Adapting a Number Sense Task to Learn More About K-5 Student Reasoning

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The fifth grade teachers are in charge of planning the annual Davis Elementary Fun Run. The teachers decide that each adult should run 6⁄4 as far as each student in grade 5 and each student in grade 1 should run 3⁄4 as far as each student in grade 5. Who has to run the longest distance? Who has to run the shortest distance? Explain your reasoning.

AS THE FIFTH GRADE STUDENTS engaged with this task, many were struggling and we wondered why. Why couldn't they access the problem? What part of the mathematics was causing them confusion? What type of prior tasks or activities would have helped develop the reasoning necessary to be successful? We thought about the task and considered the knowledge students might access to solve the problem. The following ideas came to mind:

- Do students understand the magnitude of the values, knowing that 6⁄4 is greater than 3⁄4?
- Do students use their knowledge of benchmarks, knowing that 6⁄4 is greater than 1 and 3⁄4 is less than 1?
- Do they consider the relationship between the two fractions and notice 6⁄4 is the double of 3⁄4?
- Do they see the task as a comparison task, perhaps building on comparing fraction skills developed in CCSS in grades 3 and 4?
- Do they tap into their thinking around measurement?
- Do they think of math from an arithmetic stance, relying on computation to solve, or use an algebraic lens, focusing on the relationship of the values?

When we consider the Davis Fun Run problem, the primary alignment is to standard 5.NF.B.5 (See Table 1). However, it also uses language of multiplicative comparison from 4.OA.A.1 and requires a strong understanding of 3.NF.A which establishes that fractions are in fact numbers.

Table 1

Primary CCSSM Alignment to the "Davis Fun Run" Task

Grade and Standard	
5.NF.B.5	Interpret multiplication as scaling (resizing), by explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
4.OA.A. 1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
3.NF.A	Develop understanding of fractions as numbers.

Note: This is not an exhaustive list of CCSSM expectations, but rather a focused look at the ideas related to the task.

We wondered why students did not reason by saying, “Since $\frac{6}{4} > \frac{3}{4}$, and multiplying a number by $\frac{6}{4}$ produces a larger product than multiplying the same number by $\frac{3}{4}$, it is clear that the adults ran farther.” In thinking about what students would need to understand in order to reason in that way, we considered another related task (*Figure* 1) from the Partnership for Assessment of Readiness for College and Career (PARCC).

Order the following expressions from least value to greatest value.

Drag and drop the expressions into the correct order

$19 \times \frac{3}{3}$	$19 \times \frac{1}{2}$	$19 \times \frac{3}{2}$	$19 \times \frac{2}{3}$
LEAST			GREATEST

Figure 1. PARCC task.

Retrieved 7/13/2016 from: https://prc.parcconline.org/system/files/5th%20grade%20Math%20-%20EOY%20-%20Item%20Set_April%202016.pdf

A group of teachers were analyzing this fifth grade item during a professional development session on fractions when one declared, “This item is silly. All the student needs to do is order these by the size of the fraction.” But that is precisely the understanding standard 5.NF.B.5 is targeting—that the size of the product is determined by the size of the fraction, and likely one of the key reasons why students struggle with the Davis Elementary Fun Run problem. Unlike previous state standards that often introduced fractions before students had a strong grasp of whole numbers, the CCSS require a strong foundation in whole number in K–2 before fractions are introduced, and then when fractions are introduced, the headline in Grade 3 is: “Develop understanding of fractions as numbers.” While some may argue that students inherently understand fractions better when they are presented as partitioned shapes or food items, what is missed is the understanding that 3/2 is a number between 1 and 2 and more generally that fractions are an extension of the number system that students have been learning since kindergarten or earlier.

Wanting to further investigate some ideas that students would draw upon in solving the tasks described above, we decided to focus on student understanding of comparison and number sense.

The Mathematics: Number Sense and Comparison

Number sense is a foundation for mathematics (Shumway, 2011); therefore it is important for teachers to spend time developing students’ understanding of quantitative relationships. Shumway writes about the interconnected web of components involved in number sense, and one of the specific understandings she discusses involves comparison:

