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Concentric Plithogenic Hypergraph based on Plithogenic Hypersoft sets – A Novel Outlook

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Abstract: Plithogenic Hypersoft sets (PHS) introduced by Smarandache are the extensions of soft sets and hypersoft sets and it was further protracted to plithogenic fuzzy whole Hypersoft set to make it more applicable to multi attribute decision making environment. The fuzzy matrix representation of the plithogenic hypersoft sets lighted the spark of concentric plithogenic hypergraph. This research work lays a platform for presenting the concept of concentric plithogenic hypergraph, a graphical representation of plithogenic hypersoft sets. This paper comprises of the definition, classification of concentric plithogenic hypergraphs, extended hypersoft sets, extended concentric plithogenic hypergraphs and it throws light on its application. Concentric Plithogenic hypergraphs will certainly open the new frontiers of hypergraphs and this will undoubtedly bridge hypersoft sets and hypergraphs.

Keywords: Plithogenic sets; Hypergraph; Plithogenic Hypersoft sets; Concentric Plithogenic Hypergraphs

1. Introduction

The structure of a graph comprises of vertices and edges, has a range of applications in diverse fields. In general an edge in a graph represents the relation between two vertices. Berge [1] extended this basic idea and introduced hypergraph as the generalization of graph. In a hypergraph hyperedge links one or more vertices and they are mainly used to explore configuration of the systems by clustering and segmentation, but to handle the uncertain and imprecise environment; Kaufmann [2] introduced fuzzy hypergraphs. The concept of fuzzy set was introduced by Lofti.A. Zadeh [3]. The fuzzy hypergraph introduced by Kaufmann was later generalized by Hyung and Keon [4] to overcome the limitations of inappropriate representation of fuzzy partition by redefining fuzzy hypergraph and developing many expedient concepts which finds extensive applications in system analysis, circuit clustering and pattern recognition. In a fuzzy hypergraph the hyper edges are fuzzy sets of vertices. Mordeson and Nair [5] have made significant contributions to fuzzy graphs and fuzzy hypergraphs. Parvathi et al [6] extended of intuitionistic fuzzy graphs to intuitionistic fuzzy hypergraph in which (α, β)-cut hypergraph represent intuitionistic fuzzy partition. In an intuitionistic fuzzy hypergraph the hyperedge sets are intuitionistic fuzzy sets of vertices consisting of both membership and non-membership values as Atanssov [7] introduced in Intuitionistic set. Akram and Dudek [8] discussed the properties and applications of intuitionistic fuzzy hypergraph. Akram et al [9] introduced neutrosophic hypergraphs and single valued neutrosophic hypergraphs. Neutrosophic sets introduced by Smarandache [10] deals with truth function, indeterminacy function and falsity function, based on conceptualization of neutrosophic sets, Akram.et al [11] investigated the properties of line graph of neutrosophic hypergraph, dual neutrosophic hypergraph, tempered neutrosophic hypergraph and transversal neutrosophic hypergraph with illustrations. Neutrosophic theory has extensive applications in the domain of decision- making. Abdel-Baset et al [12] introduced a novel neutrosophic approach to assess the green supply chain management practices and the recent research in multi criteria decision- making uses the neutrosophic representations. On other hand Vasantha Kanthasamy et al [13]discussed Plithogenic graph, special type of graphs based on fuzzy intuitionistic and single valued neutrosophic graphs. The characteristics of plithogenic intuitionistic fuzzy graph, plithogenic neutrosophic graphs and plithogenic complex graphs are also examined, but notion of plithogenic hypergraph was not discoursed.

The plithogenic sets introduced by Smarandache [14] deals with attributes and it is extension of crisp, fuzzy, intuitionistic and neutrosophic sets. Plithogenic sets are widely used in multi attribute decision -making systems as it plays a vital role in deriving optimal solutions to the decision-making problems. Abdel-Basset et al [15,16] has framed a novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management and formulated a hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. These proposed models are highly advantageous, compatible to make decisions as it handles multi attributes or multi criteria environment. The selection process of alternatives based on different attributes containing several attribute values becomes easier in plithogenic representation. Furthermore plithogenic hypersoft introduced by Smarandache [17] also has significant contribution in multi attribute decision-making methods. Molodtsov [18] introduced soft sets and Smarandache generalized to hypersoft set by modifying single attribute function to multi attribute function. The plithogenic hypersoft set, generalization of crisp, fuzzy , intuitionistic and neutrosophic soft sets. Shazia Rana et al [19] extended plithogenic fuzzy hypersoft set to plithogenic fuzzy whole hypersoft set ;developed Frequency Matrix Multi Attributes Decision making scheme to rank the alternatives and proposed a new ranking approach based on frequency matrix in their research work. Plithogenic hypersoft sets are finding new avenue in decision- making and in ranking process.

