

Entanglement of Rydberg atoms successively passing a thermal cavity

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Abstract

We investigated the entanglement between two identical two-level Rydberg atoms successively passing cavity and interacting with one-mode field through a one-photon and degenerate two-photon processes. For two-photon interaction, we focused our attention on the study of atomic entanglement dynamics in the presence of the Stark shift and initial atomic coherence. For one-photon case we discussed the influence of atomic coherence, detuning and cavity thermal photons on the entanglement dynamics for entangled initial atomic states.

Keywords: Rydberg atoms; one-atom maser; entanglement; Stark shift; atomic coherence

1. Introduction

Entanglement is not only one of the most surprising features of quantum theory, but also provides an important resource for various quantum information processes such as quantum information, quantum communication, and quantum cryptography [1]. Therefore, great efforts have been made to investigate entanglement characterization, entanglement control, and entanglement production in different systems [2]. One of the particular interest schemes in which entanglement can be created is a system containing two two-level atoms, since they can represent two qubits, the building blocks of the quantum gates that are essential to implement quantum protocols in quantum information processing. Two-atom entangled states have been demonstrated experimentally using ultra cold trap ions, impurity spins in solids, superconducting circuits and cavity quantum electrodynamics schemes [1]. Cavity QED has been used to generate the atom-atom and atom-photon entanglement. The entanglement between two two initially independent atoms successively passing the vacuum cavity have been demonstrated by S. Haroche et al. [3]. The entanglement procedure involves the resonant coupling, one by one, of the atoms to a high Q microwave superconducting cavity. The atoms, prepared in circular Rydberg states, exchange a single photon in the cavity and become entangled by this indirect interaction. The effect have been demonstrated with pairs of atoms separated by centimetric distances. Maximally entangled states between two modes in a single cavity have been generated using a Rydberg atom coherently interacting with each mode in turn [4]. The above entanglement investigations involved mostly the absorption or emission of a single photon in an atomic transition. Two-photon processes have also been studied in cavity QED [5]. Haroche and co-workers have demonstrated experimentally the two-photon maser action in a micromaser cavity [6]. The two-photon atomic transition process also introduces a dynamic Stark shift in the atomic transition which is related to the magnitude of the electric field of the radiation inside. Ghosh et al. [7] have investigated the entanglement properties of two Rydberg atoms with Stark-shifted two-photon atomic transitions passing through a single-mode cavity. They have shown that the Stark shift can be used to enhance the magnitude of atomic entanglement over that obtained in the resonant condition for certain parameter values. They have assumed that the two atoms are in their respective upper states before they enter the cavity empty of photons. Hu et al [8, 9] have shown that the entanglement between two atoms induced by one-mode cavity field can be manipulated by changing the initial atomic coherence. Yan [10] has investigated the entanglement properties of two atoms successively passing a cavity with Fock or thermal field but especially focused on the case when two atoms are initially in an entangled state. In our works [11]-[14] we have reexamined the dynamics of entanglement of two atoms successively passing a thermal cavity for another type of initial entangled atomic state. In this paper we have investigated the influence of initial atomic coherence on atom-atom entanglement for one-photon and two-photon transitions in two-level atom. For two-photon transitions we have taken into account in the presence of Stark shift. We also have discussed the dynamics of two previously entangled atoms taking into account the detuning and thermal cavity photon. We have found that the entanglement between two separate atoms can be induced by the thermal field, even if there is a frequency detuning between atoms and field, and the threshold time for the creation of the entanglement can be controlled by the model parameters.

2. Dynamics of atom-atom entanglement in the presence the Stark shift and initial atomic coherence

In this section we consider the system which consists of two separate identical three-level atoms passing through a cavity one after another with equal velocities. We suppose that atoms interact with cavity field via degenerate two-photon transitions and take into account the dynamical Stark shift. To obtain the effective degenerate two-photon Jaynes-Cummings model one can consider three-level atom with atomic states $|e\rangle$, $|i\rangle$, $|g\rangle$, where $|e\rangle$ is the excited, $|g\rangle$ is the ground and $|i\rangle$ is the intermediate states of atom. The model under consideration is obtained when a cascade of the atomic transitions $|e\rangle \rightarrow |i\rangle \rightarrow |g\rangle$ is resonant with twice the field frequency $\omega_{eg} = 2\omega$ whereas the intermediate transition frequencies $\omega_{ei} = \omega + \Delta$ and $\omega_{gi} = \omega - \Delta$ are strongly detuned from ω , where ω is the cavity mode frequency. After adiabatically eliminating the intermediate state, one arrives at the effective interaction

picture Hamiltonian of one atom with degenerate two-photon transitions in the RWA approximation and in the presence of the Stark shift

$$H = \hbar g(a^{+2}\sigma^- + \sigma^+ a^2) + \hbar\beta_1 a^+ a \sigma^+ \sigma^- + \hbar\beta_2 a^+ a \sigma^- \sigma^+, \quad (1)$$

where σ^+ and σ^- are the quasi-spin operators, a^+ and a are the creation and annihilation photon operators of the one-mode cavity field, g is the two-photon atom-field coupling constant and β_1, β_2 are the parameters of the Stark shift of the ground and excited states in two-level atom. We assume that the atom-field coupling is constant (thus we neglect the dependence of the spatial structure of the cavity mode). Following the exit of the first atom, a second atom enters the cavity and interacts with the field modified by interaction with the first atom. Assume that the total atom-cavity interaction time is considerably less than the cavity lifetime and that we can ignore the effects of cavity dissipation.

