ОБРАБОТКА ИЗОБРАЖЕНИЙ, РАСПОЗНАВАНИЕ ОБРАЗОВ

Time-optimal algorithms focused on the search for random pulsed-point sources

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Abstract

The article describes methods and algorithms related to the analysis of dynamically changing discrete random fields. Time-optimal strategies for the localization of pulsed-point sources having a random spatial distribution and indicating themselves by generating instant delta pulses at random times are proposed. An optimal strategy is a procedure that has a minimum (statistically) average localization time. The search is performed in accordance with the requirements for localization accuracy and is carried out by a system with one or several receiving devices. Along with the predetermined accuracy of localization of a random pulsed-point source, a significant complicating factor of the formulated problem is that the choice of the optimal search procedure is not limited to one-step algorithms that end at the moment of first pulse generation. Moreover, the article shows that even with relatively low requirements for localization accuracy, the time-optimal procedure consists of several steps, and the transition from one step to another occurs at the time of registration of the next pulse by the receiving system. In this case, the situation is acceptable when during the process of optimal search some of the generated pulses are not fixed by the receiving system. The parameters of the optimal search depending on the number of receiving devices and the required accuracy of localization are calculated and described in the paper.

Keywords: optimal search, pulsed-point source, localization accuracy, receiver.

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Introduction

The task of searching for random pulsed-point sources arises in many applied scientific and technical disciplines, for example, in applications of the classical theory of reliability and mathematical communication theory, when it is required to identify and suppress sources of local impulse noise [1–3]. The need for such algorithms appears in the efficiency analysis of modern optoelectronic sensors [3]. Similar studies are necessary for the development of methods for tech troubleshooting, appearing in a form of the alternating equipment failures [4]. In modern sections of computational mathematics these methods are required to create algorithms for detecting low-contrast and small-sized objects on aerospace images, and, for example, in signal theory, the same methods are used to estimate the reliability of random fields and point images registration [5–6].

An optimal search algorithm generally should satisfy one of two requirements: either to minimize the total search effort required to detect an object, or to maximize the total probability of its detection in the presence of a limited search effort. In this work, by a pulsed-point source we will mean the object of negligibly small angular dimensions (a mathematical point), that is placed randomly on the x axis with an a priori distribution density $f(x)$ and radiating infinitely short pulses (δ-functions) with Poisson intensity λ.

Thus, the time intervals between pulses are a random variable t with an exponential distribution density $h(t)=\lambda\exp(-\lambda t)$. The search for an object is carried out using a recording device with “view window” that can be reconstructed in any way in time. The impulse is recorded if the active object that initiated the impulse is located inside the view window of the recording device. Otherwise, the pulse is considered to be missed. After registering the pulse window narrows, as a result the position of the source becomes more accurate. It is required to find the source with accuracy ε for the minimum (in statistical terms) time.

1. Time-optimal search algorithms for pulsed-point sources for single-receiver systems

Single-step search algorithms

Introducing the binary function

$$u(x,t)=\begin{cases}1, & \text{if the point } x \text{ at the time moment } t \\ & \text{is in the view window of the receiving} \\ & \text{device,} \\ 0, & \text{otherwise,}\end{cases}$$

describing view window at time t, we obtain the ratio for the average time from the start of the search to the registration of the first pulse:

$$\langle\tau\rangle = \lambda\int_0^\infty \mathrm{d}t\int_0^\infty \mathrm{d}x\left[tf(x)u(x,t)\exp(-\lambda\int_0^t u(x,\xi)\,\mathrm{d}\xi)\right].$$