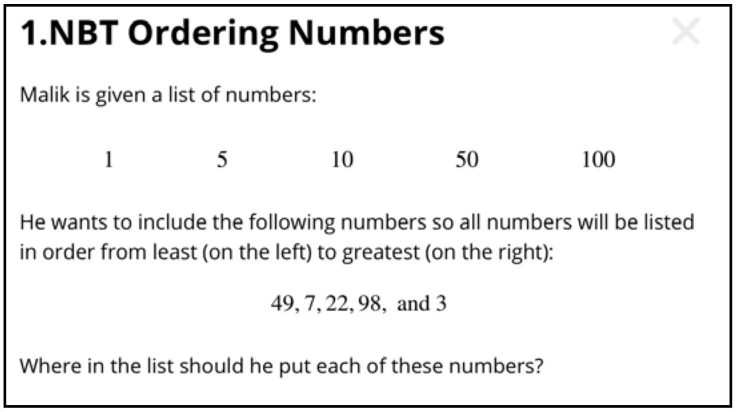
> Students have “ability to make comparisons among quantities. For example, they know that 300 is 400 away from 700 by using a mental number line ... Students with strong number sense make comparisons using their sense of quantities, using landmarks such as 10, 50, and 100, and using a mental number line (understanding where numbers fall on a number line).” (p. 9)

Ordering numbers and making comparisons helps students to develop an understanding of quantitative relationships and builds a foundation that students will use in later grades. The idea of comparison is also emphasized by the authors of the CCSS across grade levels; they write, for example, that first grade students will have opportunities to “compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. ... Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes” (CCSS, 2010, Grade 1 Introduction, para 3, http://www.corestandards.org/Math/Content/1/introduction/).

Designing the Task

Curious how number sense and reasoning developed over time, we decided to begin by looking deeper into the understandings of students in primary grades, specifically starting with students in kindergarten and first grade. We looked for a task that could help us gain insight into student thinking. Our goal was to find a task with multiple entry points that could be easily adapted for students at varying grade levels and could afford us the opportunity to understand how students reason across grade levels. A first-grade task from Illustrative Mathematics was selected (See Figure 2). (http://

s3.amazonaws.com/illustrativemathematics/attachments/000/008/467/original/public_task_6.pdf?1462386961).


1.NBT Ordering Numbers

Malik is given a list of numbers:

1 5 10 50 100

He wants to include the following numbers so all numbers will be listed in order from least (on the left) to greatest (on the right):

49, 7, 22, 98, and 3

Where in the list should he put each of these numbers?

Figure 2. Original task from Illustrative Mathematics

Using this task as a framework, we considered the list of numbers Malik is asked to reason about and wondered how these could be modified for different grade levels. Knowing that the numbers students are given within the task matter (Land, Sweeney, Johnson & Franke, 2015), we drew upon our experience working with elementary students and designed number sets that would get at important ideas and common misconceptions in the understanding of numbers and the number system and allow opportunities for student reasoning. Table 2 shows numbers we considered, as well as the rationale in modifying the task for kindergarten and first grade students. Considerations included affordances of particular numbers and the understandings that could be elicited including number sequence, magnitude, place value, and use of benchmarks. We also drew upon common misconceptions such as teen numbers, zero, and numbers that students work with less frequently or may not have conceptual understanding of. A goal was to identify numbers that would create opportunities for discussion.

As we thought about how we would use and modify the task, we thought about the way in which the task was formatted and considered the spacing of the numbers. Would this spacing elicit specific understandings and misconceptions? Would creating a scaled list provide additional insight into student understanding? Are there affordances to using the number positioning of the original task? We decided to provide students with a scaled set of numbers to gain insight into student considerations when placing numbers in the sequence, as well as their understanding of the magnitude of numbers.

Table 2

Task Adaptations and Explanations for Grades K–1

Grade Level	Task Design	Number Choice Rationale
Kindergarten students, in Common Core State Standards, work with numbers 1–20. Students often struggle with teen numbers especially numbers like 11, 12 and 13 where the number names aren't clearly connected to the number system in the way that students recognize, such as 4 in fourteen.	Malik is given a list of numbers. 1 5 10 20 (numbers to scale) Where in the list should he put each of these numbers? 2, 4, 15, 0, 11, 19	2 or 4: Do students place between 1 and 5 or think about the magnitude and relationship to anchor numbers 1 or 5? 15: Is 15 placed directly between 10 and 20 or just somewhere between the two numbers? 19: Will students recognize this as 1 less than 20? 11: A number that students are confused by. 0: What do students understand about 0?
First Grade students, in Common Core State Standards, work with numbers 1–120. They develop an understanding of the base ten system and operate on numbers to 100.	Malik is given a list of numbers. 1 5 10 50 100 (numbers to scale) Where in the list should he put each of these numbers? 49, 7, 22, 0, 98, 3	Adapt the original task by adding the number below. 0: What do students understand about 0?

