3,0.6,0.4,0.2),(u_4,0.6,0.4,0.2),(u_5,0.4,0.5,0.3)\}$$
$$\{F(p_2)=(u_1,0.6,0.7,0.2),(u_2,0.5,0.7,0.1),(u_3,0.5,0.4,0.4),(u_4,0.8,0.6,0.2),(u_5,0.6,0.4,0.2)\}$$
$$\{F(p_3)=(u_1,0.5,0.6,0.2),(u_2,0.4,0.8,0.3),(u_3,0.7,0.6,0.2),(u_4,0.6,0.4,0.4),(u_5,0.5,0.5,0.1)\}\}$$

Step 1. First we construct the respective neutrosophic soft matrix.

$$\begin{bmatrix} & p_1 & p_2 & p_3 \\ u_1 & (0.5,0.6,0.4) & (0.6,0.7,0.2) & (0.5,0.6,0.2) \\ u_2 & (0.9,0.4,0.1) & (0.5,0.7,0.1) & (0.4,0.8,0.3) \\ u_3 & (0.6,0.4,0.2) & (0.5,0.4,0.4) & (0.7,0.6,0.2) \\ u_4 & (0.6,0.4,0.2) & (0.8,0.6,0.2) & (0.6,0.6,0.4) \\ u_5 & (0.4,0.5,0.3) & (0.6,0.4,0.2) & (0.5,0.5,0.1) \end{bmatrix}$$

Step 2. Find the *object-oriented neutrosophic soft matrix* O_i for i = 1, 2, 3, 4, 5 and the *parameter- oriented neutrosophic soft matrix* P_j for j = 1, 2, 3.

$$\begin{bmatrix} & p_1 & p_2 & p_3 & O_i \\ u_1 & (0.5,0.6,0.4) & (0.6,0.7,0.2) & (0.5,0.6,0.2) & (0.533,0.633,0.2667) \\ u_2 & (0.9,0.4,0.1) & (0.5,0.7,0.1) & (0.4,0.8,0.3) & (0.6,0.633,0.1667) \\ u_3 & (0.6,0.4,0.2) & (0.5,0.4,0.4) & (0.7,0.6,0.2) & (0.6,0.4667,0.2667) \\ u_4 & (0.6,0.4,0.2) & (0.8,0.6,0.2) & (0.6,0.6,0.4) & (0.6667,0.4667,0.2667) \\ u_5 & (0.4,0.5,0.3) & (0.6,0.4,0.2) & (0.5,0.5,0.1) & (0.5,0.4667,0.2) \\ P_j & (0.6,0.46,0.24) & (0.6,0.56,0.22) & (0.54,0.58,0.24) & \end{bmatrix}$$

Now, the score matrix of *object-oriented neutrosophic soft matrix* $S(O_i)$ and *parameter- oriented neutrosophic soft matrix* $S(P_j)$ is given as:

$$\begin{bmatrix} & p_1 & p_2 & p_3 & O_i & S(O_i) \\ u_1 & (0.5,0.6,0.4) & (0.6,0.7,0.2) & (0.5,0.6,0.2) & (0.533,0.633,0.2667) & 0.364179 \\ u_2 & (0.9,0.4,0.1) & (0.5,0.7,0.1) & (0.4,0.8,0.3) & (0.6,0.633,0.1667) & 0.494479 \\ u_3 & (0.6,0.4,0.2) & (0.5,0.4,0.4) & (0.7,0.6,0.2) & (0.6,0.4667,0.2667) & 0.475531 \\ u_4 & (0.6,0.4,0.2) & (0.8,0.6,0.2) & (0.6,0.6,0.4) & (0.6667,0.4667,0.2667) & 0.542231 \\ u_5 & (0.4,0.5,0.3) & (0.6,0.4,0.2) & (0.5,0.5,0.1) & (0.5,0.4667,0.2) & 0.40666 \\ P_j & (0.6,0.46,0.24) & (0.6,0.56,0.22) & (0.54,0.58,0.24) & & \\ S(P_j) & 0.4896 & 0.4768 & 0.4008 & & \end{bmatrix}$$

Step 3. Compute the threshold element and threshold value of the neutrosophic soft matrix and its score value:

$$T=\left[(0.58,0.546,0.233)\right] \text{ and } S(T)=0.452782\,.$$

Step 4. Now, we suppress those alternatives for which $S(O_i) < S(T)$ and those parameters for which $S(P_j) > S(T)$. Thus, our new desired matrix M' is given as:

$$M^{'} = \begin{bmatrix} & p_3 \\ u_2 & 0.4945 \\ u_3 & 0.4755 \\ u_4 & 0.5422 \end{bmatrix}$$

Since the score value for supplier u_4 is highest than the other score values, therefore, the supplier u_4 is the best one to choose.

On the other hand, the same problem is studied by Sumathi and Arockiarani [38] and the solution based on their proposed methodology is as follows:

$$A_{AM} = \begin{bmatrix} 0.3664 \\ 0.4944 \\ 0.4755 \\ 0.5422 \\ 0.4067 \end{bmatrix}$$

Therefore, the supplier u_4 is best.

Comparative Remarks:

Based on the above calculations and analysis, the following are the important comparative remarks:

- Sumathi and Arockiarani [38] solved the problem of decision-making without using the concept of dimension reduction and found that the supplier u_4 is highly preferable for any other supplier.
- The proposed methodology has first dimensionally reduced the available data and then worked out that the supplier u_4 is the most suitable one.
- Hence, the proposed method is consistent and better enough for solving decision-making problems.

Advantages of the Proposed Work:

In view of the above detailed analysis, the proposed algorithm for dimensionality reduction by utilizing the concept of neutrosophic soft matrices is found to be worthy enough in contrast with the existing related literatures. The following are the major advantages of the proposed work:

- The proposed methodology has significantly reduced the amount of the data and in addition the decision is found to be equally consistent, reliable and dependable.
- The methodology involves the notions of matrices and hence will prove to be widely applicable in many real-world applications.

- In case of large data set, the proposed methodology may suitably be implemented using the matrices for which we have the built-in-tools.

5. Conclusions and Scope for Future Work

In this paper, the technique for finding threshold value of the neutrosophic soft matrix is successfully provided with the definition of object-oriented and parameter-oriented neutrosophic soft matrix. An algorithm for dimensionality reduction has been properly outlined step by step. A numerical example clearly demonstrates the proposed methodology. In order to exhibit the viability and flexibility of the proposed algorithm, an example related to the decision-making problem has also been presented in detail. The example clearly validates our contribution and demonstrates that the proposed algorithm efficiently applies for the dimension reduction process.

The proposed dimensionality reduction technique may further be applied in the following area:

- **Enhancing the performance of large-scale image retrieval:** In large multimedia databases, it may not be feasible to search through the whole database in order to retrieve the nearest neighbors for a query. For similarity search and indexing, we do need a good data structure. It is quite possible that the existing data structures do not translate well for the high dimensional multimedia descriptors. By utilizing the proposed algorithm for the dimensionality reduction, we can map the nearest neighbors in the high dimensional space to nearest neighbors in the lower dimensional space. Similarly, in the field of content-based image retrieval (CBIR), the utilization of the dimensionality reduction algorithm may be in the images on the basis of textual features and images on the basis of visual features than to apply the traditional methods where all indexes (features) to be used to compare images which will lead to a large size image collection.
- **Face Recognition Algorithm:** In the field of face recognition, a typical face recognition algorithm is 100 x 100 pixels in size i.e., 10000-dimensional vector, not all dimensions are needed. By applying the proposed algorithm for the dimensionality reduction, we can reduce the dimensional vectors. In the intrusion detection/ data mining applications, dimensionality reduction focuses on representing the data with minimum number of dimensions such that its properties are not lost and hence reducing the underlying complexity in the processing of the data. By using the proposed algorithm, we can map a given set of high dimensional data points into a surrogate low dimensional space.

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Vague –Valued Possibility Neutrosophic Vague Soft Expert Set Theory and Its Applications

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Abstract: In this paper, we first propose the concept of Vague-valued possibility Neutrosophic vague soft expert sets (VPNVSEsets in short). It is a combination of vague-valued possibility neutrosophic vague sets and soft expert sets. We also define its basic operations and study some related properties. Lastly an algorithm is proposed applied to the concept of vague-valued possibility Neutrosophic vague soft expert sets in hypothetical decision making problem. Here we associate the degree of belongingness degree of indeterminacy and non-belongingness of the elements of universe set with the vague –valued possibility set.

Keywords: Soft set, Neutrosophic soft expert set, Neutrosophic Vague soft set.

1. Introduction

Most real life problems involve data with a high level of uncertainty and imprecision. Traditionally, classical mathematical theories such as fuzzy mathematics, probability theories and interval mathematics are used to deal with uncertain and fuzziness. But all these theories have their difficulties and weakness as pointed out by Molodstov [14]. This led to the introduction of the theory of soft sets by Molodstov [14] in 1999. However, in order to handle the indeterminate and inconsistent information, neutrosophic set is defined [18]. The theory of vague set was first proposed by Gau and Buehrer [12]. It is an extension of fuzzy set theory. In 2010,W. Xu J. Ma, S. Wang and G. Hao, introduced Vague soft sets and their properties as a generalization of [12]. G. Selvachandran and A.R. salleh [19], introduced Possibility vague soft expert theory and its application in decision making.

In, [18] Smarandache talked about neutrosophic set theory. It is an important new mathematical tools for handling problems involving imprecise, indeterminacy and inconsistent data. Neutrosophic vague set was defined by S. Allehezaleh [2] in 2015. The concept of neutrosophic vague soft expert set was first introduced by Ashraf Al-Qurn and N. Hassan in 2016 [16]. It is the combination of neutrosophic vague sets and soft expert sets. In2016,[19] G.Selvachandran and Abdul Razak Salleh introduced the concept of Possibility Intuitionistic Fuzzy Soft Expert and Its Application in Decision Making . In [15], Mukherjee and Sarkar introduced the concept of possibility interval valued intuitionistic fuzzy soft expert theory which is a generalization of [20]. N. Hassan and A.Al-Quran [13], introduced Possibility Neutrosophic Vague soft expert set for decision under uncertainty. For further applications we refer the papers{[4],[5],[6],[7],[8],[9],[10],[11]}.

We first introduce the concept of vague-valued possibility neutrosophic vague soft expert set. It is a combination of vague-valued possibility neutrosophic vague set and soft expert set. The concept is to improve the reasonability of decision making in reality. Next we define its basic operation as a generalization of [13]. Finally we present an application of this concept in solving a decision making problem.

2. Preliminaries

We give some basic notions in neutrosophic vague set, neutrosophic vague soft set, soft expert set and neutrosophic soft expert set.

Definition 2.1. [2] A neutrosophic vague set A_{NV}(NVS in short) on the universe of discourse X written as A_{NV} = {<x; $\hat{T}A_{NV}$ (x); $\hat{I}A_{NV}$ (x); $\hat{F}A_{NV}$ (x)>; $x\in X$} whose truth-membership, indeterminacy-membership, and falsity-membership functions is defined as $\hat{T}A_{NV}(x)$ = [T^-, T^+], $\hat{I}A_{NV}(x)$ = [I^-, I^+] and $\hat{F}A_{NV}(x)$ = [F^-, F^+], where (1) $T^+ = 1 - F^-$, (2) $F^+ = 1 - T^-$ and (3) $^-0 \le T^- + I^- + F^- \le 2^+$.

Definition 2.2. [2] If Ψ_{NV} is a NVS of the universe U, where $\forall u_i \in U$, $\hat{T}\Psi_{NV}(x)$ = [1, 1], $\hat{I}\Psi_{NV}(x)$= [0, 0], $\hat{F}\Psi_{NV}(x)$= [0, 0], then Ψ_{NV} is called a unit NVS, where $1 \le i \le n$. If Φ_{NV} is a NVS of the universe U, where $\forall u_i \in U$, $\hat{T}\Psi_{NV}(x)$ = [0, 0], $\hat{I}\Psi_{NV}(x)$ = [1, 1], $\hat{F}\Psi_{NV}(x)$ = [1, 1], then Φ_{NV} is called a zero NVS, where $1 \le i \le$ n.

Definition 2.3.[2] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, (1) $\hat{T}A_{NV}(u_i) = \hat{T}B_{NV}(u_i)$, (2) $\hat{I}A_{NV}(u_i) = \hat{I}B_{NV}(u_i)$ and (3) $\hat{F}A_{NV}(u_i) = \hat{F}B_{NV}(u_i)$, then the NVS A_{NV} is equal to B_{NV} , denoted by $A_{NV} = B_{NV}$, where $1 \le i \le n$.

Definition 2.4. [2] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, (1) $\hat{T}A_{NV}(u_i) \le \hat{T}B_{NV}(u_i)$, (2)$\hat{I}A_{NV}(u_i) \ge \hat{I}B_{NV}(u_i)$ and (3)$\hat{F}A_{NV}(u_i) \ge \hat{F}B_{NV}(u_i)$, then the NVS A_{NV} is included by B_{NV} , denoted by $A_{NV} \subseteq B_{NV}$, where $1 \le i \le n$.

Definition 2.5. [2] The complement of a NVS A_{NV} is denoted by A^c and is defined by

$\widehat{T^c}A_{NV}$(x) = [$1 - T^+$, $1 - T^-$],

$\widehat{I^c}A_{NV}$(x) = [$1 - I^+$, $1 - I^-$], and

$\widehat{F^c}A_{NV}$(x) = [$1 - F^+$, $1 - F^-$].

Definition 2.6.[2] The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV} , written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$T_{C_{NV}}(x)$= [max ($T^-_{A_{NV_x}}$, $T^-_{B_{NV_x}}$), max ($T^+_{A_{NV_x}}$, $T^+_{B_{NV_x}}$)]

$I_{C_{NV}}(x)$= [min ($I^-_{A_{NV_x}}$, $I^-_{B_{NV_x}}$), min ($I^+_{A_{NV_x}}$, $I^+_{B_{NV_x}}$)] and

$F_{C_{NV}}(x)$= [min ($F^-_{A_{NV_x}}$, $F^-_{B_{NV_x}}$), min ($F^+_{A_{NV_x}}$, $F^+_{B_{NV_x}}$)]

Definition 2.7. [2] The intersection of two NVSs A_{NV} and B_{NV} is a *NVS* C_{NV} , written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$T_{H_{NV}}(x)$= [min($T^-_{A_{NV_x}}$, $T^-_{B_{NV_x}}$), min ($T^+_{A_{NV_x}}$, $T^+_{B_{NV_x}}$)]

$I_{H_{NV}}(x)$= [max ($I^-_{A_{NV_x}}$, $I^-_{B_{NV_x}}$), max ($I^+_{A_{NV_x}}$, $I^+_{B_{NV_x}}$)] and

$F_{H_{NV}}(x)=$ [max ($F^{-}_{A_{NV_x}}$, $F^{-}_{B_{NV_x}}$), max ($F^{+}_{A_{NV_x}}$, $F^{+}_{B_{NV_x}}$)]

Definition 2.8. [17] Let$U$ be an initial universal set . Let E be a set of parameters. Let $NV(U)$ denote the power set of all neutrosophic vague subsets of U and let $A \subseteq E$. A collection of pairs ($\hat{F}$, E) is called a neutrosophic vague soft set {*NVSset*} over U, where $\hat{F}$ is a mapping given by $\hat{F}$: A → $NV(U)$.

Let U be a universe. E a set of parameters. X a set of experts (agents), and O a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.9. [3] A pair (F, A) is called a soft expert set over U, where F is a mapping given by $F : A \to P(U)$, where $P(U)$ denotes the power set of U.

Let U be a universe, E a set of parameters, X a set of experts (agents), and O = {1 = agree, 0 = disagree} a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.10. [16] A pair (F, A) is called a neutrosophic soft expert set (NSES in short) over U, where F is a mapping given by $F : A \to PN(U)$, where $PN(U)$ denotes the power neutrosophic set of U.

Let U be a universe, E a set of parameters, X a set of experts (agents), and O = {1 = agree, 0 = disagree} a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.11. [16] A pair (F, A) is called a neutrosophic vague soft expert set over U, where F is a mapping given by $F : A \to NV^U$, where NV^U denotes the power neutrosophic vague set of U.

Suppose $F : A \to NV^U$ is a function defined as $F(a) = F(a)(u)$, $\forall u \in U$. For each $a_i \in A$, $F(a_i) = F(a_i)(u)$, where $F(a_i)$ represents the degree of belongingness, degree of indeterminacy and non-belongingness of the elements of U in $F(a_i)$. Hence $F(a_i)$ can be written as:

$F(a_i) = \left\{\frac{u_i}{F(a_i)(u_i)}\right\}$, for i = 1, 2, 3, …

Where $F(a_i)(u_i)$ = < [$T^{-}_{F(a_i)}(u_i)$, $T^{+}_{F(a_i)}(u_i)$], [$I^{-}_{F(a_i)}(u_i)$, $I^{+}_{F(a_i)}(u_i)$],[$F^{-}_{F(a_i)}(u_i)$, $F^{+}_{F(a_i)}(u_i)$] > and $T^{+}_{F(a_i)}(u_i)$ = 1- $F^{-}_{F(a_i)}(u_i)$, $F^{+}_{F(a_i)}(u_i)$ = 1- $T^{-}_{F(a_i)}(u_i)$ with [$T^{-}_{F(a_i)}(u_i)$, $T^{+}_{F(a_i)}(u_i)$], [$I^{-}_{F(a_i)}(u_i)$, $I^{+}_{F(a_i)}(u_i)$] and[$F^{-}_{F(a_i)}(u_i)$, $F^{+}_{F(a_i)}(u_i)$] representing the truth-membership function, indeterminacy-membership function and falsity-membership function of each of the elements $u_i \in U$, respectively.

Example 2.12 [16]. Suppose that a company produced new types of its products and wishes to take the opinion of some experts concerning these products. Let $U = \{u_1, u_2, u_3, u_4\}$ be a set of products. $E = \{e_1, e_2\}$ a set of decision parameters where $e_i (i = 1, 2)$ denotes the decision "easy to use," and "quality," respectively. Let $X = \{p, q\}$ be a set of experts. Suppose that the company has distributed a questionnaire to the two experts to make decisions on the company's products, and we get the following:

$F(e_1, p, 1)$

$$= \left\{\frac{u_1}{<[0.2,0.8]; [0.1,0.3]; [0.2,0.8]>}, \frac{u_2}{<[0.1,0.7]; [0.2,0.5]; [0.3,0.9] >}, \frac{u_3}{<[0.5,0.6]; [0.3,0.7]; [0.4,0.5]>}, \frac{u_4}{<[0.8,1]; [0.1,0.2]; [0,0.2]>},\right\}$$

$F(e_1, q, 1)$

$$= \left\{\frac{u_1}{<[0.8,0.9]; [0.3,0.4]; [0.1,0.2]>}, \frac{u_2}{<[0.2,0.4]; [0.2,0.4]; [0.6,0.8]>}, \frac{u_3}{<[0,0.5] ; [0.5,0.7]; [0.5,1]>}, \frac{u_4}{<[0.6,0.7]; [0.2,0.4]; [0.3,0.4]>}\right\}$$

$F(e_2, p, 1)$

$$= \left\{\frac{u_1}{<[0.3,0.9]; [0.1,0.3]; [0.1,0.7]>}, \frac{u_2}{<[0.2,0.5]; [0.2,0.5]; [0.5,0.8]>}, \frac{u_3}{<[0.6,0.9]; [0.1,0.7]; [0.1,0.4]>}, \frac{u_4}{<[0.2,0.4]; [0.2,0.2]; [0.6,0.8]>}\right\}$$

$F(e_2, q, 1)$

$= \left\{\frac{u_1}{<[0.4,0.6];\,[0.1,0.4];\,[0.4,0.6]>}, \frac{u_2}{<[0.1,0.3];\,[0.2,0.4];\,[0.7,0.9]>}, \frac{u_3}{<[0.1,0.5];\,[0.5,0.7];\,[0.5,0.9]>}, \frac{u_4}{<[0.2,0.7];\,[0.2,0.4];\,[0.3,0.8]>}\right\}$

$F(e_1, p, 0)$

$= \left\{\frac{u_1}{<[0.2,0.8];\,[0.7,0.9];\,[0.2,0.8]>}, \frac{u_2}{<[0.3,0.9];\,[0.5,0.8];\,[0.1,0.7]>}, \frac{u_3}{<[0.4,0.5];\,[0.3,0.7];\,[0.5,0.6]>}, \frac{u_4}{<[0,0.2];\,[0.8,0.9];\,[0.8,1]>}\right\}$

$F(e_1, q, 0)$

$= \left\{\frac{u_1}{<[0.1,0.7];\,[0.7,0.9];\,[0.3,0.9]>}, \frac{u_2}{<[0.5,0.8];\,[0.5,0.8];\,[0.2,0.5]>}, \frac{u_3}{<[0.5,1];\,[0.3,0.5];\,[0,0.5]>}, \frac{u_4}{<[0.3,0.4];\,[0.6,0.8];\,[0.6,0.7]>}\right\}$

$F(e_2, p, 0)$

$= \left\{\frac{u_1}{<[0.2,0.8];\,[0.7,0.9];\,[0.2,0.8]>}, \frac{u_2}{<[0.3,0.9];\,[0.5,0.8];\,[0.1,0.7]>}, \frac{u_3}{<[0.1,0.4];\,[0.3,0.9];\,[0.6,0.9]>}, \frac{u_4}{<[0.6,0.8];\,[0.8,0.8];\,[0.2,0.4]>}\right\}$

$F(e_2, q, 0)$

$= \left\{\frac{u_1}{<[0.4,0.6];\,[0.6,0.9];\,[0.4,0.6]>}, \frac{u_2}{<[0.7,0.9];\,[0.6,0.8];\,[0.1,0.3]>}, \frac{u_3}{<[0.5,0.9];\,[0.3,0.5];\,[0.1,0.5]>}, \frac{u_4}{<[0.3,0.8];\,[0.6,0.8];\,[0.2,0.7]>}\right\}$

The neutrosophic vague soft expert set (F, Z) is a parameterized family $\{F(e_i), i = 1, 2, 3, ...\}$ of all neutrosophic vague sets of U and describes a collection of approximation of an object.

Definition 2.13. [16]. The complement of a NVSE set (F,A) is denoted by (F, A) and is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \rightarrow NV^U$ is a mapping given by $F^c(\alpha) = \tilde{c}(F(\alpha))$, $\forall\alpha\in A$.

Where $\tilde{c}$is a neutrosophic vague complement.

.**Definition2.14. [15]** The union of two NVSE sets (F, A) and (G, B) over U, denoted by$(F, A)\tilde{\cup}$ (G, B), is a neutrosophic vague soft expert set (H, C), where $C = A\cup B$ and $\forall\varepsilon\in C$,

$$(H, C) = \begin{cases} F(\varepsilon), \text{if } \varepsilon \in A - B, \\ G(\varepsilon), \text{if } \varepsilon \in B - A, \\ F(\varepsilon)\ \tilde{\cup}\ G(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$ where $\tilde{\cup}$ denotes the union of the neutrosophic vague set

Definition 2.15. [16] The intersection of two neutrosophic vague soft expert sets (F, A) and (G, B) over a universe U, is a neutrosophic vague soft expert set (H, C), denoted by $(F, A)\tilde{\cap}$ (G, B) such that $C = A \cap B$ and $\forall e \in c$

$$(H, C) = \begin{cases} F(\varepsilon), \text{if } \varepsilon \in A - B, \\ G(\varepsilon), \text{if } \varepsilon \in B - A, \\ F(\varepsilon)\ \tilde{\cap}\ G(\varepsilon), \text{if } \varepsilon \in A \cap B \end{cases}$$ where$\tilde{\cap}$ denotes the intersection of neutrosophic vague set.

Definition 2.16 [16]. Let (F,A) and (G, B) be any two NVSE sets over a soft universe (U, Z).

Then "(F,A)AND(G, B)" denoted (F,A) $\tilde{\wedge}$ (G, B) is defined by $(F,A)\tilde{\wedge}$ $(G, B) = (H, A\times B)$, where $(H, A\times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where $\cap$ represents the basic intersection.

Definition 2.17[16]. Let (F,A) and (G, B) be any two neutrosophic vague soft expert sets over a soft universe (U, Z).

Then "(F,A) OR (G, B)" denoted $(F,A)\tilde{\vee}$ (G, B) is defined by $(F,A)\tilde{\vee}$ $(G, B) = (H,A \times B)$, where $(H, A \times B) = H(\alpha, \beta)$, such that $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$, for all $(\alpha, \beta) \in A \times B$, where $\cup$represents the basic union.

Definition 2.18 [16]. Let U be a Universe. E a set of parameters, X a set of experts. Q = {1 = agree, 0 = disagree} a set of opinions. Let Z = E×X×Q and A⊆Z.

Let U= {u_1,u_2, …,u_n} be a universal set of elements, let E= { e_1, e_2, e_3,------, e_m} be a universal set of parameters . Let X = {x_1,x_2, …,x_i} be a set of experts and let Q = {1 = agree, 0 = disagree} be a set of opinions. Let Z = E×X×Q and A⊆Z. Then the pair (U, Z) is called a soft universe. Let F:Z→NVSs(V), and p be a fuzzy subset of Z define by p: Z→I ^{U}, where I^U is the collection of all fuzzy subsets of U. Suppose Fp: Z→NVSs(U)×I^U be a function define by Fp= {(F(Z)(u_i), P(Z)(u_i))},for all ui∈U.. Then Fp is called a possibility neutrosophic vague soft expert set (denoted by PNVSES) over the soft universe (U, Z). For each z_i∈Z, Fp(z_i) = (F(z_i)(u_i), P(z_i)(u_i)) where F(z_i) represent the degree of belongingness degree of indeterminacy and non-belongingness of the elements of U in F(z_i) and P(z_i) represents the degree of possibility of belongingness of the elements of U in F(z_i).

3. Vague-valued possibility neutrosophic vague soft expert set

In this section we introduce the definition of a vague-valued Possibility neutrosophic vague soft expert se t(VPNVSE set)}.

Let *U* be a Universe. *E* a set of parameters. *X* a set of experts and *Q* = {1 = agree, 0 = disagree} a set of opinions. Let *Z* = *E*×*X*×*Q* and *A*⊆*Z*.

Definition3.1. Let *U*={u_1,u_2, …,u_n} be a universal set of elements, let E= { e_1, e_2,e_3,------,e_m} be a universal set of parameters . Let *X* = {x_1,x_2, …,x_i} be a set of experts and let *Q* = {1 = agree, 0 = disagree} be a set of opinions. Let *Z* = *E*×*X*×*Q* and *A*⊆*Z*. Then the pair (*U*, *Z*) is called a soft universe. Let F:Z→NVS_s(V), and p be a vague- valued subset of Z define by p: Z→V(U).
Suppose F_P: *Z*→NVS_s(*U*)×V(U) be a function define by F_P= {(*F*(*Z*)(u_i), *P*(*Z*)(u_i))}, ∀u_i∈*V*. Then F_P is called a vague- valued possibility neutrosophic vague soft expert set (denoted by V*PNVSE*s) over the soft universe (*U*, *Z*). For each z_i∈*Z*, F_P(z_i) = (*F*(z_i)(u_i), *P*(z_i)(u_i)) where *F*(z_i) represent the degree of belongingness degree of indeterminacy and non-belongingness of the elements of *U* in *F*(z_i).

So *F*(z_i)(u_i) = {$[T^-_{F(z_i)}(u_i), T^+_{F(z_i)}(u_i)]$, $[I^-_{F(z_i)}(u_i), I^+_{F(z_i)}(u_i)]$, $[F^-_{F(z_i)}(u_i), F^+_{F(z_i)}(u_i)]$}

and $T^+_{F(z_i)}(u_i) = 1 - F^-_{F(z_i)}(u_i)$, $F^+_{F(z_i)}(u_i) = 1 - T^-_{F(z_i)}(u_i)$ with [$T^-_{F(z_i)}(u_i)$, $T^+_{F(z_i)}(u_i)$] , [$I^-_{F(z_i)}(u_i)$, $I^+_{F(z_i)}(u_i)$] , [$F^-_{F(z_i)}(u_i)$, $F^+_{F(z_i)}(u_i)$] representing the truth membership function indeterminacy membership function and fails membership function of each of the elements u_i∈*U* respectively. P(z_i) represents the vague –value [t_A(x), 1-f_A(x)],indicates that the exact grade of membership of x to A(which may be unknown but it is bounded by t_A(x) and 1-f_A(x).. Hence F_P(z_i) can be written as F_P(z_i) = $\{\left\{\left(\frac{u_i}{F(z_i)(u_i)}\right), P(z_i)(u_i)\right\}$ for *i* = 1, 2, 3,……, . The V*PNVSEs*(F_P,*z*) can be written simply as F_p. If *A*⊆*Z*, it is also possible to have a V*PNVSEs*(F_P, *A*). For simplicity we take the set of opinion contains of only two values namely agree and disagree.

Suppose that a company produced new types of its products & wishes to take the opinion of some experts corresponding those products. Let U = {u_1, u_2, u_3} be a set of products. *E* = {e_1, e_2} a set of decision parameters. Here, e_i (i=1,2) denote the decision "easy to use" and "equality". Let *X* = {p, q} be a set of experts. Suppose that the company has distributed questionnaire to, the two experts to make decisions on the company products. Then we have to following.

F_p: $Z \to VNVS_s(U) \times V(U)$ is a function then

$F_p(e_1, p, 1) = \{(\frac{u_1}{[0.2,0.8]; [0.1,0.3]; [0.2,0.8]}, [0.3,0.5], (\frac{u_2}{[0.1,0.7]; [0.2,0.5]; [0.3,0.9]}, [0.5,0.7], (\frac{u_3}{[0.5,0.6]; [0.3,0.7]; [0.4,0.5]}, [0.7,0.9]\}$

$F_p(e_1, q, 1) = \{(\frac{u_1}{[0.8,0.9]; [0.3,0.4]; [0.1,0.2]}, [0.4,0.6], (\frac{u_2}{[0.2,0.4]; [0.2,0.4]; [0.6,0.8]}, [0.6,0.8], (\frac{u_3}{[0.0,0.5]; [0.5,0.7]; [0.5,1]}, [0.8,1]\}$

$F_p(e_2, p, 1) = \{(\frac{u_1}{[0.3,0.9]; [0.1,0.3]; [0.1,0.7]}, [0.5,0.7], (\frac{u_2}{[0.2,0.5]; [0.2,0.5]; [0.5,0.8]}, [0.6,0.8], (\frac{u_3}{[0.6,0.9]; [0.1,0.7]; [0.1,0.4]}, [0.3,0.5]\}$

$F_p(e_2, q, 1) = \{(\frac{u_1}{[0.4,0.6]; [0.1,0.4]; [0.4,0.6]}, [0.2,0.4], (\frac{u_2}{[0.1,0.3]; [0.2,0.4]; [0.7,0.9]}, [0.4,0.6], (\frac{u_3}{[0.1,0.3]; [0.5,0.7]; [0.7,0.9]}, [0.7,0.9]\}$

$F_p(e_1, p, 0) = \{(\frac{u_1}{[0.2,0.8]; [0.7,0.9]; [0.2,0.8]}, [0.1,0.3], (\frac{u_2}{[0.3,0.9]; [0.5,0.8]; [0.1,0.7]}, [0.3,0.6], (\frac{u_3}{[0.4,0.5]; [0.3,0.7]; [0.5,0.6]}, [0.5,0.7]\}$

$F_p(e_1, q, 0) = \{(\frac{u_1}{[0.1,0.2]; [0.6,0.7]; [0.8,0.9]}, [0.8,0.9], (\frac{u_2}{[0.6,0.8]; [0.6,0.8]; [0.2,0.4]}, [0.6,0.8], (\frac{u_3}{[0.5,1]; [0.3,0.5]; [0,0.5]}, [0.3,0.7]\}$

$F_p(e_2, p, 0) = \{(\frac{u_1}{[0.1,0.7]; [0.7,0.9]; [0.3,0.9]}, [0.2,0.5], (\frac{u_2}{[0.5,0.8]; [0.5,0.8]; [0.2,0.5]}, [0.3,0.6], (\frac{u_3}{[0.1,0.4]; [0.3,0.9]; [0.6,0.9]}, [0.3,0.7]\}$

$F_p(e_2, \mathrm{q}, 0) = \{(\frac{u_1}{[0.4,0.6]; [0.8,0.9]; [0.4,0.6]}, [0.20.5], (\frac{u_2}{[0.7,0.9]; [0.6,0.8]; [0.1,0.3]}, [0.4,0.6], (\frac{u_3}{[0.5,0.9]; [0.3,0.5]; [0.1,0.5]}, [0.5,0.7]\}$

Thus we have the VPNVSE set (F_P, Z) as follows:

$(\mathrm{F_P}, Z) = \{(e_1, p, 1) = \{(\frac{u_1}{[0.2,0.8]; [0.1,0.3]; [0.2,0.8]}, [0.3,0.5], (\frac{u_2}{[0.1,0.7]; [0.2,0.5]; [0.3,0.9]}, [0.5,0.7], (\frac{u_3}{[0.5,0.6]; [0.3,0.7]; [0.4,0.5]}, [0.7,0.9]\}$

$(e_2, p, 1) = \{(\frac{u_1}{[0.3,0.9]; [0.1,0.3]; [0.1,0.7]}, [0.5,0.7], (\frac{u_2}{[0.2,0.5]; [0.2,0.5]; [0.5,0.8]}, [0.60.8], (\frac{u_3}{[0.6,0.9]; [0.1,0.7]; [0.1,0.4]}, [0.3,0.5]\}$

$(e_1, q, 1) = \{(\frac{u_1}{[0.8,0.9]; [0.3,0.4]; [0.1,0.2]}, [0.4,0.6], (\frac{u_2}{[0.2,0.4]; [0.2,0.4]; [0.6,0.8]}, [0.6,0.8], (\frac{u_3}{[0.0,0.5]; [0.5,0.7]; [0.5,1]}, [0.8,1]\}$

$(e_2, q, 1) = \{(\frac{u_1}{[0.4,0.6]; [0.1,0.4]; [0.4,0.6]}, [0.2,0.4], (\frac{u_2}{[0.1,0.3]; [0.2,0.4]; [0.7,0.9]}, [0.4,0.6], (\frac{u_3}{[0.1,0.3]; [0.5,0.7]; [0.7,0.9]}, [0.7,0.9]\}$

$(e_1, p, 0) = \{(\frac{u_1}{[0.2,0.8]; [0.7,0.9]; [0.2,0.8]}, [0.1,0.3], (\frac{u_2}{[0.3,0.9]; [0.5,0.8]; [0.1,0.7]}, [0.3,0.5], (\frac{u_3}{[0.4,0.5]; [0.3,0.7]; [0.5,0.6]}, [0.5,0.7]\}$

$(e_2, p, 0) = \{(\frac{u_1}{[0.1,0.7]; [0.7,0.9]; [0.3,0.9]}, [0.2,0.5], (\frac{u_2}{[0.5,0.8]; [0.5,0.8]; [0.2,0.5]}, [0.3,0.6], (\frac{u_3}{[0.1,0.4]; [0.3,0.9]; [0.6,0.9]}, [0.3,0.7]\}$

$(e_1, q, 0) = \{(\frac{u_1}{[0.1,0.2]; [0.6,0.7]; [0.8,0.9]}, [0.8,0.9], (\frac{u_2}{[0.6,0.8]; [0.6,0.8]; [0.2,0.4]}, [0.6,0.8], (\frac{u_3}{[0.5,1]; [0.3,0.5]; [0,0.5]}, [0.3,0.6]\}$

$(e_2, q, 0) = \{(\frac{u_1}{[0.4,0.6]; [0.8,0.9]; [0.4,0.6]}, [0.2,0.3], (\frac{u_2}{[0.7,0.9]; [0.6,0.8]; [0.1,0.3]}, [0.4,0.6], (\frac{u_3}{[0.5,0.9]; [0.3,0.5]; [0.1,0.5]}, [0.5,0.7]\}$

The collection (F_p, Z) is a V*PNVSE* set over the soft inverse (U, Z).

Definition 3.3: Let (F_p, A) and (G_q, B) be two V*PNVSE* sets over the soft inverse (U, Z) then (F_p, A) is a V*PNVSE* sub set of (G_q, B) if $A \subseteq B$and for all $\varepsilon \in A$ the following conditions are satisfied.

(i) p(ε) is a vague sub set of q(ε).
(ii) F(ε)is a neutrosophic vague soft set of G(ε).

It is denoted by $(F_p, A) \subseteq (G_q, A)$. Then (G_q, A) is called aVague-valued possibility neutrosophic soft expert superset of (F_p, A).

Definition 3.4.Let (F_p, A) and (G_q, B) be two V*PNVSE* sets over the soft inverse (U, Z) then (F_p, A) equal to (G_q, B) if for all $\varepsilon \in A$ the following holds

(i) p(ε) = q(ε).
(ii) F(ε) = G(ε).

In other words $(F_p, A) = (G_q, B)$ if (F_p, A) is a subset of $(G_q, B)$and (G_q, B) is a subset of (F_p, A).

4. Basic Operations On Vague-Valued Possibility Neutrosophic Soft Expert Sets.

Now we introduce some basic operations on *PNVSE* sets. These are 'complement' Union & intersection. Then we study some of the properties related to these operations.

Definition 4.1 Let (F_P, A) be a VPNVSE set over the soft universe (U, Z) then the complement of (F_P, A) denoted by $(F_P, A)^c$ is defined as

$(F_P, A)^c = (\bar{c}(F\alpha), c(P(\alpha)))\ \forall\ \alpha \in A$.

Where $\bar{c}$ a neutrosophic vague complement and c is a Vague-valued set complement.

If A be a vague set over the universe U, then

A = [{x, $t_A(x)$, 1-f_A(x)}: x∈V} in this definition$t_A(x)$ is a lower bound on the grade of membership of x to A derived from the evidence for x and f_A(x) is a lower bound on the negation of x to A derived from the evidence against x. The vague value [$t_A(x)$, 1-f_A(x)] indicates that the exact grade of membership of x to A may be unknown, but it is bounded by $t_A(x)$ & 1-f_A(x). It is to be noted that every fuzzy set α correspondence to the following vague set: α = {(x, [$\alpha(x)$, 1- α(x)]): x∈ U} thus the notion of vague sets is a generalization of fuzzy sets. The complement of the vague set A is A^c = [{x, $f_A(x)$, 1-t_A(x)}: x∈U}.

Example 4.2: Consider the VPNVSE (F_P, A) over a soft universe (U, Z) as an example 3.2. Now by definition 4.1 $(F_P, A)^c$ is given as follows:

$(F_P, z)^c$ = {(e_1, p, 1) = {($\frac{u_1}{[0.2,0.8];[0.7,0.9];[0.2,0.8]}$,[0.5, 0.7]), ($\frac{u_2}{[0.3,0.9];[0.5,0.8];[0.1,0.7]}$, [0.3, 0.5]), ($\frac{u_3}{[0.4,0.5];[0.3,0.7];[0.5,0.6]}$, [0.1, 0.3])}, (e_2, p, 1) = {($\frac{u_1}{[0.1,0.7];[0.7,0.9];[0.3,0.9]}$, [0.3, 0.5]), ($\frac{u_2}{[0.5,0.8];[0.5,0.8];[0.2,0.5]}$, [0.2, 0.4]), ($\frac{u_3}{[0.1,0.4];[0.3,0.9];[0.6,0.9]}$, [0.5, 0.7])}, (e_1, q, 1) = {($\frac{u_1}{[0.1,0.2];[0.6,0.7];[0.8,0.9]}$, [0.4, 0.6]), ($\frac{u_2}{[0.6,0.8];[0.6,0.8];[0.2,0.4]}$, [0.2, 0.4]), ($\frac{u_3}{[0.5,1];[0.3,0.5];[0.0,0.5]}$, [0.0, 0.2]}, (e_2, q, 1) = {($\frac{u_1}{[0.4,0.6];[0.6,0.9];[0.4,0.6]}$, [0.6, 0.8]), ($\frac{u_2}{[0.7,0.9];[0.6,0.8];[0.1,0.3]}$, [0.4, 0.6]), ($\frac{u_3}{[0.7,0.9];[0.3,0.5];[0.1,0.3]}$, [0.1, 0.3])},(e_1, p, 0) = {($\frac{u_1}{[0.2,0.8];[0.1,0.3];[0.2,0.8]}$, [0.7, 0.9]), ($\frac{u_2}{[0.1,0.7];[0.2,0.5];[0.3,0.9]}$, [0.5, 0.7]), ($\frac{u_3}{[0.5,0.6];[0.3,0.7];[0.4,0.5]}$, [0.3, 0.5])}, (e_2, p, 0) = {($\frac{u_1}{[0.3,0.9];[0.1,0.3];[0.1,0.7]}$, [0.5, 0.8]), ($\frac{u_2}{[0.2,0.5];[0.2,0.5];[0.5,0.8]}$, [0.4, 0.7]), ($\frac{u_3}{[0.6,0.9];[0.1,0.7];[0.1,0.4]}$, [0.3, 0.7])}, (e_1, q, 0) = {($\frac{u_1}{[0.8,0.9];[0.3,0.4];[0.1,0.2]}$, [0.1, 0.2]), ($\frac{u_2}{[0.2,0.4];[0.2,0.4];[0.6,0.8]}$, [0.2, 0.4]), ($\frac{u_3}{[0,0.5];[0.5,0.7];[0.5,1]}$, [0.4, 0.7]}, (e_2, q, 0) = {($\frac{u_1}{[0.4,0.6];[0.1,0.2];[0.4,0.6]}$, [0.5, 0.8]), ($\frac{u_2}{[0.1,0.3];[0.2,0.4];[0.7,0.9]}$, [0.4, 0.6]), ($\frac{u_3}{[0.1,0.5];[0.5,0.7];[0.5,0.9]}$, [0.3, 0.5])}

Proposition 4.3: Let (F_P, A) be a VPNVSE set over the soft universe (U, Z) . Here,(F_P, A) = (F(e), p(e)) then $((F_P, A)^c)^c$ = (F_P, A).

Proof: Let $(F_P, A)^c$ = (G_q, B) then by definition (G_q, B) = (G(e), q(e))

G(e) = $\bar{C}$(F(e)) and q(e) = C(p(e)). Where $\bar{c}$ a neutrosophic vague complement and c is a Vague-valued set complement.