For the random priori distribution of the pulsed-point source on the x axis, the construction of even a single-step (ending immediately at the moment of first pulse registration) procedure of the optimal-time search causes considerable difficulties. In single-step periodic search algorithms, the relative load $\varphi(x)$ on the point x (that is, the relative time it stays in the view window) remains constant throughout the search time. With this approach, the problem is to find the function $\varphi(x)$, which minimizes the average search time

$$\langle\tau\rangle = \frac{1}{\lambda}\int\frac{f(x)}{\phi(x)}\mathrm{d}x\,, \tag{1}$$

provided that

$$\int\phi(x)\,\mathrm{d}x = \varepsilon\,, \tag{2}$$

$$0 \le \phi(x) \le 1. \tag{3}$$

Optimization of expression (1) with constraints (2)–(3) relates to non-linear programming problems [7–10]. To solve it, we use the method of Lagrange undetermined multipliers and find for the function $\varphi(x)$, which minimizes the expression

$$\int\left[\frac{f(x)}{\phi(x)} + \mu\phi(x)\right]\mathrm{d}x\,.$$

Differentiating by φ and taking into account the constraint (2), we get

$$\phi(x) = \frac{\varepsilon\sqrt{f(x)}}{\int\sqrt{f(x)}\,\mathrm{d}x}. \tag{4}$$

If condition (3) is not violated (for any x), then function (4) is a solution to the formulated extremal problem. If there are existing domains x where the solution $\varphi(x) > 1$, then inside these areas it is necessary to set $\varphi(x) = 1$, and for the remaining points to recalculate the undetermined multiplier μ taking into account the already changed conditions (2) and (3). After that, any binary function $u(x,t)$ can be selected as the optimal search strategy, satisfying the relations

$$\int u(x,t)\,\mathrm{d}x = \varepsilon;\quad \int u(x,\xi)\,\mathrm{d}\xi = \phi(x)t.$$

In the general case, the construction of the optimal (not necessarily periodic) single-step search algorithm for an unknown Poisson source is connected with finding such a function $\varphi(x,t)$ – the relative load on the point x at a time t that minimizes the average search time

$$\langle\tau\rangle = \int\mathrm{d}t\int\mathrm{d}x\, f(x)\exp\left(-\lambda\int_0^t\phi(x,\xi)\,\mathrm{d}\xi\right),$$

provided that

$$\int\phi(x,t)\,\mathrm{d}x = \varepsilon \tag{5}$$

for any t, and

$$0 \le \phi(x,t) \le 1. \tag{6}$$

To simplify further calculations, we introduce a function

$$\alpha(x,t) = \int_0^t\phi(x,\xi)\,\mathrm{d}\xi$$

corresponding to the total time spent by the point x in the viewing window from the beginning of the search to time t. To take into account the constraints (5) and (6), we introduce the undetermined Lagrange multiplier $\mu(t)$. Then the problem of constructing the optimal strategy reduces to finding the function $\alpha(x, t)$ that minimizes the functional

$$\int\mathrm{d}t\int\mathrm{d}x\left[\exp(-\lambda\alpha(x,t)f(x) + \mu(t)\alpha(x,t)\right]$$

provided that

$$\int_{-\infty}^{\infty}\alpha(x,t)\,\mathrm{d}x = \varepsilon t, \tag{7}$$

$$0 \le \alpha(x,t) \le t.$$

The solution of this variational problem is the function

$$\alpha(x,t) = \begin{cases} 0, \frac{1}{\lambda}\ln\frac{\lambda f(x)}{\mu(t)} < 0; \\ \frac{1}{\lambda}\ln\frac{\lambda f(x)}{\mu(t)}, 0 \le \frac{1}{\lambda}\ln\frac{\lambda f(x)}{\mu(t)} \le t; \\ t, t < \frac{1}{\lambda}\ln\frac{\lambda f(x)}{\mu(t)}, \end{cases} \tag{8}$$

where $\mu(t)$ is determined from the relation (7). The optimal search strategy $u(x, t)$ must belong to the class of binary functions. It is set by the equations

$$\int_0^t u(x,\xi)\,\mathrm{d}\xi = \alpha(x,t);\quad \int u(x,t)\,\mathrm{d}x = \varepsilon.$$

Practical use of optimal search algorithms encounters certain difficulties. The fact is that in cases where priori distribution density function differs from the uniform one, both of the proposed optimal single-step search algorithms cannot be realized if you try to implement it by moving an integral scanning window. Therefore, in real search procedures, one-step procedure is advisable to do in according to the following scheme.

