Development of fuzzy fractal representation of the image

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Abstract. In article the way of the fuzzy image representation with use fractal models of images is considered. According to it the image can be presented in the form of a tree range blocks which correspond to image blocks. Everyone range block represents the structure describing similarity of this block to other block of the image. As a result it turns out treelike fractal representation of the image describing display of property of self-similarity in the image. The basic possibilities of construction of algorithms of digital processing of the images which are based on given representation are listed.

Keywords: image processing, fuzzy image representation, fuzzy features of images.

1. Introduction

Most methods of digital image processing (including defectoscopic images) refer to one of two groups: spatial or frequency. Spatial methods make it possible to uniquely identify an object in an image, and frequency methods allow one to effectively analyze the spectral components of a two-dimensional signal (useful component and noise). In this case, spatial methods, as a rule, are subject to interference and require the source signal to be cleaned of noise, possibly with the loss of useful information. For frequency methods, the main problem is the localization of objects of interest in the coordinate system of the image.

The use of space-frequency approaches, for example, based on wavelet transforms, allows you to more accurately detect objects in images against background noise. This is due to the fact that the wavelet spectrum contains complete information about the original signal, including the spatial coordinates of the frequency components, and at the same time separates the useful and noise components.

However, the overwhelming majority of digital image processing algorithms are based on a clear algorithm for extracting useful information, which often does not allow the technical vision systems to solve non-trivial problems. In this regard, it is proposed to combine the mathematical theory of fuzzy sets and fuzzy logic with the proven fractal methods of digital image processing.

To develop a system of new features, a new model of digital image is needed. It is proposed to modify the fractal model developed by the project manager by using a fuzzy distance in it as a measure of similarity of the image areas. This allows to expand the hierarchy of representations of the original image, thereby increasing the amount of useful information about the original image.

With increasing complexity of the tasks being solved, it is expedient to use methods that are specially oriented to the construction of models, methods and algorithms that take into account the incompleteness and inaccuracy of the initial data. It is in such situations that the theory of fuzzy sets and fuzzy logic proves to be the most constructive. Already achieved theoretical results

allow to consider fuzzy logic as a promising tool for the analysis of heterogeneous information, in the form of complex qualitative linguistic descriptions and quantitative data. In modern works, methods and algorithms for using fuzzy logic and fuzzy-set theory to solve problems of digital image processing are considered.

Analysis of the used fuzzy image processing schemes [1, 2, 3] shows that the used fuzzy representations of images, as well as the membership functions used to translate images into fuzzy forms, do not have a single mathematical basis. Used fuzzy signs and properties of images are built on the representation of images in the form of sets. In this case, there is no possibility of sharing fuzzy technologies with classical algorithms of digital image processing. A number of papers describe fuzzy measures, fuzzy integrals that are used in segmentation and clustering problems as local statistical features that are defined on the set to which the image is represented [4]. From this it follows that for the joint use of classical features, measures and algorithms in conjunction with fuzzy imaging technologies, a system of fuzzy signs, measures and distances is needed that will allow developing new algorithms for digital image processing.

In connection with the foregoing, it is proposed to develop a fuzzy mathematical model of a digital image based on its description by the terms of fuzzy sets using fuzzy logic, which is the basis for constructing new algorithms for extracting information about features and image processing algorithms.

2. Fractal Model of Digital Image

The fractal model of the digital image developed and described in [5, 6, 7] implements the system of iterated functions and can be represented by the expression

$$f' = \sum_{k=1}^{n} \left(B_{n_i,m_i}^{r_i} \right)^* \left[s_i \left(\tilde{w} \left(B_{k_i,l_i}^{d_i} \right) [f] \right) + o_i \right]. \tag{1}$$

where

$$D = \left(B_{k_i,l_i}^{d_i} \right) [f] \tag{2}$$

is an operator of extracting a domain block with the index d_i from the original image f. The transformation that performs the change in the color characteristics of the domain block and the geometric transformation has the following form

$$D' = s_i \left(w[D] \right) + o_i. \tag{3}$$

After all the transformations are performed on the original domain block, the result is inserted into the image

$$f' = \sum_{i=1}^{M} \left(B_{n_i,m_i}^{r_i} \right)^* [D']. \tag{4}$$

Using 1, 2 and 3, formulate an expression describing the range block of the image through the domain block

$$R_i = D' = s_i \left(w[D] \right) + o_i. \tag{5}$$

For comparison of two images f, h metric used RMS:

$$g(f,h) = \sum_{i,j} \sqrt{\frac{1}{m \cdot n} [f(i,j) - h(i.j)]^2}, \tag{6}$$

where m,n - images size. For each range block of the image R_i, the domain block D_i is selected which best approximates the range block. In other words, there is a pair (R_i, D_j) for which the metric (6) is minimal.

To generate a fractal image code, a quadripet of range blocks is used. The root of the tree is the image itself.

