Digital sorting of laser beams by radial number: degenerate and non-degenerate states

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Abstract. A new technique of the digital sorting of Laguerre-Gauss (LG) modes by the radial number at a constant topological charge resulting from the perturbation of a single LG beams or their composition by a thin dielectric diaphragm with different aperture radii was proposed and experimentally implemented. The technique is based on a digital analysis of higher-order intensity moments. Two types of perturbed beams are considered: non-degenerate and degenerate beams with respect to the initial radial number of the LG composition. A diaphragm with a circle hole causes appearing a set of secondary LG modes with different radial numbers, which are characterized by an amplitude spectrum. The digital amplitude spectrum makes it possible to recover the real LG modes and find the measure of uncertainty introduced by the perturbation by means of information entropy. Measurements showed that the correlation degree of the sorted beams is 0.94. Of particular interest is the beam sorting from a composition of LG modes with different initial radial numbers. We revealed that the perturbation of a complex beam leads appearing a degenerate amplitude spectrum since a single spectral line corresponds to a set of modes generated by the initial M Laguerre-Gauss beams with different radial numbers. To decipher the spectrum, M keys were required, that were the spectra of amplitudes of nondegenerate perturbed beams in our experiment. However, the correlation degree decreased to 0.92

1. Introduction

One of the highlighted problems of data information processing for optical communications and quantum key distributions for cryptography is the vortex modes sorting of a combined vortex beams by radial n and azimuthal m quantum numbers. Solutions to this problem is mainly based on two approaches. The first approach involves the use of diffraction optical elements in conjunction with interferometric devices and modulators (see e.g. [1-5]). The second one proposes to use digital mode sorting based on higher order intensity moments technique that allows to significantly simplify optical devices and extend their capabilities (see e,g, [6,7]). Such a digital approach was developed and implemented only for sorting vortices by their topological charge. The purpose of our communication is to consider the process of digital sorting of Laguerre-Gauss beams by radial numbers.

The LG mode sorting model is based on perturbation of a standard LG beams array via a conventional hard-edged aperture with a circular hole. The beam perturbation results in a broad spectrum of secondary LG modes, the wavefield of which can be represented for a single initial LG beam as

$$\Psi_{m.n}\left(r,\varphi,R\right)=\sum_{k=0}^{\infty}C_{m.n,k}\left(R\right)r^{|m|}L_k^{|m|}\left(2r^2\right)e^{i\,m\varphi}\exp\left(-r^2\right), \qquad (1)$$

where $r=\rho/w_0$, ρ and φ are the polar coordinates, w_0 stands for a beam, R is a normalized aperture radius while the beam amplitudes are described by the equation

$$C_{m,n,k}=\int_0^R \Psi(r,\varphi)\psi^*_{m,k}(r,\varphi)r\,dr \Big/ \int_0^\infty \left|\psi_{m,k}(r,\varphi)\right|^2 r\,dr \tag{2}$$

and $\psi_{m,k}(r,\varphi)$ denotes the complex amplitude of a single non-perturbed LG beam. The analysis of the intensity distribution $\Im_{m,n}(r,\varphi)=\left|\Psi_{m,n}(r,\varphi)\right|^2$ of the perturbed beam is performed at the focal plane of the spherical lens by means of the intensity moments written in the form

$$J_{p,q}=\int_{\mathbb{R}^2} M_{p,q}(r,\varphi)\Im_{m,n}(r)\,dS, \tag{3}$$

where $M_{p,q}(r,\varphi)$ stands for the moments function, $p,q=0,1,2,\ldots$. Since the intensity distribution $\Im_{p,q}(r)$ is an axially symmetric function, the moments function can be reduced to the form $M_{p,q}(r)$. The definition of the intensity moments in Eq. (3) can be treated as a system of linear equations with respect to the squared amplitudes $C^2_{m,n,k}$ and the cross terms $2C_{m,n,k}C_{m,n,s}$. Indeed, the terms on the right-hand side of the equation (3) depend on the squared amplitudes, the cross terms and also factors