Implementing the Task and Reviewing Student Work

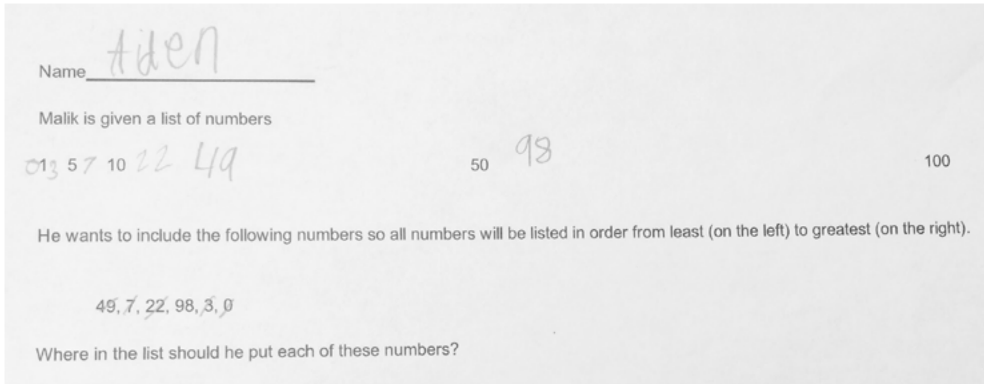
Wanting to learn more about how students reason around the numbers in the task, it was given to kindergarten and first grade students. We view kindergarten and first grade student understanding as a starting point to think about how early number sense might inform later number sense and develop over time. Our hope was to gain insight into understandings they built upon. We wondered:

- What understandings will students build upon?
- How will students think about the values of the number they are placing? Will they consider a number as being greater or less than numbers that are already given in the task?
- Will students think about placement of a number from a given benchmark or start at 1 and use some kind of counting to strategy to determine where to place a number?
- How will students articulate their thinking?

Before giving students the independent task to her first grade students, Mrs. Miller modeled a similar task on the board using numbers within 20. The numbers that needed to be added to the sequence were 0, 2, 9, 15, and 19. Mrs. Miller called on a student to share where 19 would go and to explain his thinking. He said to place the 19 in front of the 20 because 19 is right before 20 when counting. She called on another student to explain where to place 9. The student said, "between the 7 and 11." When Mrs. Miller questioned her about where 'exactly' to put it, the student said, "Exactly between the 7 and 11 because right after 7 is 8 and you have to leave room for 8. Also, right before 11 is 10 and you have to leave room for 10, so 9 would go in the exact middle." At this point students were given the task to complete independently.

A Kindergarten teacher, Mrs. Cardinale, also modeled a similar task on the board before giving the task independently to her students.

When reviewing the student work, we noticed that some students understood the concept of comparison, putting the numbers in the correct order however they didn't understand the relative value of the numbers as indicated by where they placed them in the sequence. One first grade student correctly placed 22 after 10, and then 49 after 22, but the numbers were written close together leaving a large space between 49 and 50, see Figure 3.


Name Aiden

Malik is given a list of numbers

0 1 3 5 7 10 22 49 50 98 100

He wants to include the following numbers so all numbers will be listed in order from least (on the left) to greatest (on the right).

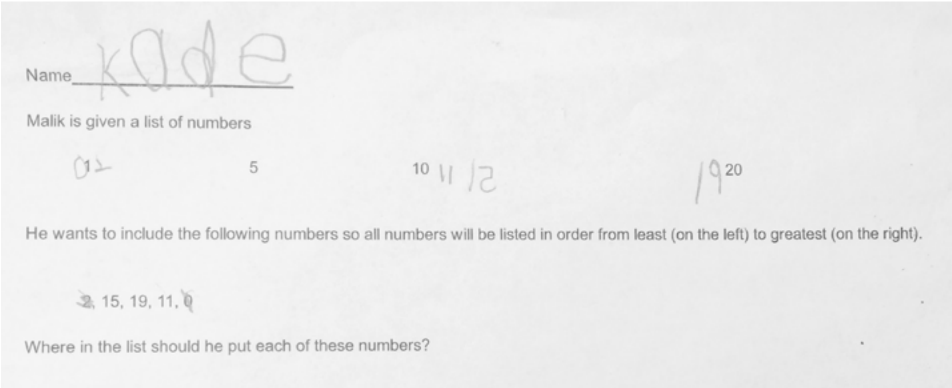
49, 7, 22, 98, 3, 0

Where in the list should he put each of these numbers?

Figure 3. First grade student's work: Aiden.

We find it interesting that these few students did not place the numbers in a correct location considering the magnitude. However, they do appear to understand the concept of greater and less. They did put the numbers in the correct order, but didn't connect to the number line, which would have helped them consider the magnitude. We wonder how teachers can help students connect to a mental number line when comparing and ordering numbers. We also wonder how we can facilitate students' reasoning of magnitude when ordering and comparing numbers.