Let the first of atoms passing the cavity is initially prepared in coherent superposition of the excited and the ground states

$$|\Psi_{A_1}(0)\rangle = \cos\theta_1|+\rangle_1 + \sin\theta_1|-\rangle_1 \quad (2)$$

and one-mode cavity field is in vacuum state $|0\rangle$. Then the initial wave-function of the system "first atom+field" has the form $|\Psi_{A_1F}(0)\rangle = \cos\theta_1|+; 0\rangle + \sin\theta_1|--; 0\rangle$.

The wave function of considered system for time moments τ when the first atom leaves the cavity, can be written in the form

$$|\Psi_{A_1F}(\tau)\rangle = X_1(\tau)|+; 0\rangle + X_2(\tau)|--; 2\rangle + X_3(\tau)|-; 0\rangle.$$

To derive the coefficients $X_i(t)$ one can solve the time-dependent Schrödinger equation. The solution of this equation has the following form

$$X_1(\tau) = e^{-ig\tau/\beta_1} \cos\theta_1 [\cos(\Omega\tau) + (t/\Omega) \sin(\Omega\tau)], \quad X_2(\tau) = (-t\sqrt{2}/\Omega)e^{-ig\tau/\beta_1} \sin(\Omega\tau), \quad X_3(\tau) = \sin\theta_1,$$

where $\Omega = g\sqrt{(\beta_1/g)^2 + 2}$. Further we will measure a new time from the moment when the second atom will begin passing through the resonator. Let us suppose that the second atom reaches the cavity being in the coherent state of the following form

$$|\Psi_{A_2}(0)\rangle = \cos\theta_2|+\rangle_2 + \sin\theta_2|-\rangle_2; 0 \quad (3)$$

Then the initial state of the system "two atoms+field" at this moment is

$$\begin{aligned} |\Psi_{A_1A_2F}(0)\rangle &= |\Psi_{A_1F}(\tau)\rangle|\Psi_{A_2}(0)\rangle = X_1(\tau) \cos\theta_2|+, +; 0\rangle + X_1(\tau) \sin\theta_2|+, -; 0\rangle + X_2(\tau) \cos\theta_2|-, +; 2\rangle \\ &+ X_2(\tau) \sin\theta_2|-, -; 2\rangle + X_3(\tau) \cos\theta_2|-, +; 0\rangle + X_3(\tau) \sin\theta_2|-, -; 0\rangle. \end{aligned}$$

As velocities of atoms are equal, the both atoms pass through the cavity during the same time. At the instant $t = \tau$ the second atom leaves the cavity. In this time the wave function of the system "two atoms+field" is

$$\begin{aligned} |\Psi_{A_1A_2F}(\tau, t)\rangle &= Y_1(t)|+, +; 0\rangle + Y_2(t)|+, -; 2\rangle + Y_2(t)|+, -; 0\rangle + Y_4(t)|-, -; 4\rangle \\ &+ Y_5(t)|-, +; 2\rangle + Y_6(t)|-, +; 0\rangle + Y_7(t)|-, -; 2\rangle + Y_8(t)|-, -; 0\rangle. \end{aligned} \quad (4)$$

The equation of motion for time-dependent coefficients $Y_i(t)$ have the form

$$\begin{aligned} i\dot{Y}_1 &= \sqrt{2}gY_2, \quad i\dot{Y}_1 = \sqrt{2}gY_1 + 2\beta_2Y_2, \quad i\dot{Y}_3 = 0, \quad i\dot{Y}_4 = \sqrt{12}gY_5 + 4\beta_2Y_4, \\ i\dot{Y}_5 &= \sqrt{12}gY_4 + 2\beta_1Y_5, \quad i\dot{Y}_6 = \sqrt{2}gY_7, \quad i\dot{Y}_7 = \sqrt{12}gY_6 + 2\beta_2Y_7, \quad i\dot{Y}_8 = \sqrt{2}gY_7 = 0. \end{aligned} \quad (5)$$

Using dressed-state representation one can obtain the exact solutions for equations (5). But these solutions are too cumbersome to present them in the paper. Therefore we have used the numerical solutions of equations (5) for entanglement dynamics modeling.

To investigate the entanglement between qubits one has to obtain the time-dependent reduced density operator by tracing the combined atoms-field density operator over the field variables:

$$\rho_{A_1A_2}(\tau, t) = \text{Tr}_F |\Psi_{A_1A_2F}(\tau, t)\rangle\langle\Psi_{A_1A_2F}(\tau, t)|.$$

For two-qubit system described by the density operator $\rho_{A_1A_2}$, a measure of entanglement or negativity can be defined in terms of the negative eigenvalues μ_i^- of partial transpose of the reduced density matrix [15, 16]

$$\varepsilon = -2 \sum_i \mu_i^-. \quad (6)$$

$\varepsilon = 0$ indicates that two qubits are separable, $\varepsilon > 0$ indicates the atom-atom entanglement and $\varepsilon = 1$ indicates maximum entanglement. Using eqs (2)-(5) and two-atom basis $|e, e\rangle, |e, g\rangle, |g, e\rangle, |g, g\rangle$ one can obtain the reduced density matrix at time t in the

$$\rho_{A_1A_2}(\tau, t) = \begin{pmatrix} \rho_{11}(\tau, t) & \rho_{12}(\tau, t) & \rho_{13}(\tau, t) & \rho_{14}(\tau, t) \\ \rho_{12}^*(\tau, t) & \rho_{22}(\tau, t) & \rho_{23}(\tau, t) & \rho_{24}(\tau, t) \\ \rho_{13}^*(\tau, t) & \rho_{23}^*(\tau, t) & \rho_{33}(\tau, t) & \rho_{34}(\tau, t) \\ \rho_{14}^*(\tau, t) & \rho_{24}^*(\tau, t) & \rho_{34}^*(\tau, t) & \rho_{44}(\tau, t) \end{pmatrix}, \quad (7)$$