So it follows that

$(G_q, B)^c = \{\bar{C}(G(e)), C(q(e)\}$

$= \{\bar{C}(\bar{C}(F(e))), C(C(p(e)))\}$

$= (F(e), p(e)) = (F_P, A)$

$((F(e), p(e)^c)^c = (F_P, A)$

Definition 4.4: Let (F_P, A) and (G_q, B) be two V PNVSE set over a soft universe (U, Z) then the intersection of (F_P, A) and (G_q, B) denoted by $(F_P, A) \bar{\cap} (G_q, B)$ is a VPNVSE set defined as $(F_P, A) \bar{\cap} (G_q, B) = (H_r, C)$,

where $C = A \cap B$ and

$r(\alpha) = p(\alpha) \cap q(\alpha) \ \forall \alpha \in C$

$H(\alpha) = F(\alpha) \bar{\cap} G(\alpha) \ \forall \alpha \in C$

$$\text{And } H(\alpha) = \begin{cases} F(\alpha) & if\ \alpha \in A - B \\ G(\alpha) & if\ \alpha \in B - A \\ F(\alpha) \cap G(\alpha) & if\ \alpha \in A \cap B \end{cases}$$

Definition 4.5 Let (F_P, A) and (G_q, B) be two VPNVSE sets over a soft universe (U, Z). Then the union of (F_P, A) and (G_q, B) denoted by $(F_P, A) \bar{\cup} (G_q, B)$ is a PNVSE set defined as $(F_P, A) \bar{\cup} (G_q, B) = (H_r, C)$, where $C = A \cup B$ and

$r(\alpha) = p(\alpha) \cup q(\alpha) \ \forall \alpha \in C$

$H(\alpha) = F(\alpha) \bar{\cup} G(\alpha) \ \forall \alpha \in C$

$$\text{And } H(\alpha) = \begin{cases} F(\alpha) & if\ \alpha \in A - B \\ G(\alpha) & if\ \alpha \in B - A \\ F(\alpha) \bar{\cup} G(\alpha) & if\ \alpha \in A \cup B \end{cases}$$

5. Application of vague-valued possibility neutrosophic vague soft expert in a decision making problem

A company is looking to have a person to fill the vacancy for a position in their company. Out of all the candidates were short listed - The three candidates form the universe of the element U = { u_1, u_2, u_3} were short listed out of all candidates. The hiring committee consists of hiring manager, head of the department and HR director of the firm . The committee is represented by the set X = { x, y, z} (a set of experts), while the set Q = { 1 = agree, 0 = disagree} represents the set of opinions of the hiring committee members. The hiring committee consider a set of parameters E = {e_1, e_2, e_3, e_4}. The parameters e_i (i = 1, 2, 3, 4) represents the characteristic or qualities that the candidates are assessed on namely "experience", "academic qualifications", "attitude towards the professionalism "and "technical knowledge" respectively. After finishing the interview of all the candidates and going through their certificates and other supporting papers. The hire committee constitutes the VPNVSE set (F_P, z) as follows:

(F_P, z) = [{(e_1, x, 1) = {($\frac{u_1}{[0.2,0.8];\,[0.1,0.3];\,[0.2,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.1,0.7];\,[0.2,0.5];\,[0.3,0.9]}$, [0.5, 0.7]), ($\frac{u_3}{[0.5,0.6];\,[0.3,0.7];\,[0.4,0.5]}$, [0.7, 0.9])}, (e_2, x, 1) = {($\frac{u_1}{[0.3,0.9];\,[0.1,0.3];\,[0.1,0.7]}$, [0.5, 0.7]), ($\frac{u_2}{[0.2,0.5];\,[0.2,0.5];\,[0.5,0.8]}$, [0.6, 0.8]), ($\frac{u_3}{[0.6,0.9];\,[0.1,0.7];\,[0.1,0.4]}$, [0.3, 0.5])},(e_3, x, 1) = {($\frac{u_1}{[0.2,0.7];\,[0.5,0.7];\,[0.3,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.1,0.7];\,[0.4,0.5];\,[0.3,0.9]}$, [0.2, 0.5]), ($\frac{u_3}{[0.2,0.6];\,[0.3,0.5];\,[0.4,0.8]}$, [0.4, 0.6])}, ($e_4$, x, 1) = {($\frac{u_1}{[0.2,0.3];\,[0.4,0.6];\,[0.7,0.8]}$, [0.5, 0.7]), ($\frac{u_2}{[0.1,0.3];\,[0.2,0.5];\,[0.7,0.9]}$, [0.3, 0.6]), ($\frac{u_3}{[0.3,0.4];\,[0.4,0.6];\,[0.6,0.7]}$, [0.6, 0.8])},(e_1, y, 1) = {($\frac{u_1}{[0.8,0.9];\,[0.3,0.4];\,[0.1,0.2]}$, [0.4, 0.6]), ($\frac{u_2}{[0.2,0.4];\,[0.2,0.4];\,[0.6,0.8]}$, [0.6, 0.8]), ($\frac{u_3}{[0.0,0.5];\,[0.5,0.7];\,[0.5,1]}$, [0.8, 1])}, (e_2, y, 1) = {($\frac{u_1}{[0.4,0.6];\,[0.1,0.4];\,[0.4,0.6]}$, [0.2, 0.4]), ($\frac{u_2}{[0.1,0.3];\,[0.2,0.4];\,[0.7,0.9]}$, [0.4, 0.6]), ($\frac{u_3}{[0.1,0.3];\,[0.5,0.7];\,[0.7,0.9]}$, [0.7, 0.9])},(e_3, y, 1) = {($\frac{u_1}{[0.2,0.5];\,[0.4,0.6];\,[0.5,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.1,0.3];\,[0.2,0.4];\,[0.7,0.9]}$, [0.6, 0.8]), ($\frac{u_3}{[0.4,0.5];\,[0.8,0.9];\,[0.5,0.6]}$, [0.5, 0.7])}, (e_4, y, 1) = {($\frac{u_1}{[0.3,0.5];\,[0.5,0.7];\,[0.5,0.7]}$, [0.5, 0.7]), ($\frac{u_2}{[0.5,0.7];\,[0.9,1];\,[0.3,0.5]}$, [0.2, 0.5]), ($\frac{u_3}{[0.6,0.9];\,[0.2,0.3];\,[0.1,0.4]}$, [0.3, 0.6])},(e_1, z, 1) = {($\frac{u_1}{[0.1,0.4];\,[0.3,0.6];\,[0.6,0.9]}$, [0.4, 0.7]), ($\frac{u_2}{[0.5,0.7];\,[0.2,0.5];\,[0.3,0.5]}$, [0.3, 0.5]), ($\frac{u_3}{[0.2,0.5];\,[0.4,0.7];\,[0.5,0.8]}$, [0.7, 0.9])}, (e_2, z, 1) = {($\frac{u_1}{[0.1,0.5];\,[0.4,0.6];\,[0.5,0.9]}$, [0.4, 0.5]), ($\frac{u_2}{[0.6,0.7];\,[0.3,0.5];\,[0.3,0.4]}$, [0.3, 0.5]), ($\frac{u_3}{[0.0,0.1];\,[0.2,0.4];\,[0.9,1]}$, [0.7, 0.9])},(e_3, z, 1) = {($\frac{u_1}{[0.3,0.5];\,[0.5,0.7];\,[0.5,0.7]}$, [0.1, 0.3]), ($\frac{u_2}{[0.3,0.4];\,[0.5,0.6];\,[0.6,0.7]}$, [0.3, 0.7]), ($\frac{u_3}{[0.4,0.6];\,[0.3,0.5];\,[0.4,0.6]}$, [0.7, 0.9])}, (e_4, z, 1) = {($\frac{u_1}{[0.1,0.5];\,[0.3,0.7];\,[0.5,0.9]}$, [0.4, 0.7]), ($\frac{u_2}{[0.5,0.7];\,[0.1,0.2];\,[0.3,0.5]}$, [0.3, 0.7]), ($\frac{u_3}{[0.4,0.5];\,[0.7,0.8];\,[0.5,0.6]}$, [0.7, 1])}, (e_1, x, 0) = {($\frac{u_1}{[0.2,0.8];\,[0.7,0.9];\,[0.2,0.8]}$, [0.1, 0.3]), ($\frac{u_2}{[0.3,0.9];\,[0.5,0.8];\,[0.1,0.7]}$, [0.3, 0.5]), ($\frac{u_3}{[0.4,0.5];\,[0.3,0.7];\,[0.5,0.6]}$, [0.5, 0.7])}, (e_2, x, 0) = {($\frac{u_1}{[0.1,0.7];\,[0.7,0.9];\,[0.3,0.9]}$, [0.2, 0.5]), ($\frac{u_2}{[0.5,0.8];\,[0.5,0.8];\,[0.2,0.5]}$, [0.3, 0.6]), ($\frac{u_3}{[0.1,0.4];\,[0.3,0.9];\,[0.6,0.9]}$, [0.3, 0.7])},(e_3, x, 0) = {($\frac{u_1}{[0.2,0.4];\,[0.3,0.6];\,[0.6,0.8]}$, [0.3, 0.5]), ($\frac{u_2}{[0.5,0.8];\,[0.3,0.6];\,[0.2,0.5]}$, [0.2, 0.5]), ($\frac{u_3}{[0.4,0.7];\,[0.7,0.8];\,[0.3,0.6]}$, [0.6, 0.9])}, (e_4, x, 0) = {($\frac{u_1}{[0.3,0.5];\,[0.7,0.9];\,[0.5,0.7]}$, [0.3, 0.5]), ($\frac{u_2}{[0.5,0.7];\,[0.8,0.9];\,[0.3,0.5]}$, [0.4, 0.7]), ($\frac{u_3}{[0.5,1];\,[0.3,0.5];\,[0,0.5]}$, [0.3, 0.6])},(e_1, y, 0) =

$\{(\frac{u_1}{[0.3,0.5];\ [0.5,0.8];\ [0.5,0.7]}$, [0.1, 0.3]), $(\frac{u_2}{[0.1,0.3];\ [0.4,0.6];\ [0.7,0.9]}$, [0.3, 0.5]), $(\frac{u_3}{[0.4,0.6];\ [0.7,0.9];\ [0.4,0.6]}$, [0.4, 0.7])},

(e_2, y, 0) = $\{(\frac{u_1}{[0.1,0.2];\ [0.6,0.7];\ [0.8,0.9]}$, [0.8, 0.9]), $(\frac{u_2}{[0.6,0.8];\ [0.6,0.8];\ [0.2,0.4]}$, [0.6, 0.8]), $(\frac{u_3}{[0.5,1];\ [0.3,0.5];\ [0,0.5]}$, [0.3, 0.6])},(e_3, y, 0) = $\{(\frac{u_1}{[0.4,0.6];\ [0.8,0.9];\ [0.4,0.6]}$, [0.2, 0.5]), $(\frac{u_2}{[0.7,0.9];\ [0.6,0.8];\ [0.1,0.3]}$, [0.4, 0.6]), $(\frac{u_3}{[0.5,0.9];\ [0.3,0.5];\ [0.1,0.5]}$, [0.5, 0.7])}, (e_4, y, 0) = $\{(\frac{u_1}{[0.3,0.5];\ [0.6,0.8];\ [0.5,0.7]}$, [0.4, 0.6]), $(\frac{u_2}{[0.1,0.3];\ [0.4,0.6];\ [0.7,0.9]}$, [0.4, 0.7]), $(\frac{u_3}{[0.2,0.6];\ [0.5,0.7];\ [0.4,0.8]}$, [0.1, 0.4])},(e_1, z, 0) = $\{(\frac{u_1}{[0.2,0.5];\ [0.4,0.7];\ [0.5,0.8]}$, [0.4, 0.8]), $(\frac{u_2}{[0.2,0.4];\ [0.3,0.5];\ [0.6,0.8]}$, [0.4, 0.7]), $(\frac{u_3}{[0.2,0.7];\ [0.5,0.7];\ [0.3,0.8]}$, [0.1, 0.5])}, (e_2, z, 0) = $\{(\frac{u_1}{[0.4,0.5];\ [0.5,1];\ [0.5,0.6]}$, [0.5, 0.8]), $(\frac{u_2}{[0.1,0.4];\ [0.3,0.5];\ [0.6,0.9]}$, [0.5, 0.8]), $(\frac{u_3}{[0.3,0.5];\ [0.7,0.9];\ [0.5,0.7]}$, [0.2, 0.5])},(e_3, z, 0) = $\{(\frac{u_1}{[0.5,0.9];\ [0.6,0.8];\ [0.1,0.5]}$, [0.4, 0.7]), $(\frac{u_2}{[0.2,0.5];\ [0.5,1];\ [0.5,0.8]}$, [0.2, 0.4]), $(\frac{u_3}{[0.4,0.7];\ [0.6,0.8];\ [0.3,0.6]}$, [0.6, 0.8])}, (e_4, z, 0) = $\{(\frac{u_1}{[0.3,0.5];\ [0.6,0.8];\ [0.5,0.7]}$, [0.4, 0.6]), $(\frac{u_2}{[0.1,0.4];\ [0.3,0.5];\ [0.6,0.9]}$, [0.6, 0.8]), $(\frac{u_3}{[0.6,0.8];\ [0.5,0.7];\ [0.2,0.4]}$, [0.5, 0.7])}.

The collection (F_P, z) is a VPNVSE set over the soft universe (U, Z). The VPNVSE set (F_P, Z) is used together with an algorithm to solve the decision making problem. The algorithm given below is taken by the committee to determine the most suitable candidate to be hired for the position. The sets of algorithm are as follows:

Step 1: Input the VPNVSE set (F_P, Z).

Step 2: Calculate the value of $\alpha_{F(a_i)}(u_i)$ = $T^{-}_{F(a_i)}(u_i)$ - $F^{-}_{F(a_i)}(u_i)$ for interval truth-membership part $[T^{-}_{F(a_i)}(u_i),\ \ T^{+}_{F(a_i)}(u_i)]$, where $T^{+}_{F(a_i)}(u_i)$ = 1- $F^{-}_{F(a_i)}(u_i)$, for each element $u_i \in U$.

Step 3: Calculate the arithmetic overage $\beta_{F(a_i)}(u_i)$ of the end points of the interval indeterminacy membership part $[I^{-}_{F(a_i)}(u_i),\ \ I^{+}_{F(a_i)}(u_i)]$, for each element $u_i \in U$.

Step 4: Find the value of $\gamma_{F(a_i)}(u_i)$ = $F^{-}_{F(a_i)}(u_i)$ - $T^{-}_{F(a_i)}(u_i)$ for interval falsity-membership part $[F^{-}_{F(a_i)}(u_i),\ \ F^{+}_{F(a_i)}(u_i)]$, where $F^{+}_{F(a_i)}(u_i)$ = 1- $T^{-}_{F(a_i)}(u_i)$, for each element $u_i \in U$.

Step 5: Find $\alpha_{F(ai)}$ (u_i)- $\beta_{F(ai)}$ (u_i) -$\gamma_{F(a_i)}(u_i)$for each element $u_i \in U$.

Step 6: Find the higher numerical grade from the agree-PNVSE set & disagree-PNVSE set.

Step 7: Take the arithmetic average of [t_A, 1-f_A] of the set corresponding vague set associated with the Neutrosophic vague soft set.

Step 8: Find the higher numerical grade for the average vague set value for the highest agree-VPNVSE set & disagree-VPNVSE set

Step 9: Compute the score of each element $u_i \in U$ by taking the sum of the product of the maximum numerical grade (λ_i) with the corresponding average numerical value of vague set μ_i for the agree VPNVSE set and disagree-VPNVSE set by A_i& D_i respectively.

Step 10: Find the value r_i = A_i - D_i, for each element $u_i \in U$.

Step 11: Determine the values of highest scores = max $u_i \in \cup \{r_i\}$. Then the decision is to choose element u_i as optimal or best solution if there are more than one element.

Table-1 Value of $\alpha_{F(a_i)}(u_i)$, $\beta_{F(a_i)}(u_i)$, $\gamma_{F(a_i)}(u_i)$The value of $\alpha_{F(a_i)}(u_i)$ - $\beta_{F(a_i)}(u_i)$ - $\gamma_{F(a_i)}(u_i)$& the average of the vague set corresponding to the highest numerical grade

	u_1	u_2	u_3		u_1	u_2	u_3
(e_1, x, 1)	0, 0.2, 0	-0.2,0.35, 0.2	0.1,0.5-0.1	(e_1, x, 0)	0,0.8,0	0.2,0.65,-0.2	-0.1,0.5,0.1
	-0.2,(0.4)	-0.75, (0.6)	-0.3,(0.8)		-0.8,(0.2)	-0.25(0.4)	-0.7,(0.6)
(e_2, x, 1)	0.2, 0.2, -0.2	-0.3,0.35,0.3	0.5,0.4-0.5	(e_2, x, 0)	-0.2,0.8,0.2	0.3,0.65,-0.3	-0.5,0.6,0.5
	0.2, (0.6)	-0.85,(0.35)	0.6,(0.4)		-1.2,(0.35)	-0.05,(0.45)	-1.6,(0.5)
(e_3, x, 1)	-0.5,0.5,0.5	-0.6,0.35,0.6	-0.3,0.5,0.3	(e_3, x, 0)	-0.4,0.45,0.4	0.3,0.45,-0.3	0.1,0.75,-0.1
	-1.5,(0.6)	-1.55,(0.45)	-0.8, (0.5)		-1.25,(0.4)	0.15,(0.35)	0.55,(0.75)
(e_4, x, 1)	-0.5,0.5,0.5	-0.6,0.35,0.6	-0.3,0.5,0.3	(e_4, x, 0)	-0.2,0.8,0.2	0.2,0.85,-0.2	0.5,0.4,-0.5
	-1.5,(0.6)	-1.55,(0.45)	-1.1, (0.7)		-1.20,(0.4)	-0.45,(0.55)	0.6,(.45)
(e_1, y, 1)	0.7,0.35,-0.7	-0.4,0.3,0.4	-0.5,0.6,0.5	(e_1, y, 0)	-0.2,0.8,0.2	-0.6,0.5,0.6	0,0.8,0
	1.05,(0.5)	-1.1,(0.7)	-0.6,(0.9)		-1.05,(0.2)	-1.7,(0.4)	-0.8,(0.55)
(e_2, y, 1)	0,0.25,0	-0.6,0.3,0.6	-0.6,0.6,0.6	(e_2, y, 0)	-0.7,0.65,0.7	0.4,0.7,-0.4	0.5,0.4,-0.5
	-0.25,(0.3)	-1.5,(0.5)	-1.8,(0.8)		-2.05,(0.85)	0.1, (0.7)	0.6, (0.45)
(e_3, y, 1)	-0.3,0.5,0.3	-0.6,0.3,0.6	-0.1,0.85,0.1	(e_3, y, 0)	0,0.85,0	0.6,0.7,-0.6	0.5,0.4,-0.5

	-1.1, (0.4)	-1.5, (0.7)	-1.05, (0.6)		-0.85,(0.35)	0.5,(0.5)	0.6,(0.45)
(e_4, y, 1)	-0.2,0.6,0.2	0.2,0.95,-0.2	0.5,0.25,-0.5	(e_4, y, 0)	-0.2,0.7,0.2	-0.6,0.5,0.6	-0.2,0.6,0.2
	-1.0,(0.6)	-0.55,(0.35)	0.75,(0.45)		-1.1,(0.5)	-1.7,(0.55)	-1.0,(0.25)
(e_1, z, 1)	-0.5,0.45,0.5	0.2,0.35,-0.2	-0.3,0.55,0.3	(e_1, z, 0)	-0.3,0.55,0.3	-0.4,0.35,0.4	-0.1,0.6,0.1
	-1.45,(0.55)	0.5,(0.4)	-1.1,(0.8)		-1.15,(0.6)	-1.15,(0.55)	-0.8,(0.3)
(e_2, z, 1)	-0.4,0.5,0.4	0.3,0.4,-0.3	-0.9,0.3,0.9	(e_2, z, 0)	-0.1,0.75,0.1	-0.5,0.4,0.5	-0.2,0.8,0.2
	-1.3,(0.45)	0.2,(0.4)	-2.1,(0.8)		-0.95,(0.65)	-1.4,(0.65)	-1.2,(0.35)
(e_3, z, 1)	-0.2,0.6,0.2	-0.3,0.55,0.3	0,0.4,0	(e_3, z, 0)	0.4,0.7,-0.4	-0.3,0.75,0.3	0.1,0.7,-0.1
	-1.0,(0.2)	-1.15,(0.5)	-0.4,(0.8)		0.1,(0.55)	-1.65,(0.3)	-0.5,(0.7)
(e_4, z, 1)	-0.4,0.5,0.4	0.2,0.15,-0.2	-0.1,0.75,0.1	(e_4, z, 0)	-0.2,0.7,0.2	-0.5,0.4,0.5	0.4,0.6,-0.4
	-1.3,(0.55)	0.25,(0.5)	-0.95,(0.85)		-1.1,(0.5)	-1.4,(0.7)	0.2,(0.6)

Table-2

	High numerical grad for agree PNVSE set (λ_i)	High numerical average value of the vague set (μ_i) corresponding to highest numerical grad	$\lambda_i \times \mu_i$		High numerical grad for disagree PNVSE set (λ_i)	High numerical average value of the vague set (μ_i) corresponding to highest numerical grad	$\lambda_i \times \mu_i$
(e_1, x, 1)	u_1(-0.2)	0.4	-0.08	(e_1, x, 0)	u_2(-0.25)	0.4	-0.1
(e_2, x, 1)	u_3(0.6)	0.4	0.24	(e_2, x, 0)	u_2(-0.05)	0.45	-0.0225
(e_3, x, 1)	u_3(0.8)	0.5	-0.40	(e_3, x, 0)	u_3(0.55)	0.75	0.4125

(e_4, x, 1)	u_3(-1.1)	0.7	-0.77	(e_4, x, 0)	u_3(0.6)	0.45	0.27
(e_1, y, 1)	u_1(1.05)	0.5	0.525	(e_1, y, 0)	u_3(-0.8)	0.55	-0.44
(e_2, y, 1)	u_1(-0.25)	0.3	-0.075	(e_2, y, 0)	u_3(0.6)	0.45	0.27
(e_3, y, 1)	u_3(-1.05)	0.6	-0.63	(e_3, y, 0)	u_3(0.6)	0.6	0.36
(e_4, y, 1)	u_3(0.75)	0.45	0.3375	(e_4, y, 0)	u_3(-1.0)	0.25	-0.25
(e_1, z, 1)	u_2(0.05)	0.4	0.02	(e_1, z, 0)	u_3(-0.8)	0.3	-0.24
(e_2, z, 1)	u_2(0.2)	0.4	0.08	(e_2, z, 0)	u_1(-0.95)	0.65	-0.6175
(e_3, z, 1)	u_3(-0.4)	0.8	-0.32	(e_3, z, 0)	u_1(0.1)	0.55	0.055
(e_4, z, 1)	u_2(0.25)	0.5	0.125	(e_4, z, 0)	u_3(0.2)	0.6	0.12

For agree

Score u_1 = -0.08+0.525+(-0.075) = 0.370

Score u_2 = 0.02+0.08+0.125 = 0.225

Score u_3 = 0.24+(-0.40)+(-0.75)+(-0.63) +0.3375+(-0.32) = -1.5225

For disagree

Score u_1 = -0.6175+0.055 = -0.5625

Score u_2 = -0.1+(-0.0225) = -0.1225,

Score u_3 = 0.4125+0.27+0.27+0.36 +(-0.25)+(-0.24)+0.12 = 0.9425

Table 3. The score $r_i = A_i - D_i$

A_i	D_i	r_i
Score u_1 = 0.37	Score u_1 = -0.5625	0.9325
Score u_2 = 0.225	Score u_2 = -0.1225	0.3475
Score u_3 = -1.5225	Score u_3 = 0.9425	-2.465

Thus S = max $_{u_i \in U}\{r_i\}$ = r_1.So, the committee is advised to hire candidate u_1 to fill the vacant position.

6. Conclusions

We give the advances of our proposal method using VPNVSE set as compared to that PVSE set as proposed by [19]. The VPNVSE set is a generalization of PVSE set. The VPNVSE set each examine the universal U in never detail with three membership functions, especially when there are many parameters involved, where PVSE set can tell us limited information about the universal U. It can only handle the incomplete information comparing both the truth-membership value and falsity-membership values with corresponding vague set. But VPNVSE set can handle problems involving imprecise, indeterminacy and incomplete data with corresponding vague set. Thus it makes more accurate and realistic than PVSE set (PNVSE set [13]). In future many applications in decision making problems can be solved with VPNSE sets- especially in medical sciences.

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Neutrosophic complex $\alpha\psi$ connectedness in neutrosophic complex topological spaces

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Abstract: Neutrosophic topological structure can be applied in many fields, viz. physics, chemistry, data science, etc., but it is difficult to apply the object with periodicity. So, we present this concept to overcome this problem and novelty of our work is to extend the range of membership, indeterminacy and non-membership from closed interval [0, 1] to unit circle in the neutrosophic complex plane and modify the existing definition of neutrosophic complex topology proposed by [17], because we can't apply the existing definition to some set theoretic operations, such as union and intersection. Also, we introduce the new notion of neutrosophic complex $\alpha\psi$ -connectedness in neutrosophic complex topological spaces and investigate some of its properties. Numerical example also provided to prove the nonexistence

Keywords: neutrosophic sets; neutrosophic complex topology; neutrosophic complex $\alpha\psi$ -closed set; neutrosophic complex $\alpha\psi$ -connectedness between neutrosophic sets.

1. Introduction

In 1965, Zadeh [25] introduced fuzzy sets, after that there have been a number of developments in this fundamental concept. Atanassov [3] introduced the notion of intuitionistic fuzzy sets, which is generalized form of fuzzy set. Using the generalized concept of fuzzy sets, D. Coker [5] introduced the notion of intuitionistic fuzzy topological spaces. F. Smarandache [21, 22] introduced and studied neutrosophic sets. Applications of neutrosophic sets has been studied by many researchers [1, 2, 14]. Shortly, Salama et.al [19] introduced and studied Neutrosophic topology. Since then more research have been identified in the field of neutrosophic topology [4, 8, 11, 15, 18, 23], neutrosophic complex topology [10], neutrosophic ideals [17], etc. Kuratowski [9] introduced connectedness between sets in general topology. Thereafter various weak and strong form of connectedness between sets have been introduced and studied, such as b-connectedness [7], p-connectedness between sets [20], GO-connectedness between sets [19]. Parimala et.al, [16] initiated and investigated the concept of neutrosophic-closed sets. Wadei Al-Omeri [24], presented the concept of generalized closed and pre-closed sets in neutrosophic topological space and extended their discussions on pre- $T_{1/2}$ space and generalized pre-$T_{1/2}$. They also initiated the concept of generalized neutrosophic connected and of their properties.

R. Devi [17] brought the concept complex topological space and investigated some properties of complex topological spaces. Topological set with real values are not sufficient for the complex plane, this led to define this proposed concept. Every neutrosophic complex set contains a membership, indeterminacy and non-membership function in neutrosophic complex topology and each membership function in neutrosophic complex set contain amplitude and phase term. Similarly, indeterminacy and non-membership functions in neutrosophic complex set contain amplitude and phase terms. The null neutrosophic complex set has 0 as amplitude and phase value in membership and indeterminacy and 1 as amplitude and phase value in non-membership. The unit neutrosophic complex set has 1 as amplitude and phase value in membership and indeterminacy and 0 as amplitude and phase value in non-membership. The only open and closed set in neutrosophic complex topological space is 0 and 1. The remaining neutrosophic complex sets are not both open and closed. If it is both open and closed sets then it can't be a connected in neutrosophic complex topology. In this work, we define the concepts of neutrosophic complex $\alpha\psi$ -connectedness between neutrosophic complex sets in neutrosophic complex topological spaces and also study some of its properties.

2. Preliminaries

We recall the following basic definitions in particular the work of R. Devi [17] which are useful for the sequel.

Definition 2.1. Let $X \neq \phi$ and I be the unit circle in the complex plane. A neutrosophic complex set (NCS) A is defined as $A = \{< x_1, P_A(x_1), Q_A(x_1), R_A(x_1) >: x_1 \in X\}$ where the mappings $P_A(x_1), Q_A(x_1), R_A(x_1)$ denote the degree of membership, the degree of indeterminacy and the degree of non-membership for each element x_1 in X to the set A, respectively, and $0 \leq P_A(x) + Q_A(x) + R_A(x) \leq 3$ for each $x_1 \in X$. Here $P_A(x_1) = T_A(x_1)e^{j\mu_A(x_1)}$, $Q_A(x_1) = I_A(x_1)e^{j\sigma_A(x_1)}$, $R_A(x_1) = F_A(x_1)e^{j\nu_A(x_1)}$ and $T_A(x_1), I_A(x_1), F_A(x_1)$ are amplitude terms, $\mu_A(x_1), \sigma_A(x_1), \upsilon_A(x_1)$ are the phase terms.

***Definition* 2.2.** Two NCSs A and B are of the form $A = \{< x_1, P_A(x_1), Q_A(x_1), R_A(x_1) >: x_1 \in X\}$ and $B = \{< x_1, P_B(x_1), Q_B(x_1), R_B(x_1) >: x_1 \in X\}$.Then

$A \subseteq B$ if and only if $P_A(x) \leq P_B(x), Q_A(x) \geq Q_B(x)$ *and* $R_A(x) \geq R_B(x)$.

$\overline{A} = \{< x_1, R_A(x_1), Q_A(x_1), P_A(x_1) >: x_1 \in X\}$.

$A \cap B = \{< x_1,\ P_A(x_1) \wedge P_B(x_1), Q_A(x_1) \vee Q_B(x_1)\ , R_A(x_1) \vee R_B(x_1) >: x_1 \in X\}$.

$A \cup B = \{< x_1,\ P_A(x_1) \vee P_B(x_1), Q_A(x_1) \wedge Q_B(x_1)\ , R_A(x_1) \wedge R_B(x_1) >: x_1 \in X\}$

Where

$P_A(x_1) \vee P_B(x_1) = (T_A \vee T_B)(x_1)e^{j(\mu_A \vee \mu_B)(x_1)}$, $\quad P_A(x_1) \wedge P_B(x_1) = (T_A \wedge T_B)(x_1)e^{j(\mu_A \wedge \mu_B)(x_1)}$,

$Q_A(x_1) \wedge Q_B(x_1) == (I_A \wedge I_B)(x_1)e^{j(\sigma_A \wedge \sigma_B)(x_1)}$,

$Q_A(x_1) \vee Q_B(x_1) == (I_A \vee I_B)(x_1)e^{j(\sigma_A \vee \sigma_B)(x_1)}$,

$$R_A(x_1) \wedge R_B(x_1) = (F_A \wedge F_B)(x_1)e^{j(\upsilon_A \wedge \upsilon_B)(x_1)}$$

$$R_A(x_1) \vee R_B(x_1) = (F_A \vee F_B)(x_1)e^{j(\upsilon_A \vee \upsilon_B)(x_1)}$$

Definition 2.3. A subset A of a neutrosophic complex topological space (X,τ) is called

i. A neutrosophic complex semi-generalized closed (briefly, NCsg-closed) set if complex semi closure of (A) $\subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ);

ii. A neutrosophic complex ψ-closed set if complex semi closure of (A) $\subseteq U$ whenever $A \subseteq U$ and U is neutrosophic complex semi-generalized open in (X,τ);

iii. A neutrosophic complex $\alpha\psi$-closed (briefly, N C $\alpha\psi$ CS) set if complex ψ closure (A) $\subseteq U$ whenever $A \subseteq U$ and U is neutrosophic complex α-open in (X,τ).

Definition 2.4. Two neutrosophic complex sets A and B of X are said to be q-complex coincident (ACqB for short) if and only if there exist an element y in X such that $A(y) \cap B(y) \neq \phi$.

Definition 2.5. For any two neutrosophic complex sets A and B of X, $A \subseteq B$ iff A and B^C are not q-coincident (B^C is the usual complement of the set B).

Remark 2.6. Every neutrosophic complex closed (resp. neutrosophic complex open) set is neutrosophic complex $\alpha\psi$- closed (resp. neutrosophic complex $\alpha\psi$- open) but the converse may not be true.

3. On neutrosophic complex $\alpha\psi$ - connectedness between neutrosophic complex sets

In this section, modified definition of neutrosophic complex topology and definition of neutrosophic complex $\alpha\psi$ - connectedness between sets are presented, some of its properties also investigated and counter examples are also provided.

Definition 3.1. A neutrosophic complex topology (NCT) on a nonempty set X is a family _ of NCSs in X satisfying the following conditions:

(T1) $0, 1 \in \tau$ where $0 = \langle x, 0e^{j0}, 1e^{j1}, 1e^{j1} \rangle, 1 = \langle x, 1e^{j1}, 0e^{j0}, 0e^{j0} \rangle$

(T2) $A \cap B \in \tau$ for any $A, B \in \tau$;

(T3) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \subseteq \tau$

Definition 3.2. A neutrosophic complex topological space (X,τ) is said to be neutrosophic complex $\alpha\psi$-connected between neutrosophic complex sets A and B if there is no neutrosophic complex $\alpha\psi$-closed neutrosophic complex $\alpha\psi$-open set F in X such that $A \subset F$ and ¬(FCqB).

Theorem 3.3. If a neutrosophic complex topological space (X,τ) is neutrosophic complex $\alpha\psi$-connected between neutrosophic complex sets A and B, then it is neutrosophic complex connected between A and B.

Proof: If (X,τ) is not neutrosophic complex connected between A and B, then there exists an neutrosophic complex closed open set F in X such that $A \subset F$ and ¬(F qB). Then every neutrosophic complex closed open set F in X is a neutrosophic complex $\alpha\psi$ closed neutrosophic complex $\alpha\psi$ open set F in X. If F is an neutrosophic $\alpha\psi$-closed $\alpha\psi$-open set in X such that $A \subset F$ and ¬(FqB) then (X,τ) is not neutrosophic $\alpha\psi$-connected between A and B, which contradicts our hypothesis. Hence (X,τ) is a neutrosophic complex connected between A and B.

Remark 3.4. Following example clears that the converse of the above theorem may be false.

Example3.5.

Let X = {a,b} and $U = \{<a, 0.5e^{0.5j}, 0.4e^{0.4j}, 0.4e^{0.4j}>, <b, 0.6e^{0.6j}, 0.4e^{0.4j}, 0.4e^{0.4j}>\}$, $A = \{<a, 0.2e^{0.2j}, 0.7e^{0.7j}, 0.7e^{0.7j}>, <b, 0.3e^{0.5j}, 0.6e^{0.6j}, 0.6e^{0.6j}>\}$ and $B = \{<a, 0.5e^{0.5j}, 0.4e^{0.4j}, 0.4e^{0.4j}>, <b, 0.4e^{0.4j}, 0.5e^{0.5j}, 0.5e^{0.5j}>\}$ be neutrosophic complex sets on X. Let $\tau = \{0_{\sim}, 1_{\sim}, U\}$ be a neutrosophic complex topology on X. Then (X,τ)

is neutrosophic complex connected between A and B but it is not neutrosophic complex $\alpha\psi$ -connected between A and B.

Theorem 3.6. A NCT (X,τ) is neutrosophic complex $\alpha\psi$ - connected if and only if it is neutrosophic complex $\alpha\psi$ - connected between every pair of its non-empty neutrosophic complex sets.

Proof: **Necessity**: Let A, B be any pair of neutrosophic complex subsets of X. Suppose (X,τ) is not neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B. Then there exists a neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set F of X such that A is a subset of F and ¬(FCqB). Since neutrosophic complex sets A and B are neutrosophic non-empty, it follows that F is a neutrosophic non - empty proper neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set of X. Hence (X,τ) is not neutrosophic complex $\alpha\psi$ - connected.

Sufficiency: Suppose (X,τ) is not neutrosophic complex $\alpha\psi$ - connected. Then there exist a neutrosophic non empty proper neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set F of X. Consequently (X,τ) is not neutrosophic complex $\alpha\psi$ - connected between F and F^C, a contradiction.

Remark 3.7. If a neutrosophic topological space (X,τ) is neutrosophic complex $\alpha\psi$ - connected between a pair of its neutrosophic complex subsets, it is not necessarily that (X,τ) is neutrosophic complex $\alpha\psi$ - connected between every pair of its neutrosophic complex subsets, as the following example shows.

Example 3.8.

Let X = {a,b} and $U=\{<a,0.5e^{0.5j},0.4e^{0.4j},0.4e^{0.4j}>,<b,0.6e^{0.6j},0.4e^{0.4j},0.4e^{0.4j}>\}$,

$A=\{<a,0.4e^{0.4j},0.3e^{0.3j},0.3e^{0.3j}>,<b,0.6e^{0.6j},0.4e^{0.4j},0.4e^{0.4j}>\}$

$B=\{<a,0.5e^{0.5j},0.2e^{0.2j},0.2e^{0.2j}>,<b,0.4e^{0.4j},0.4e^{0.4j},0.4e^{0.4j}>\}$

$C=\{<a,0.2e^{0.2j},0.7e^{0.7j},0.7e^{0.7j}>,<b,0.3e^{0.3j},0.6e^{0.6j},0.6e^{0.6j}>\}$ and

$D=\{<a,0.5e^{0.5j},0.4e^{0.4j},0.4e^{0.4j}>,<b,0.4e^{0.4j},0.5e^{0.5j},0.5e^{0.5j}>\}$ be neutrosophic sets on X. Let $\tau=\{0_\sim,1_\sim,U\}$ be a neutrosophic complex topology on X. Then (X,τ) is a neutrosophic complex connected between neutrosophic complex sets A and B but it is not neutrosophic complex connected between neutrosophic complex sets C and D. Also (X,τ) is not neutrosophic complex $\alpha\psi$ - connected.

Theorem 3.9. An NCT (X,τ) is neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B if and only if there is no neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set F in X such that $A\subset F\subset B^C$.

Proof. **Necessity**: Let (X,τ) be an neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B. Suppose on the contrary that F is an neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set in X such that $A\subset F\subset B^C$. Now $F\subset B^C$ which implies that ¬(FCqB). Therefore F is a neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set in X such that $A\subset F$ and ¬(FCqB). Hence (X,τ) is not neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B, which is a contradiction.

Sufficiency: Suppose on the contrary that (X,τ) is not a neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B. Then there is a neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set F in X such that $A\subset F$ and ¬(FCqB). Now, ¬(FCqB) which implies that $F\subset B^C$. Therefore F is a neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set in X such that $A\subset F\subset B^C$, which contradicts our assumption.

Theorem 3.10. If a NCT (X,τ) is neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B, then A and B are neutrosophic non-empty in complex plane.

Proof. Let (X,τ) be a neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B. Suppose the neutrosophic complex sets A or B or both are empty set then the intersection of A and B is empty, which is contradiction to the definition of connectedness. The only open and closed sets in neutrosophic complex sets are 0 and 1. We know that every neutrosophic complex connected space is a $\alpha\psi$ - connected between A and B. Therefore (X,τ) is not a neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B. This leads to the contradiction to the hypothesis.

Theorem 3.11. Let (X,τ) be a NCT and A,B be two neutrosophic complex sets in X. If ACqB then (X,τ) is a neutrosophic complex $\alpha\psi$ - connected between A and B.

Proof. If B is any neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set of X such that A and B^C are not q-coincident and A is a subset of B. This is contradiction to the given statement A is complex q-coincident with B. Therefore (X,τ) is neutrosophic complex $\alpha\psi$ - connected between A and B.

Remark 3.12. Example 3.13 shows that the converse of the above theorem may not hold.

Example 3.13.

Let X = {a,b} and $U=\{<a,0.2e^{0.2j},0.6e^{0.6j},0.6e^{0.6j}>,<b,0.3e^{0.3j},0.5e^{0.5j},0.5e^{0.5j}>\}$, $A=\{<a,0.4e^{0.4j},0.3e^{0.3j},0.3e^{0.3j}>,<b,0.3e^{0.3j},0.6e^{0.6j},0.6e^{0.6j}>\}$ and $B=\{<a,0.2e^{0.2j},0.5e^{0.5j},0.5e^{0.5j}>,<b,0.5e^{0.5j},0.4e^{0.4j},0.4e^{0.4j}>\}$ be neutrosophic complex sets on X. Let $\tau=\{0_\sim,1_\sim,U\}$ be a neutrosophic complex topology on X. Then (X,τ) is neutrosophic complex $\alpha\psi$ -connected between neutrosophic sets A and B but ¬(AqB).

4. On subspace of neutrosophic complex topology and subset of neutrosophic complex set

Theorem 4.1. If a NCT (X,τ) is a neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B such that A and B are subset of A_1 and B_1 respectively, then (X,τ) is a neutrosophic complex $\alpha\psi$ - connected between A_1 and B_1.

Proof. Let (X,τ) be a neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B such that A and B are subset of A_1 and B_1 respectively. Suppose (X,τ) is not a neutrosophic complex $\alpha\psi$ - connected between A1 and B1. Then there exist a set A_1 such that A_1 a subset of complement of B_1 and intersection of A and B_1 is empty. Also intersection of A and B is empty since A is a subset of A_1 and A_1 is a subset of complement of B_1. This is contradiction to the assumption that (X,τ) is a neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B. Hence (X,τ) is a neutrosophic complex $\alpha\psi$ - connected between A_1 and B_1.

Theorem 4.2. A NCT (X,τ) is a neutrosophic complex $\alpha\psi$ -connected between neutrosophic complex sets A and B if and only if it is neutrosophic complex $\alpha\psi$ - connected between NC$\alpha\psi$ cl(A) and NC$\alpha\psi$ cl(B).

Proof. **Necessity**: Let (X,τ) be a neutrosophic complex $\alpha\psi$ - connectedness between A and B. On the contrary, (X,τ) is not a neutrosophic complex $\alpha\psi$ - connected between NC$\alpha\psi$ cl(A) and NC$\alpha\psi$ cl(B). We know that every neutrosophic complex set A and B are subset of NC$\alpha\psi$ cl(A) and NC$\alpha\psi$ cl(B), respectively. Therefore there does not exist neutrosophic complex $\alpha\psi$ - connected between A and B. Follows from Theorem 4.1, because A is a subset of NC$\alpha\psi$ cl(A) and B is a subset of NC$\alpha\psi$ cl(B).

Sufficiency: Suppose (X,τ) is not a neutrosophic complex $\alpha\psi$ - connected between neutrosophic complex sets A and B. Then there is a neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ - open set F of X such that $A \subset F$ and ¬(FCqB). Since F is a neutrosophic complex $\alpha\psi$ -closed and $A \subset F$, NC$\alpha\psi$ cl(A)$\subset F$. Now, ¬(FCqB) which implies that $F \subset B^C$ Therefore F = NC$\alpha\psi$ int(F) $\subset$ NC$\alpha\psi$ int(B^C) = (NC$\alpha\psi$ cl(B))^C. Hence (FCqN $\alpha\psi$ cl(B)) and X is not a neutrosophic complex $\alpha\psi$ - connected between NC $\alpha\psi$ cl(A) and NC $\alpha\psi$ cl(B).

Theorem 4.3. Let (Y,τ_Y) be a subspace of a NCT (X,τ) and A;B be neutrosophic complex subsets of Y. If (Y,τ_Y) is a neutrosophic complex $\alpha\psi$ - connectedness between A and B then so is (X,τ)

Proof. Suppose, on the contrary, that (X,τ) is not a neutrosophic complex $\alpha\psi$ -connected between neutrosophic sets A and B. Then there exist a neutrosophic complex $\alpha\psi$ -closed complex $\alpha\psi$ -open set F of X such that $A \subset F$ and ¬(FCqB). Put $F_Y = F \cap Y$. Then F_Y is neutrosophic complex $\alpha\psi$ -closed complex $\alpha\psi$ -open set in Y such that $A \subset F_Y$ and ¬(F_YCqB). Hence (Y,τ_Y) is not a neutrosophic complex $\alpha\psi$ -connected between A and B, a contradiction.

Theorem 4.4. Let (Y,τ_Y) be a neutrosophic complex subspace of a NCT (X,τ) and A, B be neutrosophic subsets of Y. If (X,τ) is a neutrosophic complex $\alpha\psi$ -connected between neutrosophic complex sets A and B, then so is (Y,τ_Y) .

Proof. If (Y,τ_Y) is not a neutrosophic complex $\alpha\psi$ -connected between neutrosophic complex sets A and B, then there exist a neutrosophic complex $\alpha\psi$ - closed complex $\alpha\psi$ -open set F of Y such that $A \subset F$ and ¬(FCqB). Since Y is a neutrosophic complex closed open in X, F is a neutrosophic complex $\alpha\psi$ -closed complex $\alpha\psi$ -open set in X. Hence X cannot be neutrosophic complex $\alpha\psi$ -connected between neutrosophic complex sets A and B, a contradiction.

5. Conclusions

Neutrosophic topology is an extension of fuzzy topology. Neutrosophic complex topology is an extension of neutrosophic topology and complex neutrosophic set. In neutrosophic complex set, membership degree stands for truth value with periodicity, indeterminacy stands for indeterminacy with periodicity and non-membership stands for falsity with periodicity. In this paper, we modified the definition proposed by [17] and we presented the new concept of neutrosophic complex $\alpha\psi$ - connectedness between NCSs in NCTs using new definition and some properties of neutrosophic complex $\alpha\psi$ - connectedness is investigated along with numerical example. Also this work encourages that in future, this concept can be extended to various connectednesses and analyse the properties with application.