Nivetha and Pradeepa [20,21] initiated integration of hypergraphs and fuzzy hypergraphs with Fuzzy Cognitive Maps (FCM). Kosko [22] introduced FCMs, directed graphs consisting of nodes and edges which represent the casual factors and its relationship. FCM assumes simple weights such as -1 if factors have negative impact over another, 0 if no impact and 1 for positive impact. Weighted FCM assumes values from [-1,1]. The approach of FCM is analogues to the reasoning and decision -making of human and it facilitates conception of intricate social systems. Peláezand Bowles [23],Miao and Liu [24], Papageorgiou et al [25]have proposed various algorithms and methods to handle various forms of FCM. The nature of weights classifies FCMs as intuitionistic and neutrosophic FCM. One of the most difficult aspects in handling FCM is consideration of large number of study factors. Confinement of the number of inputs is essential to make optimal decision-making and this has to take place step wise. It is helpful to limit the study factors for analyzing their inter impacts, to make so, FCMs with hypergraphic and fuzzy hypergraphic approaches facilitated to formulate student's low academic performance model and assessment model of blended method of teaching. Nivetha and Pradeepa [26] also introduced concentric fuzzy hypergraphs for inclusive decision -making and this kind of hypergraph deals with hyper envelopes instead of hyper edges; examined the properties of concentric fuzzy hypergraph and justified with suitable illustrations and applications. Concentric fuzzy hypergraphs was further extended to concentric neutrosophic hypergraphs to explore the factors causing autoimmune diseases using Fuzzy Cognitive Maps (FCM).

The proposed integrated models of FCM with hypergraphs, fuzzy hypergraphs and concentric fuzzy hypergraphs focus only on the factors based on single criteria. Suppose if the factors are dependent on multi criteria then the above integrated models do not meet the need. This is limitation of the above described integrated models. Concentric plithogenic hypergraphs integrated with FCMs helps to overcome such shortcomings. As the concept of plithogenic hypergraph was not disclosed so far, this research work extends concentric hypergraphic approach of FCM to concentric plithogenic hypergraph with plithogenic hypersoft representation to make optimal decisions by ranking the study factors based on multi attribute. Such representations will he highly pragmatic and it will certainly ease the decision-making process. The frequency matrices ranks the factors represented as plithogenic hypersoft sets and the core factors considered for determining the inter relationship and inter impacts using FCM method. This approach will definitely yield optimal results with simplified computations.

The objectives of this research work are to introduce notion of concentric plithogenic hypergraph; define concentric plithogenic hypergraph based on concentric fuzzy hypergraph and plithogenic graphs; classify concentric plithogenic hypergraph based on degree of appurtenance; widens concentric plithogenic hypergraph to extended concentric plithogenic hypergraph; proposes FCM decision making model integrated with extended concentric plithogenic hypergraphic approach. But this research work concentrates on proposing concentric plithogenic hypergraphs in a more distinct way. With the brief introduction of the research work in section 1, the rest of the paper is organized with construction of concentric plithogenic hypergraph in section 2, extension and classification of concentric plithogenic hypergraphs in section 3, applications of the proposed approach in section 4, discussion of the results in section 5 and the concluding remarks in the section 6.

2. Construction of Concentric Plithogenic Hypergraphs

The concept of concentric fuzzy hypergraphs evolved at times of integrating hypergraphs with Fuzzy Cognitive Maps after integration of hypergraphic and fuzzy hypergraphic approaches. Concentric plithogenic hypergraph is an integration concentric hypergraph with plithogenity. To make it more comprehensive, the following definitions are put forward.

2.1 Hypergraph

A hypergraph H is an ordered pair H = (X,E), where

(i) X = $\{x_1, x_2, .. x_n\}$ is a finite set of vertices.

(ii) E = $\{E_1, E_2, .. E_n\}$ is a family of subsets of X and each E_j is a hyper edge.

(iii) $E_j \neq \varphi$, j = 1,2,..3 and $\cup_j E_j = X$

2.2 Concentric Fuzzy Hypergraph

A concentric fuzzy hypergraph $\mathcal{C}_H$ is defined as follows

$\mathcal{C}_H$ = (X, $\mathcal{E}$)

X- finite set of vertex set

$\mathcal{E}$- Concentric fuzzy hyper envelope – family of fuzzy sets of X

$\mathcal{E}_j = \{(x_i, \mu_j(x_i)) / \mu_j(x_i) > 0 \text{ and } \forall x_i \in X\}$ j = 1,2,…m