Preliminarily, the interval $(0, L)$ is divided into a series of discrete elements with width ε, while the a priori given density $f(x)$ is "stepwise" approximated on each of them. The value of ε is considered to be small enough (according to the high requirements for localization accuracy), so, the variation of the function $f(x)$ within one discrete can be neglected. The search should begin with "observation" of the highest "step", within which the function $f(x)$ is maximum, then after the time t_1, the window is alternately set under the two highest "peaks", then after the time t_2, three items are monitored and etc. All switching moments t_i are determined in exact correspondence with the above relation (8), which is the basis for constructing an optimal search strategy.

It should be noted that discussed search plan assumes that the intensity of the source λ is known in advance. If such a priori information is not available, a periodic procedure can be recommended that does not depend on this intensity. In accordance with it, the integrals of the density $f(x)$ in each of the discrete must be calculated. If there are m discretes, and its squares-integrals are $P_1, P_2, ..., P_m$, then the view window should cyclically "run through" all the discretes with relative load

$$\beta_i = \sqrt{P_i} \Big/ \sum_{j=1}^{m} \sqrt{P_j} \quad (j=1, ..., m) .$$

These values β_i are easily obtained if we again apply the method of undetermined Lagrange multipliers to minimize the average search time

$$\langle \tau \rangle = (1/\lambda)(P_1/\beta_1 + P_2/\beta_2 + ... + P_m/\beta_m) ,$$

provided that $\beta_1 + ... + \beta_m = 1$.

<u>*Multistep search algorithms*</u>

If we are not limited to single-step procedures, but consider the search algorithm as a multi-step process (that ends after n-th pulse registration), then the optimal strategy should deliver a minimum to the functional

$$\langle \tau \rangle = \sum_{k=1}^{n} \lambda^k \int_0^{\infty} \mathrm{d}x\, f(x) \int ... \int t_k \times \\ \times \left\{ \prod_{l=1}^{k} \left[\mathrm{d}t_l u_l \left(x, \sum_{m=1}^{l} t_m, t_1, ..., t_{l-1} \right) \times \right.\right. \\ \left.\left. \times \exp\left(-\lambda \int_{\sum_{m=1}^{l-1} t_m}^{\sum_{m=1}^{l} t_m} u_l \left(x, \xi, t_1, ... t_{l-1} \right) \mathrm{d}\xi \right) \right] \right\}, \tag{9}$$

provided that

$$\int u_n (x, t, t_1, ..., t_{n-1})\, \mathrm{d}x = \varepsilon .$$

Here $u_i (x, t, t_1, ..., t_{i-1})$ is the search strategy at the i-th step provided that the intervals between the first $(i-1)$ pulses were $t, t_1, ..., t_{i-1}$ respectively. In the general case, to find the optimal strategy $u(x, t)$ that minimizes the functional (9) is not possible. At the same time, for an important special case, $f(x) = \text{const}$, the analytic solution is rather simple. Let

$$f(x) = \begin{cases} 1/L, & x \in (0, L), \\ 0, & x \notin (0, L), \end{cases}$$

i.e. there is no a priori information about placement of the source within the interval $(0, L)$. Obviously, in the first step, the search effort should be equally distributed between all points $x \in (0, L)$. It is possible to make such a load, for example, by television scanning of the whole interval $(0, L)$ by the aperture with width l_1 (to avoid edge effects, we consider the end of the interval closed to its beginning, so, a circle with length L is scanned instead of the interval). When registering a pulse, the search continues inside the window with width l_1 using another aperture with width l_2. If we discuss an n-step search, then at the last step scanning it is done by the aperture with width ε (this is dictated by the conditions of the task). Then the average search time is

$$\langle \tau \rangle = \left(\frac{1}{\lambda} \right) \left(\frac{L}{l_1} + \frac{l_1}{l_2} + ... + \frac{l_{n-1}}{\varepsilon} \right). \tag{10}$$

