Probabilistic finite modeling of stochastic estimation of image inter-frame geometric deformations

A.G. Tashlinskii[1], G.L. Safina[2], R.O. Kovalenko[1]

[1]Ulyanovsk State Technical University, Severnii Venetz 32, Ulyanovsk, Russia, 432027
[2]National Research Moscow State University of Civil Engineering, Yaroslavskoe Shosse 26, Moscow, Russia, 129337

Abstract. The method of probabilistic finite modeling of stochastic estimation of image inter-frame geometric deformations parameters is proposed. The method is based on the discretization of the domain of deformation parameters and determination of estimates drift probability vector at each iteration (improvement, deterioration and no change of the vector with respect to coordinates of optimal values in the parameter space). The models of analyzed images and noises are given by the probability densities and the autocorrelation functions. It is a feature of the method. The usage of the probability distribution method for estimating the image deformations parameters generated by a non-identification relay procedure for a given finite number of iterations is considered. The mean square of error and the inter-frame correlation coefficient were used as cost estimation functions. Examples confirming the adequacy of the developed probabilistic mathematical model are presented.

1. Introduction

Estimation of the parameters of image sequence geometric deformations is one of the actual problems of image representation and processing. One approach to solve this problem is stochastic estimation [1–4]. Asymptotically optimal in terms of convergence rate of the stochastic approximation procedure [5, 6] have been developed. They have the highest possible convergence rate, however, they require complete a priori information and do not provide an answer to the question of errors in estimating the deformation parameters for a finite number of performed iterations. The accuracy capabilities of this class procedures are investigated only in asymptotics.

Approaches to improvement (acceleration) and analysis of the accuracy of estimates of stochastic approximation algorithms with a finite number of iterations are known [7]. As a rule acceleration is related to allowance for a priori information about the optimal solution, which is given by the finite probability density (PD). In the absence of such a priori information, optimal algorithms at finite iterations can lead to estimates that are very far from optimal. At the same time, the papers on probabilistic analysis of accuracy with a finite number of iterations is clearly not enough, it is determined the direction of this paper.

When modeling the process of deformation parameters estimation, one has to deal with the presence of a rather complex set of interfering factors, such as temporal and spatial heterogeneity of image and noise characteristics, sensitivity heterogeneity of sensors, impulse noise, etc. By their nature, these factors are of a random nature, therefore, when processing real images, both parametric and nonparametric a priori uncertainty are almost always present. In the conditions of a priori uncertainty, relay stochastic adaptive procedures [8, 9] in the form

$$\hat{\boldsymbol{\alpha}}_{t+1} = \hat{\boldsymbol{\alpha}}_t - \boldsymbol{\Lambda}_{t+1} \operatorname{sign} \boldsymbol{\beta}_{t+1}\left(Q\left(\mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}, \hat{\boldsymbol{\alpha}}_t\right)\right) \quad (1)$$

are perspective, where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_m)^T$ - vector of geometric deformations estimated parameters; $\boldsymbol{\Lambda}_t$ - gain matrix, $\boldsymbol{\beta}_t = (\beta_1, \beta_2, ..., \beta_m)^T$ - stochastic gradient of cost function (CF) $Q(\mathbf{Z}^{(1)}, \mathbf{Z}^{(2)})$ of estimation quality: t - iteration number.

Let consider that the investigated images are additively noisy: $z_{\mathbf{j}}^{(1)} = x_{\mathbf{j}}^{(1)} + \theta_{\mathbf{j}}^{(1)}$, $x_{\mathbf{j}}^{(1)} \in \mathbf{X}^{(1)}$, $\theta_{\mathbf{j}}^{(1)} \in \boldsymbol{\Theta}^{(1)}$; $z_{\mathbf{j}}^{(2)} = x_{\mathbf{j}}^{(1)}(\boldsymbol{\alpha}) + \theta_{\mathbf{j}}^{(2)}$, $x_{\mathbf{j}}^{(1)}(\boldsymbol{\alpha}) \in \mathbf{X}^{(2)}$, $\theta_{\mathbf{j}}^{(2)} \in \boldsymbol{\Theta}^{(2)}$; where $\boldsymbol{\Theta}^{(1)}$, $\boldsymbol{\Theta}^{(2)}$ - independent Gaussian random fields with zero means and identical variances σ_θ^2. $\mathbf{j} = (j_x, j_y) \in \Omega$ - coordinates of the sample grid node where images are specified; x, y - base image axes. The image $\mathbf{X}^{(2)}$ is obtained by deformation of image $\mathbf{X}^{(1)}$ with parameters $\boldsymbol{\alpha}$ of accepted deformation model. In particular, in the conducted studies the similarity model [10] is used.