Supp ($\mathcal{E}$) = X = Supp ($\mathcal{E}_j$) $\forall$ j = 1,2,…m

To illustrate concentric fuzzy hypergraph,

let $\mathcal{C}_H$ = (X, $\mathcal{E}$), where

X = $\{x_1, x_2, x_3, x_4\}$

$\mathcal{E}_1$ = {(x_1,0.4), (x_2,0.6), (x_3,0.5),(x_4,0.3)}

$\mathcal{E}_2$ = {(x_1,0.3), (x_2,0.8), (x_3,0.4),(x_4,0.5)}

$\mathcal{E}_3$ ={(x_1,0.6), (x_2,0.6), (x_3,0.7),(x_4,0.4)}

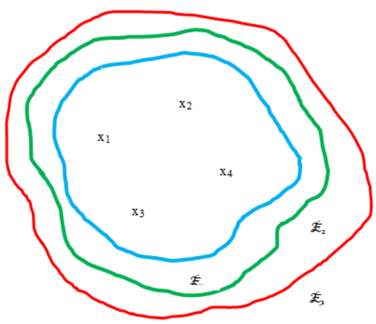

Fig.2.1. Concentric Fuzzy Hypergraph

2.3 Concentric Plithogenic Hypergraph

A hypergraph with non- empty, disjoint hyperedges and plithogenic envelopes is called as concentric plithogenic hypergraph P_{C_H}.

A concentric Plithogenic hypergraph P_{C_H} is defined as follows

P_{C_H} = (X, E, $\mathcal{P}_{\mathcal{E}}$)

- X- Finite vertex set
- E – Hyperedge set
- $\mathcal{P}_{\mathcal{E}}$ - Plithogenic envelope
- $\mathcal{P}_{\mathcal{E}j} = \{(x_{ij}, \mu(x_{ij})) / \mu(x_{ij}) > 0, x_{ij} \in E_i\}$
- $E_i \cap E_k = \emptyset$ and $\cup E_i = X$

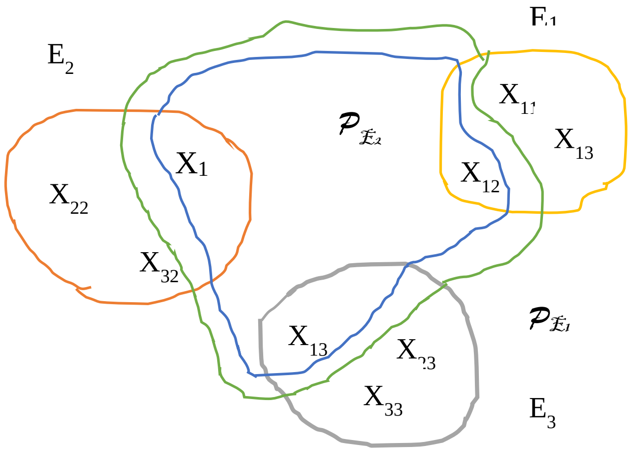

Fig.2.2. Concentric Plithogenic Hypergraph

2.4 Integration of Plithogenic Hypersoft sets and Concentric Plithogenic Hypergraph

Plithogenic hypersoft sets are extensively used in decision making situations. Let us consider the plithogenic hypersoft set presented below.

Let U = { M_1,M_2,M_3,M_4, M_5, M_6,M_7,M_8,M_9, M_{10}} be the university of discourse and set T = {M_1,M_3,M_6} ⊂ U. The attribute system is represented as follows $\mathcal{A}$ = { (A_1)**Maintenance Cost** {Maximum in the initial years of utility(A_1^1), Maximum in the latter years of utility(A_1^2), Moderate throughout (A_1^3)}, (A_2)**Reliability** {High with additional expenditure(A_2^1), Moderate with no extra expense(A_2^2), Moderate with high expense(A_2^3)}, (A_3)**Flexibility** {Single task oriented(A_3^1), Multi task oriented(A_3^2), Dual task oriented(A_3^3) }, (A_4)**Durability** {Very high in the beginning years of service(A_4^1), High in the latter years of service(A_4^2),Moderate (A_4^3) }, (A_5)**Profitability** {Moderate in the initial years(A_5^1), Maximum in the latter years(A_5^2), Moderate throughout the years (A_5^3)}}.

G: $A_1^1 \times A_2^2 \times A_3^2 \times A_4^1 \times A_5^2 \longrightarrow (\mathcal{U})$.

G(A_1^1,A_2^2 , A_3^2 , A_4^1,A_5^2) =

{M_1(0.9,0.875,0.8,0.75,0.5),M_3(0.67,0.5,0.4,0.8,0.7),M_6(0.8,0.7,0.6,0.7,0.5)}

In the below graphical representations the attributes A_1 to A_5 are the hyperedges consisting of the vertices (x_{ij}) A_i^j , i = 1,2,3,4,5 and j = 1,2,3. M_1, M_3 and M_6 are the plithogenic envelopes

