Model for discrete optimal control of enterprise's financial processes

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Abstract. The problem of financial resources planning under uncertainty of cash flow over time, coordination of inflows and outflows of financial flows of the enterprise is solved in the article. A critical theoretical and methodological analysis of foreign and domestic literature has been carried out. In analyzing the problem of financial modeling and building the operational financial strategy we used methods of system analysis, control theory and optimal control, methods of data processing under uncertainty. The model for distribution of financial flows has been developed. It uses the principles of optimal dynamic control under criterion of cumulative risks of non-payment, transaction and opportunity costs minimizing. The practical significance of the research is in developed model application, allowing to improve the financial planning quality, to increase the management efficiency and operational efficiency of an enterprise.

1. Introduction

One of the most important field of maintaining the stable functioning and sustainable development of enterprise is costs reduction associated with its industrial, investment and financial activities. Cost reducing (in terms of unreasonable costs) through effective operational management of financial resources becomes extremely important in a crisis and post-crisis periods of economic development. The inefficiency of control and financial planning being leveled during periods of favorable economic conjuncture and in growing market can lead to budget overruns during crisis periods and put enterprises on the bankruptcy. The purpose of operational management of enterprise's financial resources is the redistribution of financial resources based on the operational analysis of the current situation.

Analysis of existing approaches and tools used for the operational management of financial flows [1 – 10] allow to identify two types of problems:

- cash flow forecasting. Traditionally reserve of required monetary resources is determined enlarged and calculated using the Baumol model [11], Miller-Orr and Stone models [12, 13]. A feature of these models is the precondition about stability and determinism of cash flows and their predictability regarding these properties. All of these models are static and do not take into account the probabilistic nature of financial flows in terms time and volume. Therefore, the predictive ability of these models is not high and does not take into account the risk factors – solvency decline and liquidity, associated with the stochastic nature of cash inflows;

- cash flow building with minimal financial loss. The basic conditions for problem decision the are: a) ensuring the balance of inflows and outflows of funds in accordance with established deadlines; b) achieving a uniform value of accumulated funds balance when the fluctuations of this value in the context of the sub-periods of the planning period are minimal, which, in turn, allows the use temporarily free funds for reinvestment; c) creating the maximum possible amount of cash free for further use.

Analysis of modern software products and tools for operational financial management [14, 15, 16] showed that mathematical models for planning the financial inflow and outflow most often do not take into account its stochastic nature. Therefore, the improvement of the operational finance management mechanisms of an enterprise should be based on the use of stochastic forecasting methods (for example, simulation modeling), budget for incomes and expenses with minimal risks of losing financial recourses, taking into account their dynamic nature based on dynamic programming methods.

2. Payment calendar: features and modeling methods

A detailed operational plan for the receipt and expenditure of financial flows is scheduled in payment calendar, which interconnects all sources of cash receipts and expenses for a certain period of time [17, 18]. It fully covers the cash flow of the enterprise, makes it possible to link cash inflows and payments in cash and non-cash forms, and ensures enterprise constant solvency and liquidity. The payment calendar is a monthly breakdown of positive and negative cash flow broken down by day (weeks, decades) of the cash flow budget. The main purpose of the payment calendar is to achieve a maximum balance of positive and negative cash flows in each moment of the planned period, as well as to ensure the solvency. The system of receipt and expenditure of financial resources is stochastic, since payment flows are uncertain both in terms and resources value. Therefore, the planning of the inflow and outflow of financial resources by traditional methods based on deterministic forecasting schemes is not effective and does not reflect the nature of the system under consideration.

When modeling the payment calendar, the following assumptions are used: a) non-delimitation of cash flows through various cash accounts; b) planning the formation and repayment of accounts receivable and payable.

As a rule, these indicators are not used in the process of the payment calendar designing. However, in order to ensure a required level of enterprise’ solvency it is necessary to take into account the daily amounts of the balances of receivables and payables.

Any payment has two characteristics – the date of funds receipt and the amount of required funds. Payments (cost items) are divided into two categories: with a fixed date (salary, taxes and fees, repayment of accounts payable for raw materials, materials, electricity, water, repayment of a bank loan) and with a flexible date (premiums and fees, payments to other creditors, non-core purchases). The need for cash with a fixed date is ensured by forming the necessary minimum balance of cash. Expenditures with a moving date can be made according to the following scheme. Expenditures are fully met with free cash resources. If all items of expenditures for the current day are satisfied and the remaining amounts of money are non-zero, it is necessary to determine the direction of spending this balance. The distribution of free cash balance is based on the use of an optimization model of cash distribution. In situations in which at a certain time there is not enough free cash to cover some expenditure, the total amount of borrowed funds is formed.

