# Features of the fine structure of asymmetric TE and TM modes

**Yu.A. Egorov[1], M.V. Bretsko[1], Ya.E. Akimova[1], A.V. Volyar[1]**

[1]Physics and Technology Institute of V.I. Vernadsky Crimean Federal University, Academician Vernadsky av. 4, Simferopol, Russia, 295007

**Abstract.** This study is devoted to a new section in the field of singular optics - to beams carrying a fractional topological charge. A feature of this type of beams is structural instability, and with the slightest external perturbation, these beams form arrays of optical vortices. These vortices can be connected and, as it were, form an integral picture, or disintegrate due to the fact that during the propagation each of the vortices receives an additional phase incursion. These studies were based on a theoretical calculation and experimental study of the vector structure of beams transporting optical vortices with a fractional topological charge and the proof of the process of forming asymmetric TE (transversely electrical) and TM (transverse magnetic) modes in free space, and the study of the features of their "thin" vector structures in free space.

## 1. Introduction

The fractional-order vortex beams permit us to construct unusual wave structures with the broken axial symmetry. In contrast to the usual axial symmetric TE and TM modes with a local linear polarization in each point of the beam, the broken symmetry of the TE and TM mode beams with a fractional order $p=\pm 1/2$ vortices in each polarized component contains local elliptic polarizations at different points of the beam cross-section under the conditions $E_z=0$ for $TE$ and $H_z=0$ for $TM$ beams along the beam length. The broken symmetry of the vector field dictates the choice of the basis in the form of circularly polarized components.

From the Eq. (2) we obtain for the TE mode ($E_z=0,\ A_z=0$)

$$\partial_x E_x = -\partial_y E_y \quad \text{or} \quad \partial_x A_x = -\partial_y A_y \tag{1}$$

and

$$\partial_x H_x = -\partial_y H_y \quad \text{or} \quad \partial_x A_y = -\partial_y A_x \tag{2}$$

for the $TM$ mode $\left(H_z=0,\ A_z=0\right)$.

It is convenient to employ the circularly polarized basis

$$A_+ = A_x - iA_y,\quad A_- = A_x + iA_y \tag{3}$$

and a beam vortex structure needs new complex coordinates

$$u = x+iy = r\,e^{-i\varphi},\quad v = x-iy = r\,e^{i\varphi} \tag{4}$$

so that

$$\partial_u = \partial_x - i\partial_y = \frac{e^{-i\varphi}}{2}\left(\partial_r - \frac{i}{r}\partial_\varphi\right),$$
$$\partial_v = \partial_x + i\partial_y = \frac{e^{i\varphi}}{2}\left(\partial_r + \frac{i}{r}\partial_\varphi\right). \quad (5)$$

Then we find for $TE$ modes $A_+ = \partial_u \Psi_p$, $A_- = -\partial_v \Psi_p$ or

$$E_+ = N\left[\partial_u F_p + i\,k\frac{v}{2Z}F_p\right]G,$$
$$E_- = -N\left[\partial_v F_p + i\,k\frac{u}{2Z}F_p\right]G, \quad (6)$$

where the function $F_p$

In optical paraxial cases, where $\left|\partial_{u,v}F_p\right| << k\left|F_p\right|$, we can use the approximation

$$E_+ \approx iN\,k\frac{v}{2Z}F_pG, \quad E_+ \approx -iN\,k\frac{u}{2Z}F_pG. \quad (7)$$

Similarly we obtain $TM$ mode beams

$$E_+ \approx iN\,k\frac{v}{2Z}F_pG, \quad E_+ \approx iN\,k\frac{u}{2Z}F_pG. \quad (8)$$

## 2. Half-order vortex beams

Half-order $(2n+1)/2-$vortex-beams occupy a special place among variety of the fractional-charged optical fields because they can be easily and reliably generated at the initial plane by q-plates [1], photonic crystals [2] and arrays of microchip lasers [3]. Special types of singular beams with the fractional topological charges [4] and fractional orbital angular momentum (OAM) in the closed form (e.g, erf-G beams and others) have been recently considered in number of papers [5-7].