We noticed that several kindergarten students were successful with placing numbers correctly in the sequence, that were adjacent to numbers on the list. For instance, it appears Kade understood 11 as 1 more than 10 and 19 as 1 less than 20, because he put them right next to the adjacent numbers, see Figure 4. However, Kade placed 15 in the correct order, after 11, but not in the correct place based on magnitude. He put 15 right next to 11, instead of in-between 10 and 20.


Name kade

Malik is given a list of numbers

0 1 2 5 10 11 12 19 20

He wants to include the following numbers so all numbers will be listed in order from least (on the left) to greatest (on the right).

2, 15, 19, 11, 0

Where in the list should he put each of these numbers?

Figure 4. Kindergarten student's work: Kade.

We think it could be interesting to consider ways we might modify the task to better understand students' reasoning. In this case, the number 15 was the only non-adjacent number. If we gave the task again, we would add another non-benchmark number that's not adjacent to given numbers, such as the number 8. This would tell us more about kindergarten students' ability to reason about the magnitude of numbers when comparing and ordering.

The Sequel...Developing Student Thinking

While tasks such these are invaluable ways to look deeper into individual student thinking, there are also many classroom activities that build this same reasoning, in particular a yarn number line and guess my number. A yarn number line, or clothesline, is exactly that—a piece of yarn hung across a classroom resembling a number line. Prior to starting, the students are not told what numbers will be placed on the line. Each student, or pair of students, is given a folded index card with a number on it. The students are called on randomly to place their card on the number line and are allowed to adjust the previously placed cards if needed. For example, a 5th grade student could place 1 at the end of the yarn line, believing the line only goes to 1 as pictured below:

However, if the following student must place the card with a 2 on it, he or she must adjust the position of the 1 in relation to their placement of 2. While there are many ways to do this, one possible arrangement is given below.

This activity encourages students to think about not only the magnitude of numbers and distance between them but also their location in relation to one another. In an informal way, students reason about such things as midpoints and fractions of numbers. It is also an extremely flexible activity that can be adapted simply by changing the numbers on the cards and the distance between the starting and ending number.

Guess my number is another activity that engages students in thinking about number relationships, but in a more abstract way. Students play this game in groups of 2 with one player attempting to guess the number of their partner. These numbers can be chosen by the student or the teacher if they want to be more explicit in the numbers students are using. The partner who is guessing is allowed to ask only yes or no questions of the player whose number they are trying to guess. The conversation may go something like this in a 2nd grade classroom:

Student 1: Is your number less than 100?

Student 2: Yes

Student 1: Is your number between 30 and 50?

Student 2: No

Student 1: Do I say your number when I count by 5s?

Student 2: Yes

Student 1: Is it less than 60?

Student 2: Yes

Student 1: Is your number 55?

This activity is also very flexible in that it can be adapted by number choices and, for students who may struggle to visualize the numbers, by tools. An unmarked number line or 100 chart could be used to help students who need them.

Not only should students engage in these activities across all grade levels, but teachers should as well. Using an activity such as this in a PLC provides teachers with an opportunity to look closely at the standards at each grade level and think about the progression of these ideas in order to create a coherent learning experience for all students. We can imagine a group of teachers at various grade levels taking a task, adapting it for their grade level, and connecting the ideas of each adaptation to the learning progressions document. Table 3 includes some ways we thought about doing just that, following the line of thinking used to adapt the task for kindergarten and first grade students.

Table 3

Adapting a task to the grade level

Grade Level	Task Design	Number Choice Rationale
Second Grade students, in Common Core State Standards, work with numbers 1–1000. They extend their understanding of the base ten system and operate on numbers to 1000.	Malik is given a list of numbers. 1 250 1000 (numbers to scale) Where in the list should he put each of these numbers? 500, 0, 100, 300, 987, 5, 243, 50	500: How will students think of 500? Do they consider it directly between 1 and 1000? 0: What do students understand about 0? 100: How will students reason about the “middle-ness” between 1–250? Do they consider the relationship to 125? 300: How will students determine 50 more than 250? 987: How will students think about the value of this number? Will they consider the relation to 1000? 5: How close will students place to 1? 243: What reasoning will students use to place? Will they consider the distance to 250? 50: How will students think about 50 in relation to 250?