2. Calculating of estimate drift probability

With the accepted model of the observed images, the factors independent of the parameters of the procedure (1) are the PD and the autocorrelation functions (ACF) of the images $\mathbf{X}^{(1)}$, $\mathbf{X}^{(2)}$ and the interfering noise $\boldsymbol{\Theta}$, the type of CF Q. The characteristics of the procedure that can be influenced in its implementation include the method of calculating the stochastic gradient $\boldsymbol{\beta}$, the gain matrix $\boldsymbol{\Lambda}_t$, the number of iterations T, and the initial approximation $\hat{\boldsymbol{\alpha}}_0$ of the parameter estimates $\boldsymbol{\alpha}$. For the feasibility of the modeling procedure, it is expedient to use a minimum set of values characterizing independent factors to determine probabilistic estimates of the parameters functions $\boldsymbol{\beta}_t$, $\boldsymbol{\Lambda}_t$, t and $\hat{\boldsymbol{\alpha}}_0$. As such values, we used the estimate drift probabilities (EDP) $\boldsymbol{\rho}_i = (\rho_i^+, \rho_i^o, \rho_i^-)^T$ [11, 12], where ρ_i^+ is the probability of parameter estimate α_i change towards the optimal value α_i^*; ρ_i^- - from α_i^*; ρ_i^0 - no change of estimate. Note that if in accordance with the criterion of optimality of CF, for example, is maximized and $(\hat{\alpha}_i - \alpha_i^*) > 0$ then ρ_i^+ corresponds to the negative projection β_i of the stochastic gradient of CF:

$$\rho_i^+(\boldsymbol{\varepsilon}_t) = P\{\beta_i < 0\} = \int_{-\infty}^{0} w\left(\beta_i\left(Z_t, \hat{\alpha}_{i,t-1}\right)\right) d\beta_i \ , \quad (2)$$

where $\beta_i(Z_t, \hat{\alpha}_{i,t-1})$ is the PD of the projection β_i in space onto the axis α_i; $\boldsymbol{\varepsilon}_t = \hat{\boldsymbol{\alpha}}_t - \boldsymbol{\alpha}^*$ is the mismatch of the estimates vector on the t-th iteration; $= \{z_{\mathbf{j}t}^{(2)}, \tilde{z}_{\mathbf{j}t}^{(1)}\}$ is two-dimensional local sample of samples $z_{\mathbf{j}t}^{(2)} \in \mathbf{Z}^{(2)}$ and $\tilde{z}_{\mathbf{j}t}^{(1)} = \tilde{z}^{(1)}(\mathbf{j}, \hat{\boldsymbol{\alpha}}_t) \in \tilde{\mathbf{Z}}_t^{(1)}$, $\mathbf{j}_t \in \Omega_t \in \Omega$, according to which the stochastic gradient is calculated; $\tilde{\mathbf{Z}}_t^{(1)}$ is the resampled image, it is obtained from $\mathbf{Z}^{(1)}$ using some approximation on the base of the current estimates $\hat{\boldsymbol{\alpha}}_t$ of parameters of used deformation model.

Under the assumption that the studied images $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$ have a Gaussian brightness distribution with zero mean with the known ACF $R(\bar{j})$, expressions were found for calculating the EDP for the cases of using the mean squared error of images with the estimate at the t-th iteration:

$$q_t = \mu^{-1} \sum_{\mathbf{j}_l \in \Omega_t} \left(z_{\mathbf{j}l}^{(2)} - \tilde{z}_{\mathbf{j}l}^{(1)}\right)^2$$

and the inter-frame correlation coefficient with the estimate:

$$q_t = \left(\mu \hat{\sigma}_{Z1} \hat{\sigma}_{Z2}\right)^{-1} \left(\sum_{\mathbf{j}_l \in \Omega_t} z_{\mathbf{j}l}^{(2)} \tilde{z}_{\mathbf{j}l}^{(1)} - \mu^{-1} \sum_{\mathbf{j}_l \in \Omega_t} z_{\mathbf{j}l}^{(2)} \sum_{\mathbf{j}_l \in \Omega_t} \tilde{z}_{\mathbf{j}l}^{(1)} \right),$$

where μ - size of local sample Z_t; $\hat{\sigma}_{Z1}^2$ and $\hat{\sigma}_{Z2}^2$ - variance estimates of images $\mathbf{Z}^{(1)}$ and $\mathbf{Z}^{(2)}$.