As a basic point we rewrite it in the form of

$$F_p(R,\varphi) = Ke^{i\frac{2n+1}{2}\varphi}\int_{-\varphi/2}^{\pi-\varphi/2} \mathrm{e}^{i(2n+1)\phi}\,\mathrm{e}^{-iKR\mathrm{os}2\phi}\,d\phi. \quad (9)$$

Remember that

$$\cos(2n+1)\phi\, d\phi = \sum_{j=0}^{[n+1/2]}(-1)^j C_n^{2j}\sin^{2j}\phi\cos^{n-2j}\phi =$$
$$= \sum_{j=0}^{[n+1/2]}\sum_{m=0}^{n-j}(-1)^{n-j} C_{2n+1}^{2j}C_{n-j}^{m}\sin^{2(j+m)}\phi\;\; d(\sin\phi),$$
$$\sin(2n+1)\phi\, d\phi = \sum_{j=0}^{n}(-1)^j C_{2n+1}^{j}\sin^{2j+1}\phi\cos^{2(n-j)}\phi =$$
$$= -\sum_{j=0}^{n}\sum_{m=0}^{2j}(-1)^{m+j} C_{2n+1}^{j}C_{2j}^{m}\cos^{2(n-j+m)}\phi\;\; d(\cos\phi), \quad (10)$$

$C_n^m -$ a binomial coefficient.

For example,

$$\cos 3\phi = \left(1-4\sin^2\phi\right)d(\sin\phi),$$
$$\sin 3\phi = -\left(4\cos^2\phi - 1\right)d(\cos\phi). \quad (11)$$

After substituting Eq. (10) into Eq. (9) and integrating [8] we obtain

$$F_p = F_n = Ke^{i\frac{2n+1}{2}\varphi}\left\{\sum_{j=0}^{[n+1/2]}\sum_{m=0}^{n-j}(-1)^{n+j} C_{2n+1}^{2j}C_{n-j}^{m}F_{m,j}^{(s)} + \sum_{j=0}^{n}\sum_{m=0}^{2j}(-1)^{m+j} C_{2n+1}^{j}C_{2j}^{m}F_{m,j}^{(c)}\right\}, \quad (12)$$

where

$$F_{j,m}^{(s)} = \frac{\Gamma\left(j+m+\frac{1}{2}\right) - \Gamma\left(j+m+1/2,\, -2iKR\sin^2\frac{\varphi}{2}\right)}{\left(-2iKR\right)^{1/2+j+m}},$$
$$F_{j,m}^{(c)} = -\frac{\Gamma\left(j+m+\frac{1}{2}\right) - \Gamma\left(j+m+1/2,\, 2iKR\cos^2\frac{\varphi}{2}\right)}{\left(2iKR\right)^{1/2+j+m}}, \qquad (13)$$

$\Gamma(n,x)$ stands for the incomplete Gamma function.

For example, the fractional beam with $p=3/2$ is described by the expression

$$\Psi_{3/2} = \frac{NG}{\sigma} K\left\{F_{3/2}^{(s)} + iF_{3/2}^{(c)}\right\} e^{i\frac{3}{2}\varphi},$$
$$F_{3/2}^{(s)} = -\left\{4\sqrt{\Re}\sin\frac{\varphi}{2} e^{-\Re\sin^2\frac{\varphi}{2}} + \sqrt{\pi}\left(\Re-2\right) erf\left(\sqrt{\Re}\sin\frac{\varphi}{2}\right)\right\} / \sqrt{\Re},$$
$$F_{3/2}^{(c)} = \left\{4\sqrt{-\Re}\cos\frac{\varphi}{2} e^{-\Re\cos^2\frac{\varphi}{2}} - \sqrt{\pi}\left(\Re+2\right) erf\left(\sqrt{-\Re}\cos\frac{\varphi}{2}\right)\right\} / \sqrt{-\Re}, \qquad (14)$$
$$\Re = 2iKR.$$