Third Grade students, in Common Core State Standards, work with numbers 1–1000 and are introduced to fractions as numbers. A common misconception is that fractions are between 0 and 1. Fractions equivalent to 1 ($\frac{3}{3}$) or other whole numbers ($\frac{4}{1}$) may also be confusing.	Malik is given a list of numbers. 0 1 2 (numbers to scale) Where in the list should he put each of these numbers? $\frac{1}{4}$ $\frac{3}{3}$ $\frac{4}{1}$ $\frac{7}{6}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{1}{2}$	$\frac{1}{4}$: How do students approach this, do they eyeball, make four hash marks, break into four equal parts and then label first? $\frac{3}{3}$: Will students recognize as equivalent to 1, understand relationship of numerator and denominator? $\frac{4}{1}$: Will students understand as 4? Where will they place it distance-wise from 2? $\frac{7}{6}$: How will students think about a unit over a whole with different denominator than others so far? $\frac{3}{4}$: How will students think about $\frac{3}{4}$? Will they use a benchmark number such as $\frac{1}{2}$ or 1 to determine where to place it? $\frac{1}{3}$: How will students think about this? Will they consider the relationship to $\frac{1}{4}$ and see it as more or less? Do they use whole number thinking, a common misconception and think a bigger denominator is a bigger number? $\frac{1}{2}$: How will students think about $\frac{1}{2}$? Will they see it as a number or place it in the halfway point of all numbers (half of something)? OR Intentionally not include $\frac{1}{2}$. Will anyone add $\frac{1}{2}$ to think about where other numbers might go? What are the affordances and constraints of including one or more fractions within the list of numbers provided to students?

Fourth Grade students, in Common Core State Standards, work with numbers to 1,000,000 including fractions and decimals with denominators 10 and 100.	Malik is given a list of numbers. 0 1 2 (numbers to scale) Where in the list should he put each of these numbers? $\frac{3}{4}$ $\frac{4}{4}$ $\frac{4}{3}$ $\frac{2}{5}$ $\frac{5}{8}$ $1\frac{7}{8}$ $\frac{1}{100}$ $\frac{12}{8}$	$\frac{3}{4}$: Will students use to help them place the fraction? Will they place it equidistant from $\frac{1}{2}$ and 1? $\frac{4}{4}$: Will students know the fraction is equal to 1? $\frac{4}{3}$: Will students know this fraction is greater than 1, and how will they reason about the unit fraction over the whole? $\frac{2}{5}$: Will students reason this fraction is just shy of the benchmark fraction $\frac{1}{2}$? $\frac{5}{8}$: Will students reason about this fraction in relation to the benchmark $\frac{1}{2}$ using its equivalent fraction $\frac{4}{8}$? $1\frac{7}{8}$: Will students know this fraction is $\frac{1}{8}$ from 2? How will they reason about the size of $\frac{1}{8}$? $\frac{1}{100}$: How will students reason about really small fractions? $\frac{12}{8}$: Will students know this fraction is equivalent to the benchmark of $1\frac{1}{2}$?
Fifth Grade students, in Common Core State Standards, work with numbers to 1,000,000 including fractions and decimals to thousandths.	Malik is given a list of numbers. 0 $\frac{1}{3}$ 1 2 3 (numbers to scale) Where in the list should he put each of these numbers? .333, 0.3, .005, $\frac{1}{100}$, 1.6, $1\frac{6}{9}$, $2\frac{7}{8}$, 2.8	.333: How will students think about this? Will they think about equivalence to $\frac{1}{3}$? 0.3: How will students think about this in relation to $\frac{1}{3}$ or .333? How will they determine if it's more or less and how much more or less? .005: How will students think about this in relation to 0? $\frac{1}{100}$: Will students reason that this is almost 0? Will they consider the magnitude and relationship to .005? 1.6: Will students use benchmarks to consider where to place this? $1\frac{6}{9}$: How will students think about the relationship of $1\frac{6}{9}$ and 1.6? $2\frac{7}{8}$: Will students think about this as almost 3? 2.8: How will students think about .8 in relation to $\frac{7}{8}$?

It is amazing we can learn so much about how students engage in a 5th grade task by looking back at how they think about numbers and their values in first grade. While we had the opportunity to look deeper into how students think through a task, there is still so much more to be understood. We continue to be intrigued by student thinking and curious…

What would we learn by interviewing students? Would we learn things about their thinking that weren't visible through the paper-pencil task?

We saw things we didn't anticipate in student work. What would we learn about the thinking of students at other grade levels?

Even though the task was originally designed to build student thinking, the Davis Fun Run task has immensely contributed to our thinking as educators!

References

Land, T. J., Sweeney, M., Johnson, J. M., & Franke, N. (2015). Transforming the Task with

Number Choice. Reston, VA: National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). Common Core State Standards Mathematics. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Shumway, J. F. (2011). Number sense routines: Building numerical literacy every day in grades K–3. Portland, ME: Stenhouse Publishers.