where

$$\begin{aligned} \rho_{11}(\tau, t) &= |Y_1(\tau, t)|^2, \rho_{12} = Y_1(\tau, t)Y_3(\tau, t)^*, \rho_{13} = Y_1(\tau, t)Y_6^*, \rho_{14} = Y_1(\tau, t)Y_8(\tau, t)^*, , \\ \rho_{22}(\tau, t) &= |Y_2(\tau, t)|^2 + |Y_3(\tau, t)|^2, \rho_{23} = Y_2(\tau, t)Y_5(\tau, t)^* + Y_3(\tau, t)Y_6(\tau, t)^*, \\ \rho_{24}(\tau, t) &= Y_2(\tau, t)Y_7(\tau, t)^* + Y_3(\tau, t)Y_8(\tau, t)^*, \rho_{33} = |Y_5(\tau, t)|^2 + |Y_6(\tau, t)|^2, \\ \rho_{34}(\tau, t) &= Y_5(\tau, t)Y_7(\tau, t)^* + Y_6(\tau, t)Y_8(\tau, t)^*, \rho_{44} = |Y_4(\tau, t)|^2 + |Y_7(\tau, t)|^2 + |Y_8(\tau, t)|^2. \end{aligned}$$

Accordingly, we can write down the partial transpose matrix as

$$\rho_{A_1 A_2}^{T_1}(\tau, t) = \begin{pmatrix} \rho_{11}(\tau, t) & \rho_{12}(\tau, t) & \rho_{31}(\tau, t) & \rho_{32}(\tau, t) \\ \rho_{21}(\tau, t) & \rho_{22}(\tau, t) & \rho_{41}(\tau, t) & \rho_{42}(\tau, t) \\ \rho_{13}(\tau, t) & \rho_{14}(\tau, t) & \rho_{33}(\tau, t) & \rho_{34}(\tau, t) \\ \rho_{23}(\tau, t) & \rho_{24}(\tau, t) & \rho_{43}(\tau, t) & \rho_{44}(\tau, t) \end{pmatrix}. \quad (8)$$

Let us consider the numerical calculations of atomic entanglement for various initial states of two two-level atoms and various values of Stark shift parameters. Below we put that $\beta_1 = \beta_2 = 1$.

The results of computer modeling of negativity (6) for considered model have been presented in Fig. 1. Fig. 1(a) shows the time dependencies of atomic entanglement versus parameter $g\tau$ for different values of Stark shift parameter β in the case when both atoms successively passing a cavity are initially prepared in excited states. We will note that in a case when both atoms are prepared in the excited states and simultaneously interact with a common cavity, the entanglement does not arise. The maximum degree of entanglement when both atoms fly through the cavity is approximately equal 0.6. With increasing of the value of Stark shift parameter the maximum value of atomic entanglement decreases. Fig. 1(b) shows the time behavior of the entanglement parameters versus $g\tau$ for coherent initial atomic states of the form $|\Psi_A\rangle = 1/\sqrt{2}(|+\rangle + |-\rangle)$. In the considered case the maximum degree of entanglement when both atoms fly through the cavity is approximately equal 0.4. Thus dependence of entanglement parameter from time of atom flying through the cavity has the nonmonotonic character. For $0 \leq \beta \leq 2$ the maximum degree of entanglement increases with increasing the parameter β and for $\beta > 2$ the dependence is reverse. It is possible to sustain the entanglement over a range of interaction times by making the detuning and the Stark shift compensate each other.

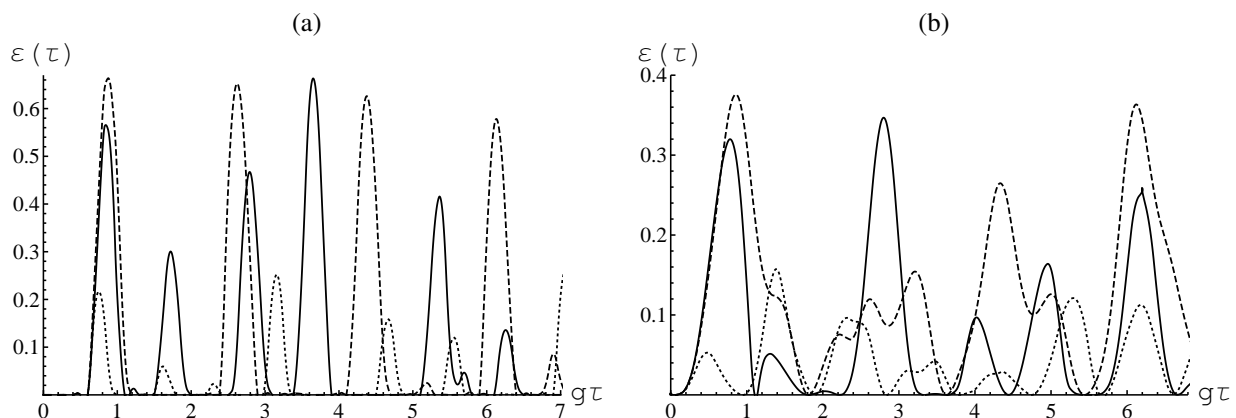


Fig. 1. Entanglement for incoherent initial atomic state $|e, g\rangle$ (a) and coherent initial atomic state $1/\sqrt{2}(|+, -\rangle + |- , +\rangle)$ (b). The Stark shift parameters are $\beta = 0$ (solid), $\beta/g = 1$ (dashed) and $\beta/g = 2$ (dotted).