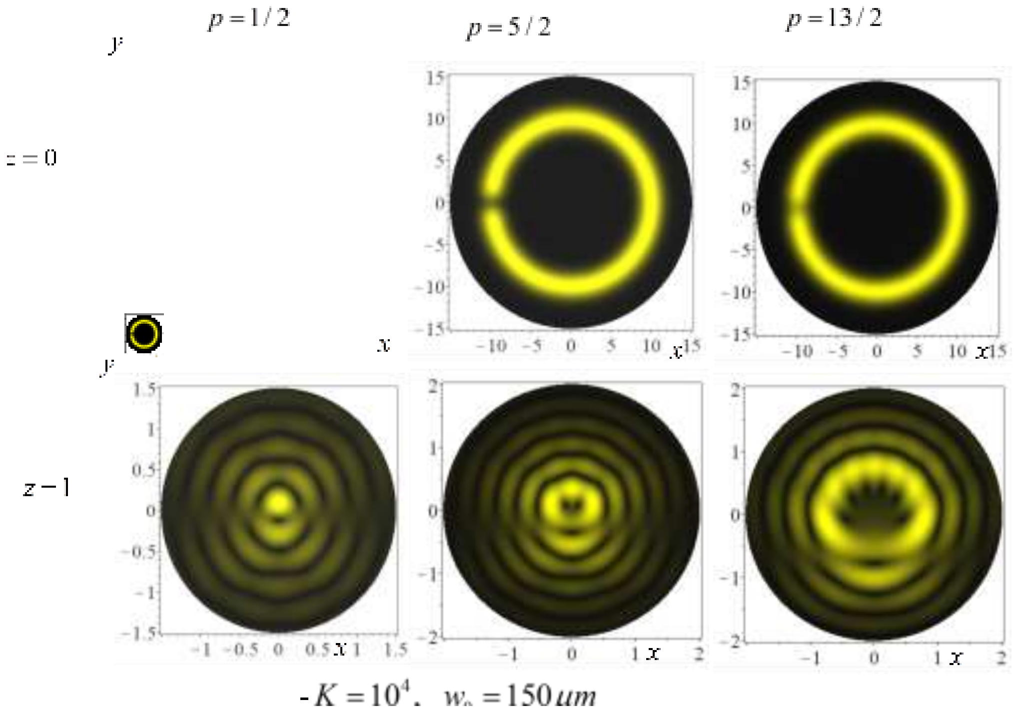


**Figure 1.** Typical representatives of the $\Gamma - G$ family of the singular beams.

It is useful to mark that the function $\Psi_{3/2}$ in Eq. (14) is a periodic one with the period $2\pi$ despite the factors $\cos\varphi/2$ and $\sin\varphi/2$ in the functions $F_{3/2}^{(c,s)}$. In order to prove it, it is necessary to take into account the factor $e^{i\frac{3}{2}\varphi}$ in the function $\Psi_{3/2}$ and oddness of the function $erf(x)$. The presented above results are of a new family of asymmetric scalar vortex beams with $p=\pm(2n+1)/2$ that we call Gamma-Gaussian beams ($\Gamma - G$ beams) referring to the complex amplitude $\Psi_p$. The $\Gamma - G$ beams are a natural generalization of the $erf-G$ beams [5] over all set of half integer-order vortex topological charges.

Typical representatives of the $\Gamma-G$ family of the singular beams are shown in Figure. 1. Thus, the field distributions at the beam cross-section depend essentially on the value of the $K-$ parameter. When the $K-$ parameter has a pure real value (see Figure. 1) the intensity distribution has a $C-$ like profile at $z=0$ with the only half-integer order vortices near the center (see e.g. [5]). However, when propagating the intensity profile is drastically transformed turning into broken Bessel beam at the length $z>>z_0$ with integer-order vortices scattering over the beam cross-section. For the pure imaginary $K-$ parameter ($|K|$ is constant), the process is reversed.