For a reasonable distribution of funds for expenditure and for sub-periods, a model of discrete optimal control for the distribution of financial flows and a stochastic model for optimization of the target cash balance have been developed.

3. The model

3.1. The statement of the problem

Solving the problem of allocating financial flows by cost as an alternative by using the priority of certain items is not correct, as enterprises always have the opportunity to change the maturity of payments, for example, use the stretch of the maturity of payables to gain additional benefits. In addition, the input payments flow has no a deterministic nature. Therefore, to solve this problem, it is necessary to involve other methods that allow in the dynamics to plan the distribution of the output payments.

The problem for managing the payment calendar is dynamic, and management should not consist in optimizing the distribution of funds at one point in time, but in determining the optimal solution over a long period (week, month, quarter). It is not enough to determine the optimal plan for resources distribution in a single period, because it is likely that in subsequent periods the distribution of funds will be suboptimal, since it did not take into account the possibilities of further development of production, the risks of non-payment and other significant factors.

Drawing up daily optimal plans is more efficient with regard to previous periods, since the monthly (annual) optimal plan will be the result of optimal decisions taken for all previous periods, and decisions taken at the previous stages should be taken into account. The ability to make a series of consecutive decisions that ensure the optimal development of the process as a whole gives the method of dynamic programming. Dynamic programming is a powerful algorithmic paradigm for optimizing sequential decision-making processes that are of a decomposition nature [19]. The algorithmic scheme of dynamic programming consists in immersing the solvable complex problem in a parameterized family of subtasks with the subsequent solution of these subtasks, using Bellman's optimality principles and the Bellman's recurrent equation that follows them.

The principle of optimal control is reflected in the Bellman functional equation, which establishes a connection between control in the t and (t-1) steps. Optimal control has the following property: whatever the initial state at any step and the control chosen at this step, subsequent controls should be chosen optimal with respect to the state to which the system will come to the end of this step. This means that the maximum benefit (risks) from the t step process is equal to the sum of the incomes (risks) from the first and (t-1) next steps, provided that the next steps left after the first step are best allocated. Such a step-by-step decision-making process is consistent and consists of three components: a set of steps (steps) of decision-making; sets of states and sets of solutions (controls).

The distribution of funds can be represented as a discrete dynamic process, for which it is necessary to find a strategy for allocating funds sequentially for each sub-period of the planning period, which ensures minimum costs. This strategy is a multi-step decision making process. In this sense, the problem of distribution of financial flows is the essence of the problem of discrete optimal control, and the method of its solution is the method of dynamic programming.

For the planning period it is necessary to propose a plan for the allocation of financial resources at the disposal of the enterprise by item of expenditure (that is, to design a payment calendar) that would ensure the minimum total costs associated with paying a fine for late payments. Only payments with a flexible date are considered.

3.2. The formal model

We introduce the following notation: $t = 1, \ldots, T$ – planning periods; $i = 1, \ldots, n$ – the number of expenditures items; u_{it} – value of funding expense in items i and in the period t; $u_t = \sum_{i=1}^{n} u_{it}$ – total funding for all expenditure items in the period t; out_{it} – value of funding requirement expense items i in the period t; in_t – value of receipts in the period t; in_0 – the initial financial fund available for financing; s_T – minimum cash balance by the end of the plan period T; c_{it} – underfinancing penalty ratio expense in items i in the period t; ξ_{it} – underfunding expense in items i in the period t; $\xi_{it} = \xi_{i,t-1} + out_{it} - u_{it}$, $\xi_t = \sum_{t=1}^{T} \xi_{it}$; ϖ_t – value of distribution of funds available by the end of the period

t, $\varpi_t = \varpi_{t-1} - \sum_{i=1}^{n} u_{it} + in_t$; $f_t(\xi_t, u_t)$ – total costs from underfinancing payment calendar items in the period t, $f_t(\xi_t, u_t) = \sum_{i=1}^{n} c_{it}\xi_{it}$.

The mathematical model of the problem has the form:

$$\sum_{t=1}^{T}\sum_{i=1}^{n} c_{it}\xi_{it} \to \min$$

$$\xi_{it} = \xi_{i,t-1} + out_{it} - u_{it},\quad i = 1,\dots,n,\quad t = 1,\dots,T-1,$$

$$\varpi_t = \varpi_{t-1} - \sum_{i=1}^{n} u_{it} + in_t,\quad i = 1,\dots,n,\quad t = 1,\dots,T,$$

$$\xi_{i,0} = 0,\ \xi_{i,T} = 0,\ \varpi_0 = in_0,\ \varpi_T \geq s_T,\ u_{it} \geq 0,\ i = 1,\dots,n,\ t = 1,\dots,T. \quad (1)$$

We use the dynamic programming method to solve the problem (1) of optimal control over the allocation of funds by expenditures over a period T. General scheme of search for optimal control $u_{it}*$, transferring the system from the initial state ξ_0 to the final ξ_T one with the best efficiency indicator $F(\xi, u) = \sum_{t=1}^{T} f_t(\xi_t, u_t)$ consists of the following steps.