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Unraveling Neutrosophic Transportation Problem Using Costs Mean and Complete Contingency Cost Table

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Abstract: As neutrosophic deal with uncertain, inconsistent and also indeterminate information, the model of NS is a significant technique to covenant with real methodical and engineering. Neutrosophic fuzzy is more generalized than intuitionistic fuzzy .The common process for unraveling the neutrosophic transportation problems involves procedures like, north-west corner method, matrix minima method and Vogel's approximation method. By determining the mean of the specified costs the optimal elucidation of the neutrosophic fuzzy transportation problem is initiated in this paper. This technique has been implemented into two phases. In first methodology, the complete contingency cost table is constructed and in the second phase and the optimum allocation is made. The significance of this technique confers a better optimal solution compared to other methods. A numerical example for the projected technique is explicated and compared along with existing techniques.

Keywords: Neutrosophic Fuzzy Transportation Problem, Complete Contingency Cost Table (CCCT), Costs Mean.

1. Introduction

The prominent fail on the charge and the pricing of raw materials and commodities is evidently owing to transportation cost. The outlay of transportation is elicited by dealer and manufacturer. Exclusive of the conservative methods like North West corner method, row minima method, least cost method, column minima method ,Vogel's approximation method and modified distribution method many researchers have endowed with new techniques to find a better initial basic feasible solution for the transportation problem.

To handle imprecise, uncertain and indeterminate problems that cannot be dealt by fuzzy and its various types, the neutrosophic set theory (NS) theory was illustrated by samarandache in 1995.NS is acquired by three autonomous mapping such as truth (T), indeterminacy (I) and falsity (F) and takes values from $]0^{-}, 1^{+}[$. The scope of neutrality is explained with the aid of NS theory. NSs can be accomplished to handle uncertainty in an enhanced way. Single valued neutrosophic acquires extra consideration and get optimized solution than other types of fuzzy sets because of accurateness, adoptability and link to a system. Vogel's approximation technique for solving the Transportation Problem was premeditated by Harvey and Shore (1970) [32].

Application of heuristics for solving Transportation Problem was proposed by Shimshak, Kaslik and Barelay (1981) [31]. Deshumukh (2012) [17] offered a pioneering technique for unraveling Transportation Problem. Sudhakar, Arunnsankar, and Karpagam (2012) [34] have given a modified approach for solving transportation problem. Transportation Problems with mixed restrictions have been resolved by Pandian and Natarajan (2010) [25]. Abdel-Basset, M., Manogaran, G., Gamal, A., & Chang, V. (2019) [2] presented an intelligent medical decision support model based on soft computing and IOT to detect and observe type-2 diabetes patients. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019) [3] discovered a hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. The proposed method is a combination of quality function deployment (QFD) with plithogenic aggregation operations.

Researchers like Md. Amirul Islam(2013)[22], Quddos et al and Sudhakar et al (2012)[26], Serder Korukoglu and Serkan Balli(2011) [30], Balakrishnan (1990)[11],Reena et al (2014,2016)[27,28], Urashikumari et al(2017) [35], Biswas.P(2016) [12] Krishna Prabha and Vimala (2016)[21,36], Palanivel and Suganya(2018) [24], Abul et al (2017) [9], Hajjari(2011)[18], Hitchcock .F.L(1947)[19], Joshua(2017)[20], Mohanaselvi et al (2012)[23], Said Broumi(2019)[15,29],Chang (1981)[16], Smarandache (2005)[33] and Wang(2010)[37] have predicted a variety of techniques for solving transportation and NS transportation problems. Real life transportation problem in neutroshopic environment is deliberated by Akansha singhet al(2017) [10]. The same numerical problem is considered. Abdel-Basset, M., Atef, A., & Smarandache, F. (2019) [6] invented a hybrid Neutrosophic multiple criteria cluster decision making approach for project selection. A novel group decision making replica based on neutrosophic sets for heart disease diagnosis was recommended by Abdel-Basset, M., Gamal, A., Manogaran, G., & Long, H. V. (2019)[7].The idea of first-and high-order NTS was suggested by Abdel-Basset, M., Chang, V., Mohamed, M., & Smarandache, F. (2019)[8].

Broumi et al. (2018)[14] proposed an innovative system and technique for the planning of telephone network using NG. Broumi et al (2019) [13] proposed SPP under interval valued neu trosophic setting. Score function is utilized in machine erudition. Abdel-Basset et al (2019) [1] have proposed a novel model for evaluation hospital medical care systems with plithogenic sets and this research stratifies the plithogenic multi criteria decision making (MCDM) technique for defining the considerable weights of assessing standards, and the VIKOR technique is applied for enhancing the serving efficiency classifications of the possible substitutes. Abdel-Basset, M., & Mohamed, M. (2019)[4] proposed a powerful framework based on neutrosophic sets to aid patients with cancer. Abdel-Basset, M., Mohamed, M., & Smarandache, F. (2019) [5] determined a Linear fractional programming based on triangular neutrosophic numbers. By means of the recommend approach, the transformed MOLFP problem is condensed to a single objective linear programming (LP) problem which can be deciphered simply, by proper linear programming method. In this paper, new unconventional technique to unravel neutrosophic Fuzzy transportation problem using Mean and CCCT is proposed and presented with numerical example. The paper is organized as follows. Section 1 confers the introduction part and section 2 deals with the preliminary. In section 3 the algorithm for unraveling is presented .A numerical example is illustrated in section 4 and the result is compared with existing methods. Finally the paper is concluded in section 5.

2. Preliminaries

Definition 2.1: Let X be a space of points with generic elements in X denoted by x .The neutrosophic set A is an object having the form, $A = \{\langle x : T_A(X), I_A(X), F_A(X)\rangle , x \in X\}$,where the functions T,I,F : X→]$^-$0,1$^+$[define respectively the truth-membership function, indeterminacy-membership function and falsity- membership function of the element $x \in X$ to the set A with the condition $^-0 \leq T_A(X) + I_A(X) + F_A(X) \leq 3^+$ .The functions are real standard or nonstandard subsets of]$^-$0,1$^+$[.

Definition 2.2 [13] Let $R_N = \langle [R_T , R_I, R_M, R_E,](T_R, I_R, F_R)\rangle$ and $S_N = \langle [S_T , S_I, S_M, S_E,](T_S, I_S, F_S)\rangle$ be two trapezoidal neutrosophic numbers (TpNNs) and $\theta \geq 0$,then

$R_N \oplus S_N = \langle [R_T + S_T , R_I + S_I, R_M + S_M, R_E + S_E](T_R + T_s - T_R T_s, I_R I_s, F_R F_S)\rangle$

$R_N \otimes S_N = \langle [R_T . S_T , R_I . S_I, R_M . S_M, R_E . S_E]\big(T_R . T_s , I_R + I_s - I_R . I_s, F_R + F_S - F_R . F_S\big)\rangle$

$\theta R_N = \langle [\boldsymbol{\theta} R_T , \boldsymbol{\theta} R_I, \boldsymbol{\theta} R_M, \boldsymbol{\theta} R_E,]\big(1 - (1 - T_R)\big)^\theta, (I_R)^\theta, (F_R)^\theta\big)\rangle$

Definition2.3 [13]: Let R= [R_T, R_I, R_M, R_E] and $R_T \leq R_I \leq R_M \leq R_E$ then the centre of gravity (COG) in R is

$$\mathrm{COG}(R) = \begin{cases} R \ if \ R_T = R_I = R_M = R_E \\ \frac{1}{3}\left[R_T + R_I + R_M + R_E - \frac{R_T R_I - R_M R_E}{R_E + R_M - R_I - R_T}\right] \\ otherwise \end{cases} \text{-------------- (1)}$$

Definition2.4 [13]: Let $S_N = \langle [S_T , S_I, S_M, S_E,](T_S, I_S, F_S)\rangle$ be a TpNN then the score, accuracy and certainty functions are as follows

$S(S_N) = COG(R) \times \frac{(2+T_S-I_S-F_S)}{3}$ -------------- (2)

$a(S_N) = COG(R) \times (T_S - I_S)$

$C(S_N) = COG(R) \times (T_S)$

Definition2.5 [12]: Let N be a trapezoidal neutrosophic number in the set of real numbers with the truth, indeterminacy and falsity membership functions are defined by

$$T_N(x) = \begin{cases} \frac{(x-a)t_N}{b-a} , & a \leq x \leq b \\ t_N , & b \leq x \leq c \\ \frac{(d-x)t_N}{d-c} , & c \leq x \leq d \\ 0 , & otherwise \end{cases}$$

$$I_N(x) = \begin{cases} \frac{b-x+(x-a)t_N}{b-a} , & a \leq x \leq b \\ i_N , & b \leq x \leq c \\ \frac{x-c+(d-x)i_N}{d-c} , & c \leq x \leq d \\ 0 , & otherwise \end{cases}$$

$$T_N(x) = \begin{cases} \frac{b-x+(x-a)f_N}{b-a} , & a \leq x \leq b \\ f_N , & b \leq x \leq c \\ \frac{x-c+(d-x)f_N}{d-c} , & c \leq x \leq d \\ 0 , & otherwise \end{cases}$$

Where t_N = [t^L, t^U] ⊂ [0,1], i_N = [i^L, i^U] ⊂ [0,1], and f_N = [f^L, f^U] ⊂ [0,1] are interval numbers.Then the number N can be denoted by ([a,b,c,d]: [t^L, t^U], [i^L, i^U] ,[f^L, f^U]) called interval valued trapezoidal neutrosophic number.

3. Customized Algorithm

The algorithm is accomplished into two phases:

1. Complete Contingency Cost Table (CCCT)
2. Optimum Allocation of Transportation Problem

3.1 Complete Contingency Cost Table – CCCT

Step 1 The slightest cost of each element in every row should be deducted and relegate it to the right-top of subsequent elements from the given Transportation Table (TT).

Step 2 The slightest cost of each element in every row should be deducted and consign them on the right-foot of the corresponding elements.

Step 3 Frame the CCCT by accumulating the right-top and right-foot elements.

3.2 Optimum Allocation of Transportation Problem

Step 1 The Row Mean Total Opportunity Cost (RMTOC) is found by calculating the row mean along every row. Column Mean Total Opportunity Cost (CMTOC) is found by calculating the column mean along every column.

Step 2 Spot the prevalent element among the RMTOCs and CMTOCs, if there is more than one prevalent element then select the prevalent element along which the least cost element is present. If there is more than one smallest element, select any one of them arbitrarily.

Step 3 Allocate x_{ij}= min (a_i, b_j) on the left top of the least entry in the (i ,j)th of the TT

Step 4

If $a_i < b_j$, leave the ith row and obtain $b_j^{|} = b_j - a_i$.

If $a_i > b_j$, leave the jth column and obtain $b_j^{|} = a_i - b_j$.

If $a_i = b_j$, leave either ith row or jth column but not both.

Step 5 Repeat the Steps 1 to 4 until all allocations are made.

Step 6 Estimate, Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}X_{ij}$, where Z is the minimum transportation cost, C_{ij} is the cost element of the TT.

4. Numerical Example

Consider the following Neutrosophic Transportation Problem,

Table 1: Neutrosophic Transportation Table

	D1	D2	D3	D4	SUPPLY
O1	(3, 5, 6,8); 0.6, 0.5, 0.4	(5, 8, 10, 14); 0.3, 0.6, 0.6	(12, 15, 19, 22); 0.6, 0.4, 0.5	(14, 17, 21, 28); 0.8, 0.2, 0.6	(22,26,28,32); 0.7, 0.3, 0.4
O2	(0, 1, 3,6); 0.7, 0.5, 0.3	(5, 7, 9, 11); 0.9, 0.7, 0.5	(15, 17, 19, 22); 0.4, 0.8, 0.4	(9, 11, 14, 16); 0.5, 0.4, 0.7	(17, 22, 27,31); 0.6, 0.4, 0.5
	(4, 8, 11, 15); 0.6, 0.3, 0.2	(1, 3, 4, 6); 0.6, 0.3, 0.5	(5, 7, 8, 10); 0.5, 0.4, 0.7	(5, 9, 14, 19); 0.3, 0.7, 0.6	(21,28,32,37);

O3					0.8, 0.2, 0.4
DEMAND	(13, 16, 18, 21); 0.5, 0.5, 0.6	(17, 21, 24, 28); 0.8, 0.2, 0.4	(24, 29, 32, 35); 0.9, 0.5, 0.3	(6, 10, 13, 15); 0.7, 0.3, 0.4	

Converting the trapezoidal neutrosophic numbers into crisp numbers by using (1) and (2), By

$\mathbf{s}(S_N)$ = COG $(R) \times \frac{(2+T_S - I_S - F_S)}{3}$, COG $(R) = \frac{1}{3}\left[R_T + R_I + R_M + R_E - \frac{R_E R_M - R_I R_T}{R_E + R_M - R_I - R_T}\right]$

(3, 5, 6, 8); 0.6, 0.5, 0.4

COG $(R) = \frac{1}{3}\left[3+5+6+8-\frac{8*6-5*3}{8+6-5-3}\right] = \frac{1}{3}\left[22-\frac{48-15}{6}\right] = \frac{1}{3}\left[22-\frac{33}{6}\right] = \frac{1}{3}[22-5.5] = \frac{16.5}{3} = 5.5$

$\mathbf{s}(S_N) = 5.5 \times \frac{(2+0.6-0.5-0.4)}{3} = 5.5 \times 0.56 = 3.116 = 3$

Similarly proceeding for all numbers we get the resulting crisp TT.

Table 2: Crisp Transportation Table

	D1	D2	D3	D4	SUPPLY
O1	3	4	8	9	26
O2	1	4	8	6	24
O3	4	2	3	5	30
	17	23	28	12	

4.1 Formation of the Complete Contingency Cost Table (CCCT)

From the given crisp transportation table, remove the least value from each of the elements of every row and consign them on the right-top of subsequent elements. In each column deduct the least value from each element and place them on the right-foot of the corresponding elements. Add the right-top and right-foot elements of Steps 1 and 2 and frame the **C**CCT.

Table 3: Complete Contingency Cost Table

	D1	D2	D3	D4	SUPPLY
O1	2	3	10	10	26
O2	0	5	12	6	24
O3	5	0	1	3	30
	17	23	28	12	

4.2 Allocation of the cost with supply and demand:

Calculate the mean of complete contingency costs of cells along each row and each column just subsequent to and beneath the supply and demand amount correspondingly inside the first brackets. By solving the given problem using the above steps, we get the following final allocation. The ( ) represents the allocations and [ ] represents the mean along each row/column.

Table 4: R /C SD Total Opportunity Cost

	D1	D2	D3	D4	SUPPLY	MEAN			
O1	2(3)	3(23)	10	10	26	[6.25]	[5]	[5]	[2.5]
O2	0(14)	5	12	6(10)	24	[5.75]	[3.7]	[3.7]	[2.5]
O3	5	0	1(28)	3(2)	30	[2.25]	[2.7]		
DEMAND	17	23	28	12					
MEAN	[2.3]	[2.6]	[7.7] MAX	[6.3]					
	[2.3]	[2.6]		[6.3] MAX					
	[1]	[4]		[8] MAX					
	[1]	[4] MAX							

The total opportunity cost is given bellow,

Table 5: CCCT Total Opportunity Cost

	D1	D2	D3	D4
O1	3(3)	4(23)	8	9
O2	1(14)	4	8	6(10)
O3	4	2	3(28)	5(2)

The optimum cost is given by (3x3)+(4x23)+(1x14)+(6x10)+(3x28)+(5x2) = 9+92+14+60+84+10 =269

Advantages and limitations of the proposed algorithm Advantages

By correlating the systematic algorithm with existing methods like North West corner, least cost and Vogel's approximation method we get the following results. This approach can be easily extended and applied to other neutrosophic networks such as Single-value, cubic, Bipolar, Interval bipolar neutrosophic numbers and so on.

Table 6: Comparison Table

NWC	MMM	VAM	PROPOSED METHOD
349	339	272	269

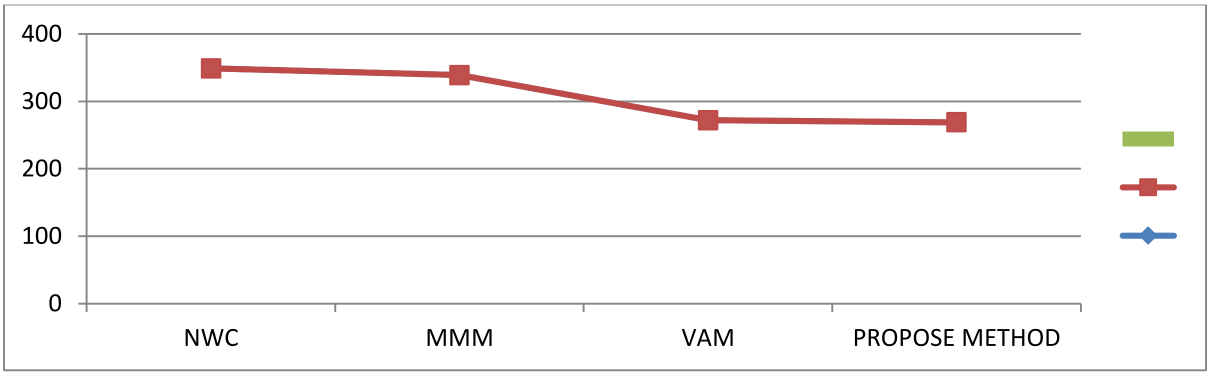

Figure 1: Comparison Chart

5. Conclusion

The advantage of using the new algorithm with CCCT is discussed in this paper. We use a numerical example to illustrate the efficiency of our proposed algorithm. The main goal of this work is to portray an algorithm for solving transportation problem, in the neutrosophic environment using CCCT. The proposed algorithm will be very effective for real-life problem. The algorithm can be extended for all kinds of neutrosophic fuzzy numbers. The new method of manipulating mean is easier and saves time. This method gives a better optimum solution when compared with other methods.

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Neutrosophic Shortest Path Problem (NSPP) in a Directed Multigraph

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Abstract: One of the important non-linear data structures in Computer Science is graph. Most of the real life network, be it a road transportation network, or airlines network or a communication network etc., cannot be exactly transformed into a graph model, but into a Multigraphs model. The Multigraph is a topological generalization of the graph where multiple links (or edges/arcs) may exist between two nodes unlike in graph. The existing algorithms to extract the neutrosophic shortest path in a graph cannot be applied to a Multigraphs. In this paper a method is developed to extract the neutrosophic shortest path in a directed Multigraph and then the corresponding algorithm is designed. The classical Dijkstra's algorithm is applicable to graphs only where all the link weights are crisp, but we borrow this concept to apply to Multigraphs where the weights of the links are neutrosophic numbers (NNs). This new method may be useful in many application areas of computer science, communication networks, transportation networks, etc. in particular in those type of networks which cannot be modelled into graphs but into Multigraphs.

Keywords: Multiset, NN, neutrosophic-min-weight arc-set, neutrosophic shortest path estimate, neutrosophic relaxation.

1. Introduction

Graph Theory [4, 13, 51] is used in huge volume of applications in various branches of Engineering, mainly in Information Technology, Computer Science, Communication Engineering, Transportation Engineering, Space Engineering, Oceanography, and also in Mathematical Sciences, Social Science, Medical Science, Economics, Optimization, Decision Sciences, etc. The Multigraph [45, 51] is an important generalization of the data structure graph in which multiple links (or edges/arcs) may exist to connect a pair of nodes. For instance, consider a communication system in an Adhoc Network or a MANET where there are many multipaths or multiroute facilities. For another example, it is common that two neighbor routers in a network may share more than one direct connections existing in the topology between them, for the purpose of reducing the bandwidth compared to the case where a single connection be used. In fact there are a number of real life instances of communication network system, airlines network, road transportation network, etc. which cannot be transformed into graphs model, but can be well transformed into multigraphs model for the purpose of various analysis and decision makings. In real life situation, in many of these type of directed multigraphs another issue is that the weights of the links are not always crisp rather neutrosophic numbers (NNs). Throughout in this paper, those multigraphs are under consideration which are not having any loop.

The NSPP problem is solved by Broumi in [32-36], but there is no work reported in the existing literature on solving neutrosophic shortest path problem (NSPP) in a multigraph. In this paper we solve the NSPP problem for a multigraph where the arc-weights are neutrosophic numbers (NNs). It is known that the very popular Dijkstra's algorithm is applicable to graphs only where the weights of the links are crisp numbers, but is not applicable to multigraphs even having crisp weights for its liunks. In this paper we extend this philosophy of Dijkstra's algorithm to apply to the case of directed multigraphs having the weights of the links as neutrosophic numbers (NNs). This problem is not solved so far in any literature, but the SPP in a multigraph having weights of the links as fuzzy or intuitionistic fuzzy numbers are solved (for example, see [46-49]). But it has been well justified in length in the pioneering works [27,28,29] about the cases where fuzzy theory fails, and intuitionistic fuzzy theory can offer soft solutions; in fact the works [27,28,29] expos the major drawbacks of the fuzzy set theory. And then in the work [8,49] it is further justified that neutrosophic theory generalizes the intuitionistic fuzzy theory. An intuitionistic fuzzy set can be viewed as a special case of a neutrosophic set, but the converse is not necessarily true. The era of improvement of various models are like:- Crisp Set → Fuzzy Set (and various types of higher order Fuzzy Sets) → IFS→ NS. And hence, by heredity the same is true for the corresponding notion of numbers too, i.e. Crisp Number → Fuzzy Number → IFN→ NN. Consequently, it is now obvious to the soft-computing researchers that the application of neutrosophic theory can surely provide better solutions [35] for ill-defined or imprecise problems.

2. Preliminaries

In this section some relevant literatures are recollected from the work of Smarandache [8-12], Salama [1, 2] and also few works of other authors [5, 14, 15, 19]. In his pioneer work, Smarandache introduced the concepts of neutrosophic trio components T, I, and F which represent respectively the membership value, indeterminacy value, and non-membership value, where $]^-0,1^+[$ stands for a non-standard unit interval.

2.1 Basic Preliminaries of the Neutrosophic Theory

This subsection contains some elements of basic notions on the theory of neutrosophic sets, in particular about the single valued neutrosophic sets out of the existing literatures.

Definition 2.1.1 Let X be a non-null set. A neutrosophic set A of the universe X is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x)>, x \in X\}$, where the trio functions $T, I, F : X \to]^-0,1^+[$ define the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively of the element $x \in X$ to the set A along with the following condition:

$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

The trio functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are three real standard (or nonstandard) subsets of non-standard unit interval $]^-0,1^+[$.

Application of the general model of NSs as defined above to the practical problems and issues may require complex computations, and consequently the authors [14, 15] suggested the notion of a

SVNS as a particular instance of a NS which can be used in real problems of scientific and engineering areas.

Definition 2.1.2 Let T, I, F be three real standard or nonstandard subsets of the non-standard unit interval $]^{-}0,1^{+}[$, with the following:

Sup_T = t_sup, inf_T = t_inf

Sup_I = i_sup, inf_I = i_inf

Sup_F = f_sup, inf_F = f_inf

n-sup = t_sup + i_sup + f_sup

n-inf = t_inf + i_inf + f_inf,

Then T, I, F are called neutrosophic trio components.

Definition 2.1.3 The NS 0_N in X is defined as follows:

(i) $0_N = \{<x, (0,0,1)> : x \in X\}$

(ii) $0_N = \{<x, (0,1,1)> : x \in X\}$

(iii) $0_N = \{<x, (0,1,0)> : x \in X\}$

(iv) $0_N = \{<x, (0,0,0)> : x \in X\}$

The NS 1_N in X is defined as follows:

(i) $1_N = \{<x, (1,0,0)> : x \in X\}$

(ii) $1_N = \{<x, (1,0,1)> : x \in X\}$

(iii) $1_N = \{<x, (1,1,0)> : x \in X\}$

(iv) $1_N = \{<x, (1,1,1)> : x \in X\}$

Definition 2.1.4 Let X be a non-null set. A single valued neutrosophic set A (SVNS A) is an object having the form $A = \{< x : T_A(x), I_A(x), F_A(x)> , x \in X\}$, where $T_A(x), I_A(x), F_A(x) \in [0,1]$ define the truth-membership function, indeterminacy-membership function, and falsity-membership function respectively of the element $x \in X$ Therefore a SVNS A could be expressed as $A = \{< x : T_A(x), I_A(x), F_A(x)> , x \in X\}$ where $T_A(x), I_A(x), F_A(x) \in [0,1]$.

Definition 2.1.5 Let $A_1 = (T_1, I_1, F_1)$ and $A_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic numbers. Then, the operations for SVNNs are defined as below:

(i) $A_1 \oplus A_2 = <T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2>$.

(ii) $A_1 \otimes A_2 = <T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2>$.

(iii) $kA_1 = < 1- (1-T_1)^k, I_1^k, F_1^k >$ where $k > 0$.

(iv) $A_1^k = < T_1^k, 1- (1-I_1)^k, 1- (1-F_1)^k>$ where $k > 0$.

Definition 2.1.6

The neutrosophic zero 0_N may be defined as follow:

$0_N = \{<x, (0,1,1)> : x \in X\}$ To compare two single valued neutrosophic numbers, one can use score function.

Definition 2.1.7 Let $A_1 = (T_1, I_1, F_1)$ be a single valued neutrosophic number. Then, the score function $s(A_1)$, accuracy function $a(A_1)$ and the certainty function $c(A_1)$ of the SVNN A_1 are defined as below :

(i) $s(A_1) = \dfrac{2 + T_1 - I_1 - F_1}{3}$

(ii) $a(A_1) = T_1 - F_1$

(iii) $c(A_1) = T_1$

Definition 2.1.8 Suppose that $A_1 = (T_1, I_1, F_1)$ and $A_2 = (T_2, I_2, F_2)$ be two single valued neutrosophic numbers. Then we define a ranking method as follows:

(i) if $s(A_1) > s(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(ii) if $s(A_1) = s(A_2)$ but $a(A_1) > a(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(iii) if $s(A_1) = s(A_2)$ but $a(A_1) = a(A_2)$ and $c(A_1) > c(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(iv) if $s(A_1) = s(A_2)$ and $a(A_1) = a(A_2)$ and $c(A_1) > c(A_2)$, then the SVNN A_1 is neutrosophic equal to the SVNN A_2 denoted by the notation $A_1 = A_2$.

However for simple cases, the following ranking method may be followed for easy applications:

(i) if $s(A_1) > s(A_2)$, then the SVNN A_1 is neutrosophic greater than the SVNN A_2 denoted by the notation $A_1 \succ A_2$.

(ii) if $s(A_1) < s(A_2)$, then the SVNN A_1 is neutrosophic less than the SVNN A_2 denoted by the notation $A_1 \prec A_2$.

(iii) if $s(A_1) = s(A_2)$, then the SVNN A_1 is neutrosophic equal to the SVNN A_2 denoted by the notation $A_1 = A_2$.

For a deep study on the Theory of Nneutrosophic Sets introduced by Smarandache, his main work [8-12] could be viewed. The notion of a neutrosophic numbers (NNs) is important to quantify an imprecise or ill-defined quantity. In this paper although, we shall use the very basic neutrosophic operations viz. neutrosophic addition $\oplus$, neutrosophic subtraction $\ominus$, and ranking of neutrosophic numbers, etc.

If we can rank n number of neutrosophic numbers, we then easily by soft-compute find out the min NN and max NN of these n number of NNs. If $A_1, A_2, A_3, \ldots., A_n$ be n neutrosophic numbers sorted in neutrosophic ascending order i.e. if $A_1 \prec A_2 \prec A_3 \prec \ldots\ldots. \prec A_n$, then A_1 and A_n can be regarded respectively as the neutrosophic-min NN and neutrosophic-max NN of these n NNs.

2.2 *Multisets: Some Preliminaries*

We present some basic preliminaries of the notion of multigraphs [45, 51]. Mathematically, a multigraph G is an ordered pair (V, E) consisting of two sets V and E, where V or V (G) is a set of vertices (or, nodes), and E or E(G) is the set of links or edges or arcs. In multigraphs, although multiple links (or edges or arcs) may exist between a pair of nodes (vertices), but in our work here we consider only those multigraphs that has no loop. The multigraphs could be classified by two types: undirected multigraphs and directed multigraphs. For any undirected multigraph if the edge (i, j) and the edge (j, i) exist, then it is obvious that they are identical unlike in the case of the directed multigraphs. A rigorous theoretical study on the algebra of multigraphs has been done in the work [45]. Figure 1 below shows a directed multigraph G = (V, E), in which the set V = {A, B, C, D} and the set E = {AB_1, AB_2, BA, AD, AC, CB, BD, DB}.

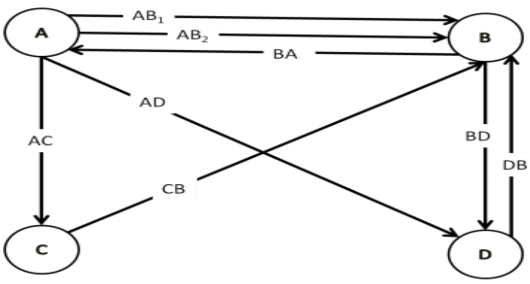

Figure 1: A directed multigraph G

A multigraph H = (W, F) is called a submultigraph of the multigraph G = (V, E) if W ⊆ V and F ⊆ E. The Figure 2 below shows a submultigraph H of the multigraph G (of Figure 1).

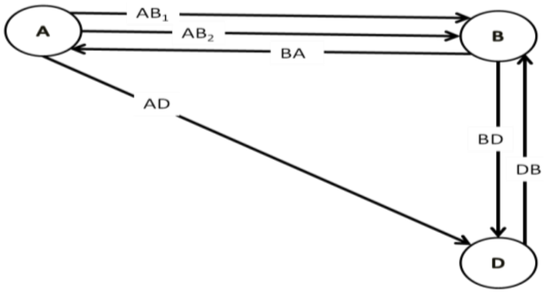

Figure 2: A submultigraph H of the directed multigraph G

It is observed that in many real life cases of various networks, be it in a communication network or road transportation network, or any such network topologies, the weights of the links are not always crisp but neutrosophic numbers. For an example, see the Figure-3 below which shows a public road transportation network multigraph for a traveler in which case the cost implication for traveling each link have been available to him as a neutrosophic number (NN). The NN of an arc in such a multigraphs is called neutrosophic weight (nw) of the arc.

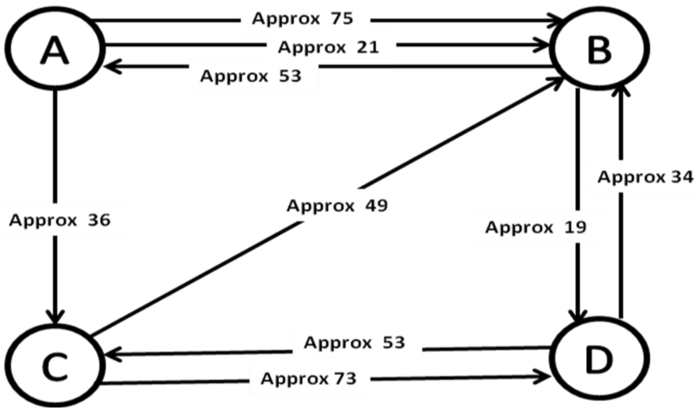

Figure 3: A directed multigraph G with neutrosophic weights of arcs.

In our work here we consider this type (as viewed in Figure 3) of real life instances of directed multigraphs of a network and then develop a soft-computing method to extract the neutrosophic shortest path from a given source node to a pre-decided destination node.

3. Neutrosophic Shortest Path in a Directed Multigraph

A good amount of work has been done on the notion of neutrosophic graph and its application by several authors [6, 7, 16, 17, 19-44, 49]. The Neutrosophic Shortest Path Problem (NSPP) has been solved for graphs by Broumi [32-36], but for the case of a directed multigraph no attempt has been reported so far in the literature for extracting a neutrosophic shortest path. In our proposed method here, we solve the NSPP for multigraphs using the style of Dijkstra's Algorithm but by soft-computing exercises. And for doing this, first of all we define the terms: Neutrosophic-Min-Weight arc-set, Neutrosophic shortest path estimate (d[v]) of a vertex, Neutrosophic relaxation of an arc, etc. In the context of the theory of multigraphs, and then develop few sub algorithms.

3.1. Neutrosophic-Min Weight Arc-set of a directed multigraph

Consider a directed multigraph G in which the links are of having neutrosophic weights. Consider two adjacent nodes u and v, and suppose that there exist n number of links arcs from the node u to the node v in G, n being a non-negative integer. Let W_{uv} denotes the ordered set consisting of the elements which are the arcs connecting the nodes u and v, but keyed & sorted in non-descending order by the values of the respective neutrosophic weights (where sorting is done by using a suitable and pre-choosen ranking method of neutrosophic numbers).

$\therefore \quad W_{uv} = \{ (uv_1 , w_{1uv}), \quad (uv_2 , w_{2uv}), \quad (uv_3 , w_{3uv}), \ldots.. , (uv_n , w_{nuv}) \}.$

Here uv_i is the arc-i from node u to nodex v and w_i is the neutrosophic weight of this arc, for i = 1, 2, 3,……, n. If two or more neutrosophic weights here happen to be neutrosophic equal then they may be placed at random at the corresponding place of non-descending array in this set with no loss of generality in our analysis.

Without any confusion, we may denote the multiset $\{ w_{1uv}, w_{2uv}, w_{3uv}, \ldots\ldots ,w_{nuv} \}$ also using the same notational name W_{uv}. Suppose that w_{uv} be the neutrosophic-min value of the members of the multiset $W_{uv} = \{w_{1uv}, w_{2uv}, w_{3uv}, \ldots. ,w_{nuv} \}$. Obviously, $w_{uv} = w_{1uv}$, because the multiset W_{uv} is already sorted.

Now construct the set $W = \{< (u,v), w_{uv} > : (u,v) \in E \}$. Then W is called the neutrosophic-min-weight arc-set of the multigraph G. Suppose that the sub algorithm NMWA(G) returns the neutrosophic-min-weight arc-set W.

3.2. Neutrosophic Shortest Path Estimate d[v] of a vertex v in a directed multigraph

Suppose that during the execution the node s is the source vertex and the currently traversed vertex is u. There is, in general, no single value of neutrosophic weight for link between the vertex u and the neighbor vertex v, rather there are multiple neutrosophic weights as there are multiple arcs between the vertex u and the neighbor vertex v. Using the value of w_{uv} from the neutrosophic-min weight multiset w of the directed multigraph G, one could now soft-compute the neutrosophic shortest path estimate i.e. d[v] of any vertex v as mentioned below:-

(Neutrosophic shortest path estimate of the vertex v) =
(Neutrosophic shortest path estimate of the vertex u) $\oplus$ (Neutrosophic-min of all the neutrosophic weights corresponding to the links from the vertex u to the vertex v).
or, $d[v] = d[u] \oplus w_{uv}$.

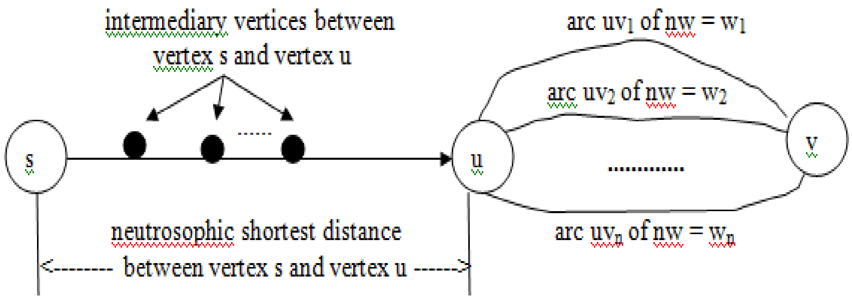

Figure 4: Neutrosophic estimation procedure for d[v]

3.3. Neutrosophic Relaxation of an Arc

In this subsection we present the next step which is 'relaxation' as introduced in the classical Dijkstra's algorithm. In our proposed method here we extend the notion of relaxation to the case of neutrosophic weighted arcs. By the term 'neutrosophic relaxation' in our work we mean the relaxation process of an arc for which the arc-weight is a neutrosophic number (as particular cases, it could be crisp or fuzzy or intuitionistic fuzzy number too as all of them could be viewed as NN). First of all we do initialization of the multigraph along with its starting vertex and neutrosophic shortest path estimate for each vertices of the multigraph G. The corresponding algorithm is called 'NEUTROSOPHIC-INITIALIZATION-SINGLE-SOURCE' as presented below:

NEUTROSOPHIC-INITIALIZATION-SINGLE-SOURCE (G, s)

1. For each vertex $v \in V[G]$
2. $d[v] = \infty$
3. $v.\pi$ = NIL
4. $d[s] = 0$

After doing the neutrosophic initialization, the process of neutrosophic relaxation of each arc starts. The following sub-algorithm NEUTROSOPHIC-RELAX will play the role to update d[v] i.e. the neutrosophic shortest distance value between the starting vertex s and the vertex v (which is a neighbor of the currently traversed vertex u).

NEUTROSOPHIC-RELAX (u, v, W)

1. IF $d[v] \succ d[u] \oplus w_{uv}$

2. THEN $d[v] \leftarrow d[u] \oplus w_{uv}$
3. $v.\pi \leftarrow u$

Where, $w_{uv} \in W$ is the neutrosophic-min weight of the arcs from vertex u to vertex v, and $v.\pi$ denotes the parent node of vertex v.

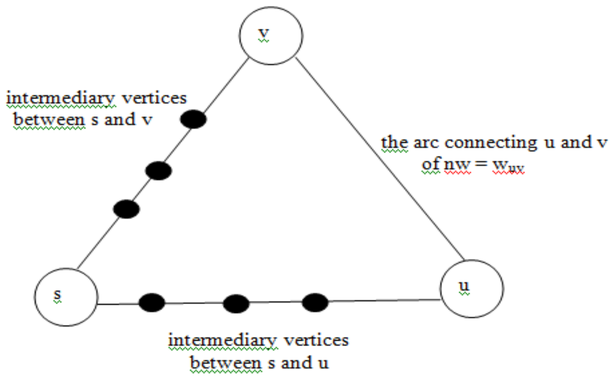

Figure 5: Diagram showing how the NEUTROSOPHIC-RELAX algorithm works.

3.4. Neutrosophic Shortest Path Algorithm (NSPA)

In this subsection we develop the main algorithm to extract the single source neutrosophic shortest path in a directed multigraph. Let us name this Neutrosophic Shortest Path Algorithm by the title NSPA. In our proposed algorithm we call the sub algorithms developed so far in this work, and also the sub algorithm EXTRACT-NEUTROSOPHIC-MIN (Q) which extracts the node u with the minimum key by using the neutrosophic ranking of NN method, and then it updates Q.

NSPA (G, s)

```
NEUTROSOPHIC-INITIALIZATION-SINGLE-SOURCE (G, s)
W ← NMWA (G)
S ← ∅
Q ← V [G]
WHILE   Q ≠ ∅
            DO    u ← EXTRACT-NEUTROSOPHIC-MIN (Q)
                  S ← S ∪ {u}
                  FOR    each vertex v ∈ Adj[u]
                              DO NEUTROSOPHIC-RELAX (u, v, W)
```

Example 3.1

Let us consider the directed Multigraph G (as in Figure 6) with neutrosophic weights of its links. The problem is to solve the single-source neutrosophic shortest paths problem over this multigraph taking the node A as the source and the node D as the destination.

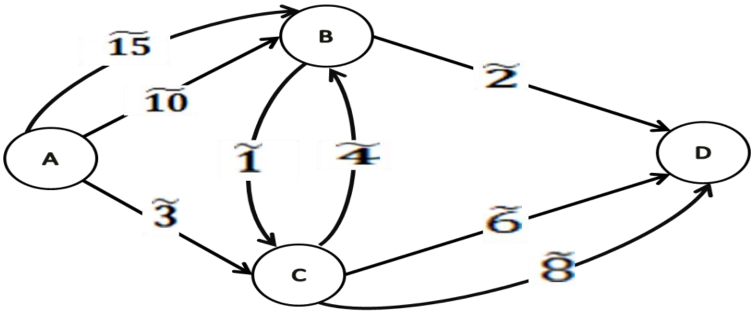

Figure 6: A directed multigraph G with neutrosophic weighted arcs.

It is clear that if the NSPA algorithm is applied to solve this NSPP, it will yield the following results:

1. $w_{AB} = \tilde{10}$, $w_{AC} = \tilde{3}$, $w_{CB} = \tilde{4}$, $w_{CD} = \tilde{6}$, and $w_{BD} = \tilde{2}$; and then

2. S = {A, C, B, D}, i.e. the extracted neutrosophic shortest path from starting the source node A to the destination node D is:

   A $\rightarrow$ C $\rightarrow$ B $\rightarrow$ D.

3. d–values i.e. Neutrosophic shortest distance estimate-values of each node From the starting node an up to the destination node D will be:

   d[A] = 0 , d[C] = NN $\tilde{3}$, d[B] = NN $\tilde{7}$, d[D] = NN $\tilde{9}$.

Here all operations are to be carried out using Definition 2.1.5. The method for ranking of n number of neutrosophic numbers is already mentioned earlier (Definition 2.1.7 and 2.1.8), and the concept of the 'neutrosophic shortest distance' is to be understood accordingly with the help of this ranking method.

Thus the result finally is A $\rightarrow$ C $\rightarrow$ B $\rightarrow$ D with minimum cost of NN $\tilde{9}$.

4. Conclusion

Multigraph is a very useful generalization of the mathematical model graph. In real life environment there are many problems of network (viz. road transportation network, communication network, circuit systems, airlines network etc.) Which cannot be mathematically modeled into 'graphs' but can be very appropriately modeled into 'multigraphs' only. And besides that, many of the directed multigraphs have the weights of the links which are not always crisp but neutrosophic number (NN). The important problem NSPP has been solved by Broumi [32-36] while it is for graphs, but not for multigraphs. In this work we have considered the NSPP for those networks which are multigraphs, and we have proposed a method to extract the neutrosophic shortest path in a directed multigraph from a given source node to one pre-choosen destination node. It is claimed by us that that our proposed method and the corresponding algorithms developed for NSPP on directed multigraphs can play an important role in many real life application areas in the fields of computer science, communication network, road transportation systems, etc. in particular for those type of networks that cannot be mathematically modeled into 'graphs' but into the multigraphs.

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$\mathcal{N}_{\alpha g^{\#}\psi}$-open map, $\mathcal{N}_{\alpha g^{\#}\psi}$-closed map and $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism in neutrosophic topological spaces

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Abstract: As a generalization of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets have been developed by Smarandache to represent imprecise, incomplete and inconsistent information existing in the real world. A neutrosophic set is characterized by a truth-value, an indeterminacy value, and a falsity-value. Salama introduced neutrosophic topological spaces by using Smarandache's neutrosophic sets. In this article, we introduce the concept of $\mathcal{N}_{\alpha g^{\#}\psi}$-open and $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mappings in neutrosophic topological spaces and studied some of their related properties. Further the work is extended to $\mathcal{N}_{\alpha g^{\#}\psi}$- homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism and $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space in neutrosophic topological spaces and establishes some of their related attributes.

Keywords: $\mathcal{N}_{\alpha g^{\#}\psi}$ -open map, $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed map, $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space, $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism.

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1. Introduction

The first successful attempt towards containing non-probabilistic uncertainty, i.e. uncertainty which is not incite by randomness of an event, into mathematical modeling was made in 1965 by L. A. Zadeh [21] through his significant theory on fuzzy sets (FST).

A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called membership value of an element in that set. This gradation concept is very well suited for applications involving vague data such as natural language processing or in artificial intelligence, handwriting and speech recognition etc. Although Fuzzy set theory is very successful in handling uncertainties arising from vagueness or partial belongingness of an element in a set, it cannot model all type of uncertainties pre-veiling in different real physical problems such as problems involving incomplete information.