In accordance with (2) for the calculation of $\rho_i^+(\bar{\varepsilon})$ it is necessary to find the PD $w(\beta_i)$ of the stochastic gradient projection β_i:

$$\beta_i = \frac{2}{\mu} \sum_{\mathbf{j}_l \in \Omega_t} \left(z_{\mathbf{j}_l}^{(2)} - \tilde{z}_{\mathbf{j}l}^{(1)} \right) \left(\frac{d\,\tilde{z}_{\mathbf{j}l}^{(1)}}{dx} \gamma_i + \frac{d\,\tilde{z}_{\mathbf{j}l}^{(1)}}{dy} \zeta_i \right), \tag{3}$$

where γ_i and ζ_i are functions depending on the parameters of the accepted model of inter-frame geometric deformations.

Since in real conditions the dependence of $\tilde{z}_{\mathbf{j}t}^{(1)}$ on $\boldsymbol{\alpha}$ and $\mathbf{j}$ is unknown a priori, it is possible to use estimates of the derivatives $d\tilde{z}_{\mathbf{j}t}^{(1)} / dx$ and $d\tilde{z}_{\mathbf{j}t}^{(1)} / dy$ in terms of finite differences. Then, for example:

$$\frac{d\,\tilde{z}_{\mathbf{j}l}^{(1)}}{dx} \approx \frac{\tilde{z}_l^{(1)}(j_{1l}-1, j_{2l}, \boldsymbol{\alpha}) - \tilde{z}_l^{(1)}(j_{1l}+1, j_{2l}, \boldsymbol{\alpha})}{2}, \quad \frac{d\,\tilde{z}_{\mathbf{j}l}^{(1)}}{dy} \approx \frac{\tilde{z}_l^{(1)}(j_{1l}, j_{2l}-1, \boldsymbol{\alpha}) - \tilde{z}_l^{(1)}(j_{1l}, j_{2l}+1, \boldsymbol{\alpha})}{2}. \tag{4}$$

It is shown that β_i quickly normalizes with increasing of μ. At $\mu = 2$ the expression (4) contains at least sixteen of the same type terms. Assuming the PD of β_i is close to the Gaussian, in accordance with (2) the probability ρ_i^+, $i = \overline{1,m}$ can be found as:

$$\rho_i^+ = 1 - F\left(\frac{M\{\beta_i\}}{\sigma\{\beta_i\}} \right), \tag{5}$$

where $F(.)$ is Laplace function.

When using the mean squared error of images for the mathematical expectation $M[\beta_i]$ and $\sigma^2[\beta_i]$ variance we obtain:

$$M[\beta_i] = -\sum_{l=1}^{\mu} \sigma_x^2 \Big(\big(R(a_l - 1; b_l) - R(a_l + 1, b_l)\big)\gamma_{i_l} + \big(R(a_l; b_l - 1) - R(a_l; b_l + 1)\big)\zeta_{i_l} \Big), \tag{6}$$

$$\begin{gathered} \sigma^2[\beta_i] = 4\sum_{l=1}^{\mu} \sigma_x^4 \Big(\big(\gamma_{i_l}{}^2 + \zeta_{i_l}{}^2\big)\big((1 - R(a_l; b_l))(1 - R(2)) + g^{-1}(2 - R(a_l; b_l) - R(2) + g^{-1})\big) + \\ + \big(\gamma_{i_l}(R(a_l - 1; b_l) - R(a_l + 1b_l;)) + \zeta_{i_l}(R(a_l; b_l - 1) - R(a_l; b_l + 1))\big)^2 \Big). \end{gathered} \tag{7}$$