3. Dynamics of atom-atom entanglement in the presence the thermal cavity photon and initial atomic coherence

In this section, we will study the effect of the initial atomic coherence on the atom-atom entanglement for a thermal cavity. The physical system under consideration consists of two separate identical two-level Rydberg atoms A_1 and A_2 passing through a cavity one after another and interacting with the cavity field through the one-photon transition. The Hamiltonian of the joint "one atom+field" system with the dipole and rotating wave approximation can be written as

$$H = \hbar g (a^+ \sigma_i^- + a \sigma_i^+), \quad (9)$$

where we use the same notation as in section 2. Let both atoms passing the cavity is initially prepared in coherent superposition of the excited and the ground states as in section 2 but field is in a thermal state

$$\rho_F(0) = \sum_n p_n |n\rangle \langle n|, \quad (10)$$

where $p_n = \bar{n}^n / (1 + \bar{n})^{n+1}$. Here \bar{n} is the mean photon number in the cavity mode $\bar{n} = (\exp[\hbar\omega/k_B T] - 1)^{-1}$ and k_B is the Boltzmann constant and T is the equilibrium temperature of the cavity mirrors.

Solving the quantum Liouville equation for two successive stages of atoms dynamics described in Section 2 one can obtain the time-dependent combined density matrix. To investigate the entanglement between qubits one has to obtain the time-dependent reduced density operator by tracing the combined atoms-field density operator over the field variables. When the second atom leaves the cavity the reduced density matrix in two-atom basis $|+, +\rangle, |+, -\rangle, |-, +\rangle, |-, -\rangle$ has the form (7) with

$$\begin{aligned} \rho_{11} &= \sum_n p_n (|X_{1n}|^2 + |X_{2n}|^2 + |X_{3n}|^2), & \rho_{12} &= \sum_n p_n (X_{1n}X_{4n}^* + X_{2n}X_{5n}^*), \\ \rho_{13} &= \sum_n p_n (X_{1n}X_{7n}^* + X_{2n}X_{9n}^*), & \rho_{14} &= \sum_n p_n X_{1n}X_{10n}^*, \\ \rho_{22} &= \sum_n p_n (|X_{4n}|^2 + |X_{5n}|^2 + |X_{6n}|^2), & \rho_{23} &= \sum_n p_n (X_{4n}X_{7n}^* + X_{5n}X_{9n}^* + X_{6n}X_{8n}^*), \\ \rho_{24} &= \sum_n p_n (X_{4n}X_{10n}^* + X_{6n}X_{11n}^*), & \rho_{33} &= \sum_n p_n (|X_{7n}|^2 + |X_{8n}|^2 + |X_{9n}|^2), \\ \rho_{34} &= \sum_n p_n (X_{7n}X_{10n}^* + X_{8n}X_{11n}^*), & \rho_{44} &= \sum_n p_n (|X_{10n}|^2 + |X_{11n}|^2 + |X_{12n}|^2), \end{aligned}$$

where

$$\begin{aligned} X_{1n}(\tau, t) &= a \cos(\sqrt{n+1}g\tau) \cos(\sqrt{n+1}gt), & X_{2n}(\tau, t) &= -i(c \sin(\sqrt{n}g\tau) \cos(\sqrt{n}gt) + b \cos(\sqrt{n+1}g\tau) \sin(\sqrt{n}gt)), \\ X_{3n}(\tau, t) &= -d \sin(\sqrt{n}g\tau) \sin(\sqrt{(n-1)gt}), & X_{4n}(\tau, t) &= -c \sin(\sqrt{n}g\tau) \sin(\sqrt{n}gt) + b \cos(\sqrt{n+1}g\tau) \cos(\sqrt{n}gt), \\ X_{5n}(\tau, t) &= -id \sin(\sqrt{n}g\tau) \cos(\sqrt{(n-1)gt}), & X_{6n}(\tau, t) &= -ia \cos(\sqrt{n+1}g\tau) \sin(\sqrt{n+1}gt), \\ X_{7n}(\tau, t) &= c \cos(\sqrt{n}g\tau) \cos(\sqrt{n+1}gt) - b \sin(\sqrt{n+1}g\tau) \sin(\sqrt{n+1}gt), \\ X_{8n}(\tau, t) &= -ia \sin(\sqrt{n+1}g\tau) \cos(\sqrt{n+2}gt), \\ X_{9n}(\tau, t) &= -id \cos(\sqrt{n}g\tau) \sin(\sqrt{n}gt), & X_{10n}(\tau, t) &= d \cos(\sqrt{n}g\tau) \cos(\sqrt{n}gt), \\ X_{11n}(\tau, t) &= -i(c \cos(\sqrt{n}g\tau) \sin(\sqrt{n+1}gt) + b \sin(\sqrt{n+1}g\tau) \cos(\sqrt{n+1}gt)), \\ X_{12n}(\tau, t) &= -a \sin(\sqrt{n+1}g\tau) \sin(\sqrt{n+2}gt). \end{aligned}$$

The partial transpose matrix for considered model has the form (8). The results for time dependence of negativity (6) for different degrees of atomic coherence and mean photon numbers have presented in Fig 2.

The results of computer modeling of negativity (6) for considered model have been presented in Fig. 2. The Fig. 2(a) clearly shows that the entanglement of the atoms occurs for any initial atomic states, in particular in the case when both atoms are initially excited. The initial atomic coherence reduces the degree of entanglement. This behavior of entanglement for atoms successively passing a resonator is fundamentally different from that for atoms interacting with a common thermal field in the resonator. In the latter case, the presence of the initial atomic coherence leads to a significant increase in atomic entanglement even in case of intensive thermal field. Fig. 2(b) shows that the quantum correlations between the atoms arises only for thermal fields of low intensity, when the mean photon number does not exceed the unity, respectively the resonator temperature does not exceed 4 K.