$$j_{pq}=\int_0^\infty r^{\,2m+1}M_{p,q}L_k^m\left(2r^2\right)L_s^m\left(2r^2\right)e^{-2r^2}dr, \tag{4}$$

that can be easily calculated. The left side of the equation can be treated as a measurable quantity. The product of the experimentally measured intensity distribution $\Im_{\exp}(r)$ and the moments function $M_{p,q}$, and therefore the entire integral (3) can be found experimentally in combination with the computer processing provided that $\Im(r)\to\Im_{\exp}(r)$. The number of variables in the equations is specified by the number of squared amplitudes $X_p=C^2_{m,n,k}$ and cross terms $Y_p=2C_{m,n,k}C_{m,n,s}$. The number of the squared amplitudes is N while the number of cross-amplitudes in in the intensity distribution is defined as the 2-combination of a N equal to the binomial coefficient $N!/(2!(N-2)!)$ while the expression (4) can be written in the matrix form $\hat{j}$. Then the vector column $\mathbf{C}$ of the mode amplitudes and the vector-column of the intensity moments $\mathbf{J}$ are related to each other by a linear matrix relation $\mathbf{C}=\hat{j}^{-1}\mathbf{J}$.
Since the circular aperture does not produce secondary modes with different topological charges m, the orbital angular moment of the perturbed beam remains constant. However multiple secondary modes of perturbed LG beam give rise to uncertainty of the wavefield state, measure of which is the information entropy (Shannon entropy) [8,9]. The information entropy can be represented in terms of squared amplitudes as

$$H_{m,n}=-\sum_{k=0}^{N}C^2_{m,n}(R,k)\log_2 C^2_{m,n}(R,k)>0\,. \tag{5}$$

Measurements of the amplitude spectra and LG mode sorting were carried out at the experimental setup depicted in Fig1 of our article [10]. Typical spectra of mode amplitudes $C_{m,n,k}$ and entropy distributions $H_{m,n}$ of a single perturbed beam are shown in Fig.1. From Fig.1a we see that the amplitude spectrum $C_{m,n}(k)$ is limited in radial mode numbers k, that enabled us to use a limited number of terms in Eq. (1) in the experiment. Besides, the sign-alternating form of the spectrum indicates that it is necessary to measure not only the squared amplitudes but also the cross-amplitudes in the intensity distribution. The oscillating distribution of the mode amplitude in Fig.1b illustrates the contribution of the mode with radial number k=0 and k = 6 to the mode with the initial radial number n = 0. The measure of uncertainty $H_{m,n=0}(R)$ of the initial state n = 0 increases as the radius of the aperture (Fig.1c) decreases due to the energy transfer into higher order modes, while the entropy $H_{m,n=6}(R)$ of the state n = 6 in Fig.1d rapidly oscillates due to the energy redistribution between modes. The broad mode spectrum of the perturbed single LG beam can make fundamental changes to the spectrum of the perturbed beam array in a data compression channel.

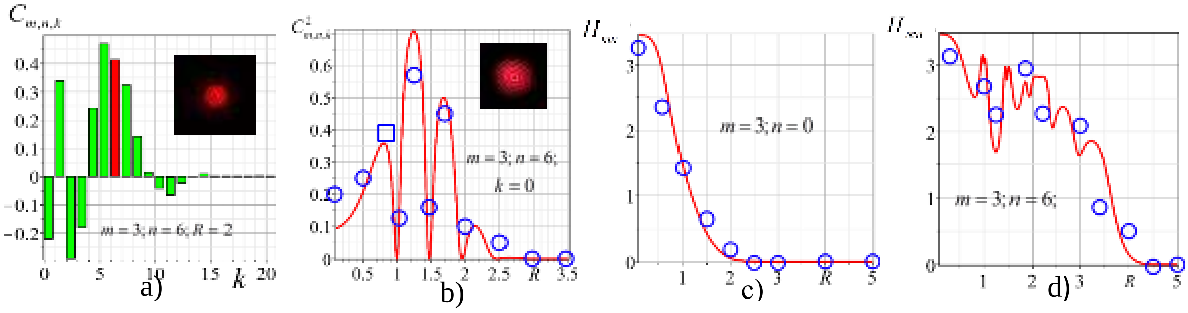

Figure 1. The experimental aamplitude spectra $C_{m,n}$ of the LG beam as a function of the radial number k ,(b) of the aperture radius R; the entropy $H_{m,n}(R)$ for the states (a) m=3, n=0 and (b) m=3, n=6; solid lines – theory, circlets - experiment.