For a fixed n, it's possible to find optimal values that minimize the expression (10):

$$\frac{L}{l_1} = \frac{l_1}{l_2} = ... = \frac{l_{n-1}}{\varepsilon} = \left(\frac{L}{\varepsilon} \right)^{1/n} .$$

Then the average time of the optimal n-step search is

$$\langle \tau_n \rangle_{opt} = \left(\frac{n}{\lambda} \right) \left(\frac{L}{\varepsilon} \right)^{1/n} . \tag{11}$$

Now (from the expression (11)) we can find the optimal number of steps n minimizes the average search time. Since the function $xa^{1/x}$ for $a > 1$ has only one minimum point $(x = \ln a)$, the optimal value n_{opt} is always either $\mathrm{entier}(\ln(L/\varepsilon))$ or $\mathrm{entier}(\ln(L/\varepsilon)) + 1$. Therefore, when $L/\varepsilon \to \propto$ the following asymptotic relations are true:

$$n_{opt} \cong \ln(L/\varepsilon) ,$$

$$\frac{L}{l_1} = \frac{l_1}{l_2} = ... = \frac{l_{n-1}}{\varepsilon} = e,$$

$$\langle \tau_{n_{opt}} \rangle_{opt} = \left(\frac{n_{opt}}{\lambda} \right) \left(\frac{L}{\varepsilon} \right)^{1/n_{\text{опт}}} = \left(\frac{e}{\lambda} \right) \ln \left(\frac{L}{\varepsilon} \right).$$

Thus, for $L/\varepsilon \to \propto$, multi-stage procedure (compared to a single-stage procedure) logarithmically reduces the average search time, so the gain increases unlimitedly as the ratio L/ε increases. Now we can compare the constructed optimal search procedure with some simplified algorithms. For example, if the search is organized according to the principle of a dichotomy (the receiving device alternately "observes" the two halves of the scanned area at every stage of the search), then the average time of the source search is

$$\langle \tau \rangle_2 = \left(\frac{2}{\lambda} \right) \log_2 \left(\frac{L}{\varepsilon} \right) = \left(\frac{2}{\lambda} \right) \ln \left(\frac{L}{\varepsilon} \right).$$

Dichotomous search has (in comparison with the optimal procedure) a small ($\approx 6\,\%$) loss in time. Trichotomic search is even closer to the optimal procedure: initial interval $(0, L)$ is divided into three subintervals, then the subinterval where impulse is fixed, in turn, is also divided into three subintervals, etc. Compared to the optimal procedure this procedure loses only

$$\frac{1}{e} \left(\frac{3}{\ln 3} - e \right) \approx 0{,}4\% .$$

It is natural to expect that in the case of arbitrariest a priori distribution $f(x)$ the multi-step search procedure (compared to one-step search) can bring a significant gain in time, especially for large values of L/ε. Since minimization of the functional (9) under the constraint (10) in each concrete case is very difficult problem, multi-step periodic search procedure seems to be more real in terms of practical implementation. In the first step, the interval $(0, L)$ is divided into three parts (three parts are chosen because for a uniform distribution $f(x)$ this procedure is closest to the optimal one). Then the values are calculated

$$P_1 = \int_0^{L/3} f(x)\,dx;\; P_2 = \int_{L/3}^{2L/3} f(x)\,dx;\; P_3 = \int_{2L/3}^{L} f(x)\,dx.$$

