Decomposition of PD-regulators design problem for systems with slow and fast modes

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Abstract. The PD-regulators design problem for the singularly perturbed control system is considered in the paper. It is shown that this problem can be reduced to the P-regulators design problems for two subsystems of lower dimension.

1. Introduction

We consider the PD-regulators design problem for the singularly perturbed control system

$$\varepsilon\ddot{x} + M(t)\dot{x} + N(t)x = B(t)u, \tag{1}$$

$x \in R^n$, $t \in R$, ε is a small positive parameter. From the formal mathematical viewpoint it necessary to construct a control law (PD-regulator) of form

$$u = Qx + R\dot{x},$$

for which the system

$$\varepsilon\ddot{x} + M(t)\dot{x} + N(t)x = B(t)(Qx + R\dot{x}) \tag{2}$$

is asymptotically stable. To simplify the solution of this problem we will use the method of decomposition the system under consideration into two independent subsystems using the splitting transformation.

2. Splitting transformation

Consider the differential system

$$\dot{x} = A_{11}x + A_{12}y + f_1, \tag{3}$$

$$\varepsilon\dot{y} = A_{21}x + A_{22}y + f_2, \tag{4}$$

$A_{ij} = A_{ij}(t,\varepsilon)$ — where $x \in R^m,\ y \in R^n,\ t \in R$.

We will use a transformation which can reduces (3)-(4) to the form

$$\begin{aligned} \dot{v} &= A_1(t,\varepsilon)v + f(t,\varepsilon), \\ \varepsilon\dot{z} &= A_2(t,\varepsilon)z. \end{aligned} \tag{5}$$

We assume that the eigenvalues $\lambda_i(t)$ of the matrix $A_{22}(t,0)$ have the property $Re\lambda_i(t) \le -2\gamma < 0$ in $t \in \mathbb{R}$ and that the matrix- and vector-functions $A_{ij},\ A_{22}^{-1}(t,0)$ and f_i are continuous and bounded as well as their partial derivatives with respect to the arguments $t \in \mathbb{R}, \varepsilon \in [0, \varepsilon_0]$. These imply than the following asymptotic representations

$$A_{ij} = \sum_{l=0}^{k} \varepsilon^l A_{ij}^{(l)}(t) + \varepsilon^{k+1} A_{ij}^{(k+1)}(t, \varepsilon),$$

$$f_i = \sum_{l=0}^{k} \varepsilon^l f_i^{(l)}(t) + \varepsilon^{k+1} f_i^{(k+1)}(t, \varepsilon)$$

take place.

Introduce new variables $v,\ z$ by formulae

$$x = v + \varepsilon P z, \quad y = z + Lx + h$$

where $L = L(t, \varepsilon),\ P = P(t, \varepsilon)$ are bounded matrix-functions and $h = h(t, \varepsilon))$ is bounded vector-function such that $v\ z$ satisfy (5), where

$$A_1 = A_{11} + A_{12}L, \quad A_2 = A_{22} - \varepsilon L A_{12}, \quad f = f_1 + A_{12}h.$$

Here $L,\ P$ and h are bounded for $t \in R$ solutions of equations

$$\varepsilon \dot{L} + \varepsilon L(A_{11} + A_{12}L) = A_{21} + A_{22}L,$$

$$\varepsilon \dot{P} + P A_2 = \varepsilon A_1 P + A_{12},$$

$$\varepsilon \dot{h} + \varepsilon L f_1 = A_2 h + f_2.$$

The hyperplane $y = Lx + h$ plays a role of slow integral manifold of (3)-(4). Note that the following representations are true

$$L = \sum_{l \geq 0} \varepsilon^l L^{(l)}(t), \quad P = \sum_{l \geq 0} \mathrm{e}^l P^{(l)}(t), \quad h = \sum_{l \geq 0} \mathrm{e}^l h^{(l)}(t)$$

with

$$L^{(0)} = -\left(A_{22}^{(0)}\right)^{-1} A_{21}^{(0)},$$

$$L^{(1)} = -\left(A_{22}^{(0)}\right)^{-1} \left[A_{21}^{(1)} + A_{22}^{(1)} L^{(0)} - \dot{L}^{(0)} - L^{(0)} A_1^{(0)}\right],$$

$$L^{(i)} = -\left(A_{22}^{(0)}\right)^{-1} [A_{21}^{(i)} + \sum_{j=1}^{i} A_{22}^{(j)} L^{(i-j)} -$$

$$\dot{L}^{(i-1)} - \sum_{j=0}^{i-1} L^{(i-j-1)} A_1^{(j)}],$$

where $A_1^{(i)} = A_{11}^{(i)} + \sum\limits_{j=0}^{i} A_{12}^{(j)} L^{(i-j)},\ i = \overline{1, k}$, and

$$P^{(0)} = A_{12}^{(0)} \left(A_{22}^{(0)}\right)^{-1},$$

$$P^{(i)} = [A_{12}^{(i)} + \sum_{j=0}^{i-1} A_1^{(j)} P^{(i-j-1)} -$$

$$\dot{P}^{(i-1)} - \sum_{j=1}^{i} P^{(i-j)} A_2^{(j)}] \left(A_{22}^{(0)}\right)^{-1}, \ i \geq 1,$$

$$h^{(0)} = -\left(A_{22}^{(0)}\right)^{-1} f_2^{(0)},$$

$$h^{(i)} = -\left(A_{22}^{(0)}\right)^{-1} [f_2^{(i)} + + \sum_{j=1}^{i} A_2^{(j)} h^{(i-j)} - \dot{h}^{(i-1)} - \sum_{j=0}^{i-1} L^{(j)} f_1^{(i-j-1)}], \ i \geq 1,$$

$$A_2^{(i)} = A_{22}^{(i)} - \sum_{j=0}^{i-1} L^{(i-j-1)} A_{12}^{(j)}.$$

3. PD-regulators

It is possible to rewrite (2) of form (3)-(4) with

$$A_{11} = 0, \quad A_{12} = I, \quad A_{21} = -N + BQ, \quad A_{22} = -M + BR, \quad f_1 = 0, \quad f_2 = 0.$$

Suppose that it is possible to choose matrix R in such a way that matrix $-M + BR$ $Re\lambda_i(t) \leq -2\gamma < 0$ in $t \in \mathbb{R}$. This means that subsystem

$$\varepsilon \dot{z} = A_2(t, \varepsilon) z$$

is asymptotically stable and the PD-regulators design problem for the original system (1) reduces to the subsystem of low dimension. It is sufficient now to choose matrix R in such a way that subsystem

$$\dot{v} = A_1(t, \varepsilon) v$$

becomes asymptotically stable.

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5. References

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