Further generalization of this fuzzy set was introduced by K. Atanassov [10] in 1986, which is known as Intuitionistic fuzzy sets (IFS). In IFS, instead of one membership value, there is also a non-membership value devoted to each element. Further there is a restriction that the sum of these two values is less or equal to unity. In IFS the degree of non-belongingness is not independent but it is dependent on the degree of belongingness. Fuzzy set theory can be considered as a special case of an IFS where the degree of non-belongingness of an element is exactly equal to 1 minus the degree of belongingness. IFS have the expertise to handle vague data of both complete and incomplete in nature. In applications like expert systems, belief systems and information fusion etc., where degree of non-belongingness is equally important as degree of belongingness, intuitionistic fuzzy sets are quite useful.

There are of course several other generalizations of Fuzzy as well as Intuitionistic fuzzy sets like L-fuzzy sets and intuitionistic L- fuzzy sets, interval valued fuzzy and intuitionistic fuzzy sets etc that have been developed and applied in solving many practical physical problems. Recently a new theory has been introduced which is known as neutrosophic logic and sets. The term neutrosophy means knowledge of impartial thought and this impartial represents the main distinction between fuzzy and intuitionistic fuzzy logic and set. Neutrosophic logic was introduced by Smarandache [14] in 1995. It is a logic in which each proposition is calculated to have a degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F). A Neutrosophic set is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies between [0, 1]*, the non-standard unit interval

Unlike in intuitionistic fuzzy sets, where the included uncertainty is dependent of the degree of belongingness and degree of non-belongingness, here the uncertainty present, i.e. the indeterminacy factor, is independent of truth and falsity values. Neutrosophic sets are indeed more general than IFS as there are no constraints between the degree of truth, degree of indeterminacy and degree of falsity. All these degrees can individually vary within [0, 1]*.

Smarandache's neutrosophic concept have wide range of real time applications for the fields of [1,2,3,4,5,6,7&8] Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, decision making. Mechanics, Electrical & Electronic, Medicine and Management Science etc.

Salama and Alblowi[18] introduced the new concept of neutrosophic topological space in 2012. The neutrosophic closed sets and neutrosophic continuous functions were introduced by Salama, Smarandache and Valeri[19] in 2014. Arokiarani et al.[9] introduced the neutrosophic α-closed set in neutrosophic topological spaces.

Parimala et al.[14] studied the concept of neutrosophic $\alpha\psi$-closed sets and neutrosophic homeomorphisms[15] in neutrosophic topological spaces. Recently Vigneshwaran et al.[13] introduced the concept of $\mathcal{N}_{\alpha g^{\#}\psi}$-closed sets in neutrosophic topological spaces and studied some of its properties and also $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous and $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute functions[12] were initiated and studied in neutrosophic topological spaces.

The focus of this article is to introduce the idea of $\mathcal{N}_{\alpha g^{\#}\psi}$ -open and $\mathcal{N}_{\alpha g^{\#}\psi}$- closed mappings in neutrosophic topological spaces and also the work is extended to $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism and $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space in neutrosophic topological spaces and obtain some of its basic properties.

2. Preliminaries

Definition 2.1.[17] A neutrosophic set $\mathcal{S}$ is an object of the following form $\mathcal{A}$={⟨s, $\mathcal{U}_{\mathcal{A}}$(s), $\mathcal{V}_{\mathcal{A}}$(s), $\mathcal{W}_{\mathcal{A}}$(s): s$\epsilon\mathcal{S}$⟩} where $\mathcal{U}_{\mathcal{A}}$(s), $\mathcal{V}_{\mathcal{A}}$(s) and $\mathcal{W}_{\mathcal{A}}$(s) denote the degree of membership, the degree of indeterminacy and the degree of non membership for each element $s\epsilon\mathcal{S}$ to the set $\mathcal{A}$, respectively.

Definition 2.2. [17] Let $\mathcal{A}$ and $\mathcal{B}$ be Neutrosophic sets of the form

$\mathcal{A}$={⟨s, $\mathcal{U}_{\mathcal{A}}(s)$, $\mathcal{V}_{\mathcal{A}}(s)$, $\mathcal{W}_{\mathcal{A}}(s)$: s$\epsilon\mathcal{S}$⟩} and
$\mathcal{B}$={⟨ s, $\mathcal{U}_{\mathcal{B}}(s)$, $\mathcal{V}_{\mathcal{B}}(s)$, $\mathcal{W}_{\mathcal{B}}(s)$: s$\epsilon\mathcal{S}$⟩}. Then
(i)$\mathcal{A} \subseteq \mathcal{B}$ if and only if $\mathcal{U}_{\mathcal{A}}(s) \leq \mathcal{U}_{\mathcal{B}}(s)$, $\mathcal{V}_{\mathcal{A}}(s) \leq \mathcal{V}_{\mathcal{B}}(s)$ and $\mathcal{W}_{\mathcal{A}}(s) \geq \mathcal{W}_{\mathcal{B}}(s)$;
(ii)$\bar{\mathcal{A}}$= {⟨$\mathcal{W}_{\mathcal{A}}(s)$, $\mathcal{V}_{\mathcal{A}}(s)$, $\mathcal{U}_{\mathcal{A}}(s)$: s$\epsilon\mathcal{S}$⟩};
(iii)$\mathcal{A} \cup \mathcal{B}$={ ⟨s, $\mathcal{U}_{\mathcal{A}}(s) \vee \mathcal{U}_{\mathcal{B}}(s)$, $\mathcal{V}_{\mathcal{A}}(s) \wedge \mathcal{V}_{\mathcal{B}}(s)$, $\mathcal{W}_{\mathcal{A}}(s) \wedge \mathcal{W}_{\mathcal{B}}(s)$: s$\epsilon\mathcal{S}$⟩};
(iv)$\mathcal{A} \cap \mathcal{B}$={ ⟨s, $\mathcal{U}_{\mathcal{A}}(s) \wedge \mathcal{U}_{\mathcal{B}}(s)$, $\mathcal{V}_{\mathcal{A}}(s) \vee \mathcal{V}_{\mathcal{B}}(s)$, $\mathcal{W}_{\mathcal{A}}(s) \vee \mathcal{W}_{\mathcal{B}}(s)$: s$\epsilon\mathcal{S}$⟩}.

Definition 2.3. [18] A neutrosophic topology in a nonempty set $\mathcal{X}$ is a family $\mathfrak{I}$ of neutrosophic sets in $\mathcal{X}$ satisfying the following axioms:

(i) 0_N, 1_N $\in \mathfrak{I}$;
(ii)$\mathcal{U} \cap \mathcal{V} \in \mathfrak{I}$ for any $\mathcal{U}, \mathcal{V} \in \mathfrak{I}$;
(iii)$\cup (\mathcal{U})_i$ for any arbitrary family $(\mathcal{U})_i$: $i \in J \subseteq \mathfrak{I}$

Definition 2.4.[18] Let $\mathcal{P}$ be a neutrosophic set in neutrosophic topological space $\mathcal{X}$. Then
$\mathcal{N}$int($\mathcal{P}$)=∪{$\mathcal{D}$: $\mathcal{D}$ is a neutrosophic open set in $\mathcal{X}$ and $\mathcal{D} \subseteq \mathcal{P}$ } is called a neutrosophic interior of $\mathcal{P}$.
$\mathcal{N}$cl($\mathcal{P}$)=∩{ $\mathcal{E}$: $\mathcal{E}$ is a neutrosophic closed set in $\mathcal{X}$ and $\mathcal{E} \supseteq \mathcal{P}$ } is called a neutrosophic closure of $\mathcal{P}$.

Definition 2.5.[12] A subset $\mathcal{A}$ of a neutrosophic space $(\mathcal{X}, \mathfrak{I})$ is called a neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set if $\mathcal{N}_{\alpha}cl(\mathcal{A}) \subseteq \mathcal{G}$ whenever $\mathcal{A} \subseteq \mathcal{G}$ and $\mathcal{G}$ is $\mathcal{N}_{g^{\#}\psi}$-open in $(\mathcal{X}, \mathfrak{I})$.

Definition 2.6. A function d: $(\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is called
(i) a $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous[13] if $d^{-1}(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set of $(\mathcal{S}, \mathfrak{I})$ for every neutrosophic closed set $\mathcal{A}$ of $(\mathcal{T}, \xi)$.
(ii) a $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute[13] if $d^{-1}(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{S}, \mathfrak{I})$ for every $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{A}$ of $(\mathcal{T}, \xi)$.

Definition 2.7.[15] A bijection g: $(\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is called a homeomorphism if g and g^{-1} are neutrosophic continuous mappings.

All over this paper neutrosophic $\alpha g^{\#}\psi$-interior and neutrosophic $\alpha g^{\#}\psi$-closure is denoted by $\mathcal{N}_{\alpha g^{\#}\psi}$-i* and $\mathcal{N}_{\alpha g^{\#}\psi}$-c* respectively.

3. $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping

Definition 3.1. A mapping $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open if image of every neutrosophic open set of $(\mathcal{S}, \mathfrak{I})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -open set in $(\mathcal{T}, \xi)$.

Theorem 3.2. Each neutrosophic open mapping is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Proof: Let $\mathcal{A}$ be a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. Since d is a neutrosophic open mapping, $d(\mathcal{A})$ is neutrosophic open in $(\mathcal{T}, \xi)$. But every neutrosophic open set is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set. Therefore, $d(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping be not a neutrosophic open map by the following example.

Example 3.3. Let $\mathcal{S} = \{u, v, w\}$, $\mathfrak{I}$ ={0_N, $\mathcal{D}_1$,$\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$ 1_N} be a neutrosophic topology on $(\mathcal{S}, \mathfrak{I})$.
$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩
$\mathcal{D}_2$ =⟨s, (0.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩
$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩
$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and
let $\mathcal{T} = \{u, v, w\}$, ξ ={0_N, $\mathcal{F}_1$,$\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ 1_N} be a neutrosophic topology on $(\mathcal{T}, \xi)$.
$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩
$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩
$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩
$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩
Define $d : (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ by $d(u) = u$, $d(v) = v$, $d(w) = w$.
$\mathcal{N}_{\alpha g^{\#}\psi}$ -open sets of $(\mathcal{T}, \xi)$ = ⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩.
Here $d(\mathcal{D}_1)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open in $(\mathcal{T}, \xi)$. Therefore d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. However, it is not a neutrosophic open mapping because $d(\mathcal{D}_1)$ is not neutrosophic open in $(\mathcal{T}, \xi)$.

Theorem 3.4. A mapping $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open iff for every neutrosophic set $\mathcal{A}$ of $(\mathcal{S}, \mathfrak{I})$, $d(i^*(\mathcal{A}) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*(d(\mathcal{A}))$.

Proof: Necessity: Let d be a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping and $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. Now, $i^*(\mathcal{A}) \subseteq \mathcal{A}$ implies $d(i^*(\mathcal{A})) \subseteq d(\mathcal{A})$. Since d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping, $d(i^*(\mathcal{A}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$ such that $d(i^*(\mathcal{A}) \subseteq d(\mathcal{A})$ therefore $d(i^*(\mathcal{A} \subseteq \mathcal{N}_{\alpha g^{\#}\psi}(i^* d(\mathcal{A}))$.

Sufficiency: Assume $\mathcal{A}$ is a neutrosophic open set of $(\mathcal{S}, \mathfrak{I})$. Then $d(\mathcal{A}) = d(i^*(\mathcal{A}) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*(d(\mathcal{A}))$. But $\mathcal{N}_{\alpha g^{\#}\psi} - (i^*(d(\mathcal{A}))) \subseteq d(\mathcal{A})$. So $d(\mathcal{A}) = \mathcal{N}_{\alpha g^{\#}\psi} - (i^*(\mathcal{A}))$ which implies $d(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{T}, \xi)$ and hence d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open.

Theorem 3.5. If $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping then $i^*(d^{-1}(\mathcal{A}) \subseteq d^{-1}(\mathcal{N}_{\alpha g^{\#}\psi}$- $(i^*(\mathcal{A}))$ for every neutrosophic set $\mathcal{A}$ of $(\mathcal{T}, \xi)$.

Proof: Let $\mathcal{A}$ is a neutrosophic set of $(\mathcal{T}, \xi)$. Then $i^*\,(d^{-1}(\mathcal{A}))$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. Since d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open $d(i^*\,(d^{-1}(\mathcal{A})))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$- open in $(\mathcal{T}, \xi)$ and hence $d(i^*(d^{-1}(\mathcal{A}))) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*\,(d(d^{-1}(\mathcal{A}))) \subseteq \mathcal{N}_{\alpha g^{\#}\psi} - (i^*\,(\mathcal{A}))$. Thus $i^*\,(d^{-1}(\mathcal{A})) \subseteq d^{-1}(\mathcal{N}_{\alpha g^{\#}\psi} - (i^*\,(\mathcal{A})))$.

Theorem 3.6. A mapping $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open iff for each neutrosophic set $\mathcal{F}$ of $(\mathcal{T}, \xi)$ and for each neutrosophic closed set $\mathcal{U}$ of $(\mathcal{S}, \mathfrak{I})$ containing $d^{-1}(\mathcal{F})$ there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ such that $\mathcal{F} \subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A}) \subseteq \mathcal{U}$.

Proof: Necessity: Assume d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. Let $\mathcal{F}$ be the neutrosophic closed set of $(\mathcal{T}, \xi)$ and $\mathcal{U}$ is a neutrosophic closed set of $(\mathcal{S}, \mathfrak{I})$ such that $d^{-1}\,(\mathcal{F}) \subseteq \mathcal{U}$. Then $\mathcal{A} = (d^{-1}(\mathcal{U}^c\,))^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{T}, \xi)$ such that $d^{-1}\,(\mathcal{A}) \subseteq \mathcal{U}$.

Sufficiency: Assume $\mathcal{G}$ is a neutrosophic open set of $(\mathcal{S}, \mathfrak{I})$. Then $d^{-1}((d(\mathcal{G}))^c \subseteq \mathcal{G}^{\,c}$ and $\mathcal{G}^{\,c}$ is neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. By hypothesis there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ such that $(d(\mathcal{G}))^c \subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A}) \subseteq \mathcal{G}^{\,c}$. Therefore $\mathcal{G} \subseteq (d^{-1}(\mathcal{A}))^c$. Hence $\mathcal{A}^c \subseteq d(\mathcal{G}) \subseteq d((d^{-1}\,(\mathcal{A}))^c) \subseteq \mathcal{A}^c$ which implies $d(\mathcal{G}) = \mathcal{A}^c$. Since $\mathcal{A}^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{T}, \xi)$. Hence $d(\mathcal{G})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open in $(\mathcal{T}, \xi)$ and thus d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Theorem 3.7. A mapping $d: (\mathcal{S}, \mathfrak{I}) \rightarrow (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open iff d^{-1} $(\mathcal{N}_{\alpha g^{\#}\psi}$-$(c^*(\mathcal{B})) \subseteq c^*(\,d^{-1}\,(\mathcal{B}))$ for every neutrosophic set $\mathcal{B}$ of $(\mathcal{T}, \xi)$.

Proof: Necessity: Assume d is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -open mapping. For any neutrosophic set $\mathcal{B}$ of $(\mathcal{T}, \xi)$, $d^{-1}(\mathcal{B}) \subseteq c^*(d^{-1}(\mathcal{B}))$. Therefore by theorem 3.3 there exists a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set $\mathcal{F}$ in $(\mathcal{T}, \xi)$ such that

$\mathcal{B} \subseteq \mathcal{F}$ and $d^{-1}(\mathcal{F}) \subseteq c^*(d^{-1}(\mathcal{B}))$. Therefore we obtain that $d^{-1}(\mathcal{N}_{\alpha g^{\#}\psi}\text{-}c^*(\mathcal{B})) \subseteq d^{-1}(\mathcal{F}) \subseteq c^*(d^{-1}(\mathcal{B}))$.

Sufficiency: Assume $\mathcal{B}$ is a neutrosophic set of $(\mathcal{T},\xi)$ and F is a neutrosophic closed set of $(\mathcal{S},\mathfrak{I})$ containing $d^{-1}(\mathcal{B})$. Put $\mathcal{W} = c^*(\mathcal{B})$, then $\mathcal{B} \subseteq \mathcal{W}$ and $\mathcal{W}$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed and $d^{-1}(\mathcal{W}) \subseteq c^*(d^{-1}(\mathcal{B})) \subseteq \mathcal{F}$. Then by theorem 3.6, d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Theorem 3.8. If $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ and $e: (\mathcal{T},\xi) \to (\mathcal{V},\omega)$ be two neutrosophic mappings and $eod: (\mathcal{S},\mathfrak{I}) \to (\mathcal{V},\omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open. If $e: (\mathcal{T},\xi) \to (\mathcal{V},\omega)$is $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute then $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Proof: Let $\mathcal{H}$ be a neutrosophic open set in $(\mathcal{S},\mathfrak{I})$. Then $eod(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{V},\omega)$ because eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. Since e is $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute and $eod(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{V},\omega)$ therefore $e^{-}(eod(\mathcal{H})) = d(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T},\xi)$. Hence d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

Theorem 3.9. If $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ is neutrosophic open and $e: (\mathcal{T},\xi) \to (\mathcal{V},\omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mappings then $eod: (\mathcal{S},\mathfrak{I}) \to (\mathcal{V},\omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open.

Proof: Let $\mathcal{H}$ be a neutrosophic open set in $(\mathcal{S},\mathfrak{I})$. Then $d(\mathcal{H})$ is a neutrosophic open set of $(\mathcal{T},\xi)$ because d is a neutrosophic open mapping. Since e is $\mathcal{N}_{\alpha g^{\#}\psi}$-open, $e(d(\mathcal{H})) = (eod)(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{V},\omega)$. Hence eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

4. $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping

Definition 4.1. A mapping $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed if image of every neutrosophic closed set of $(\mathcal{S},\mathfrak{I})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$.

Theorem 4.2. Each neutrosophic closed mapping is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Proof: Let $\mathcal{A}$ be a neutrosophic closed set in $(\mathcal{S},\mathfrak{I})$. Since d is a neutrosophic closed mapping, $d(\mathcal{A})$ is neutrosophic closed in $(\mathcal{T},\xi)$. But every neutrosophic closed set is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set. Therefore, $d(\mathcal{A})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed mapping need not be a neutrosophic closed map by the following example.

Example 4.3. Let $\mathcal{S}$ = {u, v, w}, $\mathfrak{I}$ ={0_N, $\mathcal{D}_1$,$\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$ 1_N} be a neutrosophic topology on $(\mathcal{S},\mathfrak{I})$.

$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩

$\mathcal{D}_2$ =⟨s, (10.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩

$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and

let $\mathcal{T}$ = {u, v, w}, ξ ={0_N, $\mathcal{F}_1$,$\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ 1_N} be a neutrosophic topology on $(\mathcal{T},\xi)$.

$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩

$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

Define $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ by $d(u) = u$, $d(v) = v$, $d(w) = w$.

$\mathcal{N}_{\alpha g^{\#}\psi}$-closed sets of $(\mathcal{T},\xi)$ = ⟨s, (0.3,0.5,0.5), (0.2,0.1,0.1), (0.2,0.1,0.1)⟩.

Here $d(\mathcal{D}_1)^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{T},\xi)$. Therefore d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. However, it is not a neutrosophic closed mapping because $d(\mathcal{D}_1)^c$ is not neutrosophic closed set in $(\mathcal{T},\xi)$.

Theorem 4.4. A mapping $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed iff for each neutrosophic set $\mathcal{S}$ of $(\mathcal{T}, \xi)$ and for each neutrosophic open set $\mathcal{U}$ of $(\mathcal{S}, \Im)$ containing d^{-1} $(\mathcal{S})$ there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ such that $\mathcal{S} \subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A}) \subseteq \mathcal{U}$.

Proof: Necessity: Assume d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. Let $\mathcal{S}$ be the neutrosophic closed set of $(\mathcal{T}, \xi)$ and $\mathcal{U}$ is a neutrosophic open set of $(\mathcal{S}, \Im)$ such that $d^{-1}(\mathcal{S}) \subseteq \mathcal{U}$. Then $\mathcal{A} = \mathcal{T} - d^{-1}$ $(\mathcal{U})^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set of $(\mathcal{T}, \xi)$ such that $d^{-1}(\mathcal{A}) \subseteq \mathcal{U}$.

Sufficiency: Assume $\mathcal{F}$ is a neutrosophic closed set of $(\mathcal{S}, \Im)$. Then $(d(\mathcal{F}))^c$ is a neutrosophic set of $(\mathcal{T}, \xi)$ and $\mathcal{F}^c$ is neutrosophic open set in $(\mathcal{S}, \Im)$ such that $d^{-1}((d(\mathcal{F}))^c) \subseteq \mathcal{F}^c$. By hypothesis there is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ such that $(d(\mathcal{F}))^c \subseteq \mathcal{A}$ and $d^{-1}(\mathcal{A}) \subseteq \mathcal{F}^c$. Therefore $\mathcal{F} \subseteq (d^{-1}(\mathcal{A}))^c$. Hence $\mathcal{A}^c \subseteq d(\mathcal{F}) \subseteq d((d^{-1}(\mathcal{A}))^c) \subseteq \mathcal{A}^c$ which implies $d(\mathcal{F}) = \mathcal{A}^c$. Since $\mathcal{A}^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set of $(\mathcal{T}, \xi)$. Hence $d(\mathcal{F})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{T}, \xi)$ and thus d is neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Theorem 4.5. If $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ is neutrosophic closed and $e: (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed. Then eod: $(\mathcal{S}, \Im) \to (\mathcal{V}, \omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed.

Proof: Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S}, \Im)$. Then $d(\mathcal{H})$ is neutrosophic closed set of $(\mathcal{T}, \xi)$ because d is neutrosophic closed mapping. Now $eod(\mathcal{H}) = e(d(\mathcal{H}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{V}, \omega)$ because e is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. Thus eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Theorem 4.6. If $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed map, then $\mathcal{N}_{\alpha g^{\#}\psi}$- $(c^*(d(\mathcal{A})) \subseteq d(c^*(\mathcal{A}))$.

Proof: Obvious.

Theorem 4.7. Let $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ and $e: (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$are $\mathcal{N}_{\alpha g^{\#}\psi}$- closed mappings. If every $\mathcal{N}_{\alpha g^{\#}\psi}$- closed set of $(\mathcal{T}, \xi)$ is neutrosophic α-closed then, eod: $(\mathcal{S}, \Im) \to (\mathcal{V}, \omega)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed.

Proof: Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S}, \Im)$. Then $d(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set of $(\mathcal{T}, \xi)$ because d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. By hypothesis $d(\mathcal{H})$ is neutrosophic α-closed set of $(\mathcal{T}, \xi)$. Now $e(d(\mathcal{H})) = (eod)(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{V}, \omega)$ because e is $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed mapping. Thus eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Theorem 4.8. Let $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ be a objective mapping, then the following statements are equivalent:

(a) d is a neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

(b) d is a neutrosophic $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

(c) d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mapping.

Proof: (a)⇒(b): Let us assume that d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping. By definition, $\mathcal{H}$ is a neutrosophic open set in $(\mathcal{S}, \Im)$, then $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Here, $\mathcal{H}$ is neutrosophic closed set in $(\mathcal{S}, \Im)$, then $\mathcal{S} - \mathcal{H}$ is a neutrosophic open set in $(\mathcal{S}, \Im)$. By assumption, $d(\mathcal{S} - \mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Hence, $\mathcal{T} - d(\mathcal{S} - \mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{T}, \xi)$. Therefore, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

(b) ⇒(c): Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S}, \Im)$. By (b), $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$. Hence, $d(\mathcal{H}) = (d^{-1})^{-1}(\mathcal{H})$, so d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$. Hence, d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous.

(c) ⇒(a): Let $\mathcal{H}$ be a neutrosophic open set in $(\mathcal{S}, \Im)$. By (c), $(d^{-1})^{-1}(\mathcal{H}) = d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

5. $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism

Definition 5.1. A bijection $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ is called a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism if d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous.

Theorem 5.2. Each neutrosophic homeomorphism is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Proof: Let d be neutrosophic homeomorphism, then d and d^{-1} are neutrosophic continuous. But every neutrosophic continuous function is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous. Hence, d and d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous. Therefore, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism need not be a neutrosophic homeomorphism by the following example.

Example 5.3. Let $\mathcal{S} = \{\mathrm{u, v, w}\}$, $\Im = \{0_N, \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \mathcal{D}_4\ 1_N\}$ be a neutrosophic topology on $(\mathcal{S}, \Im)$.

$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩

$\mathcal{D}_2$ =⟨s, (0.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩

$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and

let $\mathcal{T} = \{\mathrm{u, v, w}\}$, $\xi = \{0_N, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4\ 1_N\}$ be a neutrosophic topology on $(\mathcal{T}, \xi)$.

$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩

$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

Define $\mathrm{d} : (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ by $\mathrm{d(u) = u}$, $\mathrm{d(v) = v}$, $\mathrm{d(w) = w}$.

$\mathcal{N}_{\alpha g^{\#}\psi}$ -closed sets of $(\mathcal{S}, \Im)$ =$\mathcal{A}$=⟨s, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

Here $d^{-1}(\mathcal{F}_3)^c$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{S}, \Im)$. Therefore d is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous and d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous if $(\mathcal{D}_3)^c$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \Im)$, then the image $d(\mathcal{D}_3)^c$=$(\mathcal{F}_4)^c$ is neutrosophic closed in $(\mathcal{T}, \xi)$. Hence, d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous then it is a $\mathcal{N}_{\alpha g^{\#}\psi}$- homeomorphism. However, $\mathcal{A}$ is neutrosophic closed in $(\mathcal{T}, \xi)$ but it is not neutrosophic closed in $(\mathcal{S}, \Im)$. Therefore it is not neutrosophic continuous. Therefore it is not neutrosophic homeomorphism.

Theorem 5.4. Let $d: (\mathcal{S}, \Im) \to (\mathcal{T}, \xi)$ be a bijective mapping. If d is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous, then the following statements are equivalent:

(a) d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

(b) d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

(c) d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Proof: (a) ⇒(b): Assume that d is a bijective mapping and a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping. Hence, d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mapping. We know that each neutrosophic open set in $(\mathcal{S}, \Im)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open mapping.

(b) ⇒(c): Let d be a bijective and neutrosophic open mapping. Further, d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous mapping. Hence, d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous. Therefore, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism.

(c) ⇒(a): Let d be a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism, then d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous. Since each neutrosophic closed set in $(\mathcal{S}, \Im)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$, hence d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.

Definition 5.5. Let $(\mathcal{S}, \Im)$ be a neutrosophic topological spaces said to be a neutrosophic

$\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space if every $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set is neutrosophic closed in$(\mathcal{S}, \Im)$.

Theorem 5.6. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism, then d is a neutrosophic homeomorphism if $(\mathcal{S}, \mathfrak{I})$ and $(\mathcal{T}, \xi)$ are $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space.

Proof: Assume that $\mathcal{H}$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$, then d^{-1} $(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{S}, \mathfrak{I})$. Since $(\mathcal{S}, \mathfrak{I})$ is an $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space, d^{-1} $(\mathcal{H})$ is a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Therefore, d is neutrosophic continuous. By hypothesis, d^{-1} is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous. Let $\mathcal{G}$ be a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Then, $(d^{-1})^{-1}(\mathcal{G}) = d(\mathcal{G})$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$, by presumption. Since $(\mathcal{T}, \xi)$ is a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space, $d(\mathcal{G})$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$. Hence, d^{-1} is neutrosophic continuous. Hence, d is a neutrosophic homeomorphism.

Theorem 5.7. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ be a neutrosophic topological space, then the following are equivalent if $(\mathcal{T}, \xi)$ is a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space:

(a) d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed mapping.
(b) If $\mathcal{H}$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$, then $d(\mathcal{H})$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$.
(c) $d(i^*$ $(\mathcal{H}))\subseteq c^*(\,i^*(d(\mathcal{H})))$ for every neutrosophic set $\mathcal{H}$ in $(\mathcal{S}, \mathfrak{I})$.

Proof: (a) $\Rightarrow$ (b): Obvious.

(b) $\Rightarrow$ (c): Let $\mathcal{H}$ be a neutrosophic set in $(\mathcal{S}, \mathfrak{I})$. Then, $i^*(\mathcal{H})$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. Then, $d(i^*(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Since $(\mathcal{T}, \xi)$ is a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$ -space, so $d(i^*(\mathcal{H}))$ is a neutrosophic open set in $(\mathcal{T}, \xi)$. Therefore, $d(i^*(\mathcal{H})) = i^*(d(i^*(\mathcal{H})))\subseteq c^*(i^*$ $(\mathrm{d}(\mathcal{H})))$.

(c) $\Rightarrow$ (a): Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Then, $\mathcal{H}^c$ is a neutrosophic open set in $(\mathcal{S}, \mathfrak{I})$. From, $d(i^*(\mathcal{H}^c))\subseteq c^*(\,i^*(d(\mathcal{H}^c)))$. Hence, $d(\mathcal{H}^c)\subseteq c^*(int(d(\mathcal{H}^c)))$. Therefore, $d(\mathcal{H}^c)$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-open set in $(\mathcal{T}, \xi)$. Therefore, $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{S}, \mathfrak{I})$. Hence, d is a neutrosophic closed mapping.

Theorem 5.8. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ and e : $(\mathcal{T}, \xi)$ $\to$ $(\mathcal{V}, \omega)$ be $\mathcal{N}_{\alpha g^{\#}\psi}$-closed, where $(\mathcal{S}, \mathfrak{I})$ and $(\mathcal{V}, \omega)$ are two neutrosophic topological spaces and $(\mathcal{T}, \xi)$ a $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space, then the composition eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed.

Proof: Let $\mathcal{H}$ be a neutrosophic closed set in $(\mathcal{S}, \mathfrak{I})$. Since d is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed and $d(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$, by assumption, $d(\mathcal{H})$ is a neutrosophic closed set in $(\mathcal{T}, \xi)$. Since e is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed, then $e(d(\mathcal{H}))$ is $\mathcal{N}_{\alpha g^{\#}\psi}$-closed in $(\mathcal{V}, \omega)$ and $e(d(\mathcal{H})) = eod$ $(\mathcal{H})$. Therefore, eod is $\mathcal{N}_{\alpha g^{\#}\psi}$- closed.

Theorem 5.9. Let $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ and e : $(\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$be two neutrosophic topological spaces, then the following hold:

(a) If eod is $\mathcal{N}_{\alpha g^{\#}\psi}$-open and d is neutrosophic continuous, then e is $\mathcal{N}_{\alpha g^{\#}\psi}$-open.
(b) If eod is neutrosophic open and e is $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous, then d is $\mathcal{N}_{\alpha g^{\#}\psi}$-open.

Proof: Obvious

6. $\mathcal{N}_{\alpha g^{\#}\psi}$–C Homeomorphism

Definition 6.1. A bijection $d: (\mathcal{S}, \mathfrak{I}) \to (\mathcal{T}, \xi)$ is called a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism if d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mappings.

Theorem 6.2. Each $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Proof: Let us assume that $\mathcal{H}$ is a neutrosophic closed set in $(\mathcal{T},\xi)$. This shows that $\mathcal{H}$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. By assumption, $d^{-1}(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Hence, d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mapping. Hence, d and d^{-1} are $\mathcal{N}_{\alpha g^{\#}\psi}$-continuous mappings. Hence d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism.

Let a $\mathcal{N}_{\alpha g^{\#}\psi}$-homeomorphism need not be a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism by the following example.

Example 6.3. Let $\mathcal{S} = \{u, v, w\}$, $\mathfrak{I}$ ={0N, $\mathcal{D}_1$,$\mathcal{D}_2$, $\mathcal{D}_3$, $\mathcal{D}_4$ 1N} be a neutrosophic topology on $(\mathcal{S},\mathfrak{I})$.

$\mathcal{D}_1$ =⟨s, (0.2,0.1,0.1), (0.2,0.1,0.1), (0.3,0.5,0.5)⟩

$\mathcal{D}_2$ =⟨s, (0.1,0.2,0.2), (0.4,0.3,0.3), (0.3,0.3,0.3)⟩

$\mathcal{D}_3$ =⟨s, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{D}_4$=⟨s, (0.1,0.1,0.1), (0.4,0.3,0.3), (0.3,0.5,0.5)⟩, and

let $\mathcal{T} = \{u, v, w\}$, ξ ={0N, $\mathcal{F}_1$,$\mathcal{F}_2$, $\mathcal{F}_3$, $\mathcal{F}_4$ 1N} be a neutrosophic topology on $(\mathcal{T},\xi)$.

$\mathcal{F}_1$=⟨t, (0.3,0.3,0.3), (0.2,0.1,0.1), (0.2,0.2,0.2)⟩

$\mathcal{F}_2$=⟨t, (0.2,0.2,0.2), (0.1,0.1,0.1), (0.3,0.3,0.3)⟩

$\mathcal{F}_3$=⟨t, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩

$\mathcal{F}_4$=⟨t, (0.2,0.2,0.2), (0.2,0.1,0.1), (0.3,0.3,0.3)⟩

Define $d : (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ by d(u) = u, d(v) = v, d(w) = w.

Assume $\mathcal{N}_{\alpha g^{\#}\psi}$-closed sets of $(\mathcal{S},\mathfrak{I})$ =$\mathcal{A}$=⟨s, (0.3,0.3,0.3), (0.1,0.1,0.1), (0.2,0.1,0.1)⟩ is $\mathcal{N}_{\alpha g^{\#}\psi}$ -continuous then it is $\mathcal{N}_{\alpha g^{\#}\psi}$ -homeomorphism. However, it is not a $\mathcal{N}_{\alpha g^{\#}\psi}$ -C homeomorphism because it is not $\mathcal{N}_{\alpha g^{\#}\psi}$- irresolute.

Theorem 6.4. If $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism, then $\mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{H}))\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$ for each neutrosophic topological space $\mathcal{H}$ in $(\mathcal{T},\xi)$.

Proof: Let $\mathcal{H}$ be a neutrosophic topological space in $(\mathcal{T},\xi)$. Then, $\mathcal{N}_\alpha(c^*(\mathcal{H}))$ is a neutrosophic α-closed set in $(\mathcal{T},\xi)$, and every neutrosophic α-closed set is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. Assume d is $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute, d^{-1} $(\mathcal{N}_\alpha(c^*(\mathcal{B}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$, then $\mathcal{N}_{\alpha g^{\#}\psi}$ $c^*(d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Here, $\mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{H}))\subseteq \mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Therefore, $\mathcal{N}_{\alpha g^{\#}\psi}$- $c^*(d^{-1}(\mathcal{H}))\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$ for every neutrosophic set $\mathcal{H}$ in $(\mathcal{T},\xi)$.

Theorem 6.5. Let $d: (\mathcal{S},\mathfrak{I}) \to (\mathcal{T},\xi)$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism, then $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$ for each neutrosophic set $\mathcal{H}$ in $(\mathcal{T},\xi)$.

Proof: Since d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism, then d is a $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mapping. Let $\mathcal{H}$ be a neutrosophic set in $(\mathcal{T},\xi)$. Clearly, $\mathcal{N}_\alpha(c^*(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Then $\mathcal{N}_\alpha(c^*(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Since $d^{-1}(\mathcal{H})\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$, then$\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))\subseteq\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H}))))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Therefore, $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))\subseteq d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$. Let d be a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism. d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute mapping. Let us consider neutrosophic set $d^{-1}(\mathcal{H})$ in $(\mathcal{S},\mathfrak{I})$, which implies $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. Hence, $\mathcal{N}_{\alpha g^{\#}\psi}$-$c^*(d^{-1}(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S},\mathfrak{I})$. This implies that $(d^{-1})^{-1}(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))) = d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T},\xi)$. This proves $\mathcal{H} = (d^{-1})^{-1}(d^{-1}(\mathcal{H}))\subseteq (d^{-1})^{-1}(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))) = \mathrm{d}(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))$. Therefore, $\mathcal{N}_\alpha(c^*(\mathcal{H}))\subseteq \mathcal{N}_\alpha(c^*(d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))))= d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))$, since d^{-1} is a $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mapping. Hence, $d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))\subseteq d^{-1}(d(\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))))) = \mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))$. That is, $d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))\subseteq\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H})))$. Hence, $\mathcal{N}_\alpha(c^*(d^{-1}(\mathcal{H}))) = d^{-1}(\mathcal{N}_\alpha(c^*(\mathcal{H})))$.

Theorem 6.6. If $d: (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ and $e : (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$ are $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphisms, then eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphism.

Proof: Let d and e to be two $\mathcal{N}_{\alpha g^{\#}\psi}$-C-homeomorphisms. Assume $\mathcal{H}$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{V}, \omega)$. Then, $e^{-1}(\mathcal{H})$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$. Then, by hypothesis, d^{-1} $(e^{-1}(\mathcal{H}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{S}, \mathfrak{J})$. Hence, eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$ -irresolute mapping. Now, let $\mathcal{G}$ be a $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed set in $(\mathcal{S}, \mathfrak{J})$. Then, by presumption, $d(e)$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{T}, \xi)$. Then, by hypothesis, $e(d(\mathcal{G}))$ is a $\mathcal{N}_{\alpha g^{\#}\psi}$-closed set in $(\mathcal{V}, \omega)$. This implies that eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$-irresolute mapping. Hence, eod is a $\mathcal{N}_{\alpha g^{\#}\psi}$-C-homeomorphism.

7. Conclusions

In this paper, the new concept of a neutrosophic homeomorphism and a $\mathcal{N}_{\alpha g^{\#}\psi}$ - homeomorphism in neutrosophic topological spaces was discussed. Furthermore, the work was extended as the $\mathcal{N}_{\alpha g^{\#}\psi}$ -C homeomorphism, $\mathcal{N}_{\alpha g^{\#}\psi}$ -open and $\mathcal{N}_{\alpha g^{\#}\psi}$ -closed mapping and neutrosophic $\mathcal{T}_{\mathcal{N}_{\alpha g^{\#}\psi}}$-space. Further, the study demonstrated $\mathcal{N}_{\alpha g^{\#}\psi}$-C homeomorphisms and also derived some of their related attributes.

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Direct and Semi-Direct Product of Neutrosophic Extended Triplet Group

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Abstract: The object of this article is mainly to discuss the notion of neutrosophic extended triplet direct product (NETDP) and neutrosophic extended triplet semi-direct product (NETS-DP) of NET group. The purpose is to give a clear introduction that allows a solid foundation for additional studies into the field. We introduce neutrosophic extended triplet internal direct product (NETIDP) and neutrosophic extended triplet external direct products (NETEDP) of NET group. Then, we define NET internal and external semi-direct products for NET group by utilizing the notion of NET set theory of Smarandache. Moreover, some results related to NETDP and NETS-DPs are obtained.

Keywords: NET direct product; NET internal direct product; NET external direct product; NET semi-direct product; NET internal semi-direct product; NET external semi-direct product.

1. Introduction

Neutrosophy is a new branch of philosophy, presented by Florentin Smarandache [1] in 1980, which deals the interactions with different ideational spectra in our everyday life. A NET is an object of the structure $(x, e^{neut(x)}, e^{anti(x)})$, for $x \in N$, was firstly presented by Florentin Smarandache [2-4] in 2016. In this theory, the extended neutral and the extended opposites can similar or non-identical from the classical unitary element and inverse element respectively. The NETs are depend on real triads: (friend, neutral, enemy), (pro, neutral, against), (accept, pending, reject), and in general $(x, neut(x), anti(x))$ as in neutrosophy is a conclusion of Hegel's dialectics that is depend on x and $anti(x)$. This theory acknowledges every concept or idea x together along its opposite $anti(x)$ and along their spectrum of neutralities $neut(x)$ among them. Neutrosophy is the foundation of neutrosophic logic, neutrosophic set, neutrosophic probability, and neutrosophic statistics that are utilized or applied in engineering (like software and information fusion), medicine, military, airspace, cybernetics, and physics. Kandasamy and Smarandache [5] introduced many new neutrosophic notions in graphs and applied it to the case of neutrosophic cognitive and relational maps. The same researchers [6] were introduced the concept of neutrosophic algebraic structures for groups, loops, semi groups and groupoids and also their $N-$ algebraic structures in 2006. Smarandache and Mumtaz Ali [7] proposed neutrosophic triplets and by utilizing these they defined NTG and the application areas of NTGs. They also define NT field [8] and NT in physics [9]. Smarandache investigated physical structures of hybrid NT ring [10]. Zhang et al [11] examined the Notion of cancellable NTG and group coincide in 2017. Şahın and Kargın [12], [13] firstly introduced new structures called NT normed space and NT inner product respectively. Smarandache et al [14] studied new algebraic structure called NT G-module which is constructed on NTGs and NT vector spaces. The above set theories have been applied to many different areas including real decision making problems [15-39]. Additionally, Abdel Basset et al applied neutrosophic set theory to artificial intelligence in medicine [43, 44, 46, 56], decision making [45, 48, 49, 52], programming [47], forecasting [50], IoT [51], chain management [53], TOPSIS technique [54], and importing field [55].

This paper deals with direct and semi-direct products of NETGs. We give basic definitions, notations, facts, and examples about NETs which play a significant role to define and build new algebraic structures. Then, the concept of NET internal and external direct and semi-direct products are given and their difference between the classical structures are briefly discussed. Finally, some results related to NET direct and semi-direct products are obtained.

2. Preliminaries

Since some properties of NETs are used in this work, it is important to have a keen knowledge of NETs. We will point out some few NETs and concepts of NET group, NT normal subgroup, and NT costs according to what needed in this work.

Definition 2.1 [7, 9] A NT has a form $\left(a,\ neut\left(a\right),\ anti\left(a\right)\right)$, for $\left(a, neut\left(a\right), anti\left(a\right)\right)\in N$, accordingly $neut\left(a\right)$ and $anti\left(a\right)\in N$ are neutral and opposite of a, that is different from the unitary element, thus: $a*neut(a)=neut(a)*a=a$ and $a*anti(a)=anti(a)*a=neut(a)$ respectively. In general, a may have one or more than one neut's and one or more than one anti's.

Definition 2.2 [3, 9] A NET is a NT, defined as definition 1, but where the neutral of a (symbolized by $e^{neut(a)}$ and called "extended neutral") is equal to the classical unitary element. As a consequence, the "extended opposite" of a, symbolized by $e^{anti(a)}$ is also same to the classical inverse element. Thus, a NET has a form $(a, e^{neut(a)}, e^{anti(a)})$, for $a\in N$, where $e^{neut(a)}$ and $e^{anti(a)}$ in N are the extended neutral and negation of a respectively, thus: $a*e^{neut(a)}=e^{neut(a)}*a=a$, which can be the same or non-identical from the classical unitary element if any and $a*e^{anti(a)}=e^{anti(a)}*a=e^{neut(a)}$. Generally, for each a ∊ N there are one or more $e^{neut(a)}$'s and $e^{anti(a)}$'s.

Definition 2.3 [7, 9] suppose $(N,*)$ is a NT set. Subsequently $(N,*)$ is called a NTG, if the axioms given below are holds.

(1) $(N,*)$ is well-defined, i.e. for and $(a, neut(a), anti(a)),(b, neut(b), anti(b)\in N$, one has $(a, neut(a), anti(a))*(b, neut(b), anti(b))\in N$.

(2) $(N,*)$ is associative, i.e. for anyone has
$(a, neut(a), anti(a))*(b, neut(b), anti(b)*(c, neut(c), anti(c))\in N$.

Theorem 2.4 [41] Let $(N,*)$ be a commutative NET relating to $*$ an $(a, neut(a), anti(a)),(b, neut(b), anti(b))\in N$;

(i) $neut(a)*neut(b)=neut(a*b)$;

(ii) $anti(a)*anti(b)=anti(a*b)$;

Definition 2.5 [3, 9] Assume $(N,*)$ is a NET strong set. Subsequently $(N,*)$ is called a NETG, if the axioms given below are holds.