For any time interval Δt, view window with width $L/3$ must be alternately "tuned on" all three sections in such a way that $\Delta t_1 + \Delta t_2 + \Delta t_3 = \Delta t$ where $\Delta t_i/\Delta t = \beta_i$ is the relative viewing window presence time on each of the subintervals $(0, L/3)$, $(L/3, 2L/3)$, $(2L/3, L)$. When the pulse is registered, the search procedure continues similarly on the section where the pulse is fixed (i.e., this section is divided again into three parts, the coefficients $\beta_1, \beta_2, \beta_3$ are recalculated, etc.). At the first step their values are equal

$$\sqrt{P_i}\Big/\sum_{j=1}^{3}\sqrt{P_j} \quad (i = 1, 2, 3).$$

The universality of the proposed procedure is also in the fact that its realization does not require a priori information on the source intensity.

2. Time-optimal search algorithms for the multi-receiver systems

In the previous chapter of this paper optimal search algorithms for pulsed point sources were described, which assumed the using of a single receiver with arbitrarily tunable view window. Obviously, when using multiple receivers, the average localization time can be significantly reduced.

The task is to build an algorithm for finding random pulsed-point source using the system with an arbitrary but fixed number of receivers that has a minimum (statistically) average time to achieve the required localization accuracy. It is hardly possible to find an analytical solution of the problem in general, as noted in the previous section. Therefore, in the case of multi-receiver systems, we considered a special case that is very important from a practical point of view, when there is no a priori information about the probable location of the source, i.e. when the random point source is uniformly distributed over the search interval $(0, L)$. The obvious advantage of the search algorithm, which is optimal for a uniformly distributed random source, is that when it is used as a source localization procedure with an unknown a priori distribution (and this distribution can be any), the average search time will correspond to the variant of the uniform distributed source in the search interval.

The following Table 1 contains the parameters of the optimal search, that were calculated on the assumption that the receiving system has a fixed number of receiving devices n, the search is carried out in the interval $(0, L)$, and the required localization accuracy is ε. The calculated parameters of the system were: the optimal number of search steps; the size of the view windows at each step; average optimal search time. Naturally, it was considered admissible and, moreover, it was assumed that not all the pulses generated by a random source are fixed by the receiving system. Due to the limitations on the volume of this message, Table 1 shows only the final results of the calculations, and all intermediate calculations are omitted.

Table 1. Parameters of the optimal search for the random pulsed point source depending on the number of receivers n ($n \ge 2$) and the required localization accuracy ε

(ε/L) (required localization accuracy)	m_{opt}	$W_m, m=\overline{1, m_{opt}}$ (Viewing windows of the receiving system at each of m_{opt} stages of optimal search)	$<\tau>$ (average localization time)
$\frac{1}{(2^n-1)^m} \le$ $\le (\varepsilon/L) \le$ $\le \frac{1}{(2^n-1)^{m-1}} \times$ $\times\left(\frac{m-1}{m}\right)^{m-1}$	m	$W_1 = L$, $W_2 = \frac{1}{2^n-1}\times L$, … $W_m = \frac{1}{(2^n-1)^{m-1}}\times L$	$\frac{m}{\lambda}$
$\frac{1}{(2^n-1)^m}\times$ $\times\left(\frac{m}{m+1}\right)^m \le$ $\le (\varepsilon/L) \le$ $\le \frac{1}{(2^n-1)^m}$	m	$W_1 = (2^n-1)\times$ $\times(\varepsilon/L)^{1/m}\times L$, $W_2 = (2^n-1)\times$ $\times(\varepsilon/L)^{2/m}\times L$, … $W_m = (2^n-1)\times$ $\times(\varepsilon/L)^{m/m}\times L$	$\frac{m}{\lambda(2^n-1)}\times$ $\times(\varepsilon/L)^{-\frac{1}{m}}$
$\varepsilon/L \to 0$, $\frac{e^{-1}}{(2^n-1)^{m_\infty}} \le$ $\le (\varepsilon/L) \le$ $\le \frac{e^{-1}}{(2^n-1)^{m_\infty-1}}$	m_∞	$W_1 = L$, $W_2 = \frac{1}{2^n-1}\times L$, … $W_i = \frac{1}{(2^n-1)^{i-1}}\times L$, … $W_{m_\infty} = \frac{1}{(2^n-1)^{m_\infty-1}}\times$ $\times L \approx$ $\approx \frac{2^n-1}{(2^n-1)^{\frac{-\ln(\varepsilon/L)}{\ln(2^n-1)}}}\times L =$ $= (2^n-1)\varepsilon$	$\frac{m_\infty}{\lambda} \approx$ $\approx -\frac{\ln(\varepsilon/L)}{\lambda\ln(2^n-1)}$