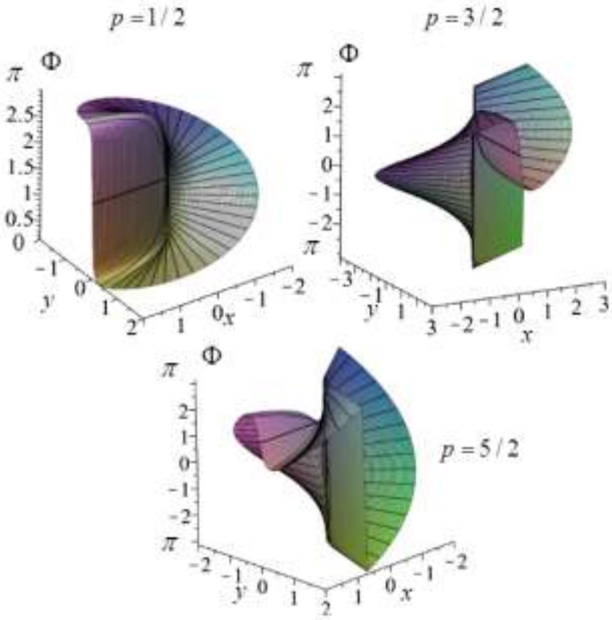


**Figure 2.** The complex phase structure for different half-order vortex topological charges.

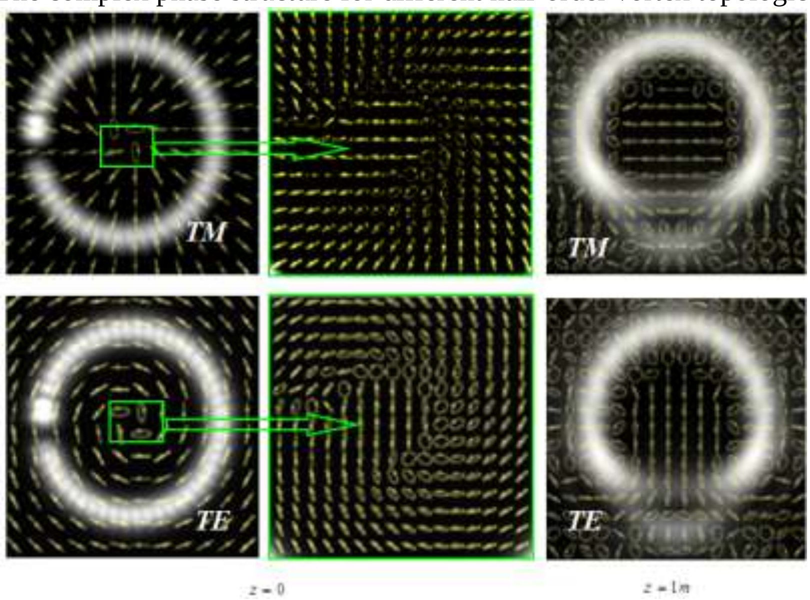


**Figure 3.** TE and TM asymmetric paraxial beam fields $p=13{,}5;\ w_0=150\mu m;\ K=10^4$.

The phase distributions shown in Figure 2 illustrate a complex phase structure for different half-order vortex topological charges. A smooth growth of the phase up to $\Phi=\pi/2$ for $p=1/2$ is replaced by the phase oscillations in the broken second branch of the two-leaved helicoid for the topological charge $p = 3/2$. The phase loss is $\Delta\Phi = \pi/2$. The same phase construction is observed for the topological charge $p = 5/2$ where the third branch of the three-leaved helicoid lacks also the phase $\Delta\Phi = \pi/2$. All phase losses are accompanied by smooth variations. The sign alternation $p \to -p$ changes the direction of the helicoid twist.

All the above equations enable us to build a great number of asymmetric transverse electric TE and transverse magnetic TM beams. Some of them are shown in Figure. 3. The fine structure of these fields is reshaped along the beam length, so that the beams are structurally unstable under propagation in free space. In contrast to standard TE and TM modes the asymmetric paraxial beam fields in Figure. 3 are elliptically polarized at each point of the beam cross-section with distinctive orientations of the ellipse axes. Near the optical axis, the field tends to form two polarization singularities of a kind (the star or lemon [8,9]). Far from the center, the directions of the linear polarization are wound into Archimedean (for TE) and logarithmic (for TM) spirals (see Figure.4).

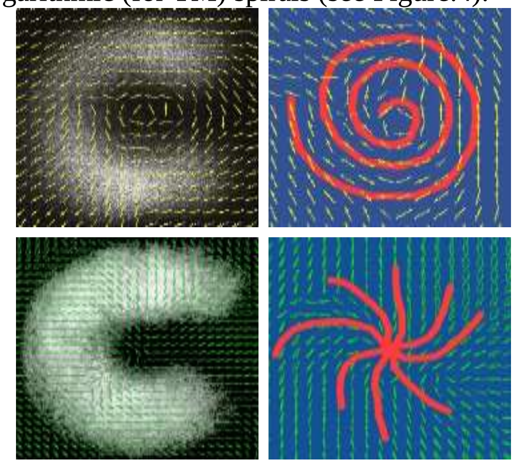

**Figure 4.** Experimental results of asymmetrical TE and TM singular modes.

The peculiar feature of the $\Gamma - G$ beams is also their capacity to gather together integer-order vortices into one with the fractional topological charge at far diffraction zone when the $K-$ parameter is a pure real value while a pure imaginary value of the $K-$ parameter induces the reverse process – the fractional vortex decays into an infinite number of integer-order vortices. Such beam behavior reflects the inherent processes in the fractional-order vortex structures in contrast to the representation of the inevitable vortex decaying.

## 3. Conclusion

It was shown that the fine structure of such fields varies along the length of the beam, so that the beams are structurally unstable when propagating in free space. In contrast to the standard TE and TM modes, the asymmetric fields of the paraxial beam are elliptically polarized at each point of the beam cross section with characteristic orientations of the ellipse axes. This behavior of the beam reflects innate processes in fractional-order vortex structures, in contrast to the notion of the inevitable decay of vortices. These studies are based on a theoretical calculation and experimental study of the vector structure of beams transporting optical vortices with a fractional topological charge, and on the proof of the formation of asymmetric TE and TM modes in free space.

## 4. Acknowledgments

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## 5. Reference

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