When using the interframe correlation coefficient:

$$M[\beta_i] \cong -0.5(1 - \mu^{-1}) \sum_{l=1}^{\mu} \big((R(a_l + 1; b_l) - R(a_l - 1; b_l))\gamma_i + (R(a_l; b_l + 1) - R(a_l; b_l - 1))\zeta_i \big), \tag{8}$$

$$\begin{gathered} \sigma^2[\beta_i] \cong 0.5(1 - \mu^{-1})^2 \left(\big(\gamma_i^2 + \zeta_i{}^2\big)(1 - R(2)) \left(1 + (\mu - 1)^{-1} \left(1 + 2\mu^{-1} \sum_{l=1}^{\mu} R^2(a_l, b_l; j_{1l}, j_{2l}) \right) \right) + \right. \\ \left. + g^{-1}\big(2 - R(2) + g^{-1}\big) \right) + 0.25\big(\gamma_i(R(a+1; b) - R(a-1; b)) + \zeta_i(R(a; b+1) - R(a; b-1))\big)^2. \end{gathered} \tag{9}$$

where $R(a;b)$ is the normalized ACF of image; a_l and b_l - the distance of the mismatch between the coordinates of the samples $z_{\mathbf{j}l}^{(2)}$ and $\tilde{z}_{\mathbf{j}l}^{(1)}$ along the axes x and y respectively. Similarly, the expressions for the probabilities $\rho_i^-(\bar{\varepsilon})$ and $\rho_i^o(\bar{\varepsilon})$ can be obtained.

For example, Figure 1a shows the graphs of $\rho_{i=h}^+$ depending of the mismatch function (estimation error) $\varepsilon_h = \hat{h}_x - h_x$ for the situation when estimation parameter is the parallel shift of the image $\mathbf{Z}^{(2)}$ along the coordinate x. The plots are calculated using expressions (6) and (7) with μ=1 (curve 1), μ=4 (curve 2) and μ=10 (curve 3). In Figure 1b, the graphs are obtained from relations (8) and (9) with μ=2 (curve 1), μ=4 (curve 2) and μ=10 (curve 3). The calculation was performed for images with the Gaussian ACF of the correlation radius 5 and the signal-to-noise ratio $g = \sigma_x^2 / \sigma_\theta^2 = 10$. The experimental results (crosses), obtained by statistical modeling on simulated images with similar parameters, synthesized using the wave model [13] are also shown. The experimental results are averaged over 150 realizations. It is shown that at μ =4 the approximation of the PD of the stochastic gradient by the Gaussian law provides satisfactory results.

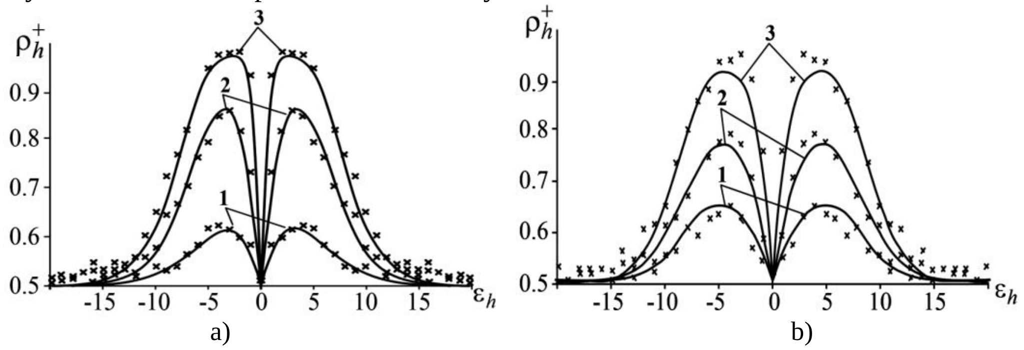

Figure 1. Examples of EDP**.**

3. Determination of PD of parameter estimates at a given estimation iteration

On the base of the method of EDP calculation, described above, a method of probabilistic finite modeling of the process of stochastic gradient estimation of inter-frame deformation parameters of a image sequence has been developed. The technique is aimed at calculating the PD of estimated parameters and other probabilistic characteristics. A special feature of the method is that it allows to store only the probability distributions of individual estimates instead of multidimensional PD of parameter estimates. This has significantly reduced the requirements for the memory required for modeling.

When forming arrays of EDP, the discretization of the parameters domain is used. In this case, with regular sampling of parameters for their domain, we obtain an already irregular discretization, at the nodes of which the values of the drift probabilities are calculated. To determine the values of EDP array at intermediate samples, a linear approximation of this irregular grid is used. It also allows to significantly reduce computational costs.

Another feature in the modeling of each estimation iteration and the calculation of the PD of the estimated parameters is the adaptive limitation of the boundaries of the modeling window in the parameter space. At the same time, in order to preserve accuracy, the EDP correction is provided in the nodes of the sampled domain near the borders of the modeling window.