Expression 5 can be represented as follows

$$R_i = D' + \xi_i, \tag{7}$$

$$\xi_i = g(R_i, D').$$

In other words, the fractal model describes the original image using quadrotree. Each leaf of a tree is a range block of the image, which corresponds to a domain block so that the approximation error is minimal

3. Intuitionistic Fuzzy Set

A fuzzy set A in a fnite set $X = x_1, x_2, , x_n$ may be represented mathematically as

$$A = \{(x, \mu_A(x)) \,|\, x \in X\}$$

where the function $\mu_A(x)\colon X \to [0,1]$ is the measure of the degree of belongingness or the membership function of an element x in the fnite set X.

An Intuitionistic Fuzzy Set A in a finite set X may be mathematically represented as

$$A = \{(x, \mu_A(x), \nu_a(x)) | x \in X\}$$

where the functions $\mu_A(x), \nu_A(x)\colon X \to [0,1]$ are, respectively, the membership function and the non-membership function of an element x in a finite set X with the necessary condition

$$0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1.$$

It is clear that every fuzzy set is a particular case of IFS:

$$A = \{(x, \mu_A(x), 1 - \mu_A(x)) | x \in X\}. \tag{8}$$

4. Development of fuzzy fractal model

To use all the features of fuzzy sets is necessary to modify the model (5) and bring it to the fuzzy mind. During the construction of the fractal image code, a set of range blocks, domain blocks and approximation error values are generated:

$$R = \{R_i\{, R_i =\} D_j, \xi_{i,j}\}. \tag{9}$$

Then, using 8, expression 9 can be expressed using a fuzzy set

$$R = \{D_j, \mu_R(D_j), 1 - \mu_R(D_j) | D_j \in D\}. \tag{10}$$

Then the membership function $\mu_R(D_j)$ may be represented as

$$\mu_R(D_j) = \xi_j / \xi_{max},$$

where ξ_{max} the maximum approximation error, which is defined as the difference between absolutely black and absolutely white images:

$$\xi_{max} = 255 / \sqrt{m \cdot n},$$

where m, n - image size.

Let the image f is a graph $G = (V, \bar{U})$, where V is the list of range blocks of the image, $\bar{U}$ is the connections between the range blocks. Then the process of forming a tree of range blocks of the image can be described as follows. There is a range block of zero level R^0, which corresponds to the whole image and $v_1 = R^0$. It contains range blocks of the first level $R_1^1, R_2^1, R_3^1, R_4^1 \in R^0$:

$$v_2 = R_1^1, v_3 = R_2^1, v_4 = R_3^1, v_5 = R_4^1,$$

$$\bar{u}_1 = (v_1, v_2), \bar{u}_2 = (v_1, v_3), \bar{u}_3 = (v_1, v_4), \bar{u}_4 = (v_1, v_5) \in \bar{U}.$$

Continuing the formation of the fractal code, a complete tree of range blocks is obtaining, from which the image is constructed. A fragment of the tree of range blocks is shown in Fig. 1.
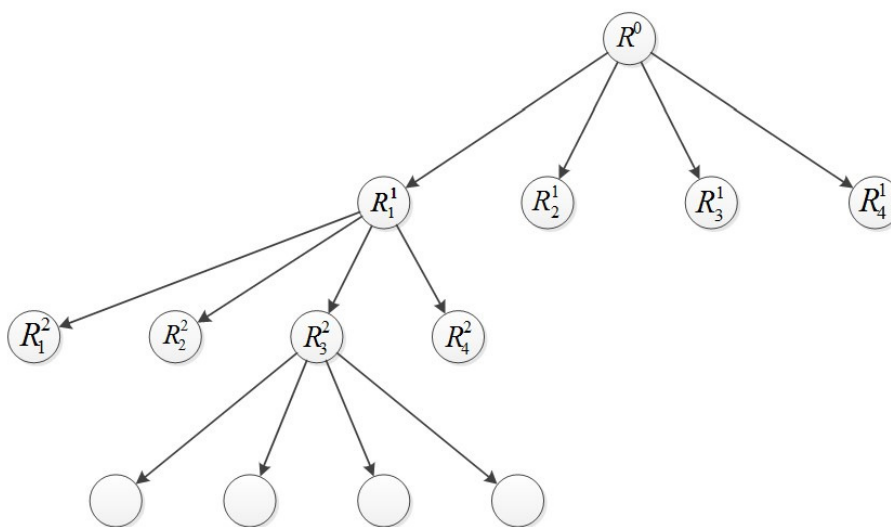

Figure 1. Fragment of the tree of range blocks.

The operator for the formation of a fractal code is called a direct fractal transformation:

$$F(f, D) = \Phi, \tag{11}$$

where Φ is the resulting fractal code of the image f by the operator F. An additional parameter of the operator is the list of domain image blocks D involved in the code generation. There is an inverse fractal transformation that forms the image by the fractal code:

$$f' = F^*(\Phi, D) = F^*(F(j, D)). \tag{12}$$

To properly restore an image from its fractal code, it is important that the transformations use the same list of domain blocks D. In the original version, the generated fractal code is complete. This means that each vertex of the graph has one parent and four descendants, but the length of the descending chain does not exceed the depth of the quad, which is specified at the initial stage of the code construction. If the full tree is used as the parameter for restoring the image by the operator 12, then the image f' will be as similar as the original image.

