Hysteretic damper based on Bouc-Wen model

A.M. Solovyov[a], M.E. Semenov[a,b,c], P.A. Meleshenko[a,b], M.A. Popov[c], O.O. Reshetova[a]

[a] *Voronezh State University, 394006, Universitetskaya sq. 1, Voronezh, Russia*
[b] *Zhukovsky-Gagarin Air Force Academy, 394064, Starykh Bolshevikov street, 54 "A", Voronezh, Russia*
[c] *Voronezh State Technical University, 394006, Dvadtsatiletiya Oktyabrya street, 84, Voronezh, Russia*

Abstract

In the presented work we consider the dynamics of the mechanical system under internal force with a damper taking into account the hysteretic nature of the damper. As a mathematical model of this hysteretic damper we consider the Bouc-Wen model. The obtained numerical results in the form of the force transfer function demonstrates the efficiency of the hysteretic damper in comparison with the nonlinear viscous damper.

Keywords: linear and nonlinear viscous damper; hysteretic damper; Bouc-Wen model; force transfer function

1. Introduction

The dampers and damping processes have a long history and especially relevant (both from the fundamental and applied points of view) in the present days due to development of the modern impact-vibrational systems (see, e.g., [1]). The damper is a device used for damping the mechanical, electrical and other modes of vibration arising in the machines and mechanical systems during its operation. Damping is an important task that has a wide range of applications. In general the damping is a process whereby the energy is taken from the vibrating system and is being absorbed by the surroundings. The examples of damping include:

- internal forces of a spring;
- viscous force in a fluid;
- electromagnetic damping in galvanometers;
- shock absorber in a car;
- anti seismic damping devices in buildings etc.

In the same time it should be noted that the damping devices are widely used in modern avionics (damper of aero-elastic vibrations, which is the electronic system for automatic cancelation of short-aircraft vibrations that inevitably arise when the flight modes change).

In the case of oscillations of mechanical systems the model of linear viscous damping (which is based on the energy dissipation due to viscous friction) is widely used. However, this type of damping has a significant drawback, namely the low efficiency outside the region of resonance of the system. One way to solve this problem is to use a nonlinear viscous damper [2,3,4,5,6,7] or a damper with hysteretic properties [8,9].

Main purpose of this work is to study the dynamics of a mechanical system under various external affections (forced oscillations) in the case of presence of a damping block. Especial interest in this case has a damper with the hysteretic properties. As a mathematical model of such a hysteretic damper we consider the Bouc-Wen phenomenological model [10,11,12].

2. Physical system

Let us consider a mechanical system under external affection and in the presence of the damping part $f_d(t)$ as is shown in Fig. 1. The system is presented as a cylinder with mass M under external affection $f(t)$ of harmonic nature. In the cylinder there is a car of mass m (moving without friction in the horizontal plane) connected to the border by a spring with stiffness k. For simplicity we assume that the system is one-dimensional.

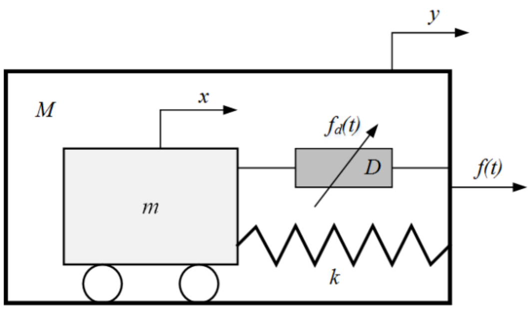

Fig. 1. The considered physical system.

Suppose that the affection force $f(t)$ is described by the following relation:

$$f(t) = Y\omega^2 \sin(\omega t), \tag{1}$$

where $Y\omega^2$ is an amplitude and ω is a frequency of the affection force. The equation of motion for considered system is

$$\begin{cases} M\ddot{y} + kz + f_d(t) = f(t), \\ m\ddot{x} - kz - f_d(t) = 0, \\ f(t) = Y\omega^2 \sin(\omega t), \quad z(t) = y(t) - x(t). \end{cases} \tag{2}$$

Here overdot determines the time derivative and $z(t)$ is a relative displacement.