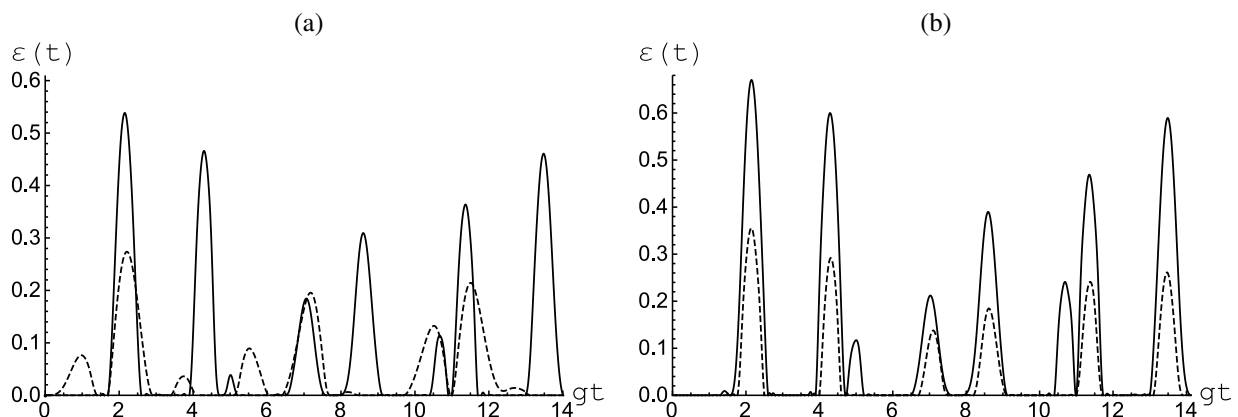


Fig. 2. Entanglement for incoherent initial and coherent atomic states. For figure 2(a) the mean photon number $\bar{n} = 0.1$ and polarized atoms amplitudes $\theta_1 = \theta_2 = 0$ (solid) and $\theta_1 = \theta_2 = \pi/4$ (dashed). For figure 2(b) $\theta_1 = \theta_2 = \pi/4$ and $\bar{n} = 0$ (solid) and $\bar{n} = 0.5$ (dashed).

4. Dynamics of atom-atom entanglement in the presence detuning

In this section, we will study the influence of initial atomic quantum correlations on the atom-atom entanglement for a thermal cavity. The physical system under consideration consists of two separate identical two-level Rydberg atoms A_1 and A_2 passing through a cavity one after another and non-resonantly interacting with the cavity field. The Hamiltonian of the joint "one atom+field" system with the dipole and rotating wave approximation can be written as

$$H = (1/2)\hbar\omega\sigma_i^z + \hbar\omega a^\dagger a + \hbar g (a^\dagger\sigma_i^- + a\sigma_i^+), \quad (11)$$

where $(1/2)\sigma_i^z$ is the inversion operator in the i -th atom ($i = 1, 2$), ω_0 is the transition frequency in two-level atoms, ω is the frequency of the cavity mode. The other notations are the same as in previous sections. We introduce the detuning as $\delta = \omega_0 - \omega$.

The evolution operator corresponding to the Hamiltonian (11) is

$$U_{A_i}(t) = e^{-\frac{i\omega_0 t}{2}} |0, -i\rangle\langle 0, -i| + \sum_{n=0}^{\infty} e^{-i\omega(n+\frac{1}{2})t} \{A_n|n+1, -i\rangle\langle n+1, -i| + B_n|n, +i\rangle\langle n, +i| + C_n(|n+1, -i\rangle\langle n, +i| + |n, +i\rangle\langle n+1, -i|)\}.$$

Here

$$A_n = \cos(\Delta_n t/2) + i\frac{\delta}{\Delta_n} \sin(\Delta_n t/2), \quad B_n = \cos(\Delta_n t/2) - i\frac{\delta}{\Delta_n} \sin(\Delta_n t/2), \quad C_n = i\frac{\Omega_n}{\Delta_n} \sin(\Delta_n t/2),$$

where $\Delta_n = \sqrt{\delta^2 + \Omega_n^2}$, $\Omega_n = 2g\sqrt{n+1}$ and $|n\rangle$ is the Fock state for the cavity mode.

Suppose that before the first atom enters the cavity the two atoms have been prepared in Bell-type entangled state of the form

$$|\Psi(0)\rangle_{A_1 A_2} = \cos \Theta |+, -\rangle + \sin \Theta |-, +\rangle, \quad (12)$$

where the parameter Θ defines the degree of initial atomic entanglement ($0 \leq \Theta \leq \pi$), and one-mode cavity field is in a thermal state (10).

The initial density matrix of the whole system is

$$\rho_{A_1 A_2 F}(0) = \rho_{A_1 A_2}(0)\rho_F(0) = \sum_n p_n |\Psi(0)\rangle_{A_1 A_2} \langle \Psi(0)| \otimes |n\rangle\langle n|,$$

The density matrix of considered system for time moment τ when the first atom leaves the cavity, can be written as

$$\rho_{A_1 A_2 F}(\tau) = U_{A_1}(\tau)\rho_{A_1 A_2 F}(0)U_{A_1}^\dagger(\tau). \quad (13)$$

The density matrix (3) on the other hand is the initial state of the system prior to entering into the cavity of the second atom. At the moment t when the second atom leaves the cavity the density matrix takes the form

$$\rho_{A_1 A_2 F}(t, \tau) = U_{A_2}(t)\rho_{A_1 A_2 F}(\tau)U_{A_2}^\dagger(t) \quad (14)$$