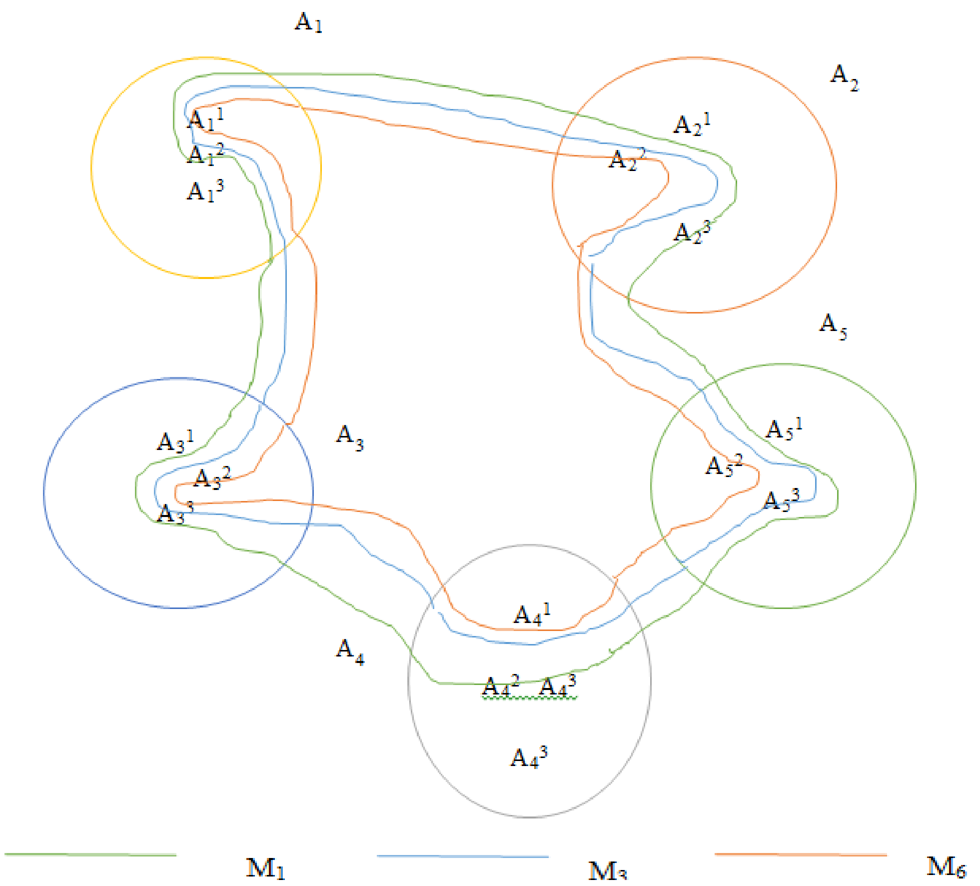

Fig 2.3 Graphical Representation of Plithogenic Hypersoft set by Concentric Plithogenic Hypergraph

To illustrate Concentric Plithogenic Hypergraph P_{C_H}based on Plithogenic Hypersoft sets, Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

$E = \{A_1, A_2, A_3, A_4, A_5\}$

$\mathcal{P}_{\mathcal{E}_1} = \{(A_1^1, 0.9), (A_2^2, 0.875), (A_3^2, 0.8), (A_4^1, 0.75), (A_5^2, 0.5)\}$

$\mathcal{P}_{\mathcal{E}_2} = \{(A_1^1, 0.67), (A_2^2, 0.5), (A_3^2, 0.4), (A_4^1, 0.8), (A_5^2, 0.7)\}$

$\mathcal{P}_{\mathcal{E}_3} = \{(A_1^1, 0.8), (A_2^2, 0.7), (A_3^2, 0.6), (A_4^1, 0.7), (A_5^2, 0.5)\}$

The formulation of the notion of concentric plithogenic hypergraph based on plithogenic hypersoft will be more rational. Also the plithogenic hypergraphs and concentric plithogenic hypergraphs can be defined based on plithogenic graphs as in the below fig.2.4 and fig.2.5

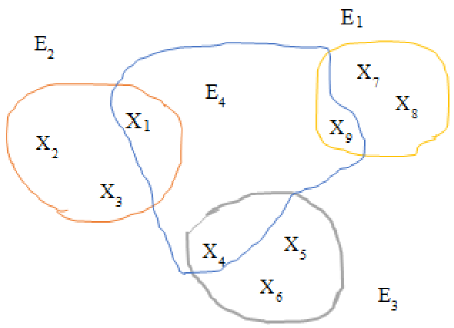

Fig.2.4 Plithogenic hypergraph

To illustrate Plithogenic Hypergraph P_{H^*}, based on Plithogenic graphs,

Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$

$E_1 = \{x_7(0.2, 0.3, 0.4), x_8\,(0.8, 0.6, 0.3), x_9\,(0.4, 0.2, 0.6)\}$

$E_2 = \{x_1(0.1, 0.5, 0.4), x_2\,(0.5, 0.6, 0.8), x_3\,(0.3, 0.2, 0.7)\}$

$E_3 = \{x_4(0.9, 0.3, 0.5), x_5\,(0.2, 0.4, 0.3), x_6(0.6, 0.2, 0.1)\}$

$E_4 = \{x_1(0.2, 0.9, 0.7), x_4\,(0.3, 0.7, 0.9), x_9\,(0.1, 0.8, 0.6)\}$

X is the vertex set and E_i, i = 1,2,3 are the plithogenic hyperedges of plithogenic hypergraph.