3. Linear and nonlinear viscous damping

Let us consider the case of viscous damping. In general case the viscous friction can be described as follows:

$$f_d(t) = c\left[1+z(t)\right]^n \dot{z}, \quad n..0, \tag{3}$$

where c is a damping coefficient. In the case n=0 we have a linear viscous friction. For n>0 the nonlinear viscous friction takes place [4,7].

The equation of motion for relative displacement $z(t)$ becomes

$$\ddot{z}(t) + \frac{M+m}{Mm}\left\{c[1+z(t)]^n \dot{z}(t) + kz(t)\right\} = \frac{Y}{M}\omega^2 \sin(\omega t). \tag{4}$$

It is more suitable to introduce the dimensionless variables

$$\Omega = \frac{\omega}{\omega_0}, \mu = \frac{mM}{m+M}, A = \frac{Y}{M}, \tau = \omega_0 t,$$
$$\omega_0 = \sqrt{\frac{k}{\mu}}, \zeta = \frac{c}{2\omega_0\mu}, u = z. \tag{5}$$

After such notations the final equation of motion has the form:

$$\frac{d^2u}{d\tau^2} + 2\zeta(1+u)^n \frac{du}{d\tau} + u = A\Omega^2 \sin(\Omega\tau). \tag{6}$$

4. Bouc-Wen model

Bouc-Wen model has been widely used to describe nonlinear hysteretic systems and this nonlinear differential equation model reflects local history dependence through introducing an extra state variable. Through appropriate choices of parameters in the model, it can represent a wide variety of softening or hardening smoothly varying or nearly bilinear hysteretic behavior. This model has also been generalized to include hysteresis pinching and stiffness/strength degradation.

The Bouc-Wen model can be described by the following equation

$$\dot{g}(t) = \dot{u}(t)\left\{B - \left[\beta sign\left(g(t)\dot{u}(t)\right) + \gamma\right]\left|g(t)\right|^p\right\}, \tag{7}$$

where B, β>0, γ and p are dimensionless quantities controlling the behaviour of the model (p=∞ retrieves the elastoplastic hysteresis).

For small values of the positive exponential parameter p the transition from elastic to the post-elastic branch is smooth, while for large values that transition is abrupt. Parameters B, β and γ control the size and shape of the hysteretic loop.

Wen assumed integer values for p; however, all real positive values of p are admissible. The parameter β is positive by assumption, while the admissible values for γ, that is γ=[-β,β], can be derived from a thermodynamical analysis [13].

Now we consider the hysteretic damper based on the Bouc-Wen phenomenological model. In this case (using the notations introduced above) the damping force can be presented as:

$$\begin{cases} f_d(\tau) = k_b g(\tau), \\ \dot{g}(\tau) = \dot{u}(\tau)\left\{B - \left[\beta sign\left(g(\tau)\dot{u}(\tau)\right) + \gamma\right]\left|g(\tau)\right|^p\right\}. \end{cases} \quad (8)$$

In this case the equation of motion for the considered system becomes:

$$\begin{cases} \ddot{u} + \xi\left(k_b g + ku\right) = A\Omega^2 \sin\left(\Omega\tau\right), \quad \xi = \dfrac{1}{\omega_0 \mu}, \\ \dot{g}_\tau = \dot{u}_\tau \left\{B - \left[\beta sign\left(g\dot{u}_\tau\right) + \gamma\right]\left|g\right|^p\right\}. \end{cases} \quad (9)$$

5. Main characteristics

Let us consider the main characteristics reflecting the efficiency of the damper in the resonance system and beyond. Such characteristics are force transmission function and “force-displacement” transmission function.