Figure 2 shows examples of PD w_ε of the error ε of parallel shift estimate at iterations of relay estimation with a gain element that varies according to the law $\lambda_t = var = \lambda_0/(1+kt)$ (Figure 2a) and constant $\lambda = const$ (Figure 2b). In both cases, the shift mismatch is 5 sample grid steps, the size of the local sample is $\mu = 4$. The figures show that for $\lambda = const$ the estimation process stabilizes starting from approximately 520 iteration. After that, a further increase in the number of iterations does not lead to an increase in the estimation accuracy. This allows for a given class of images to find, in particular, the gain matrix parameter λ that provides the required estimation accuracy, as well as the number of iterations for the vector to achieve estimates of the stabilization domain. At $\lambda_t = var$ the process of PD formation does not have an equilibrium state, and the variance of estimation theoretically constantly decreases. The accuracy of the generated estimates in this case depends on the number of iterations and the parameter k of parameter λ_t reduction. This is illustrated by the graphs of Figure 2c, which shows the dependences of the variances of estimates σ_ε^2 on the number of iterations for the same modeling conditions. Here curve 1 corresponds to $\lambda_t = var$, curve 2 corresponds to $\lambda = const$. It is shown the variance stabilizes at $\lambda = const$ and at $\lambda_t = var$ monotonously decreases with an increase in the number of iterations.

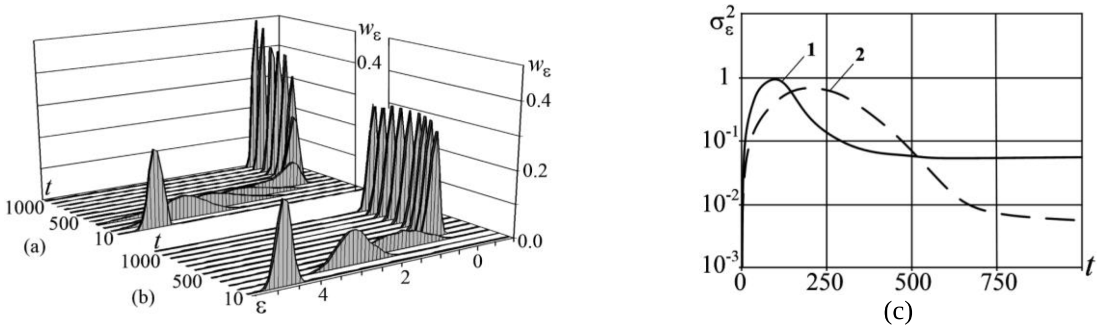

Figure 2. PD and variance of parallel shift estimate at constant and decreasing step of estimation.

4. Conclusion

When modeling the process of stochastic gradient estimation of image inter-frame geometric deformations parameters, it is necessary to take into account the complex of influencing factors. In particular, the factors independent of the parameters of stochastic gradient procedure are the PD and ACF of images and interfering noise, as well as the type of CF of estimation quality. The characteristics of the procedure that can be influenced include the method of calculating the stochastic gradient, the gain matrix, the number of iterations, and the initial approximation of the vector of parameter estimates. For the feasibility of the simulation procedure, it is advisable to use a minimum set of values, characterizing independent factors sufficient to find probabilistic parameter estimates as a function of the controlled characteristics of the procedure. As such values EDPs are used.

To obtain the calculated expressions for parameter EDPs, the normalization of the stochastic gradient of the cost function is used with increasing the size of the local sample in which it is located. Expressions are obtained for situations where stochastic gradient estimation of the mean squared error and the inter-frame correlation coefficient are used as the cost functions.

When modeling the stochastic gradient estimation process, at each iteration an adaptive limitation of the bounds of the modeling window in the parameter space is applied. At the same time, in order to preserve the accuracy of the modeling, a correction of the drift probabilities is performed at the nodes of the sampled parameters domain near the borders of the modeling window. This consideration of the probability of finding the estimates outside the modeling window made it possible to reduce computational costs while maintaining the adequacy of the model several times.

The proposed method of probabilistic finite modeling can be used to determine the accuracy and probability characteristics of stochastic algorithms for estimating image interframe geometric deformations for a given number of iterations.

5. References

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Acknowledgments

The reported study was funded by RFBR and Government of Ulyanovsk Region according to the research project № 18-41-730006.