A hypergraph with hyperedges possessing plithogenic weight representations is defined as plithogenic hypergraph otherwise Plithogenic hypergraphs can be defined as hypergraphs with plithogenic hyperedges. A hypergraph with plithogenic hyper envelopes is defined as concentric plithogenic hypergraphs.

Fig.2.5 Concentric Plithogenic hypergraph

To illustrate Concentric Plithogenic Hypergraph P_{CH^*}, based on Plithogenic graphs

Let X = {x_1,x_2,x_3}

$\mathcal{PE}_1$ = {(x_1, (0.4,0.2,0.1), (x_2,(0.2,0.3,0.4)), (x_3,(0.4,0.8,0.5))}

$\mathcal{PE}_2$ = {(x_1, (0.8,0.5,0.6), (x_2,(0.4,0.3,0.2)), (x_3,(0.7,0.6,0.1))}

$\mathcal{PE}_3$ = {(x_1, (0.7,0.6,0.3), (x_2,(0.1,0.5,0.6)), (x_3,(0.4,0.5,0.8))}

X is the vertex set and $\mathcal{PE}_i$,i = 1,2,3 are the plithogenic hyper envelopes of concentric plithogenic hypergraph.

Plithogenic hypergraphs and concentric plithogenic hypergraphs based on plithogenic graphs are distinct from the concentric plithogenic hypergraphs defined based on plithogenic hypersoft sets. The former definition doesn't take the condition of $E_i \cap E_k = \emptyset$. The graphs are called as plithogenic based on their dimension of membership values. In comparison of both kinds of representation, the latter is more feasible in nature as it incorporates the degree of appurtenance and the holistic meaning of plithogeny is reflected.

3. Classification and Extension of Concentric Plithogenic Hypergraphs

The concentric plithogenic hypergraphs is classified into crisp, fuzzy, intuitionistic and neutrosophic based on the values of the degree of appurtenance.

3.1 Crisp Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ = {(A_1^1,1),(A_2^2,1),(A_3^2,1), (A_4^1,1),(A_5^2,1)}

$\mathcal{P}_{E_2}$ = {(A_1^1,1),(A_2^2,1),(A_3^2,1), (A_4^1,1),(A_5^2,1)}

$\mathcal{P}_{E_3}$ = {(A_1^1,1),(A_2^2,1),(A_3^2,1), (A_4^1,1),(A_5^2,1)}

3.2 Fuzzy Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ = {(A_1^1,0.9),(A_2^2,0.875),(A_3^2,0.8), (A_4^1,0.75),(A_5^2,0.5)}

$\mathcal{P}_{E_2}$ = {(A_1^1,0.67),(A_2^2,0.5),(A_3^2,0.4), (A_4^1,0.8),(A_5^2,0.7)}

$\mathcal{P}_{E_3}$ = {(A_1^1,0.8),(A_2^2,0.7),(A_3^2,0.6), (A_4^1,0.7),(A_5^2,0.5)}

3.3 Intuitionistic Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ = {(A_1^1,(0.9,0.1)),(A_2^2,(0.5,0.2)),(A_3^2,(0.8,0.2)), (A_4^1,(0.75,0.5)),(A_5^2,(0.5,0.2))}

$\mathcal{P}_{E_2}$ = {(A_1^1,(0.6,0.7),(A_2^2,(0.7,0.5),(A_3^2,(0.9,0.4), (A_4^1,(0.7,0.1),(A_5^2,(0.7,0.2)}

$\mathcal{P}_{E_3}$ = {(A_1^1,(0.6,0.4),(A_2^2,(0.2,0.58),(A_3^2,(0.19,0.54), (A_4^1,(0.7,0.2),(A_5^2,(0.8,0.2)}

3.4 Neutrosophic Concentric Plithogenic Hypergraphs

Let X= { $A_1^1, A_1^2, A_1^3, A_2^1, A_2^2, A_2^3, A_3^1, A_3^2, A_3^3, A_4^1, A_4^2, A_4^3, A_5^1, A_5^2, A_5^3$}

E = {A_1, A_2,A_3,A_4, A_5}

$\mathcal{P}_{E_1}$ = {(A_1^1,(0.9,0.1,0.2),(A_2^2,(0.8,0.5,0.2),(A_3^2,(0.2,0.2,0.8), (A_4^1,(0.1,0.3,0.75),(A_5^2,(0.4,0.2,0.5)}

$\mathcal{P}_{E_2}$ = {(A_1^1,(0.7,0.1,0.2),(A_2^2,(0.6,0.2,0.1),(A_3^2,(0.3,0.2,0.5), (A_4^1,(0.4,0.3,0.5),(A_5^2,(0.6,0.2,0.3)}

$\mathcal{P}_{E_3}$ = {(A_1^1,(0.7,0.1,0.2),(A_2^2,(0.6,0.4,0.2),(A_3^2,(0.6,0.2,0.3), (A_4^1,(0.5,0.2,0.7),(A_5^2,(0.6,0.1,0.4)}

3.5 Extended Plithogenic Hypersoft sets

Extended plithogenic hypersoft sets comprises of the degree of appurtenance of the elements to the corresponding attributes along with multi expert's opinion. These sets comprise of the opinion of several experts. The representation of such kind of set is presented as follows

Let us consider a situation that the below values are given by two experts for the same example discussed under plithogenic hypersoft sets.