The force transmission function is determined by the ratio of the force applied to the cylinder *M* and the force applied to the car *m* (Fig. 1). This function reflects the efficiency of suppression of the external affection by the force transmission from an external source to the load. This characteristic is expressed as follows:

$$T_{ff} = \frac{1}{Y\omega^2} \max\left|m\omega_0^2 \frac{\mathrm{d}^2 x}{\mathrm{d}\tau^2}\right|. \quad (10)$$

The “force-displacement” transmission function is determined by the relation of the motion of car *m* relative to the cylinder *M* and the force applied to the cylinder. This quantity reflects the efficiency of vibration absorption by the ability of the damper to reduce the relative motion of the car under influence of external forces. This characteristic is expressed as

$$T_{fd} = \frac{\max\left|x(\tau)\right|}{Y\omega^2}. \quad (11)$$

During the following simulations we use these quantities for comparison of the linear viscous, nonlinear viscous and hysteretic dampers based on Bouc-Wen model.

6. Numerical results

We make the numerical simulations. In order to compare the viscous damper and hysteretic damper we present the numerical results for two characteristics of the system, namely, the force transmission function and “force-displacement” transmission function. For the nonlinear viscous damper we use the following set n={0,2,4}.

For the hysteretic damper we use the parameters k_b=100, B=1, β=0.7, γ=0.3, p=6 (It was shown experimentally that such characteristics correspond to the optimum operation of the hysteresis model). The characteristics of the mechanical system (per dimensionless units): M=1, m=1, ζ=0.8, ω_0=10; the external affection with parameters A=1, ω=0,…,30 (with a step 0.2); parameters of the difference scheme: time step h_τ=0.0167, simulation period T=10000.

The simulation results are shown in Fig 2. As it can be seen from these figures the linear viscous damper has a high efficiency in the resonance region of the system, however outside the resonance region the damping properties sharply decrease. At the same time the nonlinear viscous damper has a wide range of effective use, but loses in efficiency to linear damper in the resonance region of the system.

The hysteretic damper based on Bouc-Wen model has a high efficiency both in the resonance region and beyond. The disadvantage of the hysteretic damper is in decreasing of the ability to reduce the relative movement of car under external forces outside the resonance region of the system.

Let us consider also the phase portraits for the system both for the cases of viscous damper and for hysteretic damper. As the origin of the phase plane we use the instantaneous values of the car's coordinate inside the cylinder $x(\tau)$ and its relative speed $\dot{x}(\tau)$. The phase portraits of the system under consideration (with various kinds of dampers) are presented in the figures 3-6.

Figures show the phase portraits of the system both in the case of viscous (linear and nonlinear) damping and the hysteretic damping. The left panels show the phase portraits in the resonance region, the right panels show the far frequency domain. From the figures it is clear that the hysteretic damper has a greater efficiency (in comparison with the linear and nonlinear viscous dampers) in the resonance region, and beyond. Previously, based on the analysis of transmission functions T_{ff} and T_{fd} we have made some conclusions on the efficiency of hysteretic damper in comparison with the viscous damper. In this way the presented phase portraits are proved our conclusions.

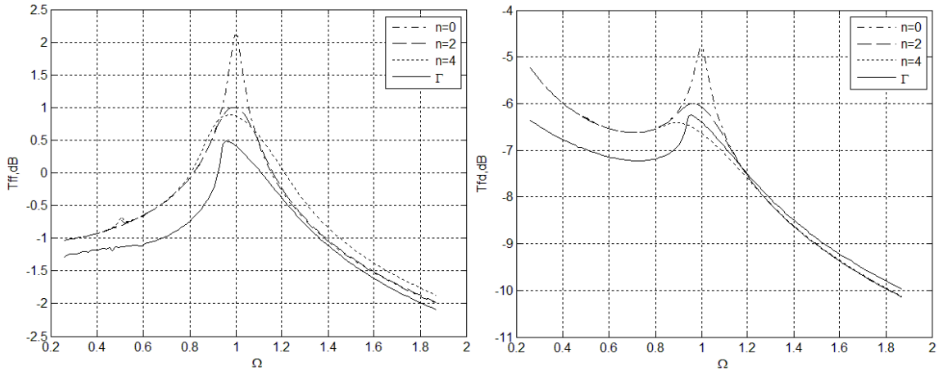

Fig. 2. Force (left panel) and "Force-displacement" (right panel) transmision function.