Taking a partial trace over the field variables one can obtain from (14) the reduced atomic density operator $\rho_{A_1 A_2}(t, \tau) = Tr_F \rho_{A_1 A_2 F}(t, \tau)$. Omitting the cumbersome calculations one can be obtain for reduced atomic density operator in the form

$$\rho_{A_1 A_2}(t, \tau) = \begin{pmatrix} U(t, \tau) & 0 & 0 & 0 \\ 0 & V(t, \tau) & H(t, \tau) & 0 \\ 0 & H(t, \tau)^* & W(t, \tau) & 0 \\ 0 & 0 & 0 & R(t, \tau) \end{pmatrix}. \quad (15)$$

We have obtained the exact formulae for the matrix elements of (15). But these are too cumbersome to present them in the paper. Using the matrix (15) one can obtain the negativity (6) in the following form

$$\epsilon(t, \tau) = \sqrt{(U(t, \tau) - R(t, \tau))^2 + 4|H(t, \tau)|^2} - U(t, \tau) - R(t, \tau); \quad (16)$$

We also consider the another Bell-type initial atomic entangled state

$$|\Psi(0)\rangle_{A_1 A_2} = \cos \Theta |+, +\rangle + \sin \Theta |-, -\rangle. \quad (17)$$

For initial atomic state (17) and a thermal cavity field the reduced atomic density operator has the form

$$\rho_A(t, \tau) = \begin{pmatrix} M(t, \tau) & 0 & 0 & F(t, \tau) \\ 0 & N(t, \tau) & S(t, \tau) & 0 \\ 0 & S(t, \tau)^* & O(t, \tau) & 0 \\ F^*(t, \tau) & 0 & 0 & P(t, \tau) \end{pmatrix}. \quad (18)$$

The matrix elements of (18) are

$$M(t, \tau) = M_1(t, \tau) + M_2(t, \tau), \quad N(t, \tau) = N_1(t, \tau) + N_2(t, \tau), \quad O(t, \tau) = O_1(t, \tau) + O_2(t, \tau),$$

$$P(t, \tau) = P_1(t, \tau) + P_2(t, \tau), \quad S(t, \tau) = S_1(t, \tau) + S_2(t, \tau), \quad F(t, \tau) = \sin \theta \cos \theta \sum_{n=0}^{\infty} p_n B_n(\tau) B_n(t) A_{n-1}^*(\tau) A_{n-1}^*(t),$$

where

$$M_1(t, \tau) = (\cos \theta)^2 \left(\sum_{n=1}^{\infty} p_n |B_n(t)|^2 |B_n(\tau)|^2 + p_0 |B_0(t)|^2 |B_0(\tau)|^2 \right),$$

$$N_1(t, \tau) = (\cos \theta)^2 \left(\sum_{n=1}^{\infty} p_n |C_n(t)|^2 |B_n(\tau)|^2 + p_0 |C_0(t)|^2 |B_0(\tau)|^2 \right),$$

$$O_1(t, \tau) = (\cos \theta)^2 \left(\sum_{n=1}^{\infty} p_n |B_{n+1}(t)|^2 |C_n(\tau)|^2 + p_0 |C_0(\tau)|^2 |B_1(t)|^2 \right),$$

$$P_1(t, \tau) = (\cos \theta)^2 \left(\sum_{n=1}^{\infty} p_n |C_{n+1}(t)|^2 |C_n(\tau)|^2 + p_0 |C_1(t)|^2 |C_0(\tau)|^2 \right),$$

$$S_1(t, \tau) = (\cos \theta)^2 \left(\sum_{n=1}^{\infty} p_n B_n(\tau) C_n(t) C_n^*(\tau) B_{n+1}^*(t) + p_0 B_0(\tau) C_0(t) C_0^*(\tau) \right) B_1^*(t),$$

$$M_2(t, \tau) = (\sin \theta)^2 \sum_{n=2}^{\infty} p_n |C_{n-1}(\tau)|^2 |C_{n-2}(t)|^2,$$

$$N_2(t, \tau) = (\sin \theta)^2 \sum_{n=1}^{\infty} p_n |C_{n-1}(\tau)|^2 |A_{n-2}(t)|^2, \quad O_2(t, \tau) = (\sin \theta)^2 \sum_{n=1}^{\infty} p_n |A_{n-1}(\tau)|^2 |C_{n-1}(t)|^2,$$

$$P_2(t, \tau) = (\sin \theta)^2 \left(\sum_{n=1}^{\infty} p_n |A_{n-1}(\tau)|^2 |A_{n-1}(t)|^2 + p[0] \right), \quad S_2(t, \tau) = (\sin \theta)^2 \sum_{n=1}^{\infty} p_n C_{n-1}(\tau) A_{n-2}(t) A_{n-1}^*(\tau) C_{n-1}^*(t).$$

The partial transpose of the reduced atomic density matrix (18) is

$$\rho_A^{T_1}(t, \tau) = \begin{pmatrix} M(t, \tau) & 0 & 0 & S^*(t, \tau) \\ 0 & N(t, \tau) & F^*(t, \tau) & 0 \\ 0 & F(t, \tau) & O(t, \tau) & 0 \\ S(t, \tau) & 0 & 0 & P(t, \tau) \end{pmatrix}. \quad (19)$$

Matrix (19) has two eigenvalues, which may take a negative value. Then, the negativity can be written as a superposition of two terms. At the same time, each term contributes to the total amount, as long as it takes a positive value. As a result the negativity is

$$\varepsilon(t, \tau) = \sqrt{(N(t, \tau) - O(t, \tau))^2 + 4|F(t, \tau)|^2} - N(t, \tau) - O(t, \tau) + \sqrt{(M(t, \tau) - P(t, \tau))^2 + 4|S(t, \tau)|^2} - M(t, \tau) - P(t, \tau). \quad (20)$$

The results of numerical calculations of entanglement parameters (16) and (20) are shown in Figs. 3-6. The curves were obtained under the assumption that $\tau = t/2$ as in [3].