G ($A_1^1, A_2^2, A_3^2, A_4^1, A_5^2$) given by the first expert =

{M_1(0.9,0.875,0.8,0.75,0.5),M_3(0.67,0.5,0.4,0.8,0.7),M_6(0.8,0.7,0.6,0.7,0.5)}

G ($A_1^1, A_2^2, A_3^2, A_4^1, A_5^2$) given by the second expert =

{M_1(0.6,0.875,0.8,0.5,0.5),M_3(0.7,0.6,0.3,0.9,0.7),M_6(0.8,0.7,0.7,0.7,0.6)}

The aggregate representation will be

$\mathcal{G}$ ($A_1^1, A_2^2, A_3^2, A_4^1, A_5^2$) = {$M_1$ {(0.9,0.875,0.8,0.75,0.5), (0.6,0.875,0.8,0.5,0.5)}, M_3 {(0.67,0.5,0.4,0.8,0.7), (0.7,0.6,0.3,0.9,0.7)}, M_6 {(0.8,0.7,0.6,0.7,0.5), (0.8,0.7,0.7,0.7,0.6)}}

The extended concentric plithogenic hypergraphs are based on the extended plithogenic hypersoft sets. The graphical representation is depicted in fig 3.1

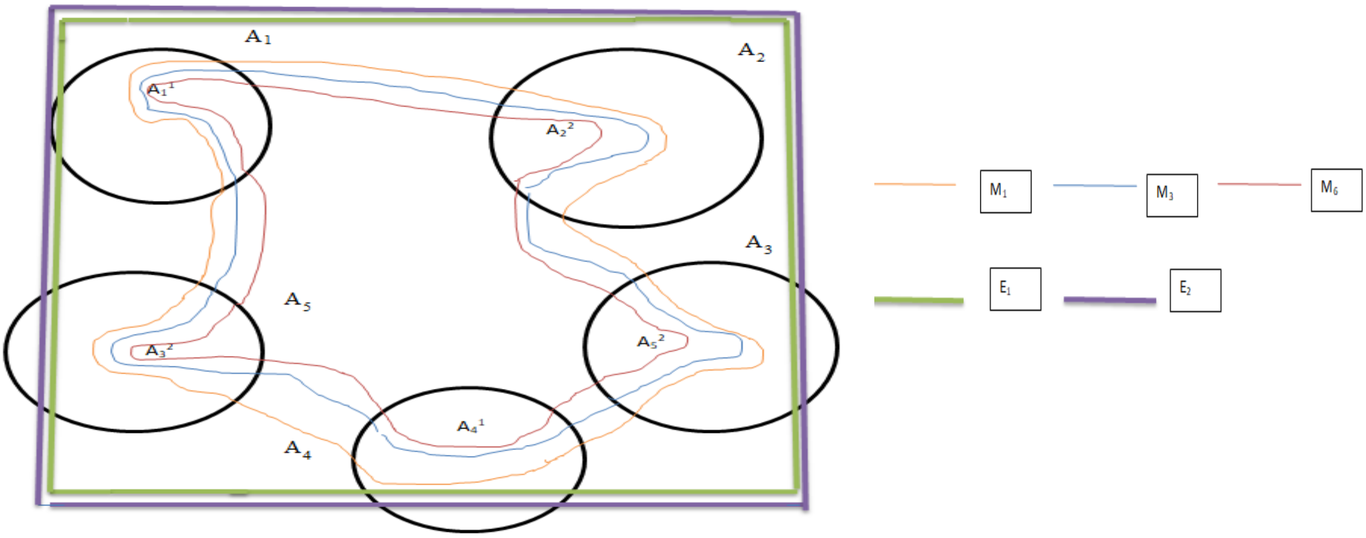

Fig.3.1. Graphical Representation of Extended Concentric Plithogenic Hypergraph

3.6 Extended Concentric Plithogenic Hypergraph

A hypergraph with non- empty, disjoint hyperedges and extended plithogenic envelopes is called as extended concentric plithogenic hypergraph $E_{P_{C_H}}$

An extended concentric Plithogenic hypergraph P_{C_H} is defined as follows

$E_{P_{C_H}}$ = (X, E, $E_{\mathcal{P}\mathcal{E}}$)

- X- finite vertex set
- E- Hyperedge set
- $E\mathcal{P}_{\mathcal{E}}$ =Extended Plithogenic hyper envelope
- $E\mathcal{P}_{\mathcal{E}j}$ = $\{(x_{ij}, \mu_{El}(x_{ij})) / \mu(x_{ij}) > 0, x_{ij} \in E_i \}$

- $E_i \cap E_k = \emptyset$ and $UE_i = X$

4. Fuzzy Cognitive Maps integrated with Extended Concentric Plithogenic Hypergraphic Approach.

Fuzzy Cognitive Maps (FCM) is a decision making tools that are predominantly used in finding the cause and effect relationship. Basically FCM is a directed graph consisting of nodes and edges representing the factors of study and its relationship respectively. The adjacency or the connection matrix is the representation of the relationship between the nodes. Let us consider a decision making problem, which comprises of several factors, then the connection matrix will be of higher order which will make the computational process complicated. In order to handle such situations the core factors can be determined by using the approach of extended concentric plithogenic hypergraph.