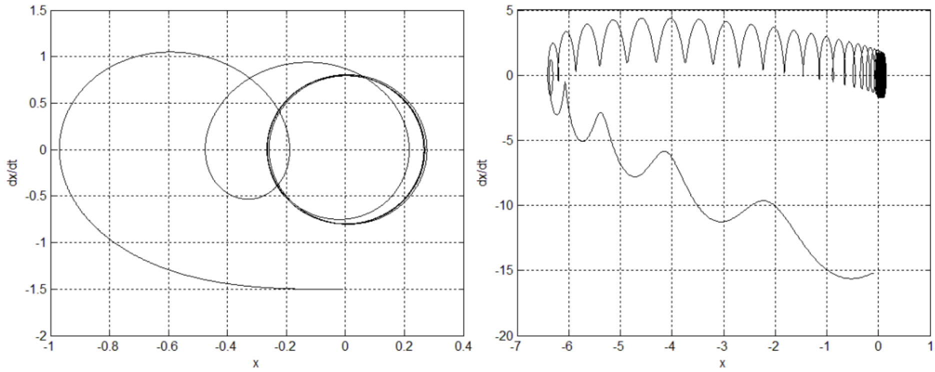

Fig. 3. Phase portraits of the system in the case of linear viscous damper n=0 at Ω=3 (left panel) and Ω =30 (right panel).

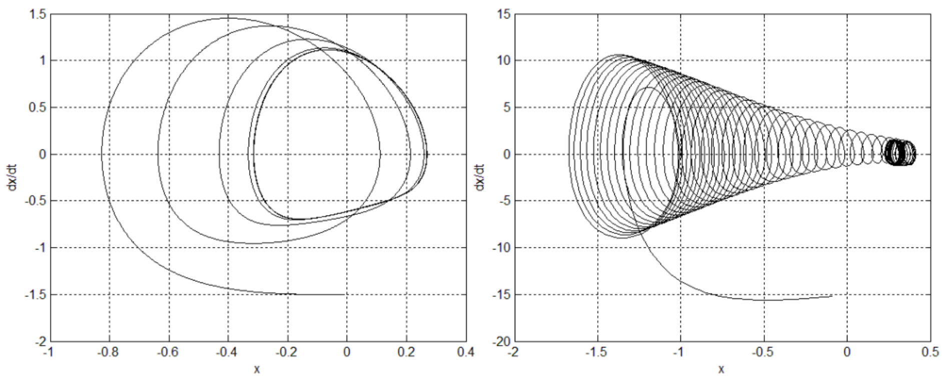

Fig. 4. Phase portraits of the system in the case of nonlinear viscous damper n=2 at Ω=3 (left panel) and Ω =30 (right panel).

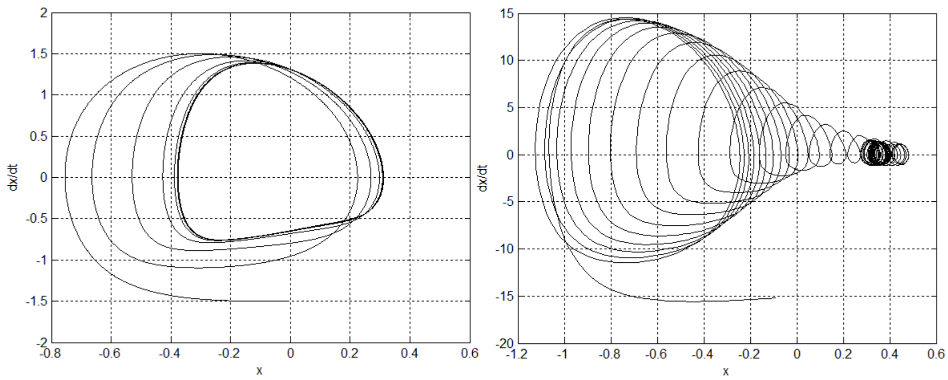

Fig. 5. Phase portraits of the system in the case of nonlinear viscous damper n=4 at Ω=3 (left panel) and Ω =30 (right panel).