In numerical calculations we have turned our attention to the effects of the detuning as for the vacuum and the thermal field. What is worth noting is that in most previous researches only the case of the resonance is considered to avoid the complexity. Without loss of generality, we consider that the detuning is necessary and meaningful. In Fig. 3 and Fig. 3 we plot the negativity as a function of a scaled time gt for entangled initial state (12) and different values of mean photon numbers and detunings. Figs.3(a) and Fig.4(a) show that for vacuum state the presence of detuning leads to decreasing of the entanglement amplitude oscillations and stabilization of the degree of entanglement. Fig. 3(b) shows how the mean photon number influences the entanglement evolution for entangled initial atomic state (12). One can see that with the increase of the mean photon number the negativity decreases, but the evolution periodicity of the entanglement does not change. One can see also from Fig. 3(b) and Fig. 4(b) that for the thermal cavity field and small detunings the effect of sudden death and birth of entanglement takes place and that for large detuning such effect vanishes. Figs 3(b) and Fig.4(b) show also that the entanglement increases as the detuning increases, meanwhile the oscillation of the negativity becomes faster and faster. Fig. 5 show the entanglement behavior for independent atoms, first of which is initially prepared in excited and the second is prepared in the ground state. Note that such initial atomic state has been used in S. Haroche et al. experiment [3]. For this initial atomic state the detuning reduces the maximum degree of entanglement both for the vacuum and the thermal field. In Fig. 6 we plot the negativity as a function of a scaled time gt for entangled initial state (12) and different values of mean photon numbers and detunings. The results for entanglement in this case is similar to that for entangled initial state (12). One can see that for vacuum state the presence of detuning leads to decreasing of the entanglement amplitude oscillations and stabilization of the degree of entanglement. For thermal fields and small detuning the sudden death of entanglement also takes place. For this cavity state the entanglement increases as the detuning increases as for the first type of Bell atomic state (12).

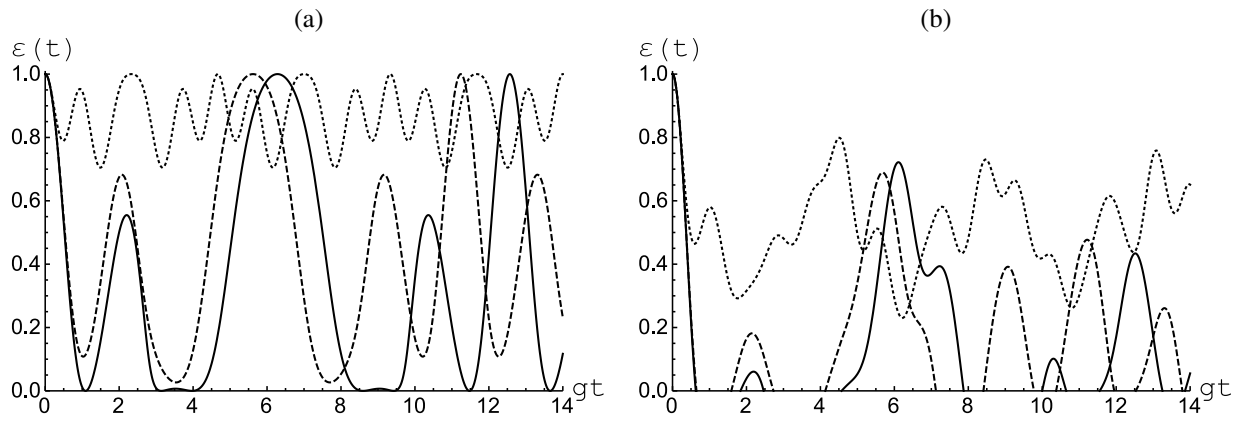


Fig. 3. The negativity as a function of a scaled time gt for the model with $\delta = 0$ (solid), $\delta = 1$ (dashed) and $\delta = 5$ (dotted). The mean photon number $\bar{n} = 0$ (a) and $\bar{n} = 0.5$ (b). The initial atomic state $|\Psi(0)\rangle_{A_1A_2} = (1/\sqrt{2})(|+, -\rangle + |-, +\rangle)$.

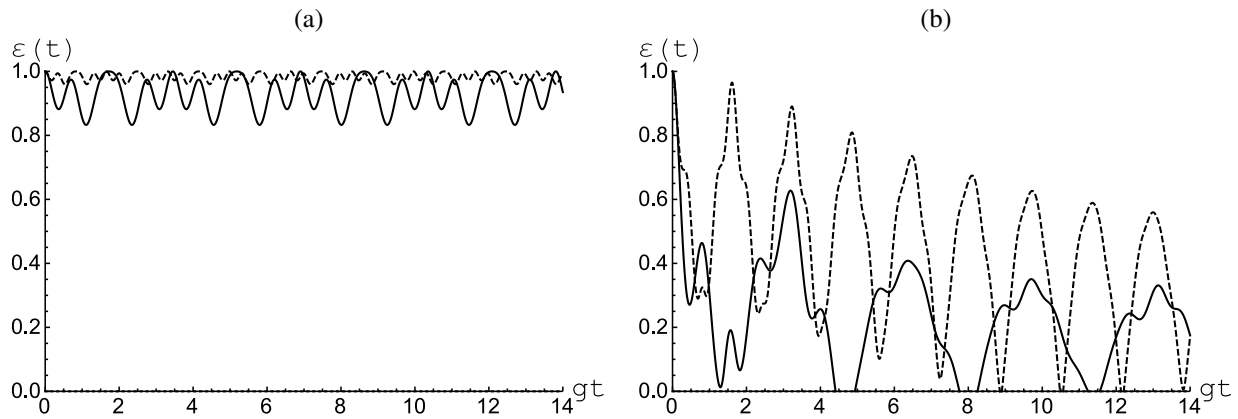


Fig. 4. The negativity as a function of a scaled time gt for the model with $\delta = 7$ (solid) and $\delta = 15$ (dashed). The mean photon number $\bar{n} = 0$ (a) and $\bar{n} = 2$ (b). The initial atomic state $|\Psi(0)\rangle_{A_1A_2} = (1/\sqrt{2})(|+, -\rangle + |-, +\rangle)$.