We carried out series of experimental and computer simulation analysis of the mode spectra and mode sorting processes of perturbed beam arrays containing several LG modes with different radial numbers n but the same topological charges m and revealed that each mode in the spectrum is degenerate while a number of degeneracy is equal to a number of modes in the initial beam array Indeed, every mode in the array of M beams experiences the same perturbation from the hard-edged aperture forming its own broad mode spectrum. As a result, each k-th mode in the perturbed spectrum contains a superposition of the M modes of the array. The spectrum becomes degenerate and a degeneracy number of each mode is M. Figure 2a shows a typical spectrum of a beam array with topological charge m = 3, containing three modes with radial numbers n = 0, n = 3 and n = 6 perturbed by the aperture with the R = 2 radius. Every mode in the spectrum is three-fold degenerated. The original modes of the array are highlighted in red. The mode degeneration affects the smoothing of the entropy distribution in Fig.1b (compare to Fig.1c,d). Since modes with different radial numbers *n* are independent, in order to decipher the spectrum in Fig. 2a, one needs to hold M=3 keys. Such keys are the mode spectra $C_{m,n,k}(R)$ of each single perturbed LG beam for the aperture radius R.

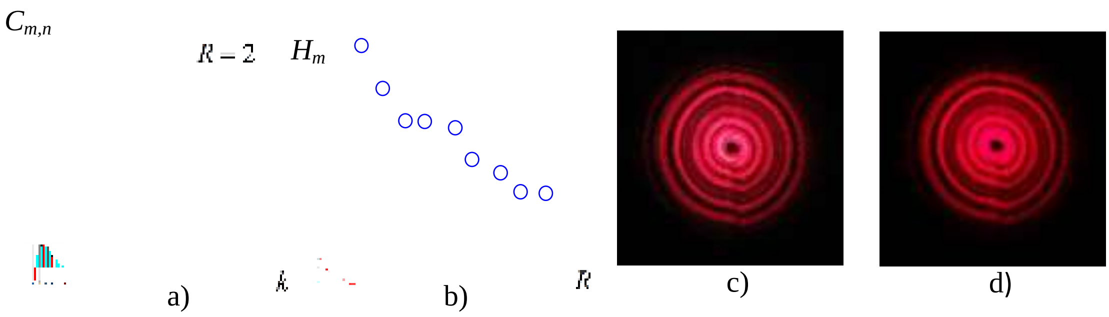

Figure 2. (a) Degenerate amplitude spectrum $C_{m,n}(k)$, (b) entropy distribution $H_m(R)$, (c) intensity distribution of the initial LG beam and (d) intensity distribution of the LG beam restored of the sorted modes.

Figure 2c,d shows two images of the complex beams: (c) before sorting and (c) the complex beam pieced together of the sorted LG modes with m=3, n = 0, n = 3 and n = 6, R=2. These images were compared with each other. The correlation degree of the intensity distributions was η=0.92 that exceeds the critical value η=0.90 while the correlation degree of a single recovered LG beam after the beam sorting is about η=0.92.

The presented new technique of digital beam sorting makes it possible to significantly simplify the existing devices for sorting beams by radial numbers, known to the authors, since it enables one to remove a number of interferometric elements from the optical devices together with auxiliary mechanical and optoelectronic gadgets. Besides, employment of degenerate perturbed beams allows one to use of new quantum key distributions in cryptography, optical communication and data processing systems.

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3. Reference

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