(1) $(N,*)$ is well-defined, i.e. for any $(a, neut(a), anti(a)),(b, neut(b), anti(b)\in N$, one has $(a, neut(a), anti(a))*(b, neut(b), anti(b)\in N$.

(2) $(N,*)$ is associative,
i.e. for any $(a, neut(a), anti(a)),(b, neut(b), anti(b)),(c, neut(c), anti(c))\in N$, one has

$$(a,neut(a),anti(a))*\big((b,neut(b),anti(b))*(c,neut(c),anti(c))\big)$$
$$=\big((a,neut(a),anti(a))*(b,neut(b),anti(b))\big)*(c,neut(c),anti(c)).$$

Definition 2.6 [42] Assume that $(N_1,*)$ and $(N_2,\circ)$ are two NETG's. A mapping $f:N_1\to N_2$ is called a neutro-homomorphism if:

(1) For any $(a,neut(a),anti(a)),(b,neut(b),anti(b)\in N_1$, we have
$$f\big((a,neut(a),anti(a))*(b,neut(b),anti(b))\big)$$
$$=f\big((a,neut(a),anti(a))\big)*f\big((b,neut(b),anti(b))\big)$$

(2) If $(a,neut(a),anti(a))$ is a NET from N_1, Then
$$f\big(neut(a)\big)=neut\big(f(a)\big) \text{ and } f\big(anti(a)\big)=anti\big(f(a)\big).$$

Definition 2.8 [40] Assume that $(N_1,*)$ is a NETG and H is a subset of N_1. H is called a NET subgroup of N if itself forms a NETG under $*$. On other hand it means:

(1) $e^{neut(a)}$ lies in H.

(2) For any $(a,neut(a),anti(a)),(b,neut(b),anti(b)\in H$,
$$(a,neut(a),anti(a))*(b,neut(b),anti(b)\in H.$$

(3) If $(a,neut(a),anti(a))\in H$, then $e^{anti(a)}\in H$.

Definition 2.9 [40] A NET subgroup H of a NETG N is called a NT normal subgroup of N if $(a,neut(a),anti(a))H=H(a,neut(a),anti(a)),\forall(a,neut(a),anti(a))\in N$ and we represent it as $H($ N.

3. Direct Products of NETG

In this section, we define NET internal and external direct products. Then, we give propositions and proof them.

Definition 3.1 Assume that we have two neutrosophic extended triplet groups H and K, and $N=H\times K$ is the NET cartesian product (NETCP) of H and K, in other words
$$N=\big((h_1,neut(h_1),anti(h_1)),(k_1,neut(k_1),anti(k_1))\big),\begin{pmatrix}(h_2,neut(h_2),anti(h_2)),\\(k_2,neut(k_2),anti(k_2))\end{pmatrix}$$
$$=\big(h_1*h_2,neut(h_1*h_2),anti(h_1*h_2)\big),\big(k_1*k_2,neut(k_1*k_2),anti(k_1*k_2)\big)\in H\times K.$$

Clearly N is closed under multiplication, it is obvious to see associativity and it has a neutral element denoted by $1_N=(1_H,1_K)$ and the anti-neutrals of $\big((h,neut(h),anti(h)),(k,neut(k),anti(k))\big)$ is $\big(anti(h)),anti(k))\big)$, respectively.

Definition 3.2 Suppose that H,K are two NETGs. The NETG $N=H\times K$ with binary operation described componentwise as denoted in definition (3.1.1) is called the "neutrosophic extended triplet direct product" of H and K.

Example 3.3 Find the NET direct product of two NETG Z_2 and z_3. Since $Z_2=\{0,1\}$ and $Z_3=\{0,1,3\}$, the NETs Z_2 is $(0,0,0),(1,0,1)$ and the NETs of Z_3 is $(0,0,0),(1,0,2),(2,0,1)$. The NET direct products are

$$Z_2 \times Z_3 = \left\{ \begin{matrix} ((0,0,0),(0,0,0)),((0,0,0),(1,0,2)),((0,0,0),(2,0,1)),((1,0,1),(0,0,0)), \\ ((1,0,1),(1,0,2)),((1,0,1),(2,0,1)) \end{matrix} \right\}.$$

Definition 3.4 If a NETG N contains neutrosophic triplet normal subgroups (NTNS-Gs) H and K as shown $N = HK$ and $H \cap K = \{1_N\}$, we call N is the "NETIDP" of H and K.

Example 3.5 Examine the NETG $(Z_6, +)$ and the following NET subgroups:

$H = \{(0,0,0),(2,0,4),(4,0,2)\}$

$K = \{(0,0,0),(3,0,3)\}$.

Note that $\left\{ \begin{matrix} (h, neut(h), anti(h)) * (k, neut(k), anti(k)) : (h, neut(h), anti(h)) \in H, \\ (k, neut(k), anti(k)) \in K \end{matrix} \right\} = N.$

That means $\{(0,0,0),(2,0,4),(4,0,2) + (0,0,0),(3,0,3)\}$

$= \{(0,0,0),(1,0,5),(2,0,4),(3,0,3),(4,0,2),(5,0,1)\}$. So the first condition is met. Also the neutral for z_6 is 0_N and $H \cap K = 0_N = \{(0,0,0)\}$ so the second condition is met. Lastly Z_6 is an abelian so the third condition is met.

Table 1. The elements of NETG $(Z_6, +)$.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

As can be seen, the formed NETs of Z_6 is $\{(0,0,0),(1,0,5),(2,0,4),(3,0,3),(4,0,2),(5,0,1)\}$.

and also all classical internal direct products are usually not NETIDPs (some do not even contain either the neutral or anti-neutral elements).

Proposition 3.6 If N is the NETIDP of H and K, subsequently N is neutro-isomorphic to the NETDP $H \times K$.

Proof to put on that N is neutro-isomorphic to $H \times K$, we describe the succeeding map $f : H \times K \to N$,

$$f\left((h, neut(h), anti(a)),(k, neut(k), anti(k))\right) = \left(h*k, neut(h*k), anti(h*k)\right) \quad (1)$$

If $((h, neut(h), anti(h)) \in H, (k, neut(k), anti(k)) \in K$, then

$$\begin{aligned} &((h*k, neut(h*k), anti(h*k)) \\ &= ((k*h, neut(k*h), anti(k*h)). \end{aligned}$$

Actually, we've utilizing that both NETGs K and H are neutrosophic triplet normal that

$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)\left(h,neut(h),anti(h)\right)^{-1}\right)\left((k,neut(k),anti(k)\right)^{-1}\in K,$$

$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)\left(h,neut(h),anti(h)\right)^{-1}\right)\left((k,neut(k),anti(k)\right)^{-1}\in H$$

Implying that

$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)\left(h,neut(h),anti(h)\right)^{-1}\right)\left((k,neut(k),anti(k)\right)^{-1}$$

$$\in K\cap H=\{1_N\}.$$

At the same time let us show that f is a NETG neutro-isomorphism.

1. This a NETG neutro-homomorphism onwards

$$f\left((h,neut(h),anti(h)),(k,neut(k),anti(k)),(h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

$$=f\left((h*h^{'},neut(h*h^{'}),anti(h*h^{'})),(k*k^{'},neut(k*k^{'}),anti(k*k^{'}))\right)\text{ by (1)}.$$
$$=(h,neut(h),anti(h))\left((h^{'}*k),neut(h^{'}*k),anti(h^{'}*k))\right)(k^{'},neut(k^{'}),anti(k^{'}))$$
$$=(h,neut(h),anti(h))\left((k*h^{'}),neut(k*h^{'}),anti(k*h^{'}))\right)(k^{'},neut(k^{'}),anti(k^{'}))$$
$$=f\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)f\begin{pmatrix}(h^{'},neut(h^{'}),anti(h^{'})),\\(k^{'},neut(k^{'}),anti(k^{'}))\end{pmatrix}.$$

2. Let us show that the map f is injective. First we have to check that its neutro-kernel is trivial. Actually, if

$$f\left((h,neut(h),anti(h)),(k,neut(k),anti(k)\right)=1_N \text{ Then}$$

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k)\right)=1_N$$

$$\Rightarrow\left(h,neut(h),anti(h)\right)=\left(k,neut(k),anti(k)\right)^{-1}$$

$$\Rightarrow\left(h,neut(h),anti(h)\right)\in K$$
$$\Rightarrow\left(h,neut(h),anti(h)\right)\in H\cap K=\{1_N\}$$

We have then that $\left(h,neut(h),anti(h)\right)_{=}\left(k,neut(k),anti(k)\right)=\{1_N\}$ which proves that the neutro-kernel is $\{(1_N,1_N)\}$.

3. Lastly it's obvious to see that f is subjective since $N=HK.$ briefly record that the definitions of NETEDP and NETIDP are assuredly unlimited to two NETGs. We can totally describe them for n NETGs as $H_1,\ldots,H_n$.

Definition 3.7 If $H_1,\ldots,H_n$ are random NETGs the NET external direct product of $H_1,\ldots,H_n$ is $N=H_1\times H_2\times\ldots\times H_n$ which is the NET cartesian product with componentwise multiplication.

Example 3.8 Let NETG $u(8)=\{1,3,5,7\}$ and $u(12)=\{1,5,7,11\}$ under multiplication modulo 8 and mudulo 12 respectively. Let's construct a NETG table for $u(12)$.

Table 2. The elements of NETG $u(12)$.

$\times$	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

The NETs of $u(8)$ are $(1,1,1),(3,1,3),(5,1,5),(7,1,7)$ and the NETs of $u(12)$ are $(1,1,1),(5,1,5),(7,1,7),(11,1,11)$.

Now let's see the NET external direct products of $u(8)\times u(12)=\big((1,1,1),(1,1,1)\big),\big((1,1,1),(5,1,5)\big),\big((1,1,1),(7,1,7)\big),\big((1,1,1),(11,1,11)\big),$
$\big((3,1,3),(1,1,1)\big),\big((3,1,3),(5,1,5)\big),\big((3,1,3),(7,1,7)\big),\big((3,1,3),(11,1,11)\big),\big((5,1,5),(1,1,1)\big),$
$\big((5,1,5),(5,1,5)\big),\big((5,1,5),(7,1,7)\big),\big((5,1,5),(11,1,11)\big),\big((7,1,7),(1,1,1)\big),\big((7,1,7),(3,1,3)\big),$
$\big((7,1,7),(5,1,5)\big),\big((7,1,7),(7,1,7)\big),\big((7,1,7),(11,1,11)\big).$

In general, all classical internal direct products are not NETEDPs (some do not even contain either the neutral or anti-neutral elements).

Definition 3.9 If N contains NETNS-Gs $H_1,\ldots,H_n$ as shown $N=H_1\ldots H_n$ and every n can be symbolized as $\big(h,neut(h),anti(h)\big)\ldots\big(h_n,neut(h_n),anti(h_n)\big)$ particularly, we call N is the neutrosophic extended triplet internal direct product of $H_1,\ldots,H_n$. There is a small distinction between neutrosophic extended triplet internal product as we see in the definition, since in this instance of two NET subgroups, the condition dedicated briefly record that each n can be symbolized particularly as $\big(h_1,neut(h_1),anti(h_1)\big)\big(h_2,neut(h_2),anti(h_2)\big)$, but alternately that the intersection of the two NET subgroups is $\{(1_N)\}$. The following proposition indicates the relation among those two points of view.

Proposition 3.10 Assume that $N=H_1\ldots H_n$ thus every H_i is a NET normal subgroup of N.

The succeeding axioms are equivalent.

I. N is the NETDP of the H_i.

II. $H_1H_2\ldots H_{i-1}\cap H_i=\{1_N\},\quad \forall i=1,\ldots,n.$

Proof Let's show I. $\Leftrightarrow$ II. Let's suppose that N is the NETIDP of the H_i, in other words all element in N can be inscribed particularly as a product of elements in H_i. Let's assume

$\left(n,neut(n),anti(n)\right)\in H_1H_2...H_{i-1}\cap H_i=\left\{\left(1_N\right)\right\}$. We obtain that

$\left(n,neut(n),anti(n)\right)\in H_1H_2...H_{i-1}$, this is particularly expressed as

$$\left(n,neut(n),anti(n)\right)=\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)...$$
$$\left(h_{i-1},neut(h_{i-1}),anti(h_{i-1})\right)1_N H_i...1_N H_n,\left(h_j,neut(h_j),anti(h_j)\right)\in H_j.$$

Moreover, $\left(n,neut(n),anti(n)\right)\in H_i$ thus $\left(n,neut(n),anti(n)\right)=\left(1_N\right)H_1n...\left(1_N\right)H_{i-1}n$ and we have $\left(h_j,neut(h_j),anti(h_j)\right)=\left(1_N\right)$ for all j and $\left(n,neut(n),anti(n)\right)=\left(1_N\right)$.

II. $\Rightarrow$ I. Conversely, let us assume that $\left(n,neut(n),anti(n)\right)\in N$ can be written either

$$\left(n,neut(n),anti(n)\right)=\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)...$$
$$\left(h_n,neut(h_n),anti(h_n)\right),\left(h_j,neut(h_j),anti(h_j)\right)\in H_j,$$

or

$$\left(n,neut(n),anti(n)\right)=\left(k_1,neut(k_1),anti(k_1)\right)\left(k_2,neut(k_2),anti(k_2)\right)...$$
$$\left(k_n,neut(k_n),anti(k_n)\right),\left(k_j,neut(k_j),anti(k_j)\right)\in H_j.$$

Remember that whereby every H_j are NET normal subgroups, subsequently

$$\left(h_i,neut(h_i),anti(h_i)\right)\left(h_j,neut(h_j),anti(h_j)\right)$$

$$=\left(h_j,neut(h_j),anti(h_j)\right)\left(h_i,neut(h_i),anti(h_i)\right),\left(h_i,neut(h_i),anti(h_i)\right)\in H_i,$$
$$\left(h_j,neut(h_j),anti(h_j)\right)\in H_j.$$

In other words, we can do the succeeding manipulations.

$$\left(h_1,neut(h_1),anti(h_1)\right)\left(h_2,neut(h_2),anti(h_2)\right)...\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(k_1,neut(k_1),anti(k_1)\right)\left(k_2,neut(k_2),anti(k_2)\right)...\left(k_n,neut(k_n),anti(k_n)\right)$$

$$\Leftrightarrow\left(h_2,neut(h_2),anti(h_2)\right)...\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(\left(h_1,neut(h_1),anti(h_1)\right)^{-1}...\left(k_1,neut(h_1),anti(h_1)\right)\right)\left(k_2,neut(k_2),anti(k_2)\right)...$$
$$\left(k_n,neut(k_n),anti(k_n)\right)$$

$$\Leftrightarrow\left(h_3,neut(h_3),anti(h_3)\right)...\left(h_n,neut(h_n),anti(h_n)\right)$$

$$=\left(\left(h_1,neut(h_1),anti(h_1)\right)^{-1}\ldots\left(k_1,neut(h_1),anti(h_1)\right)\right)\begin{pmatrix}\left(h_2,neut(h_2),anti(h_2)\right)^{-1}\\\left(k_2,neut(k_2),anti(k_2)\right)\end{pmatrix}$$

$$\left(k_3,neut(k_3),anti(k_3)\right)\ldots\left(k_n,neut(k_n),anti(k_n)\right)$$

and likewise and then so long as we achieve

$$\left(h_n,neut(h_n),anti(h_n)\right)\left(k_n,neut(k_n),anti(k_n)\right)^{-1} \quad (1)$$

$$=\left(h_1,neut(h_1),anti(h_1)\right)^{-1}\left(k_1,neut(k_1),anti(k_1)\right)\ldots\left(h_{n-1},neut(h_{n-1}),anti(h_{n-1})\right)^{-1}$$
$$\left(k_{n-1},neut(k_{n-1}),anti(k_{n-1})\right).$$

Until now the left hand side (1) refers to H_n although the right hand side refers to $H_1 \cdots H_{n-1}$,

we obtain such $\left(h_n,neut(h_n),anti(h_n)\right)\left(k_n,neut(k_n),anti(k_n)\right)^{-1} \in H_n \cap H_1 \cdots H_{n-1} = \{1_N\}$

signifying that $\left(h_n,neut(h_n),anti(h_n)\right) = \left(k_n,neut(k_n),anti(k_n)\right)$.

We end this by repeating the procedure. Let's prove this for the conditions of two NETGs. We've noticed overhead that the NET cartesian product of two NETGs H and K endowed in relation to a NETG structure by taking in mind componentwise binary operation. $\left(h_1,neut(h_1),anti(h_1)\right),\left(k_1,neut(k_1),anti(k_1)\right)$

$$=\left(h_1*h_1,neut(h_1*h_1),anti(h_1*h_1)\right),\left(k_1*k_1,neut(k_1*k_1),anti(k_1*k_1)\right) \in H \times K.$$

The preference of this binary operation of course decides the structures of $N = H \times K$, and exceptionally, we've noticed such the neutro-isomorphic duplicates of NETGs H and K in N are NETNS-Gs. Contrarily that one may describe a NETIDP, we have to suppose that we've two NETNS-Gs.

Now let's examine a further overall setting, thus the NET subgroup K doesn't need to be NET normal, for whatever we have to describe another binary operation on the NETCP $H \times K$. this'll take us to the definition of NETIS-DP and NETES-DP.

Remember that a neutro-auto orphism of a NETG H is an objective NETG neutro-homomorphism from $H \to H$. It's obvious to realize such the set of neutro-auto orphism of H shapes a NETG according to the composition of maps and identify element the neutrality map 1_H. We symbolize it by $Aut(1_H)$.

Proposition 3.11 Suppose that H and K are NETGs, and $\rho : K \to Aut(H), \left(k, neut(k), anti(k)\right) \mapsto \rho\left(k, neut(k), anti(k)\right)$ are a NETG neutro-homomorphism. Subsequently the binary operation $\left(H \times K\right) \times \left(H \times K\right) \to \left(H \times K\right)$,

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right),\begin{pmatrix}(h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),\\ anti(k^{'}))\end{pmatrix}$$

$$\rightarrow\begin{pmatrix}(h,neut(h),anti(h))\rho((k,neut(k),anti(k))\left((h^{'},neut(h^{'}),anti(h^{'}))\right)\\ (k,neut(k),anti(k))(k^{'},neut(k^{'}),anti(k^{'}))\end{pmatrix}$$

endows $H\times K$ with a NETG structure, with neutral element $\left(1_H,1_K\right)$.

Proof let's realize such the closure property is holds.

1) Neutrality: Let's prove that $\left(1_H,1_K\right)$ is the neutral element. We have

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\left(1_H,1_K\right)$$

$$=\left((h,neut(h),anti(h))\rho(k,neut(k),anti(k))\left(1_H\right),(k,neut(k),anti(k))\right)$$

$=\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)$ For all $(h,neut(h),anti(h))\in H$, $(k,neut(k),anti(k))\in K$, Whereby $\rho(k,neut(k),anti(k))$ is a NETG neutro-homomorphism. We also have

$$\left(1_H,1_K\right)\left((h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

$$=\left(\rho 1_H(h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

$$=\left((h^{'},neut(h^{'}),anti(h^{'})),(k^{'},neut(k^{'}),anti(k^{'}))\right)$$

2) Anti-neutrality : Let $\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\in H\times K$ and let's prove that

$$\left(\rho^{-1}{}_{(k,neut(k),anti(k))}\left((h,neut(h),anti(h))\right)^{-1},\left(k,neut(k),anti(k)\right)\right)^{-1}$$

is the anti-neutral of

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right).$$

We have

$$\left((h,neut(h),anti(h)),(k,neut(k),anti(k))\right)\begin{pmatrix}\rho^{-1}(k,neut(k),anti(k))\begin{pmatrix}h,neut(h)\\ ,anti(h)\end{pmatrix}^{-1}\\ \left(k,neut(k),anti(k)\right)^{-1}\end{pmatrix}$$

$$=\left(h,neut(h),anti(h)\right)\rho_{(k,neut(k),anti(k))}\begin{pmatrix}\rho^{-1}(k,neut(k),anti(k))\\ \left(h,neut(h),anti(h)\right)^{-1},1_K\end{pmatrix}$$

$$=\left(\left(h,neut(h),anti(h)\right)\left(h,neut(h),anti(h)\right)^{-1},1_K\right)=\left(1_H,1_K\right).$$

We also have

$$\left(\rho^{-1}{}_{(k,neut(k),anti(k)}\left(h,neut(h),anti(h)\right)^{-1},\left(k,neut(k),anti(k)\right)^{-1}\right)$$
$$\left((h,neut(h),anti(h))(k,neut(k),anti(k)\right)$$

$$=\begin{pmatrix}\rho^{-1}{}_{(k,neut(k),anti(k)}\left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\\ \left(h,neut(h),anti(h)\right),1_K\end{pmatrix}$$

$$=\begin{pmatrix}\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\\ \left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right),1_K\end{pmatrix}.$$

Using that

$$\rho^{-1}{}_{\left(k,neut(k),anti(k)\right)}=\rho\left(k,neut(k),anti(k)\right)^{-1}$$

Whereby ρ is a NETG neutro-homomorphism. Instantly

$$\begin{pmatrix}\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right)^{-1}\rho\left(k,neut(k),anti(k)\right)^{-1}\\ \left(h,neut(h),anti(h)\right),1_K\end{pmatrix}$$

$$=\left(\rho\left(k,neut(k),anti(k)\right)^{-1}\left(h,neut(h),anti(h)\right)^{-1}\left(h,neut(h),anti(h)\right),1_K\right)$$

$$=\left(\rho\left(k,neut(k),anti(k)\right)^{-1}\left(1_H\right),1_K\right)=\left(1_H,1_K\right)$$

using that $\rho\left(k,neut(k),anti(k)\right)^{-1}$ is a NETG neutro-homomorphism for every $\left(k,neut(k),anti(k)\right)\in K.$

3) Associativity : Lastly let's check that the following condition holds, we've

$$\begin{pmatrix}(h,neut(h),anti(h)),(k,neut(k),anti(k)),(h',neut(h'),anti(h')),\\ (k',neut(k'),anti(k'))\end{pmatrix}$$
$$\left((h'',neut(h''),anti(h'')),(k'',neut(k''),anti(k''))\right)$$

$$=\begin{pmatrix}(h,neut(h),anti(h)),\rho(k,neut(k),anti(k)),(h',neut(h'),anti(h')),\\(k,neut(k),anti(k)),(k',neut(k'),anti(k'))\end{pmatrix}$$
$$\big((h'',neut(h''),anti(h'')),(k'',neut(k''),anti(k''))\big)$$

$$=\begin{pmatrix}(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}(h',neut(h'),anti(h')),\\ \rho_{(k,neut(k),anti(k))}(k',neut(k'),anti(k'))\end{pmatrix}$$
$$\begin{pmatrix}(h'',neut(h''),anti(h'')),(k,neut(k),anti(k)),(k',neut(k'),anti(k')),\\(k'',neut(k''),anti(k''))\end{pmatrix},$$

While conversely

$$((h,neut(h),anti(h)),(k,neut(k),anti(k)))\begin{pmatrix}(h',neut(h'),anti(h')),(k',neut(k')\\,anti(k'))(h'',neut(h''),anti(h'')),\\(k'',neut(k''),anti(k''))\end{pmatrix}$$

$$=((h,neut(h),anti(h)),(k,neut(k),anti(k)))$$
$$\begin{pmatrix}(h',neut(h'),anti(h')),\rho_{(k',neut(k'),anti(k'))}(h'',neut(h''),\\anti(h'')),(k',neut(k'),anti(k'))(k'',neut(k''),anti(k''))\end{pmatrix}$$

$$=\begin{pmatrix}(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\begin{pmatrix}(h',neut(h'),anti(h'))\rho_{(k',neut(k'),anti(k'))}\\ \big((h'',neut(h''),anti(h''))\big)\end{pmatrix},\\(k,neut(k),anti(k))\big((k',neut(k'),anti(k'))(k'',neut(k''),anti(k''))\big)\end{pmatrix}$$

Whereby K is a NETG, we have

$$\big((k,neut(k),anti(k))(k',neut(k'),anti(k'))\big)(k'',neut(k''),anti(k''))$$
$$=(k,neut(k),anti(k))\big((k',neut(k'),anti(k'))(k'',neut(k''),anti(k''))\big).$$

Mark that by seeing at the first component

$$\rho_{(k,neut(k),anti(k))(k',neut(k'),anti(k'))}$$
$$=\rho_{(k,neut(k),anti(k))}\circ\rho_{(k',neut(k'),anti(k'))}$$

utilizing that ρ is a NETG neutro-homomorphism, therefore

$$(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big)$$
$$\rho_{(k,neut(k),anti(k))}(k',neut(k'),anti(k')\big((h'',neut(h''),anti(h''))\big)$$

$$=(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big)$$

$$\rho_{(k,neut(k),anti(k))}\rho_{(k,neut(k),anti(k))}\begin{pmatrix}\rho_{(k',neut(k'),anti(k'))}\\ \big((h'',neut(h''),anti(h''))\big)\end{pmatrix}.$$

Furthermore, $\rho_{(k,neut(k),anti(k))}$ is a NETG neutro-homomorphism, yielding

$$(h, neut(h), anti(h))\rho_{(k,neut(k),anti(k))}\left((h',neut(h'),anti(h'))\right)$$

$$\rho_{(k,neut(k),anti(k))}\left(\rho_{(k',neut(k'),anti(k'))}\right)\left((h'', neut(h''), anti(h''))\right)$$

$$= (h, neut(h), anti(h))\rho_{(k,neut(k),anti(k))}\begin{pmatrix}(h',neut(h'),anti(h')) \\ \rho(k',neut(k'),anti(k')) \\ \left((h'',neut(h''),anti(h''))\right)\end{pmatrix}$$

which concludes the proof. Now let's define the first NET semi-direct product.

In general, the NET direct product is not enough because the operation between elements of the two NET subgroups is always commutative. On other hand, if N is a NETG, H is a NTNS-G, K is a NET subgroup (K need not be NT normal like in a NET direct product), $K \cap N = 1_N$, then N must be a NET semi-direct product. (The operation between elements of H and K need not be commutative.) So, we can argue that the NET semi-direct product classifies all NETGs constructed in this way.

4. Semi-Direct Products of NETG

Definition 4.1 Suppose that H and K are two NETGs, and $\rho : K \to Aut(H)$ is a NETG neutro-homomorphism. The set $H \times K$ endowed in a relation to the binary operation

$$\left((h, neut(h), anti(h)), (k, neut(k), anti(k))\right)\left((h', neut(h'), anti(h')), (k', neut(k'), anti(k'))\right)$$

$$\to \begin{pmatrix}(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\left((h',neut(h'),anti(h'))\right), \\ (k,neut(k),anti(k))(k',neut(k'),anti(k'))\end{pmatrix}$$

is a NETG N called a "NET external semi-direct product of NETGs H and K" b

ρ, symbolized by $N = H \chi_\rho K.$

Example 4.2 The NET set $L = H \times N$, where H, N are NETGs and $N \leq AutH$ is the NETES-DP of H and N when equipped with the following operation, defined by the action

$$\theta : N \to AutH : \left((h_1,neut(h_1),anti(h_1)),(n_1,neut(n_1),anti(n_1))\right)$$

$$= \begin{pmatrix}(h_1,neut(h_1),anti(h_1))\theta_{(n_1,neut(n_1),anti(n_1))}\left((h_2,neut(h_2),anti(h_2))\right), \\ (n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\end{pmatrix}$$

$$= \begin{pmatrix}(h_1,neut(h_1),anti(h_1))(n_1,neut(n_1),anti(n_1))\left((h_2,neut(h_2),anti(h_2))\right), \\ (n_1,neut(n_1),anti(n_1))(n_2,neut(n_2),anti(n_2))\end{pmatrix},$$

for all $(h_1,neut(h_1),anti(h_1)),(h_2,neut(h_2),anti(h_2))\in H$ and all $(n_1,neut(n_1),anti(n_1)),$ $(n_2,neut(n_2),anti(n_2))\in N.$

Definition 4.3 Let N be a NETG in a relation to NET subgroups H and $K.$We say that N is the "NETIS-DP of H and K" if H is a NETNS-G of N, thus $HK=N$ and $H\cap K=\{1_N\}.$ It is symbolized by $N=H\rtimes K.$

Example 4.4 Let's show that the dihedral NETG D_{2n} is the NETIS-DP of two of its NET subgroups : the NET subgroup of rotations of a regular $n-$gon, and the NET subgroup generated by a single reflection of the same regular $n-$ gon. If $D_{2n}=<(a,neut(a),anti(a)),(x,neut(x),anti(x))>,$ where $(a,neut(a),anti(a))$ generates the NET subgroup $<(a,neut(a),anti(a))>$ of rotations and $(x,neut(x),anti(x))$ generates the NET subgroup $<(x,neut(x),anti(x))>,$ then we know that $(a,neut(a),anti(a))^n=1_N$ and $(x,neut(x),anti(x))^2=1_N,$ where 1_N is the neutral symmetry. We know that $\{1_N\}=<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>;$ we also know that, if x is a reflection and a a rotation, then

$$(x,neut(x),anti(x))(a,neut(a),anti(a))=(a,neut(a),anti(a))^{n-1}(x,neut(x),anti(x)).$$

Being D_{2n} the NETG of all symmetries of a regular $n-$gon, it contains all and only the rotations and reflections of the $n-$ gon itself; this fact, combined with the fact that $\{1_N\}=<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>,$ allows us to deduce

$$\left|<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>\right|=\left|D_{2n}\right|.$$

Since $<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>\leq D_{2n},$ it follows $<(a,neut(a),anti(a))>\cap<(x,neut(x),anti(x))>=D_{2n}.$ Finally, we obtain

$$\begin{aligned}&(x,neut(x),anti(x))(a,neut(a),anti(a))(x,neut(x),anti(x))^{-1}\\&=(a,neut(a),anti(a))^{n-1}\in<(a,neut(a),anti(a))>;\end{aligned}$$

thus, $<(a,neut(a),anti(a))>$ is NT normal. Therefore

$$D_{2n}=<(a,neut(a),anti(a))>\tilde{a}<(x,neut(x),anti(x))>.$$

Lemma 4.5 Assume that N is a NETG with NET subgroups H and K. Assume that $N = HK$ and $H \cap K = \{1_N\}$. Subsequently all element $(n, neut(n), anti(n))$ of N can be inscribed particularly in the form $(h, neut(h), anti(h))(k, neut(k), anti(k))$, for $(h, neut(h), anti(h)) \in H$ and $(k, neut(k), anti(k)) \in K$.

Proof Since $N = HK$, we know that $(n, neut(n), anti(n))$ can be written as $(h, neut(h), anti(h))(k, neut(k), anti(k))$. Assume it can also be inscribed $(h', neut(h'), anti(h'))(k', neut(k'), anti(k'))$. Then

$$(h, neut(h), anti(h))(k, neut(k), anti(k)) = (h', neut(h'), anti(h'))(k', neut(k'), anti(k'))$$

so

$$(h', neut(h'), anti(h'))^{-1}(h, neut(h), anti(h)) = (k', neut(k'), anti(k'))(k, neut(k), anti(k))^{-1}$$
$$\in H \cap K = \{1_N\}.$$

In case $(h, neut(h), anti(h)) = (h', neut(h'), anti(h'))$ and $(k, neut(k), anti(k)) = (k', neut(k'), anti(k'))$.

The NETIDPs and NETEDPs were two sides of the similar objects, consequently are the NETIS-DPs and NETES-PDs. If $N = H \chi_\rho K$ is the NETES-DP of NETGS H and K, subsequently $\overline{H} = H \times \{1\}$ is a NETNS-G of N and it's obvious that N is the NETIS-DP of $H \times \{1\}$ and $\{1\} \times K$. Because of this we can go from NETES-PDs to NETIS-PDs. The following conclusion goes in the another way, from NET internal to external semi-direct products.

Proposition 4.6 Assume that N is a NETG with NET subgroups H and K, and N is the NETIS-PDs of H and K. Then $N \cong H \chi_\rho K$ where $\rho : K \to Aut(H)$ is stated by

$$\rho_{(k, neut(k), anti(k))}\big((h, neut(h), anti(h))\big) = (k, neut(k), anti(k))(h, neut(h), anti(h))$$
$$\big((k, neut(k), anti(k))\big)^{-1},$$
$$(h, neut(h), anti(h)) \in H, (k, neut(k), anti(k)) \in K.$$

Proof Note that $\rho_{(k, neut(k), anti(k))}$ refers to $Aut(H)$ where H is NET normal. By the lemma 4.5 all the element $(n, neut(n), anti(n))$ of N can be inscribed particularly in terms of

$$(h, neut(h), anti(h))(k, neut(k), anti(k)),$$

with $(h, neut(h), anti(h)) \in H$ and $(k, neut(k), anti(k)) \in K$. So that, the map $\varphi : H \chi_\rho K \to N$,

$$\varphi\big((h, neut(h), anti(h))(k, neut(k), anti(k))\big) = (h, neut(h), anti(h))(k, neut(k), anti(k))$$

is a bijection. It is just to prove such this bijection is a neutro-homomorphism. Stated

$$\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big)$$

and

$$\big((h', neut(h'), anti(h')), (k', neut(k'), anti(k'))\big) \text{ in } H\chi_{\rho}K.$$

We have

$$\varphi\left(\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big)\begin{pmatrix}(h', neut(h'), anti(h')), (k', neut(k'), \\ anti(k'))\end{pmatrix}\right)$$

$$= \varphi\left(\begin{pmatrix}(h,neut(h),anti(h))\rho_{(k,neut(k),anti(k))}\big((h',neut(h'),anti(h'))\big), \\ (k,neut(k),anti(k))(k',neut(k'),anti(k'))\end{pmatrix}\right)$$

$$= \varphi\begin{pmatrix}(h, neut(h), anti(h))(k, neut(k), anti(k))(h', neut(h'), anti(h')) \\ (k, neut(k), anti(k))^{-1}, (k, neut(k), anti(k))(k', neut(k'), anti(k'))\end{pmatrix}$$

$$= (h, neut(h), anti(h))(k, neut(k), anti(k))(h', neut(h'), anti(h'))(k', neut(k'), anti(k'))$$

$$= \varphi\big((h, neut(h), anti(h)), (k, neut(k), anti(k))\big)\varphi\begin{pmatrix}(h', neut(h'), anti(h')), \\ (k', neut(k'), anti(k'))\end{pmatrix}.$$

Therefore φ is a NETG neutro-homomorphism, which ends the proof. Shortly, we obtain such all NETIS-DP is neutro-isomorphic to any NETES-DP, when φ is conjugation.

5. Conclusion

The most important point of this article is first to define the NETs and subsequently use these NETs to describe the NET internal and external direct and semi-direct products of NETG. As in classical group theory, in neutrosophic extended triplet group theory building blocks for finite NET groups is simple NET groups. One way to make this simple NETG to larger group is NET direct product. As an addition, we allow rise to a new field called NT Structures (such as neutrosophic extended triplet direct product and semi-direct product. Another researchers can work on the application of NETEDP and NETIDP and semi-direct product to NT vector spaces (representation of the NETG), module theory, number theory, analysis, geometry, zigzag products of graphs and topological spaces.

Conflicts of Interest: The authors declare no conflict of interest.

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Data Envelopment Analysis for Simplified Neutrosophic Sets

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Abstract: In recent years, there has been a growing interest in neutrosophic theory, and there are several methods for solving various problems under neutrosophic environment. However, a few papers have discussed the Data envelopment analysis (DEA) with neutrosophic sets. So, in this paper, we propose an input-oriented DEA model with simplified neutrosophic numbers and present a new strategy to solve it. The proposed method is based on the weighted arithmetic average operator and has a simple structure. Finally, the new approach is illustrated with the help of a numerical example.

Keywords: Data envelopment analysis; Neutrosophic set; Simplified neutrosophic sets (SNSs); Aggregation operator.

1. Introduction

With the advent of technology and the complexity and volume of information, senior executives have required themselves to apply scientific methods to determine and increase the productivity of the organization under their jurisdiction. Data envelopment analysis (DEA) is a mathematical technique to evaluate the relative efficiency of a set of some homogeneous units called decision-making units (DMUs) that use multiple inputs to produce multiple outputs. DMUs are called homogeneous because they all employ the same inputs to produce the same outputs. DEA by constructing an efficiency frontier measures the relative efficiency of decision making units (DMUs). Charnes et al. [1] developed a DEA model (CCR) based on the seminal work of Farrell [2] under the assumption of constant returns to scale (CRS). Banker et al. [3] extended the pioneering work Charnes et al. [1] and proposed a model conventionally called BCC to measure the relative efficiency under the assumption of variable returns to scale (VRS). DEA technique has just been effectively connected in various cases such as broadcasting companies [4], banking institutions [5-8], R&D organizations [9-10], health care services [11-12], manufacturing [13-14], telecommunication [15], and supply chain management [16-19]. However, data in the standard models are certain, but there are numerous circumstances in real life where we have to face uncertain parameters. Zadeh [20] first proposed the theory of fuzzy sets (FSs) against certain logic where the membership degree is a real number between zero and one. After this work, many researchers studied on this topic; details of some researches can be observed in [21-30]. Several researchers also proposed some models of DEA under fuzzy environment [31-42]. However, Zadeh's fuzzy sets cannot deal with certain cases in which it is difficult to define the membership degree using one specific value. To overcome this lack of knowledge, Atanassov [43] introduced an extension of the FSs that called the intuitionistic fuzzy sets (IFSs). Although the theory of IFSs can handle incomplete information in various real-world issues, it cannot address all types of uncertainty such as indeterminate and inconsistent information.

Therefore, Smarandache [44-45], proposed the neutrosophic set (NS) as a strong general framework that generalizes the classical set concept, fuzzy set [20], interval-valued fuzzy set [46], intuitionistic fuzzy set [43], and interval-valued intuitionistic fuzzy set [47]. Neutrosophic set (NS) can deal with uncertain, indeterminate and incongruous information where the indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are completely independent. It can effectively describe uncertain, incomplete and inconsistent information and overcomes some limitations of the existing methods in depicting uncertain decision information. Moreover, some extensions of NSs, including interval neutrosophic set [48-51], bipolar neutrosophic set [52-54], single-valued neutrosophic set [55-59], simplified neutrosophic sets [60-64], multi-valued neutrosophic set [65-67], and neutrosophic linguistic set [68-70] have been presented and applied to solve various problems; see [71-80].

Although there are several approaches to solving various problems under neutrosophic environment, to the best of our knowledge, there are few investigations regarding DEA with neutrosophic sets. The first attempt has been proposed by Edalatpanah in [81] and further research has been presented in [82]. So, in this paper, we design a model of DEA with simplified neutrosophic numbers (SNNs) and establish a new strategy to solve it. The proposed method is based on the weighted arithmetic average operator and has a simple structure.

This paper organized as follows: some basic knowledge, concepts and arithmetic operations on SNNs are introduced in Section 2. In Section 3, we review some concepts of DEA and the input-oriented BCC model. In Section 4, we introduce the mentioned model of DEA under the simplified neutrosophic environment and propose a method to solve it. In Section 5, an example demonstrates the application of the proposed model. Finally, some conclusions and future research are offered in Section 6.

2. Simplified neutrosophic sets

Smarandache [44-45] has provided a variety of real-life examples for possible applications of his neutrosophic sets; however, it is difficult to apply neutrosophic sets to practical problems. Therefore, Ye [60] reduced neutrosophic sets of non-standard intervals into a kind of simplified neutrosophic sets (SNSs) of standard intervals that will preserve the operations of the neutrosophic sets. In this section, we will review the concept of SNSs, which are a subclass of neutrosophic sets briefly.

Definition 1 [60]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x)$: $X \to [0,1]$, $I_A(x)$: $X \to [0,1]$, and $F_A(x)$: $X \to [0,1]$. Then, a simplification of the neutrosophic set A is denoted by $A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$, which is called a SNS. Also, SNS satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 2 [60]. For SNSs A and B, $A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$ for every x in X.

Definition 3 [63]. Let A, B be two SNSs. Then the arithmetic relations are defined as:

$$(i) A \oplus B = < T_A(x) + T_B(x) - T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x) >, \tag{1}$$

$$(ii) A \otimes B = < T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x).I_B(x), F_A(x) + F_B(x) - F_A(x).F_B(x) >, \tag{2}$$

$$(iii) \lambda A = < 1-(1-T_A(x))^{\lambda}, (I_A(x))^{\lambda}, (F_A(x))^{\lambda} >, \lambda > 0. \tag{3}$$

$$(iv) A^{\lambda} = < T_A^{\lambda}(x), 1-(1-I_A(x))^{\lambda}, 1-(1-F_A(x))^{\lambda} >, \lambda > 0. \tag{4}$$

Definition 4 [60]. Let A_j (j = 1, 2, ... , n) be a SNS. The simplified neutrosophic weighted arithmetic average operator is defined as:

$$F_{\omega}(A_1, \ldots, A_n) = \sum_{j=1}^{n} \omega_j A_j \tag{5}$$

where $W=(\omega_1,\omega_2,\ldots,\omega_n)$ is the weight vector of Aj, $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1$.

Theorem 1 [63]**.** For the simplified neutrosophic weighted arithmetic average operator, the aggregated result is as follows:

$$F_\omega(A_1,\ldots,A_n)=\left\langle 1-\prod_{j=1}^{n}(1-T_{A_j}(x))^{\omega_j},\prod_{j=1}^{n}(I_{A_j}(x))^{\omega_j},\prod_{j=1}^{n}(F_{A_j}(x))^{\omega_j}\right\rangle. \tag{6}$$

3. The input-oriented BCC model of DEA

Data envelopment analysis (DEA) is a linear programming method for assessing the efficiency and productivity of decision-making units (DMUs). In the traditional DEA literature, various well-known DEA approaches can be found such as CCR and BCC models [1, 3]. The efficiency of a DMU is established as the ratio of sum weighted output to sum weighted input, subjected to happen between one and zero. Let DMUO is under consideration, then input-oriented BCC model for the relative efficiency is as follows [3]:

$$\begin{aligned}
&Min \quad \theta_o\\
&s.t\\
&\sum_{j=1}^{n}\lambda_j x_{ij}\le\theta_o x_{i_o} \quad , \quad i=1,2,\ldots,m\\
&\sum_{j=1}^{n}\lambda_j y_{rj}\ge y_{ro} \quad , \quad r=1,2,\ldots,s\\
&\sum_{j=1}^{n}\lambda_j=1\\
&\lambda_j\ge 0 \quad , \quad j=1,2,\ldots,n
\end{aligned} \tag{7}$$

In this model, each DMU (suppose that we have n DMUs) uses m inputs x_{ij} $(i=1,2,\ldots,m)$, to obtains s outputs y_{rj} $(r=1,2,\ldots,s)$. Here $u_r\,(r=1,2,\ldots,s)$ and $v_i\,(i=1,2,\ldots,m)$, are the weights of the i th input and r th output. This model is calculated for every DMU to find out its best input and output weights. If $\theta_o^*=1$, we say that the DMU$_o$ is efficient otherwise it is inefficient.

4. Simplified Neutrosophic Data Envelopment Analysis

In this section, we establish DEA under simplified neutrosophic environment. Consider the input and output for the j th DMU as $x_{ij}^{\aleph}=(T_{x_{ij}},I_{x_{ij}},F_{x_{ij}})$, $y_{rj}^{\aleph}=(T_{y_{rj}},I_{y_{rj}},F_{y_{rj}})$ which are the simplified neutrosophic numbers (SNN). Then the simplified neutrosophic BCC model that called SNBCC is defined as follows:

$$\begin{aligned}
&Min \quad \theta_o\\
&s.t\\
&\sum_{j=1}^{n}\lambda_j x_{ij}^{\aleph}\le\theta_o x_{i_o}^{\aleph}, \quad i=1,2,\ldots,m\\
&\sum_{j=1}^{n}\lambda_j y_{rj}^{\aleph}\ge y_{r_o}^{\aleph}, \quad r=1,2,\ldots,s\\
&\sum_{j=1}^{n}\lambda_j=1,\\
&\lambda_j\ge 0, \quad j=1,2,\ldots,n.
\end{aligned} \tag{8}$$

Next, to solve the model (8) we propose the following algorithm:

Algorithm 1.