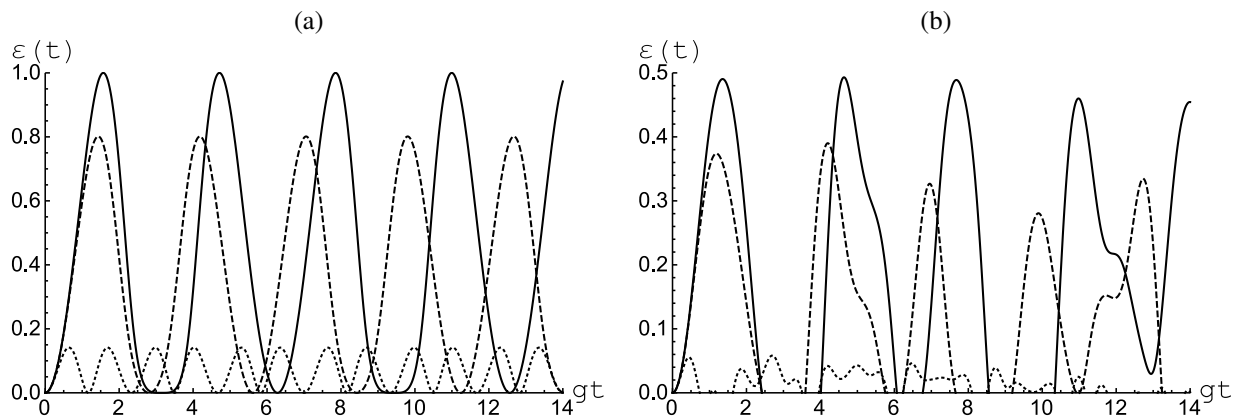


Fig. 5. The negativity as a function of a scaled time gt for the model with $\delta = 0$ (solid), $\delta = 1$ (dashed) and $\delta = 5$ (dotted). The mean photon number $\bar{n} = 0$ (a) and $\bar{n} = 0.5$ (b). The initial atomic state $|\Psi(0)\rangle_{A_1A_2} = |+, -\rangle$.

5. Conclusion

We have used negativity to study the entanglement for the system of two initially separable or entangled two-level atom successively passing a vacuum or thermal cavity of one atom maser taking into account the detuning and initial atomic coherence. We have derived the exact expressions for the reduced atomic density matrixes and calculated the analytical negativity expressions for three considered model. We have investigated the entanglement for different model parameters turning our attention to the role of detuning, thermal photon, Stark shift and initial atomic coherence or atomic quantum correlations in entanglement behavior. The all these results show that the atom-atom entanglement can be controlled by changing the system parameters, such as the mean photon numbers of thermal field, detuning, Stark shift and the degree of initial atomic entanglement or quantum correlations. We have restricted ourselves to the investigation of atom-atom entanglement. The atom-field entanglement for two atoms interacting

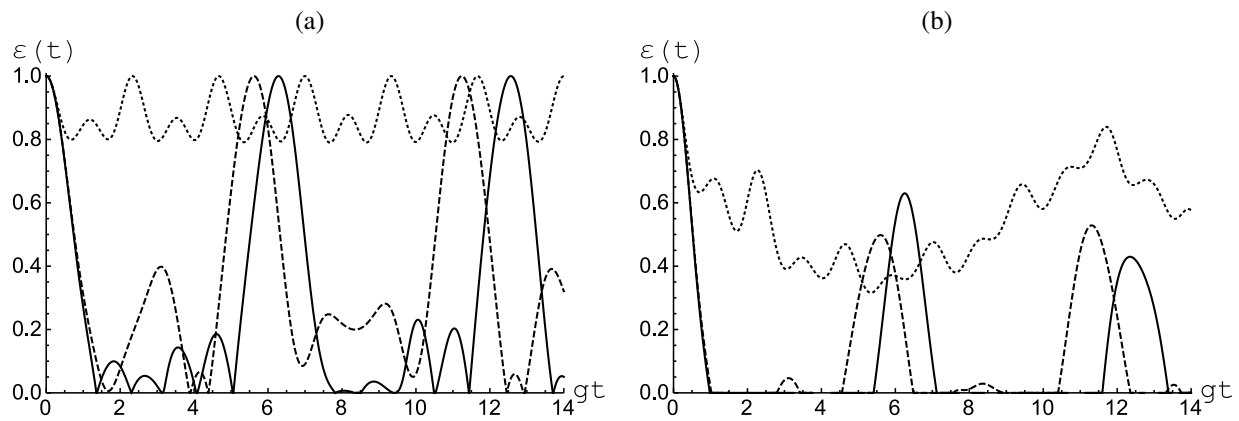


Fig. 6. The negativity as a function of a scaled time gt for the model with $\delta = 0$ (solid), $\delta = 1$ (dashed) and $\delta = 5$ (dotted). The mean photon number $\bar{n} = 0$ (a) and $\bar{n} = 0.5$ (b). The initial atomic state $|\Psi(0)\rangle_{A_1A_2} = (1/\sqrt{2})(|+, +\rangle + |-, -\rangle)$.

with cavity field have also been discussed intensively both experimentally and theoretically (see references in [17]-[19]).

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