Stage 1. The way to divide the decision-making process into steps is time intervals $t = 1,\dots,T$.

Stage 2. Status parameter ξ_{it} – underfunding expense items by the end of the period t, control variables u_{it} – value of funding expense in items i in the period t;

Stage 3. State equations:

$$\xi_{it} = \xi_{i,t-1} + out_{it} - u_{it},\ i = 1,\dots,n,\quad t = 1,\dots,T-1, \quad (2)$$

describe the change system states at each moment of time, and the system state at the current step ξ_{it} depends on the previous state $\xi_{i,t-1}$ and control u_{it}; $\xi_{i,0} = 0$, $\xi_{i,T} = 0$ – initial and final system state.

Step 4. Valid scope for control variables is as:

$$U_{it} = \left\{ u_{it} : \max\left\{0, \sum_{i=1}^{n} out_{it} - \xi_{t-1}\right\} \leq u_{it} \leq \min\left\{in_t, \xi_{it} - \xi_{i,t-1} - out_{it}\right\}\right\}. \quad (3)$$

Step 5. Whole process efficiency

$$F(\xi, u) = \sum_{t=1}^{T}\sum_{i=1}^{n} f_{it}(\xi_{it}, u_{it}) \quad (4)$$

presented as a sum of performance indicators at each step $f_{it}(\xi_{it}, u_{it})$.

Stage 6. Bellman's recurrence relations system are

$$\begin{aligned} F_t^*(\xi_{t-1}) &= \min\left\{\sum_{i=1}^{n} f_{it}(\xi_{it}, u_{it}) + F_{t+1}^*(\xi_{i,t-1} + out_{it} - u_{it})\right\}, t = 1,\dots,T-1, \\ F_T^*(\xi_{T-1}) &= \min\left\{\sum_{i=1}^{n} f_{iT}(\xi_{i,T-1}, \xi_{iT} - \xi_{i,T-1} - out_{iT})\right\}, \end{aligned} \quad (5)$$

where $F_t^*(\xi_{t-1})$ – the conditional maximum of the management efficiency indicator on the steps from t until the end of the process.

Substituting the expression for the indicator of the efficiency of the decision-making process (4) in formula (5), we receive the Bellman equations:

$$\begin{aligned} &F_t^*(\xi_{t-1}) = \min_{u_{it}\in U_{it}} \left\{ \sum_{i=1}^{n} c_{it}\left(\xi_{i,t-1} + out_{it} - u_{it}\right) + F_{t+1}^*\left(\xi_{i,t-1} + out_{it} - u_{it}\right) \right\}, t = 1,\ldots,T-1, \\ &F_T^*(\xi_{T-1}) = \min_{u_{it}\in U_{it}} \left\{ \sum_{i=1}^{n} c_{iT}\left(\xi_{i,T-1} + out_{iT} - u_{iT}\right) \right\}, \end{aligned} \tag{6}$$

whose solution allow to determine the optimal plan $u_{it}*$ for funds distribution over time and by cost items.

4. The algorithm for determine the cash balance

To ensure enterprise solvency as well as to form an insurance reserve in case of unplanned operations and to maintain compensation balances determined by agreement with credit organizations, it is necessary to establish a target cash balance. For this purpose, the problem of optimizing the average current balance of monetary assets is put and solved, providing the solution to two contradictory, but related problems to maintain the current level of solvency and liquidity, on the one hand, and to obtain additional profit from investing free assets, on the other. When planning cash flows, a balance of input and output funds must be ensured, so in modeling process of, synchronization of payments and receipts is achieved, at which the cash balance is kept at a certain acceptable level [20–24].

The value of current and insurance funds depends on the turnover of the enterprise, the uncertainty of the forecast regarding cash flows and the conditions for obtaining short-term loans. Depending on the degree of determinism-stochasticity of the values of the enterprise's assets need, the target balance can be determined using one of the methods listed below.

If cash payments are deterministic, then the intensity of cash payments *b* preserved at a certain level, then the management consists in determining the optimal amount of assets – the target balance at which the total costs *L*, associated with the shortfall in income from the provision of available credit assets (opportunity costs) c_1 и with the maintenance costs for obtaining a loan (transaction costs, or a penalty for the deficit) c_2 will be minimal. In this situation, the target residue can be defined as $s_t = \sqrt{2c_{2t}b/c_{1t}}$.