7. Conclusion

This work is considered the various kinds of damping processes that occur in the oscillations of real mechanical systems with damping blocks. Namely we consider the peculiarities of the linear and nonlinear viscous damping as well as the fractional damping, which can be considered as an effective model for the viscoelastic medium. Especial attention in this work we are paid to the hysteretic damper, which constructive model is based on the phenomenological Bouc-Wen model. We presented the numerical results for the observable characteristics of the system under consideration such as force transmission function and "force-displacement" transmission function both for the cases of hysteretic and viscous (linear and nonlinear) dampers. The phase portraits of the system with various kinds of damping are plotted and analyzed. The obtained results allow to compare the various types of viscous dampers (linear and nonlinear) and hysteretic damper. Based on the obtained numerical results we can formulate the following concluding notes:

- *Linear viscous damper* has a high efficiency in the resonance region of the system, however, outside the resonance region the damping properties sharply decrease.
- *Nonlinear viscous* damping has a wide range of effective use, but loses in efficiency to linear damper in the resonance region of the system.
- *Hysteretic damper* based on the Bouc-Wen model has a high efficiency both in the resonance region and beyond. The disadvantage of the hysteretic damper is in decreasing of the ability to reduce the relative movement of the car under external forces outside the resonance region of the system.

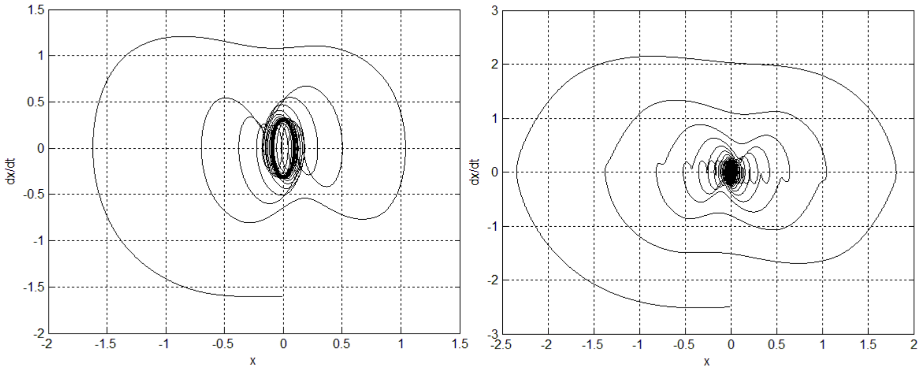

Fig. 6. Phase portraits of the system in the case of hysteretic damper at Ω=3 (left panel) and Ω =30 (right panel).

Acknowledgements

This work is supported by the RFBR grant No 16-08-00312, 17-01-00251.

References

[1] Babitsky, V., Krupenin, V. Vibration of Strongly Nonlinear Discontinuous Systems / V. Babitsky, V. Krupenin – Berlin: Springer, 2001.

[2] Rigaud, E., Perret-Liaudet, J. Experiments and numerical results on non-linear vibrations of an impacting hertzian contact. part 1: harmonic excitation / E. Rigaud , J. Perret-Liaudet // Journal of Sound and Vibration – 2003. – Vol. 265(2). – P. 289–307.

[3] Lang, Z., Billings, S., Yue, R., Li, J. Output frequency response function of nonlinear Volterra systems / Z. Lang, S. Billings, R. Yue, J. Li // Automatica – 2007. – Vol. 43(5). – P. 805–816.