Step 1. Consider the DEA model (8) that the inputs and outputs of each DMU are SNN.

Step 2. Using the Definition 3 and Theorem 1, the SNBCC model of Step 1 can be transformed into the following model:

$$
\begin{aligned}
&Min \quad \theta_o \\
&s.t \\
&\left(1-\prod_{j=1}^{n}(1-T_{x_{ij}})^{\lambda_j}, \prod_{j=1}^{n}(I_{x_{ij}})^{\lambda_j}, \prod_{j=1}^{n}(F_{x_{ij}})^{\lambda_j}\right) \le \left(1-(1-T_{x_{io}})^{\theta_o}, (I_{x_{io}})^{\theta_o}, (F_{x_{io}})^{\theta_o}\right) \\
&\left(1-\prod_{j=1}^{n}(1-T_{y_{rj}})^{\lambda_j}, \prod_{j=1}^{n}(I_{y_{rj}})^{\lambda_j}, \prod_{j=1}^{n}(F_{y_{rj}})^{\lambda_j}\right) \ge \left(T_{y_{ro}}, I_{y_{ro}}, F_{y_{ro}}\right) \\
&\sum_{j=1}^{n}\lambda_j = 1, \\
&\lambda_j \ge 0, \qquad j = 1, 2, ..., n.
\end{aligned} \tag{9}
$$

Step 3. Using Definition 2, the SNBCC model of Step 2 can be transformed into the following model:

$$
\begin{aligned}
&Min \quad \theta_o \\
&s.t \\
&\prod_{j=1}^{n}(1-T_{x_{ij}})^{\lambda_j} \ge (1-T_{x_{io}})^{\theta_o}, \qquad i = 1, 2, \ldots, m \\
&\prod_{j=1}^{n}(I_{x_{ij}})^{\lambda_j} \ge (I_{x_{io}})^{\theta_o}, \qquad i = 1, 2, \ldots, m \\
&\prod_{j=1}^{n}(F_{x_{ij}})^{\lambda_j} \ge (F_{x_{io}})^{\theta_o}, \qquad i = 1, 2, \ldots, m \\
&\prod_{j=1}^{n}(1-T_{y_{rj}})^{\lambda_j} \le (1-T_{y_{ro}}), \qquad r = 1, 2, \ldots, s \\
&\prod_{j=1}^{n}(I_{y_{rj}})^{\lambda_j} \le I_{y_{ro}}, \qquad r = 1, 2, \ldots, s \\
&\prod_{j=1}^{n}(F_{y_{rj}})^{\lambda_j} \le F_{y_{ro}}, \qquad r = 1, 2, \ldots, s \\
&\sum_{j=1}^{n}\lambda_j = 1, \\
&\lambda_j \ge 0, \qquad j = 1, 2, \ldots, n.
\end{aligned} \tag{10}
$$

Step 4. Using the natural logarithm, transform the nonlinear model of (10) into the following linear model:

$$Min \quad \theta_o \tag{11}$$

$$s.t$$

$$\sum_{j=1}^{n}\lambda_j \ln(1-T_{x_{ij}}) \ge \theta_o \ln(1-T_{x_{io}}), \qquad i = 1, 2, ..., m \tag{12}$$

$$\sum_{j=1}^{n}\lambda_j \ln(I_{x_{ij}}) \ge \theta_o \ln(I_{x_{io}}), \qquad i = 1, 2, ..., m \tag{13}$$

$$\sum_{j=1}^{n}\lambda_j \ln(F_{x_{ij}}) \ge \theta_o \ln(F_{x_{io}}), \qquad i = 1, 2, ..., m \tag{14}$$

$$\sum_{j=1}^{n}\lambda_j \ln(1-T_{y_{rj}}) \le \ln(1-T_{y_{ro}}), \qquad r = 1, 2, ..., s \tag{15}$$

$$\sum_{j=1}^{n}\lambda_j \ln(I_{y_{rj}}) \le \ln(I_{y_{ro}}), \qquad r = 1, 2, ..., s \tag{16}$$

$$\sum_{j=1}^{n} \lambda_j \ln(F_{y_{rj}}) \le \ln(F_{y_{ro}}), \qquad r = 1, 2, \ldots, s \tag{17}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \tag{18}$$

$$\lambda_j \ge 0, \qquad j = 1, 2, \ldots, n.$$

Step 5. Run model (11) and obtain the optimal solution.

5. Numerical example

In this section, an example of DEA problem under simplified neutrosophic environment is used to demonstrate the validity and effectiveness of the proposed model.

Example 5.1. Consider 10 DMUs with three inputs and outputs where all the input and output data are designed as SNN (see tables 1 and 2).

Table 1. DMUs with three SNN inputs

DMUS	Inputs 1	Inputs 2	Inputs 3
DMU1	<0.75, 0.1, 0.15>	<0.75,0.1, 0.15>	<0.8, 0.05, 0.1>
DMU2	<0.85, 0.2,0.15>	<0.6, 0.05,0.05>	<0.9, 0.1, 0.2>
DMU3	<0.9, 0.01, 0.05>	<0.95, 0.01, 0.01>	<0.98, 0.01, 0.01>
DMU4	<0.7,0.2, 0.1>	<0.65, 0.2, 0.15>	<0.8, 0.05, 0.2>
DMU5	<0.9, 0.05, 0.1>	<0.95, 0.05, 0.05>	<0.7, 0.2, 0.4>
DMU6	<0.85, 0.2, 0.1>	<0.7, 0.05, 0.1>	<0.6, 0.2, 0.3>
DMU7	<0.8, 0.3, 0.1>	<0.9, 0.5, 0.1>	<0.8, 0.1, 0.3>
DMU8	<0.55, 0.3, 0.35>	<0.65, 0.2, 0.25>	<0.5, 0.35, 0.4>
DMU9	<0.8, 0.05, 0.1>	<0.9, 0.01, 0.05>	<0.8, 0.05, 0.1>
DMU10	<0.6, 0.1, 0.3>	<0.8. 0.3. 0.1>	<0.65, 0.2, 0.1>

Table 2. DMUs with three SNN outputs.

DMUS	Outputs 1	Outputs 2	Outputs 3
DMU1	<0.7, 0.15, 0.2>	<0.7,0.15, 0.2>	<0.65, 0.2, 0.25>
DMU2	<0.15, 0.2,0.25>	<0.15, 0.2,0.25>	<0.25, 0.15, 0.05>
DMU3	<0.75, 0.1, 0.15>	<0.7, 0.15, 0.2>	<0.8, 0.05, 0.1>
DMU4	<0.5,0.35, 0.4>	<0.6, 0.25, 0.3>	<0.55, 0.3, 0.35>
DMU5	<0.6, 0.2, 0.25>	<0.6, 0.15, 0.4>	<0.3, 0.5, 0.5>
DMU6	<0.55, 0.3, 0.35>	<0.5, 0.5, 0.5>	<0.6, 0.25, 0.3>
DMU7	<0.8, 0.1, 0.2>	<0.3, 0.01, 0.05>	<0.9, 0.05, 0.05>
DMU8	<0.8, 0.1, 0.3>	<0.8, 0.25, 0.3>	<0.85, 0.2, 0.2>
DMU9	<0.65, 0.2, 0.25>	<0.7, 0.15, 0.2>	<0.75, 0.1, 0.15>
DMU10	<0.6, 0.1, 0.5>	<0.75. 0.1. 0.3>	<0.8, 0.3, 0.5>

Next, we use Algorithm.1 to solve the mentioned performance assessment problem. For example, The Algorithm.1 for DMU_1 can be used as follows:

Step 1. Obtain the SNBCC model (8):

$$
\begin{aligned}
& Min \quad \theta_1 \\
& s.t \\
& \begin{pmatrix}
\lambda_1 < 0.75, 0.1, 0.15 > \oplus \lambda_2 < 0.85, 0.2, 0.15 > \oplus \lambda_3 < 0.9, 0.01, 0.05 > \oplus \\
\lambda_4 < 0.7, 0.2, 0.1 > \oplus \lambda_5 < 0.9, 0.05, 0.1 > \oplus \lambda_6 < 0.85, 0.2, 0.1 > \oplus \\
\lambda_7 < 0.8, 0.3, 0.35 > \oplus \lambda_8 < 0.8, 0.05, 0.1 > \oplus \lambda_9 < 0.6, 0.1, 0.3 > \oplus \\
\lambda_{10} < 0.6, 0.1, 0.3 >
\end{pmatrix} \leq \left(\theta_1 < 0.75, 0.1, 0.15 >\right), \\
& \begin{pmatrix}
\lambda_1 < 0.7, 0.1, 0.2 > \oplus \lambda_2 < 0.6, 0.05, 0.05 > \oplus \lambda_3 < 0.95, 0.01, 0.01 > \oplus \\
\lambda_4 < 0.65, 0.2, 0.15 > \oplus \lambda_5 < 0.95, 0.05, 0.05 > \oplus \lambda_6 < 0.7, 0.05, 0.1 > \oplus \\
\lambda_7 < 0.9, 0.5, 0.1 > \oplus \lambda_8 < 0.65, 0.2, 0.25 > \oplus \lambda_9 < 0.9, 0.01, 0.05 > \oplus \\
\lambda_{10} < 0.8, 0.3, 0.1 >
\end{pmatrix} \leq \left(\theta_1 < 0.7, 0.1, 0.2 >\right), \\
& \begin{pmatrix}
\lambda_1 < 0.8, 0.05, 0.1 > \oplus \lambda_2 < 0.9, 0.1, 0.2 > \oplus \lambda_3 < 0.98, 0.01, 0.01 > \oplus \\
\lambda_4 < 0.8, 0.05, 0.2 > \oplus \lambda_5 < 0.7, 0.2, 0.4 > \oplus \lambda_6 < 0.6, 0.2, 0.3 > \oplus \\
\lambda_7 < 0.8, 0.1, 0.3 > \oplus \lambda_8 < 0.5, 0.35, 0.4 > \oplus \lambda_9 < 0.7, 0.05, 0.1 > \oplus \\
\lambda_{10} < 0.65, 0.2, 0.1 >
\end{pmatrix} \leq \left(\theta_1 < 0.8, 0.05, 0.1 >\right), \\
& \begin{pmatrix}
\lambda_1 < 0.7, 0.15, 0.2 > \oplus \lambda_2 < 0.15, 0.2, 0.25 > \oplus \lambda_3 < 0.75, 0.1, 0.15 > \oplus \\
\lambda_4 < 0.5, 0.35, 0.4 > \oplus \lambda_5 < 0.6, 0.2, 0.25 > \oplus \lambda_6 < 0.55, 0.3, 0.35 > \oplus \\
\lambda_7 < 0.8, 0.1, 0.2 > \oplus \lambda_8 < 0.8, 0.1, 0.3 > \oplus \lambda_9 < 0.65, 0.2, 0.25 > \oplus \\
\lambda_{10} < 0.6, 0.1, 0.5 >
\end{pmatrix} \geq \left(< 0.7, 0.15, 0.2 >\right), \\
& \begin{pmatrix}
\lambda_1 < 0.6, 0.1, 0.3 > \oplus \lambda_2 < 0.2, 0.1, 0.3 > \oplus \lambda_3 < 0.7, 0.15, 0.2 > \oplus \\
\lambda_4 < 0.6, 0.25, 0.3 > \oplus \lambda_5 < 0.6, 0.15, 0.4 > \oplus \lambda_6 < 0.5, 0.5, 0.5 > \oplus \\
\lambda_7 < 0.3, 0.01, 0.05 > \oplus \lambda_8 < 0.8, 0.25, 0.3 > \oplus \lambda_9 < 0.7, 0.15, 0.2 > \oplus \\
\lambda_{10} < 0.75, 0.1, 0.3 >
\end{pmatrix} \geq \left(< 0.6, 0.1, 0.3 >\right), \\
& \begin{pmatrix}
\lambda_1 < 0.65, 0.2, 0.25 > \oplus \lambda_2 < 0.25, 0.15, 0.05 > \oplus \lambda_3 < 0.8, 0.05, 0.1 > \oplus \\
\lambda_4 < 0.55, 0.3, 0.35 > \oplus \lambda_5 < 0.3, 0.5, 0.5 > \oplus \lambda_6 < 0.6, 0.25, 0.3 > \oplus \\
\lambda_7 < 0.9, 0.05, 0.05 > \oplus \lambda_8 < 0.85, 0.2, 0.2 > \oplus \lambda_9 < 0.75, 0.1, 0.15 > \oplus \\
\lambda_{10} < 0.8, 0.3, 0.5 >
\end{pmatrix} \geq \left(< 0.65, 0.2, 0.25 >\right), \\
& \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} = 1, \\
& \lambda_j \geq 0, \qquad j = 1, 2, \ldots, 10.
\end{aligned}
$$

Step 2. Using the Step 4 of Algorithm 1, we have:

$$
\begin{aligned}
& Min \quad \theta_1 \\
& s.t
\end{aligned}
$$

(Using Eq. (12))

$$
\begin{aligned}
& \lambda_1 \ln(0.25) + \lambda_2 \ln(0.15) + \lambda_3 \ln(0.1) + \lambda_4 \ln(0.3) + \lambda_5 \ln(0.1) + \\
& \lambda_6 \ln(0.15) + \lambda_7 \ln(0.2) + \lambda_8 \ln(0.2) + \lambda_9 \ln(0.4) + \lambda_{10} \ln(0.4) \geq \theta_1 \ln(0.25),
\end{aligned}
$$

$$\lambda_1 \ln(0.3) + \lambda_2 \ln(0.4) + \lambda_3 \ln(0.05) + \lambda_4 \ln(0.35) + \lambda_5 \ln(0.05) + \lambda_6 \ln(0.3) + \lambda_7 \ln(0.1) + \lambda_8 \ln(0.35) + \lambda_9 \ln(0.1) + \lambda_{10} \ln(0.2) \geq \theta_1 \ln(0.3)$$

$$\lambda_1 \ln(0.2) + \lambda_2 \ln(0.1) + \lambda_3 \ln(0.02) + \lambda_4 \ln(0.2) + \lambda_5 \ln(0.3) + \lambda_6 \ln(0.4) + \lambda_7 \ln(0.2) + \lambda_8 \ln(0.5) + \lambda_9 \ln(0.3) + \lambda_{10} \ln(0.35) \geq \theta_1 \ln(0.2)$$

(Using Eq. (13))

$$\lambda_1 \ln(0.1) + \lambda_2 \ln(0.2) + \lambda_3 \ln(0.01) + \lambda_4 \ln(0.2) + \lambda_5 \ln(0.05) + \lambda_6 \ln(0.2) + \lambda_7 \ln(0.3) + \lambda_8 \ln(0.05) + \lambda_9 \ln(0.1) + \lambda_{10} \ln(0.1) \geq \theta_1 \ln(0.1)$$

$$\lambda_1 \ln(0.1) + \lambda_2 \ln(0.05) + \lambda_3 \ln(0.01) + \lambda_4 \ln(0.2) + \lambda_5 \ln(0.05) + \lambda_6 \ln(0.05) + \lambda_7 \ln(0.5) + \lambda_8 \ln(0.2) + \lambda_9 \ln(0.01) + \lambda_{10} \ln(0.3) \geq \theta_1 \ln(0.1)$$

$$\lambda_1 \ln(0.05) + \lambda_2 \ln(0.05) + \lambda_3 \ln(0.01) + \lambda_4 \ln(0.05) + \lambda_5 \ln(0.2) + \lambda_6 \ln(0.2) + \lambda_7 \ln(0.1) + \lambda_8 \ln(0.35) + \lambda_9 \ln(0.05) + \lambda_{10} \ln(0.2) \geq \theta_1 \ln(0.05)$$

(Using Eq. (14))

$$\lambda_1 \ln(0.15) + \lambda_2 \ln(0.15) + \lambda_3 \ln(0.05) + \lambda_4 \ln(0.1) + \lambda_5 \ln(0.1) + \lambda_6 \ln(0.1) + \lambda_7 \ln(0.35) + \lambda_8 \ln(0.1) + \lambda_9 \ln(0.3) + \lambda_{10} \ln(0.3) \geq \theta_1 \ln(0.15)$$

$$\lambda_1 \ln(0.2) + \lambda_2 \ln(0.05) + \lambda_3 \ln(0.01) + \lambda_4 \ln(0.15) + \lambda_5 \ln(0.05) + \lambda_6 \ln(0.1) + \lambda_7 \ln(0.1) + \lambda_8 \ln(0.25) + \lambda_9 \ln(0.05) + \lambda_{10} \ln(0.1) \geq \theta_1 \ln(0.2)$$

$$\lambda_1 \ln(0.1) + \lambda_2 \ln(0.2) + \lambda_3 \ln(0.01) + \lambda_4 \ln(0.2) + \lambda_5 \ln(0.4) + \lambda_6 \ln(0.3) + \lambda_7 \ln(0.3) + \lambda_8 \ln(0.4) + \lambda_9 \ln(0.1) + \lambda_{10} \ln(0.1) \geq \theta_1 \ln(0.1)$$

(Using Eq. (15))

$$\lambda_1 \ln(0.3) + \lambda_2 \ln(0.85) + \lambda_3 \ln(0.25) + \lambda_4 \ln(0.5) + \lambda_5 \ln(0.4) + \lambda_6 \ln(0.45) + \lambda_7 \ln(0.2) + \lambda_8 \ln(0.2) + \lambda_9 \ln(0.35) + \lambda_{10} \ln(0.4) \leq \ln(0.3),$$

$$\lambda_1 \ln(0.4) + \lambda_2 \ln(0.8) + \lambda_3 \ln(0.3) + \lambda_4 \ln(0.4) + \lambda_5 \ln(0.4) + \lambda_6 \ln(0.5) + \lambda_7 \ln(0.7) + \lambda_8 \ln(0.2) + \lambda_9 \ln(0.3) + \lambda_{10} \ln(0.25) \leq \ln(0.4),$$

$$\lambda_1 \ln(0.35) + \lambda_2 \ln(0.75) + \lambda_3 \ln(0.2) + \lambda_4 \ln(0.45) + \lambda_5 \ln(0.7) + \lambda_6 \ln(0.4) + \lambda_7 \ln(0.1) + \lambda_8 \ln(0.15) + \lambda_9 \ln(0.25) + \lambda_{10} \ln(0.2) \leq \ln(0.35),$$

(Using Eq. (16))

$$\lambda_1 \ln(0.15) + \lambda_2 \ln(0.2) + \lambda_3 \ln(0.1) + \lambda_4 \ln(0.35) + \lambda_5 \ln(0.2) + \lambda_6 \ln(0.3) + \lambda_7 \ln(0.1) + \lambda_8 \ln(0.1) + \lambda_9 \ln(0.2) + \lambda_{10} \ln(0.1) \leq \ln(0.15),$$

$$\lambda_1 \ln(0.1) + \lambda_2 \ln(0.1) + \lambda_3 \ln(0.15) + \lambda_4 \ln(0.25) + \lambda_5 \ln(0.15) + \lambda_6 \ln(0.5) + \lambda_7 \ln(0.01) + \lambda_8 \ln(0.25) + \lambda_9 \ln(0.15) + \lambda_{10} \ln(0.1) \leq \ln(0.1),$$

$$\lambda_1 \ln(0.2) + \lambda_2 \ln(0.15) + \lambda_3 \ln(0.05) + \lambda_4 \ln(0.3) + \lambda_5 \ln(0.5) +$$
$$\lambda_6 \ln(0.25) + \lambda_7 \ln(0.05) + \lambda_8 \ln(0.2) + \lambda_9 \ln(0.1) + \lambda_{10} \ln(0.3) \le \ln(0.2),$$

(Using Eq. (17))

$$\lambda_1 \ln(0.2) + \lambda_2 \ln(0.25) + \lambda_3 \ln(0.15) + \lambda_4 \ln(0.4) + \lambda_5 \ln(0.25) +$$
$$\lambda_6 \ln(0.35) + \lambda_7 \ln(0.2) + \lambda_8 \ln(0.3) + \lambda_9 \ln(0.25) + \lambda_{10} \ln(0.5) \le \ln(0.2),$$

$$\lambda_1 \ln(0.3) + \lambda_2 \ln(0.3) + \lambda_3 \ln(0.2) + \lambda_4 \ln(0.3) + \lambda_5 \ln(0.4) +$$
$$\lambda_6 \ln(0.5) + \lambda_7 \ln(0.05) + \lambda_8 \ln(0.3) + \lambda_9 \ln(0.2) + \lambda_{10} \ln(0.3) \le \ln(0.3),$$

$$\lambda_1 \ln(0.25) + \lambda_2 \ln(0.05) + \lambda_3 \ln(0.1) + \lambda_4 \ln(0.35) + \lambda_5 \ln(0.5) +$$
$$\lambda_6 \ln(0.3) + \lambda_7 \ln(0.05) + \lambda_8 \ln(0.2) + \lambda_9 \ln(0.15) + \lambda_{10} \ln(0.5) \le \ln(0.25),$$

(Using Eq. (18))

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} = 1,$$
$$\lambda_j \ge 0, \qquad j = 1, 2, \dots, 10.$$

Step 3. After computations with Lingo, we obtain $\theta_1^* = 0.9068$ for *DMU1*.

Similarly, for the other DMUs, we report the results in Table 3.

Table 3. The efficiencies of the other DMUs

DMUs	1	2	3	4	5	6	7	8	9	10
θ^*	0.9068	0.9993	0.5153	0.9973	0.6382	0.6116	1	1	0.6325	1
Rank	4	2	8	3	5	7	1	1	6	1

By these results, we can see that DMUs 7, 8, and 10 are efficient and others are inefficient.

6. Conclusions and future work

There are several approaches to solving various problems under neutrosophic environment. However, to the best of our knowledge, the Data Envelopment Analysis (DEA) has not been discussed with neutrosophic sets until now. This paper, therefore, plans to fill this gap and a new method has been designed to solve an input-oriented DEA model with simplified neutrosophic numbers. A numerical example has been illustrated to show the efficiency of the proposed method. The proposed approach has produced promising results from computing efficiency and performance aspects. Moreover, although the model, arithmetic operations and results presented here demonstrate the effectiveness of our approach, it could also be considered in other DEA models and their applications to banks, police stations, hospitals, tax offices, prisons, schools and universities. As future researches, we intend to study these problems.

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Neutrosophic Vague Binary Sets

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Abstract: Vague sets and neutrosophic sets play an inevitable role in the developing scenario of mathematical world. In this modern era of artificial intelligence most of the real life situations are found to be immersed with unclear data. Even the newly developed concepts are found to fail with such problems. So new sets like Plithogenic and new combinations like neutrosophic vague arose. Classical set theory dealt with single universe and can be studied by taking it's subsets. Situations demand two universes instead of a unique one in certain problems. In this paper two universes are introduced simultaneously and under consideration in a neutrosophic vague environment. It's basic operations, topology and continuity are also discussed with examples. A real life example is also discussed.

Keywords: binary set, fuzzy binary set, vague binary set, neutrosophic vague binary sets, neutrosophic vague binary topology, neutrosophic vague binary continuity

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1. Introduction

Functions are tightly packed but relations are not. They are more general than functions. Decimal system deals with ten digits while binary with two - only with 0 and 1. For detecting electrical signal's on or off state binary system can be used more effectively. It is the prime reason of selecting binary language in computers. Binary operations in algebra will give another idea! After a binary operation, 'operands' produce an element which is also a member of the parent set - means 'domain and co-domain' are in the same set. But binary relations are quite different from the ideas mentioned above. They are subsets of the cartesian product of the sets under consideration, taken in a special way. It is clear that binary stands for two. In point-set topology information from elements of topology will give information about subsets of the universal set under consideration. But real life can't be confined into a single universal set. It may be two or more than two. Being an extension of classical sets [George Cantor, 1874-1897] [27], fuzzy sets (FS's) [Zadeh, 1965] [29] can deal with partial membership. In intuitionistic fuzzy sets (IFS's) [Attanassov, 1986] [12] two membership grades are there - truth and false. As an extension of fuzzy sets Gau and Buehrer [9] introduced vague sets in 1993. Neutro-sophy means knowledge of neutral thought. It is a new branch of philosophy introduced by Florentin Smarandache [6] in 1995 - by giving an additional component - indeterminacy. Movement of paradoxism was set up by him in early 1980's. New concept dealt with the principle of using non-artistic elements to set artistic. Within no time so many hybrid structures developed by using the merits of the newly developed theory. In 2014, Alblowmi. S. A and Mohmed Eisa [1] gave some new concepts of neutrosophic sets. In 1996, Dontchev [5] developed Contra-continous functions and strongly s-closed spaces. In 2014, Salama A.A, Florentin Smarandache and Valeri Kromov [25] developed neutrosophic closed set and neutrosophic continous functions.

Shawkat Alkhazaleh [26] introduced the concept of neutrosophic vague in 2015. To loosen the hard structure of classical sets, Molodtsov [15] introduced soft set theory in 1999. In 2017, Gulfam Shahzadi, Muhammad Akram and Arsham Borumand Saeid [10] gave an application via 3 different methods of single-valued neutrosophic sets in medical field. Mai Mohamed et al., [19] developed a critical path problem in network diagrams under uncertain activity time. Later in 2018, Mohamed Abdel Basset et al., [23] developed a project selection method using TOPSIS and trapezoidal neutrosophic number. Mai Mohamed et al., [21] made a medical application to aid cancer patients based on neutrosophic set theory. As an extenstion to crisp, fuzzy, intuitionistic and neutrosophic sets, Florentin Smarandache [7] introduced plithogenic sets in 2018. In 2018, Mary Margaret A, Trinita Pricilla M [14] developed neutrosophic vague generalized Pre-continous and irresolute mappings. In 2018, Mohamed Abdel-Basset, Asmaa Atef, Florentin Smarandache [18] introduced a hybrid neutrosophic multiple criteria group decision making approach for project selection. In 2018, Vildan Cetkin and Halis Ayg$\ddot{u}n$ [28] developed an approach to neutrosophic ideals. Later in 2019, Mohamed Abdel-Basset et al., [22] applied plithogenic aggregation operators to a decision making method for projects viz., 'supply chain sustainability'. In 2019, Mohamed Abdel-Basset and Mai-Mohamed [20] introduced linear fractional programming based on triangular neutrosophic numbers. In 2019, Mohamed Abdel- Basset, Gunasekaran Manogaran et al., [17] developed a neutrosophic multi criteria decision making method for type 2 diabetic patients. In 2019, Hazwani Hashim, Lazim Abdullah and Asharaf Al-Quran [11] developed interval neutrosophic vague sets. In 2019, Mohamed Abdel-Basset, El-hosney, M., Gamal., & Smarandache.F [16] gave a new model for evaluation hospital medical care systems based on plithogenic sets. In 2019 Banu Priya et al., [2] investigated neutrosophic αgs continuity and neutrosophic αgs irresolute maps. In 2019, Dhavaseelan et al., [3] introduced neutrosophic α^m-closed setsand discussed it's continuity, strongly continuity and irresoluteness. In 2019 Dhavaseelan, Subash Moorthy and S. Jafari [4] introduced gNcompact open topology and discussed on generalized neutrosophic exponential map. In 2019, Muhammad Akram et al., [24] proposed the notion of neutrosophic Soft topological K-Algebras and discussed it's several terms like C_5- connectedness, super connectedness, compactness etc. In 2019, Mary Margaret A, Trinita Pricilla M and Shawkat Alkhazaleh [13] developed neutrosophic vague topological spaces. Vague binary soft set theory was developed by Dr. Francina Shalini. A [8] and Remya.P.B in 2018. In this paper a new concept neutrosophic vague binary set is developed by using two universes. It's topology, continuity and various types of continuities are also under concern.

2. Preliminaries

Definition 2.2. [26] (Neutrosophic vague set)

A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X can be written as $A_{NV} = \{\langle x\,;\, \hat{T}_{AN}(X)\,;\, \hat{I}_{AN}(X), \hat{F}_{AN}(X)\rangle; x \in X\}$ whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as

$\hat{T}_{A_{NV}}(x)$=$[T^-, T^+]$, $\hat{I}_{A_{NV}}(x)$= $[I^-, I^+]$ and $\hat{F}_{A_{NV}}(x)$= $[F^-, F^+]$

where (1) T^+= $1-F^-$; $F^+ = 1- T^-$ and

(2) $^-0 \le T^- + I^- + F^- \le 2^+$

$^-0 \le T^+ + I^+ + F^+ \le 2^+$

Definition 2.3. [26] (Unit Neutrosophic Vague Set)

Let Ψ_{NV} be a neutrosophic vague set (NVS in short) of the universe U where $\forall\ u_i \in U$,

$\hat{T}_{\Psi_{NV}}(x) = [1, 1]$, $\hat{I}_{\Psi_{NV}}(x) = [0, 0]$, $\hat{F}_{\Psi_{NV}}(x) = [0, 0]$, then Ψ_{NV} is called a unit NVS, where $1 \le i \le n$

Definition 2.4. [26] (Zero Neutrosophic Vague Set)

Let Φ_{NV} be a neutrosophic vague set (NVS in short) of the universe U where $\forall\ u_i \in U$,

$\hat{T}_{\Phi_{NV}}(x) = [0, 0]$, $\hat{I}_{\Phi_{NV}}(x) = [1, 1]$, $\hat{F}_{\Phi_{NV}}(x) = [1, 1]$, then Φ_{NV} is called a zero NVS, where $1 \le i \le n$

Definition 2.5. [26] (Neutrosophic vague subset)

Let A_{NV} and B_{NV} be two $NVS's$ of the universe U.

If $\forall\ u_i \in$; [1 ≤ i ≤ n]

1. $\hat{T}_{A_{NV}}(u_i) \le \hat{T}_{B_{NV}}(u_i)$

2. $\hat{I}_{A_{NV}}\ (u_i)\ \geq\ \hat{I}_{B_{NV}}\ (u_i)$ and
3. $\hat{F}_{A_{NV}}\ (u_i) \geq\ \hat{F}_{B_{NV}}\ (u_i)$

then the $NVS\ A_{NV}$ are included by B_{NV} denoted by $A_{NV}\ \subseteq\ B_{NV}$

Definition 2.6. [26] (Complement of a Neutrosophic vague set)

The complement of a $NVS\ A_{NV}$ is denoted by $A^c{}_{NV}$ and is defined by

$\hat{T}^c{}_{A_{NV}}(x)$ = [1−T^+, 1−T^-], $\hat{I}^c A_{NV}\ (x)$ = [1−I^+, 1−I^-] and $\hat{F}^c A_{NV}\ (x)$ = [1−F^+, 1−F^-]

Definition 2.7. [26] (Union of Neutrosophic vague sets)

Union of two $NVS's$ A_{NV} and B_{NV} is a NVS C_{NV} written as $C_{NV} = A_{NV} \cup B_{NV}$ whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$\hat{T}_{C_{NV}}(x) =$ [max $(T^-\ A_{NV}(x), T^- B_{NV}(x))$, max $(T^+ A_{NV}(x), T^+\ B_{NV}\ (x))$]

$\hat{I}_{C_{NV}}(x) =$ [min $(I^-\ A_{NV}(x),\ I^- B_{NV}(x))$, min $(I^+ A_{NV}(x),\ \ I^+\ B_{NV}\ (x))$]

$\hat{F}_{C_{NV}}(x) =$ [min $(F^-\ A_{NV}(x), F^- B_{NV}(x))$, min $(F^+ A_{NV}(x), F^+\ B_{NV}\ (x))$]

Definition 2.8. [26] (Intersection of Neutrosophic vague sets)

Intersection of two $NVS's$ A_{NV} and B_{NV} is a NVS C_{NV} written as $D_{NV} = A_{NV} \cap B_{NV}$ whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$\hat{T}_{D_{NV}}\ (x) =$ [min $(T^-\ A_{NV}(x), T^- B_{NV}(x))$, min $(T^+ A_{NV}(x), T^+\ B_{NV}\ (x))$]

$\hat{I}_{D_{NV}}\ (x) =$ [max $(I^-\ A_{NV}(x),\ I^- B_{NV}(x))$, max $(I^+ A_{NV}(x),\ \ I^+\ B_{NV}\ (x))$]

$\hat{F}_{D_{NV}}(x) =$ [max $(F^-\ A_{NV}(x), F^- B_{NV}(x))$, max $(F^+ A_{NV}(x), F^+\ B_{NV}\ (x))$]

Definition 2.9.[14]

Let (X, τ) be a topological space. A subset A of X is called:

(i) Semi-closed set if $int(cl(A)) \subseteq A$
(ii) Pre-closed set if $cl(int(A)) \subseteq A$
(iii) Semi-pre closed set if $int(cl(int(A))) \subseteq A$
(iv) Regular-closed set if $A = cl(int(A))$
(v) Generalized semi-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X

Definition 2.10. [4] (Image and Pre-image of neutrosophic vague sets)

Let X_{NV} and Y_{NV} be two non-empty neutrosophic vague sets and f : $X_{NV} \longrightarrow Y_{NV}$ be a function, then the following statements hold:

(1)If $B_{NV}\ = \{\langle x,\ \hat{T}_B(x);\ \hat{I}_B(x);\ \hat{F}_B(x)\rangle; x \in X_{NV}\}$ is a NVS in Y_{NV}, then the preimage of B_{NV} under f , denoted

by $f^{-1}(B_{NV})$, is the NVS in X_{NV} defined by

$$f^{-1}(B_{NV}) = \left\{\langle x,\ f^{-1}\left(\hat{T}_B(x)\right)\ ;\ f^{-1}\left(\hat{I}_B(x)\right);\ f^{-1}\left(\hat{F}_B(x)\right)\rangle\ ; x \in X_{NV}\right\}$$

(2)If $A_{NV}\ = \{\langle x,\ \hat{T}_A(x);\ \hat{I}_A(x);\ \hat{F}_A(x)\rangle; x \in X_{NV}\}$ is a NVS in X_{NV}, then the image of A_{NV} under f , denoted

by $f(A_{NV})$, is the NVS in Y_{NV} defined by

$$f(A_{NV}) = \left\{\langle y,\ f_{sup}\left(\hat{T}_A(y)\right)\ ;\ f_{inf}\left(\hat{I}_A(y)\right);\ f_{inf}\left(\hat{F}_A(y)\right)\rangle\ ; y \in Y_{NV}\right\}$$

where

$$f_{sup}\left(\hat{T}_A(y)\right) = \begin{cases} sup_{x\in f^{-1}(\mathrm{y})}\hat{T}_A(x), & if\ f^{-1}(\mathrm{y}) \neq \emptyset \\ 0, & otherwise \end{cases}$$

$$f_{inf}\left(\hat{I}_A(y)\right) = \begin{cases} sup_{x\in f^{-1}(\mathrm{y})}\hat{I}_A(x), & if\ f^{-1}(\mathrm{y}) \neq \emptyset \\ 0, & otherwise \end{cases}$$

$$f_{inf}\left(\hat{F}_A(y)\right) = \begin{cases} sup_{x\in f^{-1}(\mathrm{y})}\hat{T}_A(x), & if\ f^{-1}(\mathrm{y}) \neq \emptyset \\ 0, & otherwise \end{cases}$$

for each $y \in Y_{NV}$

Definition 2.11.[5] (Strongly continous functions)

A function f : (X, τ) $\rightarrow$(Y, σ) to be strongly continous if $f(\bar{A}) \subset f(A)$, $\forall$ subset A of X or equivalently, if the inverse image of every set in Y is clopen in X.

Definition 2.12. [14] (Neutrosophic Vague Continuous Mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague continuous (NV continuous) if $f^{-1}(V)$ is neutrosophic vague closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.13 [14] (Neutrosophic Vague semi-continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague semi-continous if $f^{-1}(V)$ is neutrosophic vague semi-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.14 [14] (Neutrosophic Vague pre-continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague pre-continous if $f^{-1}(V)$ is neutrosophic vague pre-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.15 [14] (Neutrosophic Vague regular continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague regular continous if $f^{-1}(V)$ is neutrosophic vague regular-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.16 [14] (Neutrosophic Vague semi pre-continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague semi pre-continous if $f^{-1}(V)$ is neutrosophic vague semi pre-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

3. Neutrosophic Vague Binary Sets

In this section neutrosophic vague binary sets are discussed with examples. For this as a preliminary tool fuzzy binary sets and vague binary sets are discussed as a general case by taking all members instead of taking a subset of cartesian product in a confined manner.

Definition 3.1. (Binary Set)

Binary set A over a common universe $\left\{U_1 = \{x_j/\ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\right\}$ is an object of the form $\check{A} = \{\langle x_j\rangle, \langle y_k\rangle\}$

Definition 3.2. (Fuzzy Binary Set)

Fuzzy binary set A over a common universe $\left\{U_1 = \{x_j/\ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\right\}$ is an object of the form

$\check{A}_F = \left\{\langle \frac{\mu_A(x_j)}{x_j}; \forall\, x_j \in U_1\rangle, \langle \frac{\mu_A(y_k)}{y_k}; \forall\, y_k \in U_2\rangle\right\}$ where $\mu_A(x_j)\colon\ U_1 \longrightarrow [0,\ 1]$ gives the truth membership value of the elements x_j in U_1; $\mu_A(y_k)\colon\ U_2 \longrightarrow [0,\ 1]$ gives the truth membership values of the elements y_k in U_2

Example 3.3.

$\check{A}_F = \left\{\langle \frac{0.2}{h_1^N}, \frac{0.4}{h_2^N}, \frac{0.1}{h_3^N}\rangle, \langle \frac{0.6}{h_1^S}, \frac{0.3}{h_2^S}\rangle\right\}$ represents the fuzzy binary set

Definition 3.4. (Vague Binary Set)

Vague binary set A over a common universe $\left\{U_1 = \{x_j/\ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\right\}$ is an object of the form

$\check{A}_V = \left\{\langle \frac{V_A(x_j)}{x_j}; \forall\, x_j \in U_1\rangle, \langle \frac{V_A(y_k)}{y_k}; \forall\, y_k \in U_2\rangle\right\} = \left\{\langle \frac{[t_A(x_j), 1 - f_A(x_j)]}{x_j}; \forall\, x_j \in U_1\rangle, \langle \frac{[t_A(y_k), 1 - f_A(y_k)]}{y_k}; \forall\, y_k \in U_2\rangle\right\}$;

$V_A(x_j)\ :\ U_1 \longrightarrow [0, 1]$; $V_A(y_k)\colon\ U_2 \longrightarrow [0, 1]$

Example 3.5.

$\check{A}_V = \left\{\langle \frac{[0.2, 0.6]}{h_1^N}, \frac{[0.4, 0.7]}{h_2^N}, \frac{[0.1, 0.9]}{h_3^N}\rangle, \langle \frac{[0.6, 0.9]}{h_1^S}, \frac{[0.3, 0.4]}{h_2^S}\rangle\right\}$ is a vague binary set where $U_1 = \left\{h_1^N, h_2^N, h_3^N\right\}$, $U_2 = \left\{h_1^S, h_2^S\right\}$

Definition 3.6. (Neutrosophic binary set)

Neutrosophic binary set $\check{A}_N$ over a common universe $\{U_1 = \{x_j / 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\}$ is an object of the form

$$\check{A}_N = \left\{ \langle \frac{\left(T_A(x_j),\ I_A(x_j),\ F_A(x_j)\right)}{x_j} /\ \forall\, x_j \in U_1 \rangle, \langle \frac{\left(T_A(y_k),\ I_A(y_k),\ F_A(y_k)\right)}{y_k} /\ \forall\, y_k \in U_2 \rangle \right\}$$

$T_A(x_j),\ I_A(x_j),\ F_A(x_j)\ :\ U_1 \longrightarrow [0, 1]$ gives the 'truth, indeterminacy and false' membership values of the elements x_j in U_1 and $T_A(y_k), I_A(y_k), F_A(y_k): U_2 \longrightarrow [0, 1]$ gives the 'truth, indeterminacy and false' membership values of the elements y_k in U_2

Example 3.7.

$\check{A}_N = \left\{ \langle \frac{(0.2,\ 0.3,\ 0.4)}{h_1^N}, \frac{(0.4,\ 0.1,\ 0.3)}{h_2^N}, \frac{(0.1,\ 0.3,\ 0.1)}{h_3^N} \rangle, \langle \frac{(0.6,\ 0.2,\ 0.1)}{h_1^S}, \frac{(0.3,\ 0.5,\ 0.6)}{h_1^S} \rangle \right\}$ is a neutrosophic binary set where $U_1 = \{h_1^N, h_2^N, h_3^N\}$, $U_2 = \{h_1^S, h_2^S\}$

Definition 3.8. (Neutrosophic vague binary set)

A neutrosophic vague binary set M_{NVB} ($NVBS$ in short) over a common universe $\{U_1 = \{x_j / 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\}$ is an object of the form

$$M_{NVB} = \left\{ \langle \frac{\hat{T}_{M_{NVB}}(x_j),\ \hat{I}_{M_{NVB}}(x_j),\ \hat{F}_{M_{NVB}}(x_j)}{x_j};\ \forall\, x_j \in U_1 \rangle\ \langle \frac{\hat{T}_{M_{NVB}}(y_k),\ \hat{I}_{M_{NVB}}(y_k),\ \hat{F}_{M_{NVB}}(y_k)}{y_k};\ \forall\, y_k \in U_2 \rangle \right\}$$

is defined as

$\hat{T}_{M_{NVB}}(x_j) = [T^-(x_j), T^+(x_j)]$, $\hat{I}_{M_{NVB}}(x_j) = [I^-(x_j),\ I^+(x_j)]$ and $\hat{F}_{M_{NVB}}(x_j) = [F^-(x_j),\ F^+(x_j)]$; $x_j \in U_1$ and $\hat{T}_{M_{NVB}}(y_k) = [T^-(y_k), T^+(y_k)]$, $\hat{I}_{M_{NVB}}(y_k) = [I^-(y_k),\ I^+(y_k)]$ and $\hat{F}_{M_{NVB}}(y_k) = [F^-(y_k),\ F^+(y_k)]$; $y_k \in U_2$

where (1) $T^+(x_j) = 1 - F^-(x_j)$; $F^+(x_j) = 1 - T^-(x_j)$; $\forall\, x_j \in U_1$ and
$T^+(y_k) = 1 - F^-(y_k)$; $F^+(y_k) = 1 - T^-(y_k)$; $\forall\, y_k \in U_2$

(2) $^-0 \le T^-(x_j) + I^-(x_j) + F^-(x_j) \le 2^+$; $^-0 \le T^-(y_k) + I^-(y_k) + F^-(y_k) \le 2^+$

or

$^-0 \le T^-(x_j) + I^-(x_j) + F^-(x_j) + T^-(y_k) + I^-(y_k) + F^-(y_k) \le 4^+$

and

$^-0 \le T^+(x_j) + I^+(x_j) + F^+(x_j) \le 2^+$; $^-0 \le T^+(y_k) + I^+(y_k) + F^+(y_k) \le 2^+$

or

$^-0 \le T^+(x_j) + I^+(x_j) + F^+(x_j) + T^+(y_k) + I^+(y_k) + F^+(y_k) \le 4^+$

(3) $T^-(x_j),\ I^-(x_j),\ F^-(x_j)\ :\ V(U_1) \longrightarrow [0,1]$ and $T^-(y_k),\ I^-(y_k),\ F^-(y_k)\ :\ V(U_2) \longrightarrow [0,1]$
$T^+(x_j),\ I^+(x_j),\ F^+(x_j)\ :\ V(U_1) \longrightarrow [0,1]$ and $T^+(y_k),\ I^+(y_k),\ F^+(y_k)\ :\ V(U_2) \longrightarrow [0,1]$

Here $V(U_1),\ V(U_2)$ denotes power set of vague sets on $U_1,\ U_2$ respectively.