For obtaining modified images, incomplete trees of range blocks are used. To form an incomplete tree, use the operator, that can be represented as:

$$\Phi^* = \Gamma(\Phi). \tag{13}$$

The form of the operator Γ depends on the required result. For example, by removing some branches of a fractal tree, you can obtain an image in which the areas for which remote branches respond are smoothed and the remaining areas are clear. Using 10, the threshold operator Γ can be represented as:

$$\Gamma(\Phi) = \{R', D\}, R' = \{R_i|_R(D_j) > \epsilon\}. \tag{14}$$

The operator 14 allows us to use only range blocks whose value of the membership function is greater than a given threshold ϵ.
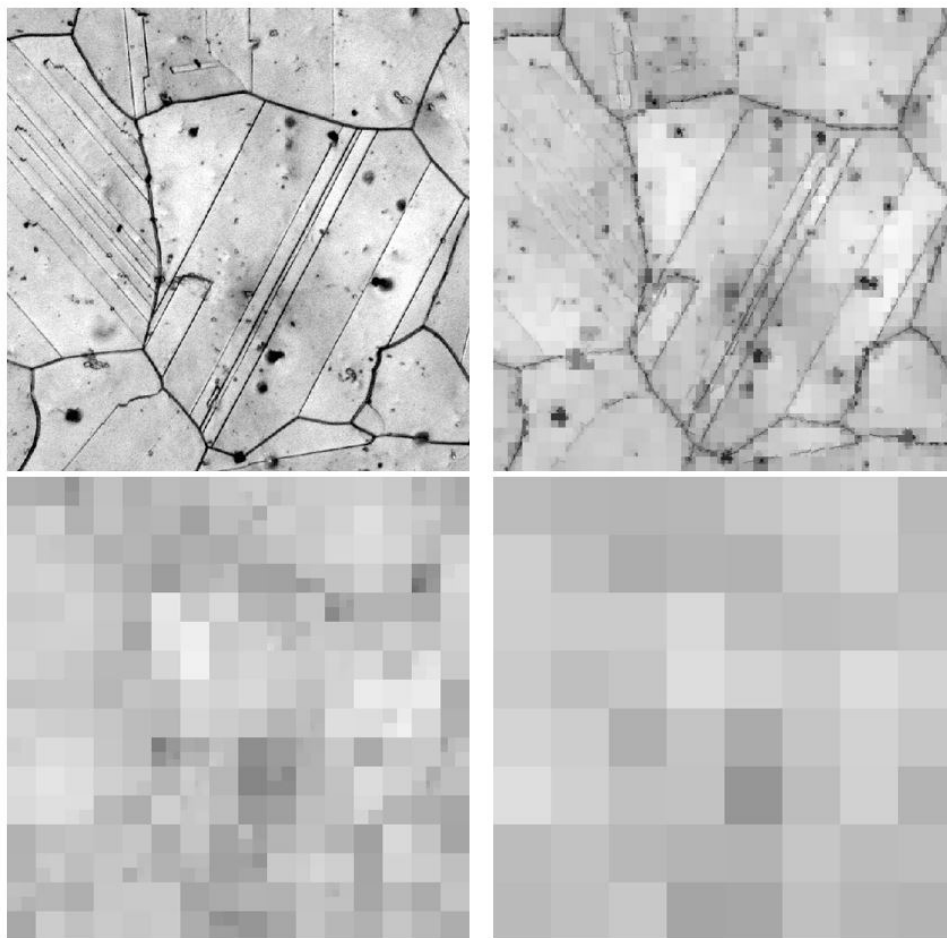

Figure 2. Fractal fuzzy representation of a test image.

To obtain a fractal representation of the image from the fractal code, the leaves of the tree are removed. The result is a smoothed, fractal image.

In Fig. 2 shows an example of such a decomposition of a test image. The resulting set of images allows for a more detailed analysis of the original image. The transition to other scales allows you to move away from small and random details, it is better to reveal the "internal" structure. It is of interest to consider the structural elements allocated at different scales of the representation of the image.

The reason for this approach is the following. Structural elements of the image, such as contours, segments, skeleton, basically carry all the meaningful information about the original image. At the same time, they have a much more compact representation and are better suited

for analysis, both automatic and subjective, than the original halftone image. In the case of automatic analysis, this is determined by the possibility of a formal description of the structural elements, for example, the parameters of the curve (for the contour, the skeleton) or the area geometric object (for segmentation).

5. Conclusion

The fractal image code can be represented in the form of a graph G, in which the vertices of V are range blocks R, and the arcs are the nestedness relations of range blocks. At the same time, the structure of the code remains unchanged, this approach changes only the form of representation and the mathematical apparatus of research.

The described approach to the fractal representation of images can be used to form a set of features reflecting the internal structure of the image, since small details are discarded. Further on, based on the proposed description, algorithms for isolating contours, image segmentation, skeletonization, etc. can be constructed. This support the usability of the proposed algorithm in practical problems of nondestructive testing.

6. References

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