Let us consider a decision making situation to find inter relational impacts of the factors contributing towards the sales promotion of a manufacturing firm. The promotion of sales generally depends on major attributes of a company such as Customers, Pricing stratagem and marketing strategies. Different companies follow various other aspects to foster their sales promotion, but the above three attributes play predominant roles.

If customer (A_1), pricing stratagem (A_2) and marketing strategies (A_3) are considered as the attribute sets then A_1 = {**Potential**, Impulsive, Novel, Loyal}, A_2 = {Competition- Based, Skimming, **Penetration**, Dynamic}, A_3 = {**Social**, Service, Green, Holistic, Direct} are the attribute values. A company has decided to launch a new product in the market with the focus towards potential customers by following penetration pricing strategy through social marketing, the ultimate target of the company to increase the sales and attain the target within the stipulated time, so it has called the experts to present the aspects the company has to concentrate in deep. The expert's perception is presented as factors.

F_1 The touch of innovation in the entire lifecycle of the product

F_2 Extensive design of the product

F_3 Customer centric approach

F_4 Product improvisation suiting the contemporary needs

F_5 Widening of the distribution channels

F_6 Implementation of Price breaks

F_7 Enriching the portals of communication

F_8 Employment of self- assessment tools

F_9 Placement of suitable personnel

F_{10} Counter actions to the competitors

The extended concentric plithogenic hyper envelopes with linguistic representations in accordance to the expert's opinion are presented below in Table 4.1

Table 4.1. Representation of Expert's opinion

Factors	$E\mathcal{P}_{\mathcal{E}_1}$	$E\mathcal{P}_{\mathcal{E}_2}$
F_1	(H,M,M)	(M,H,VH)
F_2	(VH,H,H)	(H,H,M)
F_3	(VH,H,VH)	(H,H,H)
F_4	(H,VH,H)	(VH,VH,M)
F_5	(VH,H,VH)	(VH,VH,VH)
F_6	(H,H,L)	(H,VH,M)
F_7	(H,H,VH)	(H,H,VH)
F_8	(H,H,H)	(H,H,VH)
F_9	(VH,VH,H)	(VH,VH,VH)
F_{10}	(H,H,H)	(H,H,H)

The linguistic representations of the experts are quantified using hexagonal fuzzy numbers based on the below values in Table 4.2.

Table 4.2. Hexagonal Quantification of values

Very Low (VL)	(0,0.05,0.1,0.15,0.2,0.25)	0.125
Low (L)	(0.15,0.2,0.25,0.3,0.35,0.4)	0.275
Moderate (M)	(0.3,0.35,0.4,0.45,0.5,0.55)	0.425
High (H)	(0.45,0.5,0.55,0.6,0.65,0.7)	0.575
Very High (VH)	(0.65,0.7,0.75,0.8,0.9,1)	0.8

The quantified representations of the experts are presented in Table 4.3

Table 4.3. Modified representation of Expert's opinion

Factors	$E\mathcal{P}_{\mathcal{E}_1}$	$E\mathcal{P}_{\mathcal{E}_2}$
F_1	(0.575,0.425,0.425)	(0.425,0.575,0.8)
F_2	(0.8,0.575,0.575)	(0.575,0.575,0.425)
F_3	(0.8,0.575,0.8)	(0.575,0.575,0.575)
F_4	(0.575,0.8,0.575)	(0.8,0.8,0.425)
F_5	(0.8,0.575,0.8)	(0.8,0.8,0.8)
F_6	(0.575,0.575,0.275)	(0.575,0.8,0.425)
F_7	(0.575,0.575,0.8)	(0.575,0.575,0.8)
F_8	(0.575,0.575,0.575)	(0.575,0.575,0.8)
F_9	(0.8,0.8,0.575)	(0.8,0.8,0.8)
F_{10}	(0.575,0.575,0.575)	(0.575,0.575,0.575)

The combined values of each factor are presented in below Table 4.4.