Example 3.9.

Let $U_1 = \{x_1,\ x_2,\ x_3\}$, $U_2 = \{y_1,\ y_2\}$ be the common universe under consideration.

A $NVBS$ is given below:

$$M_{NVB} = \left\{ \begin{matrix} \langle \frac{[0.2,\ 0.3],\ [0.6, 0.7],\ [0.7, 0.8]}{x_1}; \frac{[0.3,\ 0.7],\ [0.5, 0.6],\ [0.3, 0.7]}{x_2}; \frac{[0.1,\ 0.9],\ [0.4, 0.8],\ [0.1, 0.9]}{x_3} \rangle \\ \langle \frac{[0.6,\ 0.8],\ [0.5, 0.7],\ [0.2, 0.4]}{y_1}; \frac{[0.2,\ 0.7],\ [0.6, 0.9],\ [0.3, 0.8]}{y_2} \rangle \end{matrix} \right\}$$

Definition 3.10. (Zero neutrosophic vague binary set and Unit Neutrosophic vague binary set)

Let $\{U_1 = \{x_j / 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\}$ be two universes under consideration.

(i) A zero $NVBS$ denoted as Φ_{NVB} over this common universe is given by,

$$\Phi_{NVB} = \left\{ \langle \frac{[0,\ 0],\ [1,\ 1],\ [1, 1]}{x_j};\ \forall\, x_j \in U_1 \rangle, \langle \frac{[0,\ 0],\ [1,\ 1],\ [1, 1]}{y_k};\ \forall\, y_k \in U_2 \rangle \right\}$$

(ii) A unit $NVBS$ denoted as Ψ_{NVB} over this common universe is given by,

$$\Psi_{NVB} = \left\{ \langle \frac{[1,1],\ [0,0],\ [0,0]}{x_j};\ \forall\, x_j \in U_1 \rangle, \langle \frac{[1,1],\ [0,0],\ [0,0]}{y_k};\ \forall\, y_k \in U_2 \rangle \right\}$$

4. Operations on Neutrosophic Vague Binary sets

In this section some usual set theoretical operations are developed for $\boldsymbol{NVBS's}$

Definition 4.1. (Subset of Neutrosophic vague binary sets)
Let $\boldsymbol{M_{NVB}}$ and $\boldsymbol{P_{NVB}}$ be two $\boldsymbol{NVBS's}$ on a common universe $\boldsymbol{U_1}, \boldsymbol{U_2}$. Then $\boldsymbol{M_{NVB}}$ is included by $\boldsymbol{P_{NVB}}$ denoted by $\boldsymbol{M_{NVB}} \subseteq \boldsymbol{P_{NVB}}$ if the following conditions found true :
If $\forall\, \boldsymbol{x_j} \in \boldsymbol{U_1}$ and $1 \le j \le n$
(1) $\widehat{\boldsymbol{T}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{x_j}) \le \widehat{\boldsymbol{T}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{x_j})$ (2) $\widehat{\boldsymbol{I}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{x_j}) \ge \widehat{\boldsymbol{I}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{x_j})$ and (3) $\widehat{\boldsymbol{F}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{x_j}) \ge \widehat{\boldsymbol{F}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{x_j})$
and $\forall\, \boldsymbol{y_k} \in \boldsymbol{U_2}$ and $1 \le k \le p$
(1) $\widehat{\boldsymbol{T}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{y_k}) \le \widehat{\boldsymbol{T}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{y_k})$ (2) $\widehat{\boldsymbol{I}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{y_k}) \ge \widehat{\boldsymbol{I}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{y_k})$ and (3) $\widehat{\boldsymbol{F}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{y_k}) \ge \widehat{\boldsymbol{F}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{y_k})$

Example 4. 2.
Let $\boldsymbol{U_1} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}, \boldsymbol{U_2} = \{\boldsymbol{y_1}\}$ be a common universe. Let

$$M_{NVB} = \left\{\langle\frac{[0.1,\ 0.2],\ [0.6, 0.7],\ [0.8, 0.9]}{x_1}; \frac{[0.2,\ 0.6],\ [0.5, 0.6],\ [0.4, 0.8]}{x_2}\rangle \langle\frac{[0.1,\ 0.3],\ [0.6, 0.7],\ [0.7, 0.9]}{y_1}\rangle\right\}$$

$$P_{NVB} = \left\{\langle\frac{[0.2,0.3],\ [0.5,0.6],\ [0.7,0.8]}{x_1}; \frac{[0.3,0.7],\ [0.4,0.5],\ [0.3,0.7]}{x_2}\rangle \langle\frac{[0.2,0.4],\ [0.5,0.6],\ [0.6,0.8]}{y_1}\rangle\right\}.$$

Clearly, $M_{NVB} \subseteq P_{NVB}$

Definition 4.3. (Union of two neutrosophic vague binary sets)
Let M_{NVB} and P_{NVB} are two $NVBS's$
(i) Union of two $NVBS's$, M_{NVB} and P_{NVB} is a $NVBS$, given as

$$M_{NVB} \cup P_{NVB} = S_{NVB} = \left\{\langle\frac{\hat{T}_{S_{NVB}}(x_j),\ \hat{I}_{S_{NVB}}(x_j),\ \hat{F}_{S_{NVB}}(x_j)}{x_j}; \forall\, x_j \in U_1\rangle \langle\frac{\hat{T}_{S_{NVB}}(y_k),\ \hat{I}_{S_{NVB}}(y_k),\ \hat{F}_{S_{NVB}}(y_k)}{y_k}; \forall\, y_k \in U_2\rangle\right\}$$

whose truth-membership, indeterminacy-membership and false-membership functions are related to those of M_{NVB} and P_{NVB} is given by

$$\hat{T}_{S_{NVB}}(x_j) = \left[\max\left(T^- M_{NVB}(x_j), T^- P_{NVB}(x_j)\right), \max\left(T^+ M_{NVB}(x_j), T^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{I}_{S_{NVB}}(x_j) = \left[\min\left(I^- M_{NVB}(x_j),\ I^- P_{NVB}(x_j)\right), \min\left(I^+ M_{NVB}(x_j),\ I^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{F}_{S_{NVB}}(x_j) = \left[\min\left(F^- M_{NVB}(x_j), F^- P_{NVB}(x_j)\right), \min\left(F^+ M_{NVB}(x_j), F^+ P_{NVB}(x_j)\right]\right.$$

and

$$\hat{T}_{S_{NVB}}(y_k) = [\max(T^- M_{NVB}(y_k), T^- P_{NVB}(y_k)), \max(T^+ M_{NVB}(y_k), T^+ P_{NVB}(y_k))]$$
$$\hat{I}_{S_{NVB}}(y_k) = [\min(I^- M_{NVB}(y_k),\ I^- P_{NVB}(y_k)), \min(I^+ M_{NVB}(y_k),\ I^+ P_{NVB}(y_k))]$$
$$\hat{F}_{S_{NVB}}(y_k) = [\min(F^- M_{NVB}(y_k), F^- P_{NVB}(y_k)), \min(F^+ M_{NVB}(y_k), F^+ P_{NVB}(y_k)]$$

Example 4. 4.
In example 4. 2.

$$S_{NVB} = \left\{\langle\frac{[0.2,\ 0.3],\ [0.5, 0.6],\ [0.7, 0.8]}{x_1}; \frac{[0.3,\ 0.7],\ [0.4, 0.5],\ [0.3, 0.7]}{x_2}\rangle \langle\frac{[0.2,\ 0.4],\ [0.5, 0.6],\ [0.6, 0.8]}{y_1}\rangle\right\}$$

Definition 4. 5. (Intersection of two neutrosophic vague binary sets)
Let M_{NVB} and P_{NVB} are two $NVBS's$
(i) Intersection of two $NVBS's$, M_{NVB} and P_{NVB} is a $NVBS$, given as

$$M_{NVB} \cap P_{NVB} = R_{NVB} = \left\{\langle\frac{\hat{T}_{R_{NVB}}(x_j),\ \hat{I}_{R_{NVB}}(x_j),\ \hat{F}_{R_{NVB}}(x_j)}{x_j}; \forall\, x_j \in U_1\rangle \langle\frac{\hat{T}_{R_{NVB}}(y_k),\ \hat{I}_{R_{NVB}}(y_k),\ \hat{F}_{R_{NVB}}(y_k)}{y_k}; \forall\, y_k \in U_2\rangle\right\}$$

whose truth-membership, indeterminacy-membership and false-membership functions are related to those of M_{NVB} and P_{NVB} is given by

$$\hat{T}_{R_{NVB}}(x_j) = \left[\min\left(T^- M_{NVB}(x_j), T^- P_{NVB}(x_j)\right), \min\left(T^+ M_{NVB}(x_j), T^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{I}_{R_{NVB}}(x_j) = \left[\max\left(I^- M_{NVB}(x_j),\ I^- P_{NVB}(x_j)\right), \max\left(I^+ M_{NVB}(x_j),\ I^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{F}_{R_{NVB}}(x_j) = \left[\max\left(F^- M_{NVB}(x_j), F^- P_{NVB}(x_j)\right), \max\left(F^+ M_{NVB}(x_j), F^+ P_{NVB}(x_j)\right]\right.$$

and

$$\hat{T}_{R_{NVB}}(y_k) = [\min(T^- M_{NVB}(y_k), T^- P_{NVB}(y_k)), \min(T^+ M_{NVB}(y_k), T^+ P_{NVB}(y_k))]$$
$$\hat{I}_{R_{NVB}}(y_k) = [\max(I^- M_{NVB}(y_k),\ I^- P_{NVB}(y_k)), \max(I^+ M_{NVB}(y_k),\ I^+ P_{NVB}(y_k))]$$
$$\hat{F}_{R_{NVB}}(y_k) = [\max(F^- M_{NVB}(y_k), F^- P_{NVB}(y_k)), \max(F^+ M_{NVB}(y_k), F^+ P_{NVB}(y_k)]$$

Example 4. 6.

In example 4. 2. $R_{NVB} = \left\{ \langle \frac{[0.1,0.2],\ [0.6,0.7],\ [0.7,0.8]}{x_1} ; \frac{[0.2,0.6],\ [0.5,0.6],\ [0.4,0.8]}{x_2} \rangle \langle \frac{[0.1,0.3],\ [0.6,0.7],\ [0.7,0.9]}{y_1} \rangle \right\}$

Definition 4. 7. (Complement of a NVBS)

Let M_{NVB} is defined as in definition 3.1. It's complement is denoted by M_{NVB}^C and is given by

$M^c{}_{NVB} = \left\{ \langle \frac{\hat{T}^c{}_{M_{NVB}}(x_j),\ \hat{I}^c{}_{M_{NVB}}(x_j),\ \hat{F}^c{}_{M_{NVB}}(x_j)}{x_j} ; \forall\, x_j \in U_1 \rangle \langle \frac{\hat{T}^c{}_{M_{NVB}}(y_k),\ \hat{I}^c{}_{M_{NVB}}(y_k),\ \hat{F}^c{}_{M_{NVB}}(y_k)}{y_k} ; \forall\, y_k \in U_2 \rangle \right\}$

is defined as

$\hat{T}^c{}_{M_{NVB}}\ (x_j) = [\ 1 - T^+(x_j),\ 1 - T^-(x_j)],$

$\hat{I}^c{}_{M_{NVB}}(x_j) = [1 - I^+(x_j), 1 - I^-(x_j)]$ and

$\hat{F}^c{}_{M_{NVB}}(x_j)\ = [1\ - F^+(x_j),\ 1 - F^-(x_j)]\ ;\ \ \forall\, x_j \in U_1$

and

$\hat{T}^c{}_{M_{NVB}}\ (y_k) = [\ 1 - T^+(y_k),\ 1 - T^-(y_k)],$

$\hat{I}^c{}_{M_{NVB}}(y_k) = [1 - I^+(y_k), 1 - I^-(y_k)]$ and

$\hat{F}^c{}_{M_{NVB}}(y_k)\ = [1\ - F^+(y_k),\ 1 - F^-(y_k)]\ ;\ \forall\, y_k \in U_2$

Example 4. 8.

Let M_{NVB} is defined as in example 3.2. It's complement is given by,

$$M_{NVB}^c = \left\{ \begin{array}{c} \langle \frac{[0.7,\ 0.8],\ \ [0.3, 0.4],\ \ [0.2, 0.3]}{x_1} ; \frac{[0.3,\ 0.7],\ \ [0.4, 0.5],\ \ [0.3, 0.7]}{x_2} ; \frac{[0.1,\ 0.9],\ \ [0.2, 0.6],\ \ [0.1, 0.9]}{x_3} \rangle \\ \langle \frac{[0.2,\ 0.4],\ \ [0.3, 0.5],\ \ [0.6, 0.8]}{y_1} ; \frac{[0.3,\ 0.8],\ \ [0.1, 0.4],\ \ [0.2, 0.7]}{y_2} \rangle \end{array} \right\}$$

5. Neutrosophic vague binary topology

In this section neutrosophic vague binary topology ($NVBT$ in short) is developed for $NVBS's$. It's various concepts are also discussed.

Definition 5.1. (Neutrosophic vague binary topology)

A neutrosophic vague binary topology on a common universe U_1, U_2 is a family τ_Δ^{NVB} of neutrosophic vague binary sets in $U_1,\ U_2$ satisfying the following axioms:

(1) $\Phi_{NVB},\ \Psi_{NVB}\ \in\ \tau_\Delta^{NVB}$

(2) For any $M_{NVB},\ P_{NVB}\ \in\ \tau_\Delta^{NVB},\ M_{NVB}\ \cap\ P_{NVB}\ \in\ \tau_\Delta^{NVB}$

i.e., finite intersection of $NVBS's$ of τ_Δ^{NVB} is again a member of τ_Δ^{NVB}

(3) Let $\left\{ M_{NVB}^i ; i \in I \right\} \subseteq \tau_\Delta^{NVB}$ then $\cup_{i \in I}\, \tau_\Delta^{NVB}\ \subseteq\ \tau_\Delta^{NVB}$

i.e., arbitrary union of neutrosophic vague binary sets in τ_Δ^{NVB} is again a member of τ_Δ^{NVB}

Example 5.2.

Let $U_1 = \{x_1,\ x_2\}\ ;\ U_2 = \{y_1\}$. Following is a neutrosophic vague binary topology ;

$$\tau_\Delta^{NVB} = \left\{ \begin{array}{c} \Phi_{NVB} = \left(\left\{ \langle \frac{[0,0],[1,1],[1,1]]}{x_1} \rangle,\ \ \langle \frac{[0,0],[1,1],[1,1]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0,0],[1,1],[1,1]}{y_1} \rangle \right\} \right), \\ M_{NVB} = \left(\left\{ \langle \frac{[0.2,0.4],[0.6,0.8],[0.6\ 0.8]]}{x_1} \rangle,\ \ \langle \frac{[0.3,0.6],[0.7,0.8],[0.4,0.7]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.8],[0.7.9],[0.2,0.4]}{y_1} \rangle \right\} \right), \\ P_{NVB} = \left(\left\{ \langle \frac{[0.6,0.7],[0.1,0.9],[0.3,0.4]]}{x_1} \rangle,\ \ \langle \frac{[0.7,0.8],[0.3,0.7],[0.2,0.3]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.7],[0.2,0.5],[0.3,0.4]}{y_1} \rangle \right\} \right), \\ K_{NVB} = M_{NVB} \cap P_{NVB} = \left(\left\{ \langle \frac{[0.2,0.4],[0.6,0.9],[0.6,0.8]]}{x_1} \rangle,\ \ \langle \frac{[0.3,0.6],[0.7,0.8],[0.4,0.7]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.7],[0.7,0.9],[0.3,0.4]}{y_1} \rangle \right\} \right), \\ H_{NVB} = M_{NVB} \cup P_{NVB} = \left(\left\{ \langle \frac{[0.6,0.7],[0.1,0.8],[0.3,0.4]]}{x_1} \rangle,\ \ \langle \frac{[0.7,0.8],[0.7,0.8],[0.2,0.3]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.8],[0.2,0.5],[0.2,0.4]}{y_1} \rangle \right\} \right), \\ \Psi_{NVB} = \left(\left\{ \langle \frac{[1,1],[0,0],[0,0]]}{x_1} \rangle,\ \ \langle \frac{[1,1],[0,0],[0,0]}{x_2} \rangle \right\}, \left\{ \langle \frac{[1,1],[0,0],[0,0]}{y_1} \rangle \right\} \right) \end{array} \right\}$$

Definition 5.3. (Neutrosophic vague binary open set)

Every elements of a $NVBT$ is known as a neutrosophic vague binary open set ($NVBOS$ in short)

Example 5.4.

In example 5.2. $\Phi_{NVB},\ M_{NVB},\ P_{NVB},\ K_{NVB},\ H_{NVB},\ \Psi_{NVB}$ are all $NVBOS$'s

Definition 5.5. (Neutrosophic vague binary closed set)

Complement of a $NVBOS$ is known as a neutrosophic vague binary closed set ($NVBCS$ in short)

Example 5.6.

In example 5.2. Φ^{c}_{NVB}, M^{C}_{NVB}, P^{C}_{NVB}, K^{C}_{NVB}, H^{C}_{NVB}, Ψ^{c}_{NVB} are all $NVBCS$'s, where

$\Phi^{c}_{NVB}= \left(\left\{\langle\frac{[1,1],[0,0],[0,0]}{x_1}\rangle,\ \langle\frac{[1,1],[0,0],[0,0]}{x_2}\rangle\right\},\left\{\langle\frac{[1,1],[0,0],[0,0]}{y_1}\rangle\right\}\right) = \Psi_{NVB}$

$M^{C}_{NVB}= \left(\left\{\langle\frac{[0.6,0.8],[0.2,0.4],[0.2\ 0.4]}{x_1}\rangle,\ \langle\frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2}\rangle\right\},\left\{\langle\frac{[0.2,0.4],[0.1.3],[0.6,0.8]}{y_1}\rangle\right\}\right)$

$P^{C}_{NVB}= \left(\left\{\langle\frac{[0.3,0.4],[0.1,0.9],[0.6,0.7]}{x_1}\rangle,\ \langle\frac{[0.2,0.3],[0.3,0.7],[0.7,0.8]}{x_2}\rangle\right\},\left\{\langle\frac{[0.3,0.4],[0.5,0.8],[0.6,0.7]}{y_1}\rangle\right\}\right)$

$K^{C}_{NVB}=\left(\left\{\langle\frac{[0.6,0.8],[0.1,0.4],[0.2,0.4]}{x_1}\rangle,\ \langle\frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2}\rangle\right\},\left\{\langle\frac{[0.3,0.4],[0.1,0.3],[0.6,0.7]}{y_1}\rangle\right\}\right)$

$H^{C}_{NVB}=\left(\left\{\langle\frac{[0.3,0.4],[0.2,0.9],[0.6,0.7]}{x_1}\rangle,\ \langle\frac{[0.2,0.3],[0.2,0.3],[0.7,0.8]}{x_2}\rangle\right\},\left\{\langle\frac{[0.2,0.4],[0.5,0.8],[0.6,0.8]}{y_1}\rangle\right\}\right)$

$\Psi^{c}_{NVB}=\left(\left\{\langle\frac{[0,0],[1,1],[1,1]}{x_1}\rangle,\ \langle\frac{[0,0],[1,1],[1,1]}{x_2}\rangle\right\},\left\{\langle\frac{[0,0],[1,1],[1,1]}{y_1}\rangle\right\}\right) = \Phi_{NVB}$

Remark 5.7.

Φ_{NVB} and Ψ_{NVB} will both acts as $NVBOS$ and $NVBCS$

Definition 5.8. (Neutrosophic vague binary topological space)

The triplet $(U_1, U_2, \tau^{NVB}_{\Delta})$ is known as a neutrosophic vague binary topological space ($NVBTS$ in short), where τ^{NVB}_{Δ} is a neutrosophic vague binary topology defined as in definition 5.1.

Example 5.9.

If $U_1 = \{x_1,\ x_2\}$; $U_2 = \{y_1\}$; $\tau^{NVB}_{\Delta}= \{\Phi_{NVB}, M_{NVB}, P_{NVB}, K_{NVB}, H_{NVB}, \Psi_{NVB}\}$ defined as in example 5.2. then the triplet $(U_1, U_2, \tau^{NVB}_{\Delta})$ is clearly a $NVBTS$.

Definition 5.10. (Neutrosophic vague binary discrete topology and Neutrosophic vague binary discrete topological Space)

A topology consisting of only empty and unit $NVBS$'s is known as a neutrosophic vague binary discrete topology ($NVBDT$ in short) and the corresponding neutrosophic vague binary topological space is known as a neutrosophic vague binary discrete topological space ($NVBDTS$ in short).

i.e., $\tau^{NVB}_{\Delta}= \{\Phi_{NVB},\ \Psi_{NVB}\}$

Example 5.11.

In example 5.2.

$$\tau^{NVB}_{\Delta}= \left\{\begin{array}{l}\Phi_{NVB} = \left(\left\{\langle\frac{[0,0],[1,1],[1,1]}{x_1}\rangle,\ \langle\frac{[0,0],[1,1],[1,1]}{x_2}\rangle\right\},\left\{\langle\frac{[0,0],[1,1],[1,1]}{y_1}\rangle\right\}\right),\\ \Psi_{NVB} = \left(\left\{\langle\frac{[1,1],[0,0],[0,0]}{x_1}\rangle,\ \langle\frac{[1,1],[0,0],[0,0]}{x_2}\rangle\right\},\left\{\langle\frac{[1,1],[0,0],[0,0]}{y_1}\rangle\right\}\right)\end{array}\right\}$$

is clearly a $NVBDT$ and the corresponding neutrosophic vague topological space is the $NVBDTS$.

Definition 5.12. (Neutrosophic vague binary indiscrete topology and Neutrosophic vague binary discrete topological Space)

A $NVBT$ defined by it's power set is known as neutrosophic vague binary indiscrete topology ($NVBIDT$ in short) and the corresponding neutrosophic vague binary topological space is known as a neutrosophic vague binary indiscrete topological space ($NVBIDTS$ in short).

Definition 5.13. (Neutrosophic vague binary interior and Neutrosophic vague binary closure)

Let $(U_1, U_2, \tau^{NVB}_{\Delta})$ be a $NVBTS$ and also

let $M_{NVB} = \left\{\langle\frac{\hat{T}_{M_{NVB}}(x_j),\ \hat{I}_{M_{NVB}}(x_j),\ \hat{F}_{M_{NVB}}(x_j)}{x_j};\ \forall\ x_j \in U_1\rangle\ \langle\frac{\hat{T}_{M_{NVB}}(y_k),\ \hat{I}_{M_{NVB}}(y_k),\ \hat{F}_{M_{NVB}}(y_k)}{y_k};\ \forall\ y_k \in U_2\rangle\right\}$

is a $NVBS$ over a common universe U_1, U_2 defined as in definition 3.1. Then it's neutrosophic vague binary interior (denoted as M^{0}_{NVB}) and neutrosophic vague binary closure (denoted as $\overline{M}_{NVB}$) are defined as follows:

$M^{0}_{NVB}= \cup\ \left\{M^{i}_{NVB}; i \in I \middle| M^{i}_{NVB}\ is\ a\ NVBOS\ over\ U_1, U_2\ with\ M^{i}_{NVB} \subseteq M_{NVB}\ ;\ \forall\ i\right\}$

$\overline{M}_{NVB}= \cap \left\{M^{i}_{NVB}; i \in I \middle| M^{i}_{NVB}\ is\ a\ NVBCS\ over\ U_1, U_2\ with\ M_{NVB} \subseteq M^{i}_{NVB}\ ;\ \forall\ i\right\}$

Example 5.14.

In example 5.2.

$H^{0}_{NVB}= \left(\left\{\langle\frac{[0.6,0.7],[0.1,0.8],[0.3,0.4]}{x_1}\rangle,\ \langle\frac{[0.7,0.8],[0.7,0.8],[0.2,0.3]}{x_2}\rangle\right\},\left\{\langle\frac{[0.6,0.8],[0.2,0.5],[0.2,0.4]}{y_1}\rangle\right\}\right) = H_{NVB}$

From example 5.6.

$\overline{M^{C}_{NVB}}= \left(\left\{\langle\frac{[0.6,0.8],[0.2,0.4],[0.2\ 0.4]}{x_1}\rangle,\ \langle\frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2}\rangle\right\},\left\{\langle\frac{[0.2,0.4],[0.1.3],[0.6,0.8]}{y_1}\rangle\right\}\right)= M^{C}_{NVB}$

Proposition 5.15.

(i) M_{NVB} is a $NVBOS \Leftrightarrow M_{NVB}^{0} = M_{NVB}$
(ii) M_{NVB} is a $NVBCS \Leftrightarrow \overline{M}_{NVB} = M_{NVB}$

Proof

Proof is clear

Proposition 5.16.

(i) $M_{NVB}^1 \subseteq M_{NVB}^2$ and $P_{NVB}^1 \subseteq P_{NVB}^2 \Rightarrow (M_{NVB}^1 \cup P_{NVB}^1) \subseteq (M_{NVB}^2 \cup P_{NVB}^2)$ and $(M_{NVB}^1 \cap P_{NVB}^1) \subseteq (M_{NVB}^2 \cap P_{NVB}^2)$

(ii) $M_{NVB} \subseteq M_{NVB}^1$ and $M_{NVB} \subseteq M_{NVB}^2 \Rightarrow M_{NVB} \subseteq (M_{NVB}^1 \cap M_{NVB}^2)$
$M_{NVB}^1 \subseteq M_{NVB}$ and $M_{NVB}^2 \subseteq M_{NVB} \Rightarrow (M_{NVB}^1 \cup M_{NVB}^2) \subseteq M_{NVB}$

(iii) $\overline{\overline{M}}_{NVB} = M_{NVB}$

(iv) $M_{NVB} \subseteq P_{NVB} \Rightarrow \overline{P}_{NVB} \subseteq \overline{M}_{NVB}$

(v) $\overline{\emptyset}_{NVB} = \psi_{NVB}$

(vi) $\overline{\Psi}_{NVB} = \Phi_{NVB}$

Proof

Proof is clear

6. Continuous mapping for $NVBS'$s

Continuity plays vital role in any topology. In this section image, pre-image and continuity are developed for $NVBS'$s.

Definition 6.1. (Image and Pre-image of neutrosophic vague binary sets)

Let M_{NVB} and P_{NVB} be two non-empty $NVBS'$s defined on two common universes U_1, U_2 and V_1, V_2 respectively. Define a function f : $M_{NVB} \longrightarrow P_{NVB}$, then the following statements hold:

(1)If $D_{NVB} = \left\{ \langle \frac{\hat{T}_{D_{NVB}}(s_i);\, \hat{I}_{D_{NVB}}(s_i);\, \hat{F}_{D_{NVB}}(s_i)}{s_i};\ s_i \in V_1 \rangle ; \langle \frac{\hat{T}_{D_{NVB}}(t_r);\, \hat{I}_{D_{NVB}}(t_r);\, \hat{F}_{D_{NVB}}(t_r)}{t_r};\ t_r \in V_2 \rangle \right\}$ is a $NVBS$ in P_{NVB}, then the preimage of D_{NVB} under f , denoted by $f^{-1}(D_{NVB})$, is a $NVBS$ in M_{NVB} defined by

$$f^{-1}(D_{NVB}) = \left\{ \langle \frac{f^{-1}\left(\hat{T}_{D_{NVB}}(s_i)\right);\ f^{-1}\left(\hat{I}_{D_{NVB}}(s_i)\right);\ f^{-1}\left(\hat{F}_{D_{NVB}}(s_i)\right)}{s_i};\ s_i \in V_1 \rangle ; \langle \frac{f^{-1}\left(\hat{T}_{D_{NVB}}(t_r)\right);\ f^{-1}\left(\hat{I}_{D_{NVB}}(t_r)\right);\ f^{-1}\left(\hat{F}_{D_{NVB}}(t_r)\right)}{t_r};\ t_r \in V_2 \rangle \right\}$$

(2)If $A_{NVB} = \left\{ \langle \frac{\hat{T}_{A_{NVB}}(x_j);\, \hat{I}_{A_{NVB}}(x_j);\, \hat{F}_{A_{NVB}}(x_j)}{x_j};\, x_j \in U_1 \rangle ; \langle \frac{\hat{T}_{A_{NVB}}(y_k);\, \hat{I}_{A_{NVB}}(y_k);\, \hat{F}_{A_{NVB}}(y_k)}{y_k};\ y_k \in U_2 \rangle \right\}$ is a $NVBS$ in M_{NVB}, then the image of A_{NVB} under f , denoted by $f(A_{NVB})$, is a $NVBS$ in P_{NVB} defined by

$$f(A_{NVB}) = \left\{ \langle \frac{f_{sup}\left(\hat{T}_{A_{NVB}}(s_i)\right);\ f_{inf}\left(\hat{I}_{A_{NVB}}(s_i)\right);\ f_{inf}\left(\hat{F}_{A_{NVB}}(s_i)\right)}{s_i};\ s_i \in V_1 \rangle ; \langle \frac{f_{sup}\left(\hat{T}_{A_{NVB}}(t_r)\right);\ f_{inf}\left(\hat{I}_{A_{NVB}}(t_r)\right);\ f_{inf}\left(\hat{F}_{A_{NVB}}(t_r)\right)}{t_r};\ t_r \in V_2 \rangle \right\}$$

where

$$\begin{cases} f_{sup}\left(\hat{T}_{A_{NVB}}(s_i)\right) = \begin{cases} sup_{x_j \in f^{-1}(s_i)} \hat{T}_{A_{NVB}}(x_j), & if\ f^{-1}(s_i) \neq \emptyset \\ 0, & otherwise \end{cases} \\ f_{sup}\left(\hat{T}_{A_{NVB}}(t_r)\right) = \begin{cases} sup_{y_k \in f^{-1}(t_r)} \hat{T}_{A_{NVB}}(y_k), & if\ f^{-1}(t_r) \neq \emptyset \\ 0, & otherwise \end{cases} \end{cases}$$

$$\begin{cases} f_{inf}\left(\hat{I}_{A_{NVB}}(s_i)\right) = \begin{cases} inf_{x_j \in f^{-1}(s_i)} \hat{I}_{A_{NVB}}(x_j), & if\ f^{-1}(s_i) \neq \emptyset \\ 0, & otherwise \end{cases} \\ f_{inf}\left(\hat{I}_{A_{NVB}}(t_r)\right) = \begin{cases} inf_{y_k \in f^{-1}(t_r)} \hat{I}_{A_{NVB}}(y_k), & if\ f^{-1}(t_r) \neq \emptyset \\ 0, & otherwise \end{cases} \end{cases}$$

$$\begin{cases} f_{inf}\left(\hat{F}_{A_{NVB}}(s_i)\right) = \begin{cases} inf_{x_j \in f^{-1}(s_i)} \hat{F}_{A_{NVB}}(x_j), & if\ f^{-1}(s_i) \neq \emptyset \\ 0, & otherwise \end{cases} \\ f_{inf}\left(\hat{F}_{A_{NVB}}(t_r)\right) = \begin{cases} inf_{y_k \in f^{-1}(t_r)} \hat{F}_{A_{NVB}}(y_k), & if\ f^{-1}(t_r) \neq \emptyset \\ 0, & otherwise \end{cases} \end{cases}$$

for each $s_i \in V_1$ and for each $t_r \in V_2$

Definition 6.2. (Neutrosophic Vague strongly continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) →(Y, σ) is said to be neutrosophic vague strongly continous if inverse image of every neutrosophic vague set in (Y, σ) is neutrosophic vague clopen set [a set which acts simultaneously as neutrosophic vague open set and neutrosophic vague closed set] in (X, τ)

Definition 6.3.

(i) Neutrosophic Vague Binary Continuity:

Let $(U_1, U_2, \tau_\Delta^{NVB})$ and $(V_1, V_2, \sigma_\Delta^{NVB})$ be any two $NVBTS'$s.

A map f : $(U_1, U_2, \tau_\Delta^{NVB}) \longrightarrow (V_1, V_2, \sigma_\Delta^{NVB})$ is said to be neutrosophic vague binary **continuous** (NVB continuous) if forevery $NVBOS$ (or $NVBCS$) M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$, $f^{-1}(M_{NVB})$ is a $NVBOS$ (or $NVBCS$) in $(U_1, U_2, \tau_\Delta^{NVB})$

(ii)Various kinds of Continuities for $NVBS'$s

Let $(U_1, U_2, \tau_\Delta^{NVB})$ and $(V_1, V_2, \sigma_\Delta^{NVB})$ be any two $NVBTS'$s. A map f : $(U_1, U_2, \tau_\Delta^{NVB}) \longrightarrow (V_1, V_2, \sigma_\Delta^{NVB})$ is said to be

(1) Neutrosophic vague binary **semi-continuous** ($NVBSC$):

if forevery neutrosophic vague binary open set ($NVBOS$ in short) [or neutrosophic vague binary closed set ($NVBCS$ in short)] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$, $f^{-1}(M_{NVB})$ is a neutrosophic vague binary semi-open set ($NVBSOS$ in short) [or neutrosophic vague binary semi - closed set ($NVBSCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

(2) Neutrosophic vague binary **pre-continuous** ($NVBPC$ continuous):

if forevery $NVBOS$ [or $NVBCS$] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$ $f^{-1}(M_{NVB})$ is a neutrosophic vague binary pre-open set ($NVBPOS$ in short) [or neutrosophic vague binary pre-closed set ($NVBPCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

(3) Neutrosophic vague binary **strongly-continuous** ($NVBSC$ continuous):

if inverse image of every neutrosophic vague binary set in $(V_1, V_2, \sigma_\Delta^{NVB})$ is neutrosophic vague binary clopen set [a set which acts simultaneously as neutrosophic vague binary open set and neutrosophic vague binary closed set] in $(U_1, U_2, \tau_\Delta^{NVB})$

(4) Neutrosophic vague binary **regular-continuous** ($NVBRC$ continuous):

if forevery $NVBOS$ [or $NVBCS$] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$ $f^{-1}(M_{NVB})$ is a neutrosophic vague binary regular- open set ($NVBROS$ in short) [or neutrosophic vague binary regular-closed set ($NVBRCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

(5) Neutrosophic vague binary **semi-pre-continuous** ($NVBRC$ continuous):

if forevery $NVBOS$ [or $NVBCS$] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$ $f^{-1}(M_{NVB})$ is a neutrosophic vague binary generalized semi- open set ($NVBGSOS$ in short) [or neutrosophic vague binary generalized semi-closed set ($NVBGSCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

Example 6.4.

Let $f = (g,h): M_{NVB} \longrightarrow P_{NVB}$ be a function defined as , $f(\Phi_{NVB}^1) = \Phi_{NVB}^2$, $f(M_{NVB}^1) = P_{NVB}^1$, $f(M_{NVB}^2) = P_{NVB}^1$, $f(\Psi_{NVB}^1) = \Psi_{NVB}^2$ where $g: U_1 \longrightarrow V_1$ and $h: U_2 \longrightarrow V_2$ be two functions with $g(x_1) = s_2$, $g(x_2) = s_1$ and $h(y_1) = t_1$, where $U_1 = \{x_1, x_2\}$, $U_2 = \{y_1\}$ and $V_1 = \{s_1, s_2\}$, $U_2 = \{t_1\}$.

Let $\tau_\Delta^{NVB} = \{\Phi_{NVB}^1, M_{NVB}^1, M_{NVB}^2, M_{NVB}^3, M_{NVB}^4, M_{NVB}^5, M_{NVB}^6, M_{NVB}^7, M_{NVB}^8, M_{NVB}^9, M_{NVB}^{10}, M_{NVB}^{11}, \Psi_{NVB}^1\}$ and $\sigma_\Delta^{NVB} = \{\Phi_{NVB}^2, P_{NVB}^1, \Psi_{NVB}^2\}$ be their respective $NVBT'$s.

Here

$$\Phi_{NVB}^1 = \left\{\langle \frac{[0,0],\ [1,1],\ [1,1]}{x_1};\ \frac{[0,0],\ [1,1],\ [1,1]}{x_2} \rangle,\ \ \langle \frac{[0,0],\ [1,1],\ [1,1]}{y_1} \rangle\right\}$$

$$M_{NVB}^1 = \left\{\langle \frac{[0.3,\ 0.4],\ [0.7, 0.8],\ [0.6, 0.7]}{x_1};\ \frac{[0.2,\ 0.7],\ [0.1, 0.5],\ [0.3, 0.8]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle\right\}$$

$$M_{NVB}^2 = \left\{\langle \frac{[0.1,\ 0.6],\ [0.6, 0.9],\ [0.4, 0.9]}{x_1};\ \frac{[0.6,\ 0.8],\ [0.3, 0.7],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M_{NVB}^3 = \left\{\langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1};\ \frac{[0.3,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.7]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M_{NVB}^{4} = \left\{ \langle \frac{[0.1,\ 0.4],\ [0.7, 0.9],\ [0.6, 0.9]}{x_1};\ \frac{[0.2,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.8]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle \right\}$$

$$M_{NVB}^{5} = \left\{ \langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1};\ \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle \right\}$$

$$M_{NVB}^{6} = \left\{ \langle \frac{[0.1,\ 0.4],\ [0.7, 0.9],\ [0.6, 0.9]}{x_1};\ \frac{[0.2,\ 0.7],\ [0.3, 0.7],\ [0.3, 0.8]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle \right\}$$

$$M_{NVB}^{7} = \left\{ \langle \frac{[0.3,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.7]}{x_1};\ \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle \right\}$$

$$M_{NVB}^{8} = \left\{ \langle \frac{[0.1,\ 0.6],\ [0.6, 0.9],\ [0.4, 0.9]}{x_1};\ \frac{[0.3,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.7]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle \right\}$$

$$M_{NVB}^{9} = \left\{ \langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1};\ \frac{[0.6,\ 0.8],\ [0.3, 0.7],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle \right\}$$

$$M_{NVB}^{10} = \left\{ \langle \frac{[0.3,\ 0.4],\ [0.7, 0.8],\ [0.6, 0.7]}{x_1};\ \frac{[0.2,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.8]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle \right\}$$

$$M_{NVB}^{11} = \left\{ \langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1};\ \frac{[0.3,\ 0.7],\ [0.1, 0.5],\ [0.3, 0.7]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle \right\}$$

$$\Psi_{NVB}^{1} = \left\{ \langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{x_1};\ \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{x_2} \rangle,\ \langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{y_1} \rangle \right\}$$

and $V_1 = \{s_1,\ s_2\}$, $V_2 = \{t_1\}$ be a common universe with

$$\Phi_{NVB}^{2} = \left\{ \langle \frac{[0,\ 0],\ [1, 1],\ [1, 1]}{s_1};\ \frac{[0,\ 0],\ [1, 1],\ [1, 1]}{s_2} \rangle,\ \langle \frac{[0,\ 0],\ [1, 1],\ [1, 1]}{t_1} \rangle \right\}$$

$$P_{NVB}^{1} = \left\{ \langle \frac{[0.2,\ 0.3],\ [0.5, 0.6],\ [0.7, 0.8]}{s_1};\ \frac{[0.3,\ 0.7],\ [0.4, 0.5],\ [0.3, 0.7]}{s_2} \rangle \langle \frac{[0.2,\ 0.4],\ [0.5, 0.6],\ [0.6, 0.8]}{t_1} \rangle \right\}$$

$$\Psi_{NVB}^{2} = \left\{ \langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{s_1};\ \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{s_2} \rangle,\ \langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{t_1} \rangle \right\}$$

It is got that, $f^{-1}(\Phi_{NVB}^{2}) = \Phi_{NVB}^{1}$, $f^{-1}(\mathrm{P}_{NVB}) = M_{NVB}^{3}$, $f^{-1}(\Psi_{NVB}^{2}) = \Psi_{NVB}^{1}$. Then clearly f is a neutrosophic vague binary contiuous mapping.

7. Distance Measures for $NVBS's$

Let $U_1 = \{x_1,\ x_2, ----,\ x_n\}$; $U_2 = \{y_1,\ y_2, ----,\ y_p\}$ be the common universe. Also let

M_{NVB} and P_{NVB} be two $NVBS's$.

(i) Hamming distance between them is defined as

$$d_{NVB}^{H}(M_{NVB}, P_{NVB}) =$$

$$\frac{1}{6}\Big[\textstyle\sum_{j=1}^{n}\{[|T_{MNB}^{-}(x_j) - T_{PNB}^{-}(x_j)| + |I_{MNB}^{-}(x_j) - I_{PNB}^{-}(x_j)| + |F_{MNB}^{-}(x_j) - F_{PNB}^{-}(x_j)|] + [|T_{MNB}^{+}(x_j) - T_{PNB}^{+}(x_j)| + |I_{MNB}^{+}(x_j) - I_{PNB}^{+}(x_j)| + |F_{MNB}^{+}(x_j) - F_{PNB}^{+}(x_j)|]\}\Big]$$

$$+ \frac{1}{6}\Big[\textstyle\sum_{k=1}^{p}\{[|T_{MNB}^{-}(y_k) - T_{PNB}^{-}(y_k)| + |I_{MNB}^{-}(y_k) - I_{PNB}^{-}(y_k)| + |F_{MNB}^{-}(y_k) - F_{PNB}^{-}(y_k)|] + [|T_{MNB}^{+}(y_k) - T_{PNB}^{+}(y_k)| + |I_{MNB}^{+}(y_k) - I_{PNB}^{+}(y_k)| + |F_{MNB}^{+}(y_k) - F_{PNB}^{+}(y_k)|]\}\Big]$$

(ii) Normalized Hamming distance between them is defined as

$$d_{NVB}^{nH}(M_{NVB}, P_{NVB}) =$$

$$\frac{1}{6n}[\sum_{j=1}^{n}\{[|T_{MNB}^{-}(x_j) - T_{PNB}^{-}(x_j)| + |I_{MNB}^{-}(x_j) - I_{PNB}^{-}(x_j)| + |F_{MNB}^{-}(x_j) - F_{PNB}^{-}(x_j)|] + [|T_{MNB}^{+}(x_j) - T_{PNB}^{+}(x_j)| + |I_{MNB}^{+}(x_j) - I_{PNB}^{+}(x_j)| + |F_{MNB}^{+}(x_j) - F_{PNB}^{+}(x_j)|]\}]$$

$$+ \frac{1}{6p}[\sum_{k=1}^{p}\{[|T_{MNB}^{-}(y_k) - T_{PNB}^{-}(y_k)| + |I_{MNB}^{-}(y_k) - I_{PNB}^{-}(y_k)| + |F_{MNB}^{-}(y_k) - F_{PNB}^{-}(y_k)|] + [|T_{MNB}^{+}(y_k) - T_{PNB}^{+}(y_k)| + |I_{MNB}^{+}(y_k) - I_{PNB}^{+}(y_k)| + |F_{MNB}^{+}(y_k) - F_{PNB}^{+}(y_k)|]\}]$$

(iii)Euclidean distance between them is defined as

$$d_{NVB}^{E}(M_{NVB}, P_{NVB}) =$$

$$\sqrt{\begin{matrix}\left[\frac{1}{6}\left[\sum_{j=1}^{n}\{[|T_{MNB}^{-}(x_j) - T_{PNB}^{-}(x_j)|^2 + |I_{MNB}^{-}(x_j) - I_{PNB}^{-}(x_j)|^2 + |F_{MNB}^{-}(x_j) - F_{PNB}^{-}(x_j)|^2] + [|T_{MNB}^{+}(x_j) - T_{PNB}^{+}(x_j)|^2 + |I_{MNB}^{+}(x_j) - I_{PNB}^{+}(x_j)|^2 + |F_{MNB}^{+}(x_j) - F_{PNB}^{+}(x_j)|^2]\}\right]\right] \\ + \\ \left[\frac{1}{6}\left[\sum_{k=1}^{p}\{[|T_{MNB}^{-}(y_k) - T_{PNB}^{-}(y_k)|^2 + |I_{MNB}^{-}(y_k) - I_{PNB}^{-}(y_k)|^2 + |F_{MNB}^{-}(y_k) - F_{PNB}^{-}(y_k)|^2] + [|T_{MNB}^{+}(y_k) - T_{PNB}^{+}(y_k)|^2 + |I_{MNB}^{+}(y_k) - I_{PNB}^{+}(y_k)|^2 + |F_{MNB}^{+}(y_k) - F_{PNB}^{+}(y_k)|^2]\}\right]\right]\end{matrix}}$$

(iv)Normalized Euclidean distance between them is defined as

$$d_{NVB}^{nE}(M_{NVB}, P_{NVB}) =$$

$$\sqrt{\begin{matrix}\left[\frac{1}{6n}\left[\sum_{j=1}^{n}\{[|T_{MNB}^{-}(x_j) - T_{PNB}^{-}(x_j)|^2 + |I_{MNB}^{-}(x_j) - I_{PNB}^{-}(x_j)|^2 + |F_{MNB}^{-}(x_j) - F_{PNB}^{-}(x_j)|^2] + [|T_{MNB}^{+}(x_j) - T_{PNB}^{+}(x_j)|^2 + |I_{MNB}^{+}(x_j) - I_{PNB}^{+}(x_j)|^2 + |F_{MNB}^{+}(x_j) - F_{PNB}^{+}(x_j)|^2]\}\right]\right] \\ + \\ \left[\frac{1}{6p}\left[\sum_{k=1}^{p}\{[|T_{MNB}^{-}(y_k) - T_{PNB}^{-}(y_k)|^2 + |I_{MNB}^{-}(y_k) - I_{PNB}^{-}(y_k)|^2 + |F_{MNB}^{-}(y_k) - F_{PNB}^{-}(y_k)|^2] + [|T_{MNB}^{+}(y_k) - T_{PNB}^{+}(y_k)|^2 + |I_{MNB}^{+}(y_k) - I_{PNB}^{+}(y_k)|^2 + |F_{MNB}^{+}(y_k) - F_{PNB}^{+}(y_k)|^2]\}\right]\right]\end{matrix}}$$

8. $NVBS's$ in Medical Diagonosis

This section deals with an application of $NVBS's$ in medical diagnosis.