*when $\varepsilon/L \to 0$, the asymptotic number of stages m_∞ is equal

$$m_\infty \approx \frac{-\ln(\varepsilon/L)}{\ln(2^n-1)}.$$

For comparison, Table 2 presents the parameters of the optimal search, calculated for a system with one receiver. Taking into account, that

$$\lim_{m\to\infty}\left(\frac{M}{M+1}\right) = e^{-1},$$

under high localization accuracy requirements, i.e., for $(\varepsilon/L) \to 0$, we have the following asymptotic relations for a system with one receiver:

$$m_{opt} \approx \ln(L/\varepsilon); \quad W_i \approx e^{-i} \times L, \ i = \overline{1, m_{opt}}; \quad <\tau>_{opt} \approx \frac{e \ln(L/\varepsilon)}{\lambda}.$$

Table 2. Parameters of optimal search for a random uniformly distributed pulsed point source (for a system with one receiver)

(ε/L) (required accuracy of localization)	M_{opt} *	W_m, $m=\overline{1,m_{opt}}$ (system's search windows at each of M_{opt} steps of optimal search)	<τ> (average time of localization)
$\frac{1}{4} \le (\varepsilon/L) < 1$	1	$W_1 = \varepsilon$	$\frac{1}{\lambda}(\varepsilon/L)^{-1}$
$\left(\frac{2}{3}\right)^6 \le (\varepsilon/L) \le \frac{1}{4}$	2	$W_1 = (\varepsilon/L)^{\frac{1}{2}} \times L$ $W_2 = (\varepsilon/L) \times L = \varepsilon$	$\frac{2}{\lambda}(\varepsilon/L)^{-\frac{1}{2}}$
$\left(\frac{3}{4}\right)^{12} \le (\varepsilon/L) \le \left(\frac{2}{3}\right)^6$	3	$W_1 = (\varepsilon/L)^{\frac{1}{3}} \times L$ $W_2 = (\varepsilon/L)^{\frac{2}{3}} \times L$ $W_3 = (\varepsilon/L) \times L = \varepsilon$	$\frac{3}{\lambda}(\varepsilon/L)^{-\frac{1}{3}}$
⋮	⋮	⋮	⋮
$\left(\frac{M}{M+1}\right)^{M(M+1)} \le (\varepsilon/L) \le$ $\le \left(\frac{M-1}{M}\right)^{M(M-1)}$	M	$W_1 = (\varepsilon/L)^{\frac{1}{M}} \times L$ $W_2 = (\varepsilon/L)^{\frac{2}{M}} \times L$ ⋮ $W_m = (\varepsilon/L)^{\frac{m}{M}} \times L$ ⋮ $W_M = (\varepsilon/L)^{\frac{M}{M}} \times L = \varepsilon$	$\frac{M}{\lambda}(\varepsilon/L)^{-\frac{1}{M}}$

**Optimal number of stages for given localization accuracy*

Conclusion

The proposed search strategies offer the prospect of constructing optimal localization algorithms in the case when the probability density function of a random source of pulses is different from the uniform one, and the search is performed by a multi-window system. Another interesting and little explored direction of the problem is the construction of optimal search procedures aimed at simultaneously localizing several random sources.

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