In the case of stochastic cash flow, the assessment of the minimum cash balance becomes more complicated. The dynamic problem of optimal control of the remainder is not posed, since the magnitude of the need for resources has different values of distribution parameters at different points in time. If they had the same distribution density, it would be possible to find such an optimal strategy for the formation of a cash balance during T time units $t_1, t_2, \ldots, t_T$, to meet the need for outflow with minimal cost. Therefore, the problem of managing the cash balance with a discrete random nature of the need for a money resource is solved by the criterion of minimum costs associated with a shortage or resources excess.

For stochastic cash flows, an estimate of the target aasets balance in a specific period is proposed based on the following scheme.

Step 1. The loss function L is as a piecewise linear function of the required assets out_t:

$$L(s_t, out_t) = \begin{cases} c_{1t}(s_t - out_t), \text{ если } s_t \geq out_t; \\ c_{2t}(out_t - s_t), \text{ если } s_t < out_t, \end{cases} \tag{7}$$

where out_t – variable unknown in advance.

Step 2. Based on statistical observations, we obtain the a posteriori distribution of the required assets out_t. In case of discrete out_t and distribution law $f(out_t)$, expectation of total costs is as:

$$M(L(s_t, out_t)) = c_{1t} \sum_{out_t=0}^{s_t} (s_t - out_t) p(out_t) + c_{2t} \sum_{out_t = s_t+1}^{\infty} (out_t - s_t) p(out_t). \tag{8}$$

In (8) the first term takes into account the costs for the shortfall in income from $(s_t - out_t)$ assetes, and the second – the losses associated with insufficient resource $(out_t - s_t)$. The problem of managing the cash balance is to find such balance s_t, at which the mathematical expectation (7) takes its minimum value.

Step 3. Using the distribution function $F(out_t)$ define the quartile $\frac{c_{2t}}{c_{1t}+c_{2t}}$, where $F\left(s_t^*\right)=\frac{c_{2t}}{c_{1t}+c_{2t}}$ – a posteriori resource allocation function, s_t^* – optimal assets, or quartile $\frac{c_{2t}}{c_{1t}+c_{2t}}$ a posteriori distribution of out_t. If $c_{1t}=c_{2t}$ then optimal resource level s_t^* corresponds to equality $F\left(s_t^*\right)=\frac{1}{2}$, that is the cash balance represents the median in the posterior assets distribution out_t.

5. Results and conclusion

Managing the cash balance is important in business administration and in practice. The literature review shows the importance of cash balance within enterprises, but the development of cash management models are still bound to formulations developed over nearly five decades, without improving the used model. Furthermore, the view of the cash balance is still limited and not regarded as an investment, which has a negative profitability (defined by total cost of the cash), immediate liquidity, and risk associated with cash deficit. Thus, it is necessary to understand the cash balance together with other financial investments as a portfolio investment and examines the investment choices in financial products according to their variable liquidity, profitability, and associated risk. Another relevant aspect is the methodology in developing cash management models. The literature shows a clear preference for stochastic models, and the researchers do not use computer models.

The use of evolutionary computational algorithms, not only genetic algorithms, can reduce limitations when developing more complex models, reducing the constraints presented in this work and making computational implementation easier in accounting and financial management systems within enterprises.

Analysis of the distribution of article in the journal’s areas shows that originally, the articles were presented in journals of economics and finance, but, with the evolution of methods and computer applications in 2000s, the major area of publication has been the operational research and computational optimization. This demonstrates a greater concern about the method, but not with the problem’s formulation.

This is a classic problem in business, involving economics, accounting, and finance, and it should return to be the focus of discussions in these areas, as the existing limitations concerning the models and methods can be eliminated. The cash balance problem not only about the method involved in optimization but also in practical application.

The system of financial flows receipt and expenses is stochastic since payment flows are uncertain both in time and in terms of resources. Therefore, the planning financial resources inflow and outflow by traditional methods based on deterministic predicting schemes is not effective and does not correspond the nature of enterprises’ financial system.

To ensure the synchronization of cash flows and to develop a rational strategy in which cash outflows are combined with inflows, a discrete model of optimal control of cash has been developed. This model provides, firstly, the determination of the necessary and sufficient amount of cash resources in the enterprise’s accounts, secondly, it gives the necessary recommendations to financial analysts to ensure control over cash payments, to cooperate with creditors and debtors, and thirdly, to minimize the risks of financial losses due to inefficient management of the enterprise's financial flows.

A stochastic model for optimizing the cash balance into the enterprise’s accounts has been developed, taking into account the random nature of cash flows according to the criterion of cost minimization. An analytical dependence of the optimal cash balance from alternative costs, related to short-received income from other use remaining after repayment all expenses and transaction costs for service an additional loan for the normal distribution law of a discrete random variable of financial resource demand is obtained.

6. References

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