Following table describes datas collected from three patients after conducting liver function test. First set of sample is collected before treatment which describes the first universe. Second set of sample is collected after treatment which describes the second universe. P_{NVB}^1, P_{NVB}^2, P_{NVB}^3 are three NVBS's formed, based on the datas of the three patients under consideration

Before Treatment (BT)	P_1	P_2	P_3
Albumin	[0.042, 0.052]	[0.025, 0.052]	[0.052, 0.064]
Globulin Serum	[0.035, 0.045]	[0.033, 0.035]	[0.011, 0.035]
Bilirubin Total	[0.045, 0.100]	[0.070, 0.100]	[0.093, 0.100]

After Treatment (AT)	P_1	P_2	P_3
Albumin	[0.031, 0.052]	[0.036, 0.052]	[0.052, 0.064]
Globulin Serum	[0.021, 0.035]	[0.035, 0.042]	[0.019, 0.035]
Bilirubin Total	[0.025, 0.100]	[0.017, 0.100]	[0.099,0.100]

Data collected from 3 persons are converted to $NVBS's$ as given below:

$$P_{NVB}^{1} = \left\{\begin{matrix}\langle\frac{[0.042, 0.052],\ [0.948, 0.958],\ [0.948, 0.958]}{P_{BT}^{Albumin}}, \frac{[0.035, 0.045],\ [0.955, 0.965],\ [0.955, 0.965]}{P_{BT}^{Globulin\ Serum}}, \frac{[0.045, 0.100],\ [0.900, 0.955],\ [0.900, 0.955]}{P_{BT}^{Bilirubin\ Total}}\rangle, \\ \langle\frac{[0.031, 0.052],\ [0.948, 0.969],\ [0.948, 0.969]}{P_{AT}^{Albumin}}, \frac{[0.021, 0.035],\ [0.965, 0.979],\ [0.965, 0.979]}{P_{AT}^{Globulin\ Serum}}, \frac{[0.025, 0.100],\ [0.900, 0.975],\ [0.900, 0.975]}{P_{AT}^{Bilirubin\ Total}}\rangle\end{matrix}\right\}$$

$$P^2_{NVB} = \left\{ \begin{array}{l} \langle \frac{[0.025, 0.052],\ [0.948, 0.975],\ [0.948, 0.975]}{P^{Albumin}_{BT}}, \frac{[0.033, 0.035],\ [0.965, 0.967],\ [0.965, 0.967]}{P^{Globulin\ Serum}_{BT}}, \frac{[0.070, 0.100],\ [0.900, 0.930],\ [0.900, 0.930]}{P^{Bilirubin\ Total}_{BT}} \rangle, \\ \langle \frac{[0.036, 0.052],\ [0.948, 0.964],\ [0.948, 0.964]}{P^{Albumin}_{AT}}, \frac{[0.035, 0.042],\ [0.958, 0.965],\ [0.958, 0.965]}{P^{Globulin\ Serum}_{AT}}, \frac{[0.017, 0.100],\ [0.900, 0.983],\ [0.900, 0.983]}{P^{Bilirubin\ Total}_{AT}} \rangle \end{array} \right\}$$

$$P^3_{NVB} = \left\{ \begin{array}{l} \langle \frac{[0.052, 0.064],\ [0.936, 0.948],\ [0.936, 0.948]}{P^{Albumin}_{BT}}, \frac{[0.011, 0.035],\ [0.965, 0.989],\ [0.965, 0.989]}{P^{Globulin\ Serum}_{BT}}, \frac{[0.011, 0.035],\ [0.965, 0.989],\ [0.965, 0.989]}{P^{Bilirubin\ Total}_{BT}} \rangle, \\ \langle \frac{[0.052, 0.064],\ [0.936, 0.948],\ [0.936, 0.948]}{P^{Albumin}_{AT}}, \frac{[0.019, 0.035],\ [0.965, 0.981],\ [0.965, 0.981]}{P^{Globulin\ Serum}_{AT}}, \frac{[0.099, 0.100],\ [0.900, 0.901],\ [0.900, 0.901]}{P^{Bilirubin\ Total}_{AT}} \rangle \end{array} \right\}$$

D^{LFT}_{NVB} is a NVBS formed, based on the actual range fixed for a liver function test. Ranges for D^{LFT}_{NVB} under a liver function test for albumin, Globulin serum and Bilirubin Total is given as follows:

Before Treatment (BT)	D^{LFT}_{NVB}
Albumin	[0.034, 0.052], [0.948, 0.966], [0.948, 0.966]
Globulin Serum	[0.015, 0.035], [0.965, 0.985], [0.965, 0.985]
Bilirubin Total	[0.000, 0.100], [0.900, 0.100], [0.900, 0.100]

After Treatment (AT)	D^{LFT}_{NVB}
Albumin	[0.034, 0.052], [0.948, 0.966], [0.948, 0.966]
Globulin Serum	[0.015, 0.035], [0.965, 0.985], [0.965, 0.985]
Bilirubin Total	[0.000, 0.100], [0.900, 0.100], [0.900, 0.100]

Above datas are converted to NVBS as below.

$$D^{LFT}_{NVB} = \left\{ \begin{array}{l} \langle \frac{[0.034, 0.052],\ [0.948, 0.966],\ [0.948, 0.966]}{D^{Albumin}_{BT}}, \frac{[0.015, 0.035],\ [0.965, 0.985], [0.965, 0.985]}{D^{Globulin-Serum}_{BT}}, \frac{[0.000, 0.100],\ [0.900, 0.100],\ [0.900, 0.100]}{D^{Bilirubin\ Total}_{BT}} \rangle, \\ \langle \frac{[0.034, 0.052],\ [0.948, 0.966],\ [0.948, 0.966]}{D^{Albumin}_{AT}}, \frac{[0.015, 0.035],\ [0.965, 0.985], [0.965, 0.985]}{D^{Globulin-Serum}_{AT}}, \frac{[0.000, 0.100],\ [0.900, 0.100],\ [0.900, 0.100]}{D^{Bilirubin\ Total}_{AT}} \rangle \end{array} \right\}$$

Neutrosophic vague binary euclidean distance measure can be used to diagonise which patient is more suffering with liver problems even after treatment. Following table gives the neutrosophic vague binary euclidean difference between each of the patients from D^{LFT}_{NVB}

$d^{ED}_{NVBS}(P^1_{NVB}, D^{LFT}_{NVB})$	$d^{ED}_{NVBS}(P^2_{NVB}, D^{LFT}_{NVB})$	$d^{ED}_{NVBS}(P^3_{NVB}, D^{LFT}_{NVB})$
0.014856	0.277330	0.745502

Lowest neutrosophic vague binary euclidean difference is for patient I. So patient I suffers more with liver problems even after treatment

9. Conclusions

Neutrosophic vague binary sets are developed in this paper with some examples and basic concepts. Real life situations demand binary and higher dimensional universes than a unique one. Being the vital concept to homeomorphism - 'which is the underlying principle to any topology' – continuity has an important role in topology. It is also developed for this new concept. Practical applications are tremendous for binary concept in day today life. One real life example in medical diagonosis is discussed above. Several situations demand combined result than 'a unique separate one'- to compare and deal situations in a more fast manner. Neutrosophic vague binary sets is a good tool for comparison in such cases. It could be made use in surveys, case studies and in some other sort of similar situations. Topology are special type of subsets to a universal set- based on which study of all other subsets of the universal set is possible. New study will produce a combined result or net effect than taking a single result. This work could be extended by taking subsets of the common universe.

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Multi-level linear programming problem with neutrosophic numbers: A goal programming strategy

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Abstract: In the paper, we propose an alternative strategy for multi-level linear programming (MLP) problem with neutrosophic numbers through goal programming strategy. Multi-level linear programming problem consists of *k* levels where there is an upper level at the first level and multiple lower levels at the second level with one objective function at every level. Here, the objective functions of the level decision makers and constraints are described by linear functions with neutrosophic numbers of the form [u + v*I*], where u, v are real numbers and *I* signifies the indeterminacy. At the beginning, the neutrosophic numbers are transformed into interval numbers and consequently, the original problem transforms into MLP problem with interval numbers. Then we compute the target interval of the objective functions via interval programming procedure and formulate the goal achieving functions. Due to potentially conflicting objectives of *k* decision makers, we consider a possible relaxation on the decision variables under the control of each level in order to avoid decision deadlock. Thereafter, we develop three new goal programming models for MLP problem with neutrosophic numbers. Finally, an example is solved to exhibit the applicability, feasibility and simplicity of the proposed strategy.

Keywords: neutrosophic numbers; interval numbers; multi-level linear programming; goal programming

1. Introduction

Multi-level programming (MLP) programming problem consists of multi-levels with single objective function at each level where each level decision maker (DM) tries to get maximum benefit over a common feasible region. In the paper, we consider an MLP problem with neutrosophic numbers information where the objective functions and common constraints are linear functions and each DM independently controls a set of decision variables. In 1988, Anandalingam [1] proposed Stackelberg solution concept for MLP problem in crisp environment and extended the concept to solve decentralized bi-level programming problem.

Goal programming (GP) [2, 3, 4, 5, 6, 7, 8] is one of the popular mathematical tools for solving multi-objective mathematical programming problems with multiple and conflicting objectives to obtain optimal compromise solutions. In 1991, Inuiguchi and Kume [9] incorporated the notion of interval GP.

In 1998, Smarandache [10] incorporated a novel concept called neutrosophic set to tackle with inconsistent, incomplete, indeterminate information where indeterminacy is an independent and important factor. Roy and Das [11] developed a computational algorithm for solving multi-objective linear programming problem by utilizing neutrosophic optimization technique. Das and Roy [12] used neutrosophic optimization method for obtaining optimal solution for multi-objective non linear programming problem. Hezam et al. [13] used first order Taylor polynomial series approximation method for neutrosophic multi-objective programming problem. Abdel-Baset et al. [14] developed two models for neutrosophic goal programming problems and applied the concept to industrial design problem. In 2016, Pramanik [15] proposed three novel neutrosophic GP models for optimization problem by minimizing indeterminacy membership functions for practical neutrosophic optimization. Pramanik [16] also proposed the framework of neutrosophic linear goal programming for multi-objective optimization with uncertainty and indeterminacy simulteneously.

Smarandache [17, 18] introduced the concept of neutrosophic number and presented its fundamental properties. Jiang and Ye [19] presented a general neutrosophic number optimization model for solving optimal design of truss structures. Deli and Şubaş [20] developed a ranking method for single valued neutrosophic numbers and applied the concept to solve a multi-attribute decision making problem. Ye [21] discussed a neutrosophic number linear programming technique for neutrosophic number optimization problems where objective functions and constraints are described by neutrosophic numbers. Ye et al. [22] presented general solutions of neutrosophic number non-linear optimization models for unconstrained and constrained problems.

In 2018, Pramanik and Banerjee [23] discussed a solution methodology for single-objective linear programming problem where the coefficients of objective functions and the constraints are neutrosophic numbers. Pramanik and Banerjee [24] also studied GP technique for multi-objective linear programming problem with neutrosophic coefficients. Recently, Pramanik and Dey [25] proposed novel GP models for solving bi-level programming problem with neutrosophic numbers by minimizing deviational variables. In this paper, we extend the concept of Pramanik and Dey [25] to solve MLP problem with neutrosophic numbers based on GP strategy.

We organize the paper in the following way. In section 2, some definitions concerning interval numbers, neutrosophic numbers and their essential properties are given. In section 3, we present the mathematical formulation of MLP problem described by neutrosophic numbers. In section 4, the GP strategies for MLP problem with neutrosophic numbers is discussed by considering upper (superior) and lower (inferior) preference bounds on the decision vectors of the level DMs. In section 5, an application of the developed strategy for MLP problem is demonstrated. Finally, conclusion with some future scope of research is provided in the last section.

2. Preliminaries

In the section, we provide some basic definitions regarding interval numbers, neutrosophic numbers.

2.1 Interval number [26]

An interval number is defined by $P = [P^L, P^U] = \{p: P^L \le p \le P^U, p \in \Re\}$, where P^L, P^U are left and right limit of the interval P on the real line $\Re$.

Definition 2.1: Let γ (*P*) and δ (*P*) be the midpoint and the width of an interval number, respectively.

Then, $\gamma(P) = \frac{1}{2}[P^L + P^U]$ and $\delta(P) = [P^U - P^L]$

The scalar multiplication of *P* by μ is defined as given below.

$$\mu P = \begin{cases} [\mu P^L, \mu P^U], \mu \geq 0, \\ [\mu P^U, \mu P^L], \mu \leq 0 \end{cases}$$

The absolute value of *P* is defined as given below.

$$|P| = \begin{cases} [P^L, P^U], P^L \geq 0, \\ [0, \max\{-P^L, P^U\}], P^L < 0 < P^U \\ [-P^U, -P^L], P^U \leq 0 \end{cases}$$

The binary operation * between $P_1 = [P_1^L, P_1^U]$ and $P_2 = [P_2^L, P_2^U]$ is defined as follows:

$P_1 * P_2 = \{p_1 * p_2\text{: } P_1^L \leq p_1 \leq P_1^U, P_2^L \leq p_2 \leq P_2^U, p_1, p_2 \in \Re\}$.

2.2 Neutrosophic number [17, 18]

A neutrosophic number is represented by $E = m + nI$, where m, n are real numbers where m is determinate part and n*I* is indeterminate part and $I \in [I^L, I^U]$ represents indeterminacy.

Therefore, $E = [m + nI^L, m + nI^U] = [E^L, E^U]$, (say)

Example: Suppose a neutrosophic number $E = 2 + 3I$, where 2 is determinate part and 3*I* is indeterminate part. Here, we take $I \in [0.2, 0.7]$. Then, *E* becomes an interval number of the form *N* = [2.6, 4.1].

Now, we define some properties regarding neutrosophic numbers as follows:

Suppose that $E_1 = [m_1 + n_1I_1] = [m_1 + n_1 I_1^L, m_1 + n_1 I_1^U] = [E_1^L, E_1^U]$ and $E_2 = [m_2 + n_2 I_2] = [m_2 + n_2 I_2^L, m_2 + n_2 I_2^U] = [E_2^L, E_2^U]$ be two neutrosophic numbers where $I_1 \in [I_1^L, I_1^U]$, $I_2 \in [I_2^L, I_2^U]$, then

(i). $E_1 + E_2 = [E_1^L + E_2^L, E_1^U + E_2^U]$,

(ii). $E_1 - E_2 = [E_1^L - E_2^U, E_1^U - E_2^L]$,

(iii). $E_1 \times E_2 = [\text{Min}\{E_1^L \times E_2^L, E_1^L \times E_2^U, E_1^U \times E_2^L, E_1^U \times E_2^U\}, \text{Max}\{E_1^L \times E_2^L, E_1^L \times E_2^U, E_1^U \times E_2^L, E_1^U \times E_2^U\}]$

(iv). $E_1 / E_2 = [\text{Min}\{E_1^L / E_2^L, E_1^L / E_2^U, E_1^U / E_2^L, E_1^U / E_2^U\}, \text{Max}\{E_1^L / E_2^L, E_1^L / E_2^U, E_1^U / E_2^L, E_1^U / E_2^U\}]$, if $0 \notin E_2$.

3. Formulation of MLP problem for minimization-type objective function with neutrosophic numbers

Mathematically, an MLP problem with neutrosophic numbers for minimization-type objective function at every level can be formulated as given below.

$$\underset{x_1}{Min}\ Z_1(x) = [A_{11} + B_{11}I_{11}]\, x_1 + [A_{12} + B_{12}\, I_{12}]\, x_2 + \ldots + [A_{1k} + B_{1k}\, I_{1k}]\, x_k + [G_1 + H_1I_1] \quad (1)$$

$$\underset{x_2}{Min}\ Z_2(x) = [A_{21} + B_{21}I_{21}]\, x_1 + [A_{22} + B_{22}\, I_{22}]\, x_2 + \ldots + [A_{2k} + B_{2k}\, I_{2k}]\, x_k + [G_2 + H_2I_2] \quad (2)$$

$$\underset{x_k}{Min}\ Z_k(x) = [A_{k1} + B_{k1}I_{k1}]\, x_1 + [A_{k2} + B_{k2}\, I_{12}]\, x_2 + \ldots + [A_{kk} + B_{kk}\, I_{kk}]\, x_k + [G_k + H_kI_k] \quad (3)$$

Subject to

$$x \in X = \{x = (x_1, x_2, \ldots, x_k) \in R^N \mid [C_1 + D_1 I_1^{/}]\, x_1 + [C_2 + D_2 I_2^{/}]\, x_2 + \ldots + [C_k + D_k I_k^{/}]\, x_k \leq \rho + \sigma I^{/}, x \geq 0\}. \quad (4)$$

Here, $x_i = (x_{i1}, x_{i2}, \ldots, x_{iN_i})^T$: Decision vector under the control of i-th level DM, i = 1, 2, .., *k*. A_{i1}, B_{i1} (i = 1, 2, ..., *k*) are N_1- dimension row vectors; A_{i2}, B_{i2} (i = 1, 2, ..., *k*) are N_2- dimension row vectors;

and similarly, A_{ik}, B_{ik} (i = 1, 2, ..., k) are N_k - dimension row vectors where N = $N_1 + N_2 + \ldots + N_k$; and G_i, H_i (i = 1, 2, ..., k) are constants. C_i, D_i (i = 1, 2, ..., k) are $M \times N_i$ (i = 1, 2, ..., k) constant matrix and ρ , σ are M dimensional constant column matrix. X ($\neq \Phi$) is considered compact and convex in R^N. Also, we have $I_{ij} \in [I_{ij}^L, I_{ij}^U]$, i = 1, 2, ..., k; j = 1, 2, ..., k; $I_i \in [I_i^L, I_i^U]$, $I_i^{/} \in [I_i^{/L}, I_i^{/U}]$, i = 1, 2, ..., k. Representation of an MLP problem is shown in Figure 1 as follows.

First level

Second level

k-th level

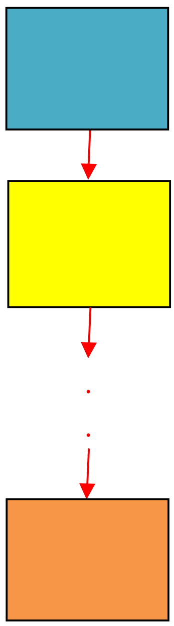

Figure 1. Depiction of an MLP problem

4. Goal programming strategy for solving MLP problem involving neutrosophic numbers

The MLP problem with neutrosophic numbers that is defined in Section 3 can be restated as follows:

First level:

$$\underset{x_1}{Min}\ Z_1(x) = [A_{11} + B_{11} I_{11}]\, x_1 + [A_{12} + B_{12} I_{12}]\, x_2 + \ldots + [A_{1k} + B_{1k} I_{1k}]\, x_k + [G_1 + H_1 I_1]$$

$$= \{[A_{11} + B_{11} I_{11}^L]\, x_1 + [A_{12} + B_{12} I_{12}^L]\, x_2 + \ldots + [A_{1k} + B_{1k} I_{1k}^L]\, x_k + [G_1 + H_1 I_1^L],\ [A_{11} + B_{11} I_{11}^U]\, x_1 + [A_{12} + B_{12} I_{12}^U]\, x_2 + \ldots + [A_{1k} + B_{1k} I_{1k}^U]\, x_k + [G_1 + H_1 I_1^U]\} = [S_1^L(x), S_1^U(x)] \text{ (say);} \quad (5)$$

Second level:

$$\underset{x_2}{Min}\ Z_2(x) = [A_{21} + B_{21} I_{21}]\, x_1 + [A_{22} + B_{22} I_{22}]\, x_2 + \ldots + [A_{2k} + B_{2k} I_{2k}]\, x_k + [G_2 + H_2 I_2]$$

$$= \{[A_{21} + B_{21} I_{21}^L]\, x_1 + [A_{22} + B_{22} I_{22}^L]\, x_2 + \ldots + [A_{2k} + B_{2k} I_{2k}^L]\, x_k + [G_2 + H_2 I_2^L],\ [A_{21} + B_{21} I_{21}^U]\, x_1 + [A_{22} + B_{22} I_{22}^U]\, x_2 + \ldots + [A_{2k} + B_{2k} I_{2k}^U]\, x_k + [G_2 + H_2 I_2^U]\} = [S_2^L(x), S_2^U(x)] \text{ (say);} \quad (6)$$

and similarly, for

k- th level:

$$\underset{x_k}{Min}\ Z_k(x) = [A_{k1} + B_{k1} I_{k1}]\, x_1 + [A_{k2} + B_{k2} I_{k2}]\, x_2 + \ldots + [A_{kk} + B_{kk} I_{kk}]\, x_k + [G_k + H_k I_k]$$

$$= \{[A_{k1} + B_{k1} I_{k1}^L]\, x_1 + [A_{k2} + B_{k2} I_{k2}^L]\, x_2 + \ldots + [A_{kk} + B_{kk} I_{kk}^L]\, x_k + [G_k + H_k I_k^L],\ [A_{k1} + B_{k1} I_{k1}^U]\, x_1 + [A_{k2} + B_{k2} I_{k2}^U]\, x_2 + \ldots + [A_{kk} + B_{kk} I_{kk}^U]\, x_k + [G_k + H_k I_k^U]\} = [S_k^L(x), S_k^U(x)] \text{ (say);} \quad (7)$$

and the system constrains reduce to

$$[C_1 + D_1 I_1^{/}]\, x_1 + [C_2 + D_2 I_2^{/}]\, x_2 + \ldots + [C_k + D_k I_k^{/}]\, x_k \ge \rho + \sigma I^{/}$$

$\Rightarrow$ {[$C_1 + D_1 I_1^{/L}$] x_1 + [$C_2 + D_2 I_2^{/L}$] x_2 + ...+ [$C_k + D_k I_k^{/L}$] x_k, {[$C_1 + D_1 I_1^{/U}$] x_1 + [$C_2 + D_2 I_2^{/U}$] x_2 + ...+ [C_k + $D_k I_k^{/U}$] x $\geq [\rho + \sigma\ I^{/L}, \rho + \sigma\ I^{/U}] = [R^L, R^U]$ (say)

$\Rightarrow [W^L(x), W^U(x)] \geq [R^L, R^U]$. (8)

Proposition 1. [27]

If $\sum_{j=1}^{n} [\alpha_1^j, \alpha_2^j] z_j \geq [q_1, q_2]$, then $\sum_{j=1}^{n} [\alpha_2^j] z_j \geq q_1$, $\sum_{j=1}^{n} [\alpha_1^j] z_j \geq q_2$ are the maximum and minimum value range inequalities for the constraint condition, respectively.

According to the proposition 1 of Shaocheng [27], the interval inequality of the system constraints (8) transform to the following inequalities as follows:

[$C_1 + D_1 I_1^{/L}$] x_1 + [$C_2 + D_2 I_2^{/L}$] $x_2 \geq R^U$, [$C_1 + D_1 I_1^{/U}$] x_1 + [$C_2 + D_2 I_2^{/U}$] $x_2 \geq R^L$, $x_i \geq 0$, i = 1, 2,

i.e. $W^L(x) \geq R^U$, $W^U(x) \geq R^L$, $x \geq 0$.

Hence, the minimization-type MLP problem can be re-formulated as follows:

First level: $\underset{x_1}{Min}\ Z_1(x) = [S_1^L(x), S_1^U(x)]$,

Second level: $\underset{x_2}{Min}\ Z_2(x) = [S_2^L(x), S_2^U(x)]$,

.

.

k-th level: $\underset{x_k}{Min}\ Z_k(x) = [S_k^L(x), S_k^U(x)]$,

Subject to

$[W^L(x), W^U(x)] \geq [R^L, R^U]$, $x \geq 0$. (9)

For getting the best optimal solution of Z_i, (i = 1, 2, ..., k), the following problem is solved owing to Ramadan [28] as follows:

$\underset{x \in X}{Min}\ Z_i(x) = S_i^L(x)$, i = 1, 2, ..., k

Subject to

$W^U(x) \geq R^L$, $x \geq 0$, i = 1, 2, ..., k. (10)

We solve the Eq. (10) and let $x_i^B = (x_{i1}^B, x_{i2}^B, ..., x_{iN_1}^B, x_{iN_{1+1}}^B, ..., x_{iN}^B)$, (i = 1, 2, ..., k) be the individual best solution of i-th level DM and $S_i^L(x_i^B)$, (i = 1, 2, ..., k) be the individual best objective value of i-th level DM, (i = 1, 2, ..., k).

For obtaining the worst optimal solution of Z_i, (i = 1, 2, ..., k), we solve the following problem due to Ramadan [28] as given below.

$\underset{x \in X}{Min}\ Z_i(x) = S_i^U(x)$, i = 1, 2, ..., k

Subject to

$W^L(x) \geq R^U$, $x \geq 0$. (11)

Let $x_i^* = (x_{i1}^*, x_{i2}^*, ..., x_{iN_1}^*, x_{iN_{1+1}}^*, ..., x_{iN}^*)$, (i = 1, 2, ..., k) be the individual worst solution of i-th level DM subject to the given constraints and $S_i^U(x_i^*)$, (i = 1, 2, ..., k) be the individual worst objective value of i-th level DM, (i = 1, 2, ..., k).

Therefore, [$S_i^L(x_i^B)$, $S_i^U(x_i^*)$] be the optimal value of i-th level DM, (i = 1, 2, ..., k) in the interval form. Let [T_i^+, U_i^+] be the target interval of i-th objective functions set by level DMs.

The target level of i-th objective function can be formulated as follows:

$S_i^U(x) \geq T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) \leq U_i^+$, (i = 1, 2, ..., k).

Hence, the goal achievement functions are formulated as follows:

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

where d_i^U, d_i^L, (i = 1, 2, ..., k) are deviational variables.

In a large hierarchical organization, the individual benefit of the level DMs are not same, cooperation between k level DMs is necessary to arrive at a compromise optimal solution.

Suppose that $x_i^B = (x_{i1}^B, x_{i2}^B, ..., x_{iN_i}^B, x_{iN_i+1}^B, ..., x_{iN}^B)$, (i = 1, 2, ..., k) be the individual best solution of i-th level DM. Suppose ($x_i^B - \eta_i$) and ($x_i^B + \tau_i$), (i = 1, 2, ..., k) be the lower and upper bounds of decision vector provided by i-th level DM where η_i and τ_i, (i = 1, 2, ..., k) are the negative and positive tolerance variables which are not essentially equal [25, 29-41].

Now by considering the preference bounds of the decision variables, we propose three alternative GP models for MLP problem with neutrosophic numbers as follows:

GP Model I.

Min $\sum_{i=1}^{k} (d_i^U + d_i^L)$

Subject to

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

$W^L(x) \geq R^U$, $W^U(x) \geq R^L$,

$(x_i^B - \eta_i) \leq x_i \leq (x_i^B + \tau_i)$, (i = 1, 2, ..., k)

$d_i^L, d_i^U, x \geq 0$, (i = 1, 2).

GP Model II.

Min $\sum_{i=1}^{k} (w_i^U d_i^U + w_i^L d_i^L)$

Subject to

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

$W^L(x) \geq R^U$, $W^U(x) \geq R^L$,

$(x_i^B - \eta_i) \leq x_i \leq (x_i^B + \tau_i)$, (i = 1, 2, ..., k)

$w_i^U \geq 0$, $w_i^L \geq 0$, (1, 2, ..., k)

$d_i^L, d_i^U, x \geq 0$, (1, 2, ..., k).

GP Model III.

Min ψ

Subject to

$-S_i^U(x) + d_i^U = -T_i^+$, (i = 1, 2, ..., k)

$S_i^L(x) + d_i^L = U_i^+$, (i = 1, 2, ..., k)

$W^L(x) \geq R^U$, $W^U(x) \geq R^L$,

$(x_i^B - \eta_i) \leq x_i \leq (x_i^B + \tau_i)$, (i = 1, 2, ..., k)

$\psi \geq d_i^U, \psi \geq d_i^L$, (i = 1, 2)

$d_i^L, d_i^U, x \geq 0$, (1, 2, ..., k).

A flowchart of the proposed strategy for MLP problem with neutrosophic coefficients is shown in Figure 2.

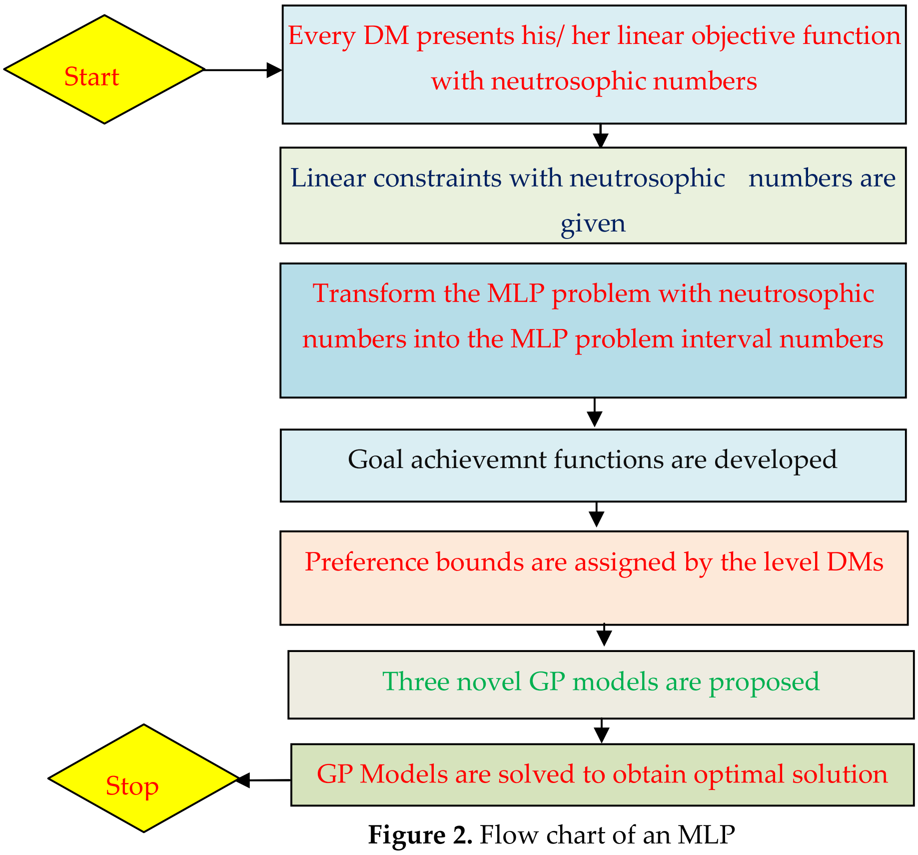

Figure 2. Flow chart of an MLP

5. Numerical Example

We consider the following MLP problem with neutrosophic numbers to demonstrate the proposed GP procedure. Without any loss of generality we consider $I \in [0, 1]$.

First level:

$$\underset{x_1}{Min}\ Z_1(x) = [11 + 2I]\, x_1 + [7 + 3I]\, x_2 + [3 + I]\, x_3,$$

Second level:

$$\underset{x_2}{Min}\ Z_2(x) = [1 + 2I]\, x_1 + [2 + I]\, x_2 + [2 + 3I]\, x_3 + [4 + I],$$

Third level:

$$\underset{x_3}{Min}\ Z_3(x) = [1 + 2I]\, x_1 + [2 + I]\, x_2 + 0.5\, x_3 + [5 + I],$$

Subject to

$[3 + 2I]\, x_1 + [1 + I]\, x_2 + [1 + 2I]\, x_3 \geq [5 + 2I]$,

$[4 + I]\, x_1 + [2 + 3I]\, x_2 - [2 + I]\, x_3 \geq [4 + 3I]$,

$[1 + I]\, x_1 + [2 + 2I]\, x_2 + [2 + I]\, x_3 \geq [3 + 2I]$,

$x_1, x_2, x_3 \geq 0$.

Using interval programming technique, the transformed problem of first level DM can be presented as follows (see Table 1):

Table 1. First level DM's problem for best and worst solutions

First level DM's problem to obtain best solution	First level DM's problem to obtain worst solution
Min $S_1^L(x) = 11x_1 + 7x_2 + 7x_3$	Min $S_1^U(x) = 13x_1 + 10x_2 + 4x_3$
Subject to	Subject to
$5x_1 + 2x_2 + 3x_3 \geq 5$,	$3x_1 + x_2 + x_3 \geq 7$,
$5x_1 + 5x_2 - 3x_3 \geq 4$,	$4x_1 + 2x_2 - 2x_3 \geq 7$,
$2x_1 + 4x_2 + 3x_3 \geq 3$,	$x_1 + 2x_2 + 2x_3 \geq 5$,
$x_1, x_2, x_3 \geq 0$.	$x_1, x_2, x_3 \geq 0$.

The best and worst solutions of First level DM are calculated as follows (see Table 2):

Table 2. First level DM's best and worst solutions

The best solution	The worst solution
S_1^B = 10.536 at (0.78, 0.171, 0.252)	S_1^* = 34.3 at (1.8, 0.75, 0.85)

The transformed problem of second level DM can be presented as follows (see Table 3):

Table 3. Second level DM's problem for best and worst solutions

Second level DM's problem to get best solution	Second level DM's problem to get worst solution
Min $S_1^L(x) = x_1 + 2x_2 + 2x_3 + 4$	Min $S_2^U(x) = 3x_1 + 3x_2 + 5x_3 + 5$
Subject to	Subject to
$5x_1 + 2x_2 + 3x_3 \geq 5$,	$3x_1 + x_2 + x_3 \geq 7$,
$5x_1 + 5x_2 - 3x_3 \geq 4$,	$4x_1 + 2x_2 - 2x_3 \geq 7$,
$2x_1 + 4x_2 + 3x_3 \geq 3$,	$x_1 + 2x_2 + 2x_3 \geq 5$,
$x_1, x_2, x_3 \geq 0$.	$x_1, x_2, x_3 \geq 0$.

The best and worst solutions of second level DM are determined as given below (see Table 4)

Table 4. Second level DM's best and worst solutions

The best solution	The worst solution
S_2^B = 5.5 at (0.875, 0.312, 0)	S_2^* = 15.2 at (1.8, 1.6, 0)

Similarly, the transformed problem of third level DM can be shown as follows (see Table 5):

Table 5. Third level DM's problem for best and worst solutions

Third level DM's problem to get best solution	Third level DM's problem to get worst solution
Min $S_3^L(x) = x_1 + 2x_2 + 0.5x_3 + 5$	Min $S_3^U(x) = 3x_1 + 3x_2 + 0.5x_3 + 6$
Subject to	Subject to
$5x_1 + 2x_2 + 3x_3 \geq 5$,	$3x_1 + x_2 + x_3 \geq 7$,
$5x_1 + 5x_2 - 3x_3 \geq 4$,	$4x_1 + 2x_2 - 2x_3 \geq 7$,
$2x_1 + 4x_2 + 3x_3 \geq 3$,	$x_1 + 2x_2 + 2x_3 \geq 5$,
$x_1, x_2, x_3 \geq 0$.	$x_1, x_2, x_3 \geq 0$.

The best and worst solutions of third level DM are computed as given below (see Table 6)

Table 6. Third level DM DM's best and worst solutions

The best solution	The worst solution
$S_3^B = 6.167$ at (1, 0, 0.333)	$S_3^* = 13.85$ at (2.4, 0, 1.3)

The objective function of first level DM with specified targets can be presented as follows:

$11x_1 + 7x_2 + 3x_3 \leq 35$, $13x_1 + 10x_2 + 4x_3 \geq 11$,

The goal achievement functions of first level DM with specified targets can be presented as follows:

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35$, $-13x_1 - 10x_2 - 4x_3 + d_1^U = -11$,

The objective function of second level DM with specified targets can be presented as follows:

$x_1 + 2x_2 + 2x_3 \leq 16$, $3x_1 + 3x_2 + 5x_3 + 5 \geq 6$,

Also, the goal achievement functions of LDM with specified targets can be developed as follows:

$x_1 + 2x_2 + 2x_3 + d_2^L = 16$, $-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6$,

Similarly, the objective function of third level DM with specified targets can be presented as follows:

$x_1 + 2x_2 + 0.5x_3 + 5 \leq 14$, $3x_1 + 3x_2 + 0.5x_3 + 6 \geq 7$,

Also, the goal achievement functions of third level DM with specified targets can be established as follows:

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14$, $-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7$,

Let, the first level DM assigns preference bounds on the decision variable x_1 as $0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8$, the second level DM offers preference bounds on the decision variable x_2 as $0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5$, and the third level DM provides preference bounds on the decision variable x_3 as $0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5$, in order to get optimal compromise solution.

Therefore, the GP models for MLP problem involving neutrosophic coefficients can be developed as follows:

GP Model I.

Min ($d_1^L + d_1^U + d_2^L + d_2^U + d_3^L + d_3^U$)

Subject to

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35,$

$-13x_1 - 10x_2 - 4x_3 + d_1^U = -11,$

$x_1 + 2x_2 + 2x_3 + d_2^L = 16,$

$-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6,$

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14,$

$-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7,$

$5x_1 + 2x_2 + 3x_3 \geq 5,$

$5x_1 + 5x_2 - 3x_3 \geq 4,$

$2x_1 + 4x_2 + 3x_3 \geq 3,$

$3x_1 + x_2 + x_3 \geq 7,$

$4x_1 + 2x_2 - 2x_3 \geq 7,$

$x_1 + 2x_2 + 2x_3 \geq 5,$

$0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8,$

$0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5,$

$0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5$

$d_i^L, d_i^U \geq 0$, (i = 1, 2, 3)

$x_1, x_2, x_3 \geq 0.$

GP Model II.

Min $\frac{1}{6}(d_1^L + d_1^U + d_2^L + d_2^U + d_3^L + d_3^U)$

Subject to

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35,$

$-13x_1 - 10x_2 - 4x_3 + d_1^U = -11,$

$x_1 + 2x_2 + 2x_3 + d_2^L = 16,$

$-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6,$

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14,$

$-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7,$

$5x_1 + 2x_2 + 3x_3 \geq 5,$

$5x_1 + 5x_2 - 3x_3 \geq 4,$

$2x_1 + 4x_2 + 3x_3 \geq 3,$

$3x_1 + x_2 + x_3 \geq 7,$

$4x_1 + 2x_2 - 2x_3 \geq 7,$

$x_1 + 2x_2 + 2x_3 \geq 5,$

$0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8,$

$0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5,$

$0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5$

$d_i^L, d_i^U \geq 0$, (i = 1, 2, 3)

$x_1, x_2, x_3 \geq 0.$

GP Model III.

Min ψ

Subject to

$11x_1 + 7x_2 + 3x_3 + d_1^L = 35,$

$-13x_1 - 10x_2 - 4x_3 + d_1^U = -11,$

$x_1 + 2x_2 + 2x_3 + d_2^L = 16,$

$-3x_1 - 3x_2 - 5x_3 + 5 + d_2^U = -6,$

$x_1 + 2x_2 + 0.5x_3 + 5 + d_3^L = 14,$

$-3x_1 - 3x_2 - 0.5x_3 - 6 + d_3^U = -7,$

$5x_1 + 2x_2 + 3x_3 \geq 5,$

$5x_1 + 5x_2 - 3x_3 \geq 4,$

$2x_1 + 4x_2 + 3x_3 \geq 3,$

$3x_1 + x_2 + x_3 \geq 7,$

$4x_1 + 2x_2 - 2x_3 \geq 7,$

$x_1 + 2x_2 + 2x_3 \geq 5,$

$0.78 - 0.7 \leq x_1 \leq 0.78 + 0.8,$

$0.312 - 0.3 \leq x_2 \leq 0.312 + 1.5,$

$0.333 - 0.3 \leq x_3 \leq 0.333 + 1.5,$

$\psi \geq D_i^L, \psi \geq D_i^U$, (i = 1, 2, 3)

$d_i^L, d_i^U \geq 0$, (i = 1, 2, 3)

$x_1, x_2, x_3 \geq 0.$

The solutions of the developed GP models are shown in the Table 7 as follows:

Table 7. The solutions of the MLP problem involving neutrosophic numbers

GP Model	**Solution point** (x_1, x_2, x_3)	**Objective values** Z_1	Z_2	Z_3
GP Model I	**(1.58, 1.3, 0.96)**	**(29.36, 37.38)**	**(10.10, 18.44)**	**(9.66, 15.12)**
GP Model II	**(1.58, 1.3, 0.96)**	**(29.36, 37.38)**	**(10.10, 18.44)**	**(9.66, 15.12)**
GP Model III	**(1.58, 1.3, 0.96)**	**(29.36, 37.38)**	**(10.10, 18.44)**	**(9.66, 15.12)**

Note: It is observed that the three GP models produce the same optimal compromise solution set.

6. Conclusion

In the paper, we have proposed three new goal programming models for multi-level linear programming problem where objective and constraints are linear functions with neutrosophic coefficients. By applying interval programming procedure, we transform the multi-level linear programming problem into interval programming problem. Then, we determine best and worst solutions for all k -level decision makers and establish the goal achievement functions. We consider preference upper and lower bounds on the decision variables under the control of all k - level decision makers in order to achieve optimal compromise solution of the multi-level system. Finally, goal programming models are proposed to solve multi-level linear programming problem by minimizing deviational variables. A multi-level linear programming under neutrosophic numbers environment is finally solved to show the applicability and feasibility of the proposed GP strategy.

In future, we hope to utilize the proposed GP strategy to solve multi-objective decentralized bi-level linear programming, multi-objective decentralized multi-level linear programming problems, and other real world decision-making problems with neutrosophic numbers information.

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Abstract

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