Table 4.4 Combined values of the factors

Factors	EP_E
F_1	(1,1,1.225)
F_2	(1.375,1.15,1)
F_3	(1.375,1.15,1.375)
F_4	(1.375,1.6,1)
F_5	(1.6,1.375,1.6)
F_6	(1.15,1.375,0.7)
F_7	(1.15,1.15,1.6)
F_8	(1.15,1.15,1.375)
F_9	(1.6,1.6,1.375)
F_{10}	(1.15,1.15,1.15)

These factors are to be ranked and the above values corresponding to each factor are represented in matrix form

	A_1^1	A_2^3	A_3^1
F_1	1	1	1.225
F_2	1.375	1.15	1
F_3	1.375	1.15	1.375
F_4	1.375	1.6	1
F_5	1.6	1.375	1.6
F_6	1.15	1.375	0.7
F_7	1.15	1.15	1.6
F_8	1.15	1.15	1.375
F_9	1.6	1.6	1.375
F_{10}	1.15	1.15	1.15

By using the procedure of ranking as discussed by Shazia Rana et.al [19] the factors are ranked

The frequency matrix F representing the ranking of the factors is

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
F_1	0	0	0	0	0	0	1	0	2	0
F_2	0	0	0	0	1	0	2	0	0	0
F_3	0	0	1	1	1	0	0	0	0	0
F_4	1	0	1	0	0	0	1	0	0	0
F_5	3	0	0	0	0	0	0	0	0	0
F_6	0	0	0	0	1	0	0	0	1	1
F_7	1	0	1	1	0	0	0	0	0	0
F_8	0	0	1	0	1	1	0	0	0	0
F_9	3	0	0	0	0	0	0	0	0	0
F_{10}	0	0	1	0	0	0	0	1	0	1

Based on the percentage measure of authenticity of ranking of the factors, the following factors F_3, F_4, F_5, F_8, F_9 are considered for analyzing their inter relation using fuzzy cognitive maps (FCM). These factors are taken as C_1, C_2, C_3, C_4, C_5 and by using the procedure of finding the cause and effect relationship, the limit points are presented in Table 4.5.

The core factors considered for the study are

C_1 Customer centric approach

C_2 Product improvisation suiting the contemporary needs

C_3 Widening of the distribution channels

C_4 Employment of self- assessment tools

C_5 Placement of suitable personnel

The connection matrix M representing the degree of association between the core factors and the graphical representation in Fig 4.1 is presented as follows

	C_1	C_2	C_3	C_4	C_5
C_1	0	1	1	0	1
C_2	1	0	0	0	0
C_3	1	0	0	0	0
C_4	1	1	1	0	1
C_5	1	1	1	1	0

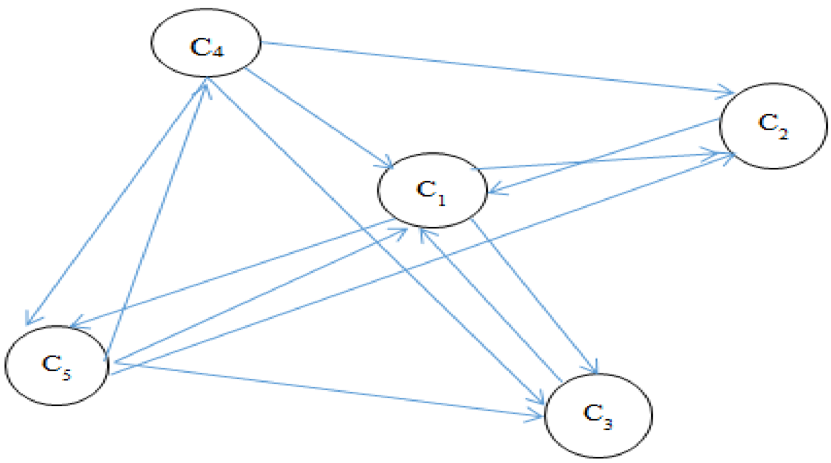

Fig 4.1 Graphical Representation of the association of core factors

Table 4.5. Limit points of the Core Factors

Core Factors in On Position	Limit Points
(10000)	(11111)
(01000)	(11111)
(00100)	(11111)
(00010)	(11111)
(00001)	(11111)

5. Discussion

The integration of FCM with extended concentric plithogenic hypergraphic approach is an innovative effort in minimizing the number of factors considered for studying interrelationship. In the decision making problem as discussed in section 4, the factors which are to be concentrated deeply for sales promotion are considered based on the fulfilment of the three attributes. The consideration of the factors are very specific, but if the company wishes to find the association and impact between the factors using FCM, then the proposed ten factors are to be considered and the computation of the limit points using higher order matrix will be tedious and time consuming, but if the core factors are only considered, the process becomes compatible and the intervention of extended concentric plithogenic hypergraphic approach makes it highly objective and reliable.

6. Conclusion

This research article presents the evolution of the concept of concentric plithogenic hypergraphs based on plithogenic hypersoft sets and plithogenic graphs. This work also introduces extended hypersoft sets and extended concentric plithogenic hypergraphs. The integration of extended concentric plithogenic hypergraphs with fuzzy cognitive maps using linguistic expert's representation is presented with a decision making problem and such integrations are highly advantageous in dealing with several multi attribute factors of study using FCM approach. The linguistic representation of expert's opinion is a new initiative made in this research work and the proposed model of making decisions can be extended using refined plithogenic sets in representations.

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