[4] Milovanovic, Z., Kovacic, I., Brennan, M. On the displacement transmissibility of a base excited viscously damped nonlinear vibration isolator / Z. Milovanovic, I. Kovacic, M. Brennan // Journal of Vibration and Acoustics – 2009. – Vol. 131(5).

[5] Felix, J., Balthazar, J., Brasil, R., Pontes, B. On lugre friction model to mitigate nonideal vibrations / J. Felix, J. Balthazar, R. Brasil, B. Pontes // Journal of Computional and Nonlinear Dynamics – 2009. – Vol. 4(3).

[6] Peng, Z., Meng, G., Lang, Z., Zhang, W., Chu, F. Study of the effects of cubic nonlinear damping on vibration isolations using harmonic balance method / Z. Peng, G. Meng, Z. Lang, W. Zhang, F. Chu // International Journal of Non-Linear Mechanics – 2012. – Vol. 47(10). – P. 1073–1080.

[7] Lv, Q., Yao, Z. Analysis of the effects of nonlinear viscous damping on vibration isolator / Q.Lv, Z. Yao // Nonlinear Dynamics – 2015. – Vol. 79(4). – P. 2325–2332.

[8] Richards, R. Comparison of linear, nonlinear, hysteretic, and probabilistic mr damper models / R. Richards – Blacksburg, Virginia: Master's thesis, Faculty of the Virginia Polytechnic Institute and State University, 2007.

[9] Latour, M. Theoretical and Experimental Analysis of Dissipative Beam-to-Column Joints in Moment Resisting Steel Frames / M. Latour – BrownWalker Press, 2014.

[10] Lv, Q., Yao, Z. Analysis of the effects of nonlinear viscous damping on vibration isolator / Qibao Lv, Zhiyuan Yao // Nonlinear Dynamics – 2015. – Vol. 79. – P. 2325–2332.

[11] Ikhouane, F, Manosa, V, Rodellar, J. Dynamic properties of the hysteretic Bouc-Wen model / F. Ikhouane, V. Manosa, J. Rodellar // Systems & Control Letters – 2007. – Vol. 56. – P. 197–205.

[12] Ikhouane, F, Rodellar, J. On the Hysteretic Bouc-Wen Model / F. Ikhouane, J. Rodellar // Nonlinear Dynamics – 2005. – Vol. 42. – P. 63–78.

[13] Baber, T.T. and Wen, Y.K. Random vibrations of hysteretic degrading systems / T.T. Baber and Y.K. Wen // Journal of Engineering Mechanics. ASCE – 1981. – Vol. 107(EM6). – P. 1069–1089.

[14] Semenov, M.E., Solovyov, A.M., Meleshenko, P.A. Elastic inverted pendulum with hysteretic nonlinearity in suspension: stabilization problem / M.E. Semenov, A.M. Solovyov, P.A. Meleshenko // Nonlinear Dynamics – 2015. – Vol. 82(1). – P. 677-688.

[15] Semenov, M.E., Solovyov, A.M., Semenov, A.M., Gorlov, V.A., Meleshenko, P.A. Elastic inverted pendulum under hysteretic nonlinearity in suspension: stabilization and optimal control / M.E. Semenov, A.M. Solovyov, A.M. Semenov, V.A. Gorlov, P.A. Meleshenko // Proceedings of the 5th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering Held in Crete Greece 25-27 – 2015. – Vol. 2. – P. 2995-3004.

[16] Semenov, M.E., Meleshenko, P.A., Solovyov, A.M., Semenov, A.M. Hysteretic Nonlinearity in Inverted Pendulum Problem / M.E. Semenov, P.A. Meleshenko, A.M. Solovyov, A.M. Semenov // Springer Proceedings in Physics – 2015. – Vol. 168. – P. 463-507.

[17] Mortell, M.P., O'malley, R.E., Pokrovskii, A., Sobolev, V. Singular perturbations and hysteresis / M.P. Mortell, R.E. O'malley, A. Pokrovskii, V. Sobolev – Philadelphia: SIAM, 2005.